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Rough Set Techniques in Wireless Sensor Networks using Membership Function and Rough Labelling Graphs for Energy Aware Routing

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Abstract—Rough set theory and rough graphs are employed for data analysis. Rough graphs utilize approximations, and this paper presents a method for implementing rough graphs using rough membership functions and graph labeling for data structures and reduction. This paper proposes a rough graph labeling method, termed rough ζ-labeling similarity graph, that utilizes a similarity measure for vertex and edge labeling. This method aims to minimize boundary regions in rough graphs and is applicable to wireless sensor networks (WSNs). In WSN, the proposed algorithms such as PSO-LSTM, COA-LSTM, LOA-LSTM integrates with rough set theory based rough ζ-labeling for boundary region identification. The proposed method incorporates the membership functions encapsulates the rough labeling graphs for cluster boundary region identification for WSN. The cluster boundary for WSN is based on rough set membership and rough labelling is termed as rough set membership boundary region (RMB). RMB in WSN and implementation of proposed routing algorithm such as LOA-LSTM provides high throughput and energy saving when compared to existing cluster boundary structure methods such as Voronoi, Spectrum and Chain.

Keywords- Rough Graph; Rough Membership function; Rough ζ labeling similarity graph; LOA- LSTM; PSO-LSTM; COA-LSTM.

I. Introduction

WSN play an important role in wireless communication. WSN monitors and collects physical or environmental data. In WSN, node consists of sensors and measures the parameters, gather data from remote locations. The major problem in WSN is limited energy resources, communication bandwidth and time complexity. Clustering is a technique used in WSNs for network resources and prolong the node management lifetime. In a clustered WSN, network boundary regions are grouped and clustered with sensor nodes. For each cluster boundary, a node is designated as cluster head. The cluster head aggregates and transmits data from each member nodes with in the boundary and send data to base station. Clusters formation and cluster heads selection perform with various algorithms such as LEACH (low-energy Adaptive Clustering Hierarchy), HEED (Hybrid Energy-Efficient Distributed clustering) and PEGASIS (Power-Efficient Gathering in sensor information systems). Cluster head selection and boundary structure plays a vital role in WSNs for energy reduction and efficient data collection. Cluster boundary structure optimizes network performance and extends the lifespan of the network. The clustering structures such as Voronoi, Spectrum and Chain are formed to the nodes in flat clustering, nodes are partitioned without any hierarchical organization. k-means is an example of flat clustering, hierarchical clustering such as tree-like structure clusters allows for multiple levels of granularity. Fuzzy clustering is based on membership values of nodes which indicates the degree of each cluster.

Density based cluster node is based on high density and low density. Grid based clustering divides the node into grid of cells. Cluster is formed based on grouping cells with significant number of nodes. This cluster handles large datasets and reduces computational complexity. Model-based clustering is based on probability distributions. Clusters are formed based on Gaussian Mixture Models (GMMs). Similarity metrics is used for partition and effective for complex structures and nonlinear relationships. Conceptual clustering is based on shared attributes or features focus on semantics of node. Voronoi partition is based on geometric structure, which is based on the distance cluster head. Each region has nodes closer to each cluster head. Voronoi structure clustering is based on spatial proximity to specific cluster head. Spectrum structure refers to the patterns and relationships that emerge from the eigenvalues and Eigen vectors during clustering. Chain structure refers to the arrangement of nodes in linear fashion, looks like chain. The

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distance between different cluster heads defines the inter-cluster distance. This distance is higher for well-separated clusters, indicating a better quality of clustering. Intra-cluster distance refers to the average or total distance between data points within the same cluster. It measures the compactness of a cluster, indicating how closely grouped the data points are within a cluster. A smaller intra-cluster distance suggests that the data points within a cluster are tightly clustered together.

Rough set theory resolves ambiguous problems and it is based on low and high approximations on boundary regions and membership functions [5,6]. By using boundary regions, Zhouming M provided a family of innovative definitions of approximation operators based on a covering [20]. The relationship between the reductions, the granularity of the boundary region, the rules variance and the uncertainty measure were the four areas in which Ying Wang and Nan Zhang evaluated their uncertainty analysis for five different reductions [21]. To address uncertain problems in an abstract manner, rough set theory was enhanced with graph theory. Subsequently, Tong He and K Shi introduced the rough graph, a concept that utilizes binary relations to define its structure [7], describing the rough graph's representation [8] and its properties, following that vertex rough graph was demonstrated [10,12] and an extensive types of graph labelling was implemented on rough approximations [24]. Likewise, Anitha and Arunadevi constructed rough graph using rough membership function and also introduced rough graphs with additional metrics based on the neighbourhood concept and its mathematical features [9,11,22].

The labeling of classical graphs finds extensive applications in Network analysis, Data Compression, Optimization, Image Processing, and Cryptography. On the other hand, the labeling of rough graphs and fuzzy graphs is designed to handle data with partial truth and uncertain knowledge bases. Pappis and Karacapilidis proposed a subsequent ratio of similarity measures that are specifically applicable to fuzzy sets [23]. Zadeh laid the groundwork for utilizing similarity measures for fuzzy sets, proving to be an effective strategy for managing uncertainty. Both rough sets and fuzzy sets address these types of data with their respective boundary values and degrees of membership. Numerous researchers are putting the theoretical and real-time applications of these sets into practice [14-19]. In a previous study, we delved into the concept of ζ-graceful labeling for even vertices in various fundamental rough graphs [13]. This current work focuses on demonstrating the application of rough ζ-labeling similarity measures to rough graphs. Our primary objective is to minimize boundary regions and highlight the significance of labeling in rough graphs, particularly in the context of wireless sensor networks.
For fault diagnosis in WSN, rough set and artificial neural network ensemble (RS-ANNE) is used. In WSN, rough set theory solves the problems such as complex calculation and high energy consumption better than existing algorithm [1,2]. In WSN, nodes attribute reduction is performed using discernibility matrix based on rough set theory. For load balancing in WSN, nodes are switched during various rounds using RF-LEACH. Cluster head selection is based on fuzzy logic and partitioning is performed using rough fuzzy c means (RFCM) [3]. In cluster sensor network, radial basis function neural networks (RBFNN) training learning process speeds by using rough set (RS) which reduces the parameters of samples [4]. The boundary is more significant in WSN clustering applications.

A. *Inferences from literature survey*

Progress in Cluster Boundary Region Management for Wireless Sensor Networks
A comprehensive literature review reveals noteworthy findings and methodologies employed in wireless sensor networks (WSNs) for cluster boundary region management.
Fault Diagnosis
Fault diagnosis in WSNs has been effectively implemented using rough set theory and artificial neural networks (ANNs), leading to enhanced reliability and reduced energy consumption. Rough set theory plays a crucial role in reducing attributes in fault diagnosis models.
Load Balancing
Load balancing has been achieved using fuzzy logic and rough fuzzy c means (RFCM) algorithms, coupled with cluster management and data optimization techniques.
Healthcare Applications
In the realm of WSN-based Internet of Things (IoT), multi-agent systems (MAS) and rough set theory-based frameworks have enriched healthcare WSN sensor decision-making through knowledge sharing and conflict resolution.
Challenges and Future Directions
Despite significant advancements, WSNs continue to face major challenges, including:

- Energy Conservation: Ensuring efficient energy consumption to prolong network lifespan.
- Interference Prevention: Mitigating interference to maintain reliable data transmission.
- Robust Integration: Guaranteeing seamless integration with other IoT devices and networks.

Addressing these challenges and exploring novel solutions will pave the way for more robust, efficient, and scalable WSN applications in diverse domains.

B. *Contributions*

- To apply rough set theory with rough graphs and membership function for cluster boundary formation based on each WSN node data.
- To apply labelling techniques in cluster structure/ boundary formation during WSN node to complex data structures which enables accurate extraction and data analysis.
- To develop a prototype test bed or WSN with rough set graph labelling with membership functions for cluster structure and implement PSO-LSTM, COA-LSTM and LOA-LSTM algorithm in WSN for routing, the research combines rough set theory with contemporary methods, while introducing membership functions for precise data relationship representation.

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- To reduce energy consumption in nodes of WSN, RMB based cluster structure and LOA-LSTM algorithm for routing performance compared with existing algorithm.

II. METHODOLOGY

The proposed RMB algorithm is used for WSN cluster boundary/structure and explained the boundary structure formation using rough set theory:
An information system is a pair $f = (\Omega, \mathcal{P}, V_a, f)$, Let Ω be a universe set and $\mathcal{P}$ is a non-empty finite set of attributes, there is a mapping $f: \Omega \times \mathcal{P} \to V_a$ for $a \in \mathcal{P}$ where V_a is called the value set of a.
Let the attribute set $\mathcal{R} \subseteq \Omega \times \Omega$ with the equivalence class $[v]_{\mathcal{R}}$ and $\mathcal{R} \subseteq \mathcal{P}$, then Indiscernibility relation is defined by

$$[v]_{\mathcal{R}}=\mathrm{IND}(\mathcal{R})=\{(v, v') \in \Omega^{2} \mid a \in \mathcal{R}, a(x) = a(x')\}.$$

For a subset $\mathcal{Y} \subset \Omega$, the lower and upper approximations of $\mathcal{Y}$ are defined as

$$\underline{\mathcal{R}}(\mathcal{Y})=U_{v \in \Omega}\{\mathcal{R}(\mathcal{Y}): \mathcal{R}(\mathcal{Y}) \subseteq \mathcal{Y}\}$$

$$\overline{\mathcal{R}}(\mathcal{Y}) = U_{v \in \Omega}\{\mathcal{R}(\mathcal{Y}): \mathcal{R}(\mathcal{Y}) \cap \mathcal{Y} \neq \phi\} \qquad Bn(\mathcal{Y}) = \overline{\mathcal{R}}(\mathcal{Y}) - \underline{\mathcal{R}}(\mathcal{Y})$$

The non-empty boundary region forms rough set with respect to the target set. The Decision system for any boundary formation is as follows:
For an information system $I = (\Omega, \mathcal{P} = \mathcal{C} \cup \mathcal{D})$ is called a decision system where $\mathcal{C}$ is the set of condition attributes and $\mathcal{D}$ be the set of decision attributes where

$$\Omega/_{\mathcal{D}} = \{\mathcal{Y}_1, \mathcal{Y}_2, \dots . \mathcal{Y}_r\}$$

then the positive region, boundary region and negative region of the decision system is defined as

Positive region: $Pos_{\mathcal{C}}(\mathcal{D}) = \cup \underline{R_{\mathcal{C}}(\mathcal{Y}_i)}$,

Negative region: $Neg(\mathcal{D}) = \Omega - \overline{R_{\mathcal{C}}(\mathcal{Y}_i)}$,

Boundary region: $Bn_{\mathcal{C}}(\mathcal{D}) = \overline{R_{\mathcal{C}}(\mathcal{Y}_i)} - \underline{R_{\mathcal{C}}(\mathcal{Y}_i)}$.

The following Fig. 1 depicts the rough set boundary approximations.

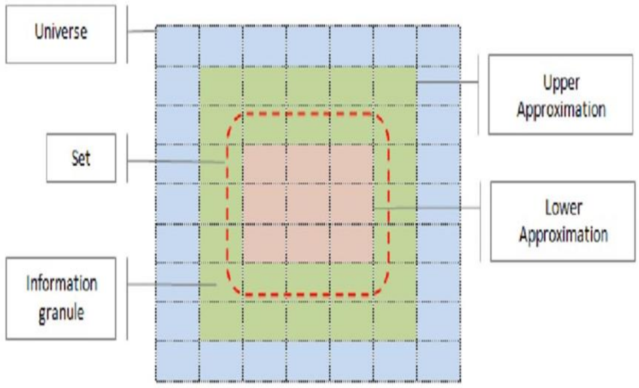

Fig 1: Rough set diagram for boundary region based on approximation

A. Axioms and Principles Governing Approximations

(i) $\underline{\mathcal{R}}\mathcal{Y} \subset \mathcal{Y} \subset \overline{\mathcal{R}}\mathcal{Y}$

(ii) $\underline{\mathcal{R}}\Omega=\overline{\mathcal{R}}\Omega = \Omega$ & $\underline{\mathcal{R}}\emptyset = \overline{\mathcal{R}}\emptyset = \emptyset$

(iii) $\underline{\mathcal{R}}(\mathcal{Y} \cap \mathcal{F}) = \underline{\mathcal{R}}\mathcal{Y} \cap \underline{\mathcal{R}}\mathcal{F}$

(iv) $\underline{\mathcal{R}}(\mathcal{Y} \cap \mathcal{F}) \supset \underline{\mathcal{R}}\mathcal{Y} \cap \underline{\mathcal{R}}\mathcal{F}$

(v) $\overline{\mathcal{R}}(\mathcal{Y} \cup \mathcal{F}) = \overline{\mathcal{R}}\mathcal{Y} \cup \overline{\mathcal{R}}\mathcal{F}$

(vi) $\underline{\mathcal{R}}(\mathcal{Y} - \mathcal{F}) = \overline{\mathcal{R}}\mathcal{Y} - \overline{\mathcal{R}}\mathcal{F}$

(vii) $\sim \underline{\mathcal{R}}\mathcal{Y} = \overline{\mathcal{R}}(\sim \mathcal{Y})$

B. Rough Membership Function (RMB)

Rough membership function (RMB) is described through the function $f_{\mathcal{R}}: \mathcal{Y} \to [0, 1]$ and defined by

$$\omega_{\mathcal{Y}}^{\mathcal{R}}(v) = \frac{|[v]_{\mathcal{R}} \cap \mathcal{Y}|}{|[v]_{\mathcal{R}}|}, \forall x \in \Omega .$$

It measures the degree of attributes at which degree it belongs to the set with following mathematical qualities,

(i) $\omega_{\mathcal{Y}}^{\mathcal{R}}(v)=1$ iff $v \in \underline{\mathcal{R}}(\mathcal{Y})$

(ii) $\omega_{\mathcal{Y}}^{\mathcal{R}}(v) = 0$ iff $v \in \Omega - \overline{\mathcal{R}}\mathcal{Y}$

(iii) $0 < \omega_{\mathcal{Y}}^{\mathcal{R}}(x) < 1$ iff $v \in Bn_{\mathcal{R}}(\mathcal{Y})$.

(iv) If $IND_{\mathcal{T}}(\mathcal{R})=\{(v, v') \in \Omega^2 \mid a \in \mathcal{R}, a(v)=a(v')$ then $\omega_{\mathcal{Y}}^{\mathcal{R}}(v)$ is the characteristic function of X .

(v) If $v IND(\mathcal{T}) u$ then $\omega_{\mathcal{Y}}^{\mathcal{R}}(v)=\omega_{\mathcal{Y}}^{\mathcal{R}}(u)$.

(vi) $\omega_{\mathcal{Y}}^{\mathcal{R}}(v) - \mathcal{Y}(v) = 1 - \omega_{\mathcal{Y}}^{\mathcal{R}}(v)$ for any $v \in \mathcal{Y}$.

(vii) $\omega_{\mathcal{Y} \cup \mathcal{F}}^{\mathcal{R}}(x) \geq max\left(\omega_{\mathcal{Y}}^{\mathcal{R}}(x), \omega_{\mathcal{F}}^{\mathcal{R}}(x)\right)$ for any $v \in \Omega$.

(viii) $\omega_{\mathcal{Y} \cap \mathcal{F}}^{\mathcal{R}}(x) \leq min\left(\omega_{\mathcal{Y}}^{\mathcal{R}}(x), \omega_{\mathcal{F}}^{\mathcal{R}}(x)\right)$ for any $v \in \Omega$.

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C. Boundary Regions Based for Proposed RBM Algorithm:

a) Definition: Let $(\Omega, \mathcal{R})$ be the approximation space and $\mathcal{Y} \subset \Omega$ then the boundary regions be defined as follows:

Inner boundary region: $\underline{Bn(\mathcal{Y})} = \mathcal{Y} - \underline{R(\mathcal{Y})}$,

Outer boundary region: $\overline{Bn(\mathcal{Y})} = \overline{R(\mathcal{Y})} - \mathcal{Y}$,
Boundary region: $Bn(\mathcal{Y}) = \overline{Bn(\mathcal{Y})} - \underline{Bn(\mathcal{Y})}$.

Approaches and boundary regions of a set $\mathcal{Y}$ are defined by the approximate membership function:

$\underline{R(\mathcal{Y})} = \{x \in \Omega \ : \ \omega_{\mathcal{Y}}^{\mathcal{R}}(x) = 1\}$

$\overline{R(\mathcal{Y})} = \{x \in \Omega : \ \omega_{\mathcal{Y}}^{\mathcal{R}}(x) > 0\}$

$Bn(\mathcal{Y}) = \{x \in \Omega : 0 < \omega_{\mathcal{Y}}^{\mathcal{R}}(x) < 1\}$

D. Properties of boundary region for proposed RBM method:

For any $\mathcal{Y}, \mathcal{F} \subseteq \Omega$, the boundary Bn($\mathcal{Y}$) satisfies the following properties:

(i) $Bn(\Omega) = \emptyset$
(ii) $Bn(\emptyset) = \emptyset$
(iii) $Bn(\mathcal{Y}) = Bn(\sim\mathcal{Y})$
(iv) $Bn(\mathcal{Y} \cup \mathcal{F}) \subseteq Bn(\mathcal{Y}) \cup Bn(\mathcal{F})$
(v) $Bn(\mathcal{Y} \cap \mathcal{F}) \subseteq Bn(\mathcal{Y}) \cup Bn(\mathcal{F})$

Theorem 1: $Bn_{\mathcal{B}}(\mathcal{v}) \subseteq Bn_{\mathcal{P}}(\mathcal{v})$

Proof: Since $[\mathcal{v}]_{\mathcal{B}} \subseteq [\mathcal{v}]_{\mathcal{P}}$, We have $\omega_{\mathcal{B}}(x) = \omega_{\mathcal{P}}(x)$
Let $\mathcal{v} \in \Omega$ and $\subset \Omega$,
then $[\mathcal{v}]_{\mathcal{B}} \cap \mathcal{Y} \neq \emptyset$ iff $\omega_{\mathcal{B}}^{R}(\mathcal{v}) > 0$ such that $\mathcal{v} \in \overline{R_{\mathcal{B}}}(\mathcal{Y})$
And $[\mathcal{v}]_{\mathcal{B}} \cap (\Omega - \mathcal{Y}) \neq \emptyset$ iff $\omega_{\mathcal{B}}^{R}(\mathcal{v}) < 1$ such that $\mathcal{v} \in \underline{R_{\mathcal{B}}}(\mathcal{Y})$
We know that $\underline{R_{\mathcal{B}}}(\mathcal{Y}) = \{x \in \Omega : \omega_{\mathcal{B}}^{R}(\mathcal{v}) = 1\}$
$\overline{R_{\mathcal{B}}}(\mathcal{Y}) = \{x \in \Omega : \omega_{\mathcal{B}}^{R}(\mathcal{v}) > 0\}$
$Bn_{\mathcal{B}}(\mathcal{Y}) = \overline{R_{\mathcal{B}}}(\mathcal{Y}) - \underline{R_{\mathcal{B}}}(\mathcal{Y})$
$= \{\mathcal{v} \in \Omega : 0 < \omega_{\mathcal{B}}^{R}(\mathcal{v}) < 1\}$
Then we have $\mathcal{v} \in \overline{R_{\mathcal{B}}}(\mathcal{Y}) - \underline{R_{\mathcal{B}}}(\mathcal{Y})$
This implies $\mathcal{v} \in Bn_{\mathcal{B}}(\mathcal{Y})$
Similarly, we prove that $\mathcal{v} \in Bn_{\mathcal{B}}(\mathcal{Y})$
Therefore, $Bn_{\mathcal{B}}(\mathcal{v}) \subseteq Bn_{\mathcal{P}}(\mathcal{v})$

Theorem 2: $\overline{R_{\mathcal{B}}}(\mathcal{D}) = \Omega$

Proof: Let $\mathcal{D}_i \subseteq \overline{R_{\mathcal{B}}(\mathcal{Y}_i)}$
$\underline{R_{\mathcal{B}}}(\mathcal{D}) = \Omega_{i=1}^{L} R_{\mathcal{B}}(\mathcal{Y}_i)$ iff $\Omega_{i=1}^{L} \omega_{\mathcal{Y}_i}^{R_{\mathcal{B}}}(\mathcal{v}) = 1$

$\overline{R_{\mathcal{B}}}(\mathcal{D}) = \Omega_{i=1}^{L} R_{\mathcal{B}}(\mathcal{Y}_i)$ iff $\Omega_{i=1}^{L} \omega_{\mathcal{Y}_i}^{R_{\mathcal{B}}}(\mathcal{v}) > 0$
We have $\Omega \subseteq \overline{R_{\mathcal{B}}}(\mathcal{D})$ and $\overline{R_{\mathcal{B}}}(\mathcal{D}) \subseteq \Omega$ which penetrates $\overline{R_{\mathcal{B}}}(\mathcal{D}) = \Omega$

Theorem 3:
$Pos_{\mathcal{B}}(\mathcal{D}) \cap Bn_{\mathcal{B}}(\mathcal{D}) = \emptyset$ and $Pos_{\mathcal{B}}(\mathcal{D}) \cup Bn_{\mathcal{B}}(\mathcal{D}) = \overline{R_{\mathcal{B}}}(\mathcal{D})$
Proof: We know that $\mathcal{v} \in \Omega$ and $\mathcal{B} \subseteq \mathcal{P}$ and let $\mathcal{v} \in [\mathcal{v}]_{\mathcal{B}}$ and $\mathcal{Y} \subset \Omega$.
If $[\mathcal{v}]_i \subseteq \mathcal{Y}$ then $\omega_{X_i}^{R_{\mathcal{B}}}(x) = 1 \Longrightarrow \mathcal{v} \in POS_{\mathcal{B}}(\mathcal{D})$
(By the definition, $Pos_{\mathcal{B}}(\mathcal{D}) = \Omega_{i=1}^{L} R_{\mathcal{B}}(\mathcal{Y}_i)$)
If $[\mathcal{v}]_{\mathcal{B}} \cap \mathcal{Y} = \emptyset$ then $\omega_{\mathcal{B}}^{R}(\mathcal{v}) = 0 \Longrightarrow \mathcal{v} \in \Omega - \overline{R_{\mathcal{B}}}(\mathcal{Y})$ such that $\mathcal{v} \in Neg_{\mathcal{B}}(D)$
If $[\mathcal{v}]_{\mathcal{B}} \cap \mathcal{Y} \neq \emptyset$ then $0 < \omega_{\mathcal{B}}^{R}(\mathcal{v}) < 1 \Longrightarrow \mathcal{v} \in [0,1]$ $\mathcal{v} \in \{\overline{R_{\mathcal{B}}}(\mathcal{Y}_i), \underline{R_{\mathcal{B}}}(\mathcal{Y}_i)\}$
Such that this implies $\mathcal{v} \in Bn_{\mathcal{B}}(\mathcal{D})$
By the definition,

$$Bn_{\mathcal{B}}(\mathcal{D}) = \overline{R_{\mathcal{B}}}(\mathcal{D}) - \underline{R_{\mathcal{B}}}(\mathcal{D})$$
$$= \Omega_{i=1}^{L} \overline{R_{\mathcal{B}}}(\mathcal{Y}_i) - \Omega_{i=1}^{L} \underline{R_{\mathcal{B}}}(\mathcal{Y}_i)$$

Therefore, we conclude that by the above statement,

$$Pos_{\mathcal{B}}(\mathcal{D}) \cap Bn_{\mathcal{B}}(\mathcal{D}) = \emptyset$$

Also $Pos_{\mathcal{B}}(\mathcal{D}) \cup Bn_{\mathcal{B}}(\mathcal{D}) = \overline{R_{\mathcal{B}}}(\mathcal{D})$

Preposition: $Bn_{\mathcal{B}}(\mathcal{D}) = \overline{R_{\mathcal{B}}}(\mathcal{D}) - \underline{R_{\mathcal{B}}}(\mathcal{D})$
Proof: we have $\mathcal{B} \subseteq \mathcal{A}$ *and* $\mathcal{Y} \subset \Omega$ and let $\mathcal{v} \in \Omega$ and $[\mathcal{v}]_{\mathcal{B}} \subset \Omega$ then $\omega_{\mathcal{B}}^{R}(x) = 1$ iff $[\mathcal{v}]_{\mathcal{B}} \subseteq \mathcal{Y} \Longrightarrow \mathcal{v} \in \overline{R_{\mathcal{B}}}(\mathcal{Y}_i)$
Since $\left(\overline{R}(\mathcal{Y}^c)\right)^c = \underline{R}(\mathcal{Y})$ we have the boundary region,
$Bn_{\mathcal{B}}(\mathcal{D}) = \overline{R_{\mathcal{B}}}(\mathcal{D}) \cap \underline{R_{\mathcal{B}}}(\mathcal{D}^c)$
$= U_{i=1}^{L} \overline{R_{\mathcal{B}}}(\mathcal{Y}_i) - U_{i=1}^{L} (\overline{R}(\overline{\mathcal{Y}_i^c})^c$
$Bn_{\mathcal{B}}(\mathcal{D}) = U_{i=1}^{L} \overline{R_{\mathcal{B}}}(\mathcal{Y}_i) - U_{i=1}^{L} \underline{R_{\mathcal{B}}}(\mathcal{Y}_i)$
$= \overline{R_{\mathcal{B}}}(\mathcal{D}) - \underline{R_{\mathcal{B}}}(\mathcal{D})$

Note:

$$Bn_{\mathcal{B}}\left(\underline{R_{\mathcal{B}}}(\mathcal{D})\right) \leq Bn_{\mathcal{B}}(\mathcal{D})$$
$$Bn_{\mathcal{B}}\left(\overline{R_{\mathcal{B}}}(\mathcal{D})\right) \leq Bn_{\mathcal{B}}(\mathcal{D})$$
$$\overline{R_{\mathcal{B}}}(\mathcal{D}) \leq \mathcal{Y} - Bn_{\mathcal{B}}(\mathcal{D})$$

E. Boundary regions of clusters for WSN based on node parameter using decision system:

An information table consists of six nodes and battery, distance between nodes microprocessor temperature, energy and throughput are condition attributes. Label is the decision attribute is given. The table 1 shows the decision system of nodes. RBM based boundary region of cluster for nodes are shown in table 2 and table 3 and 4 shows the outside region and the positive region of cluster region.

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TABLE 1: DECISION SYSTEM OF NODES IN WSN

Nodes	Battery (volts)	Distance between nodes (m)	Micro processor temp	Energy	Throughput	Label
$n1$	3.0	200	65	876	1792	F
$n2$	2.8	300	70	876	1792	F
$n3$	2.7	150	45	834	1754	F
$n4$	2.8	100	55	890	1828	Y
$n5$	2.8	100	55	890	1828	Y
$n6$	2.7	150	45	834	1754	Y

Universal set: $\Omega = \{n_1, n_2, n_3, n_4, n_5, n_6\}$
Equivalence classes:
$[n_1]_R = \{n_1\}$, $[n_2]_R = \{n_2\}$, $[n_3]_R = \{n_3, n_6\}$ $[n_4]_R = \{n_4, n_5\}$=$[n_5]_R$, $[n_6]_R = \{n_3, n_6\}$
Target set : $\mathcal{Y} = \{n_4, n_5, n_6\}$={decision=yes}
$\underline{\mathcal{R}}(\mathcal{Y}) = \{n_4, n_5\}$ (lower approximation),
$\overline{\mathcal{R}}(\mathcal{Y}) = \{n_3, n_4, n_5, n_6\}$ (upper approximation)
$Bn(\mathcal{Y}) = \{n_3, n_6\}$ {boundary region}

TABLE 2: BOUNDARY REGION FOR WSN

Nodes	Battery (volts)	Distance between nodes (m)	Microprocessor temp	Energy	Through put	Label
$n3$	2.7	150	45	834	1754	F
$n6$	2.7	150	45	834	1754	Y

Outside region: $\Omega - \overline{\mathcal{R}}(\mathcal{Y})$
$=\{n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8\}$-$\{n_3, n_4, n_5, n_6\}$ $= \{n_1, n_2\}$

TABLE 3: OUTSIDE REGION BASED ON NODE PARAMETERS

Nodes	Battery (volts)	Distance between nodes (m)	Microprocessor temp	Energy	Throughput	Label
$n1$	3.0	200	65	876	1792	F
$n2$	2.8	300	70	876	1792	F

TABLE 4: POSITIVE REGION BASED ON WSN PARAMETERS

Nodes	Battery (volts)	Distance between nodes (m)	Microprocessor temp	Energy	Throughput	Label
$n4$	2.8	100	55	890	1828	Y
$n5$	2.8	100	55	890	1828	Y

F. Rough graph[10]:

Let $\mathbb{U} = \{V, E, \omega\}$ be a triple consisting of non-empty set $V = \{v_1, v_2, \ldots. v_n\}$,
where $\mathbb{U}$ is a universe, $E = \{e_1, e_2, \ldots\ldots e_n\}$ be a set of unordered pairs of distinct elements of V and ω be a function $\omega: V \to [0,1]$. A Rough graph is defined as

$$\mathrm{U}(v_i, v_j) = \begin{cases} max\left(\omega_G^V(v_i), \omega_G^V(v_j)\right) > 0, \text{edge} \\ max\left(\omega_G^V(v_i), \omega_G^V(v_j)\right) = 0, \text{no edge.} \end{cases}$$

1) Rough ζ -Labelling Similarity Graph[19]:

A Rough graph $\mathcal{R}_{\mathcal{L}}^{\varphi} = (V, E, \rho^{\varphi}, \sigma^{\varphi}, \omega)$ is said to be rough ζ -Labelling Similarity Graph if V= $\{\rho^{\varphi}(v_i)\}$ for $i = 1,2, \ldots. n$ and E=$\{\sigma^{\varphi}(v_i, v_j)\}$ for $i = 1,2, \ldots. n$ and $\omega: V * V \to [0,1]$ is bijection such that edges and vertices can be labelled using similarity classes and measures if it satisfies the following requirements:

If $\mathcal{R}_{\mathcal{L}}^{\varphi} = \max(\omega(v_i^{\varphi}), \omega(v_j^{\varphi})) > 0$ then edge exists for $v_i, v_j \in V$.

Vertex labelling: $\rho^{\varphi}(v_i) = \frac{[v_i]_{\mathcal{S}_r}}{n}$ where
$[v_i]_{\mathcal{S}_r} = \{v_j / v_i \mathcal{S}_r v_j\}$

Edge labelling: $\sigma^{\varphi}(v_i, v_j) = Sim_{\zeta}(v_i, v_j) = \frac{\zeta}{\zeta + \eta}$

where $\eta = \frac{|[v_i]_{\mathcal{S}_r}| * |[v_j]_{\mathcal{S}_r}|}{|[v_i]_{\mathcal{S}_r}| + |[v_j]_{\mathcal{S}_r}|}$ and $\zeta_{ij} = \rho^{\varphi}(v_i) + \rho^{\varphi}(v_j) + m$ where $\rho^{\varphi}(v_i)$ and $\rho^{\varphi}(v_j)$ represents the vertex labelling of v_i,v_j and m represents the total no. of edges.

G. Calculation of Similarity class:

From the decision system, Similarity table is constructed and shown in Table which brings the relationship between objects w.r.t its five attributes. The value '0' represents that there are no remaining attribute overlaps between the two objects and '5' denotes that two objects are identical.

TABLE 5: SIMILARITY CLASS OF MACHINE DATA

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$\mathcal{S}_r$	n_1	n_2	n	n_4	n_5	n_6
n_1	5	2	0	0	0	0
n_2	2	5	0	1	1	0
n_3	0	0	5	0	0	5
n_4	0	1	0	5	5	0
n_5	1	1	0	5	5	0
n_6	0	0	5	0	0	5

From Table 5, the following similarity classes have been identified

$[n_1]_{\mathcal{S}_r}=\{n_1, n_2\}$; $\left|[n_1]_{\mathcal{S}_r}\right| = 2$
$[n_2]_{\mathcal{S}_r}=\{n_1, n_2, n_4, n_5\}$; $\left|[n_2]_{\mathcal{S}_r}\right| = 4$
$[n_3]_{\mathcal{S}_r}=\{n_3, n_6\}$; $\left|[n_3]_{\mathcal{S}_r}\right| = 2$
$[n_4]_{\mathcal{S}_r}=\{n_2, n_4, n_5\}$; $\left|[n_4]_{\mathcal{S}_r}\right| = 3$
$[n_5]_{\mathcal{S}_r}=\{n_1, n_2, n_4, n_5\}$; $\left|[n_5]_{\mathcal{S}_r}\right| = 4$
$[n_6]_{\mathcal{S}_r}=\{n_3, n_6\}$; $\left|[n_5]_{\mathcal{S}_r}\right| = 2$

H. RMB- LABELING:

Vertex Labelling (Fig 2):

$$\rho^{\varphi}(v_i) = \frac{\left|[v_i]_{\mathcal{S}_r}\right|}{n}$$

$\rho^{\varphi}(n_1) = 0.3,$

$\rho^{\varphi}(n_2) = 0.6,$

$\rho^{\varphi}(n_3) = 0.3,$

$\rho^{\varphi}(n_4) = 0.5,$

$\rho^{\varphi}(n_5) = 0.6,$

$\rho^{\varphi}(n_6) = 0.3$

Edge Labelling (Fig 2): $E_G^{\varphi}(v_i, v_j) = Sim(v_i, v_j)$

$Sim_{\zeta}(v_i, v_j) = \frac{\zeta}{\zeta+\eta}$ where $\eta = \frac{\left|[v_i]_{\mathcal{S}_r}\right| * \left|[v_j]_{\mathcal{S}_r}\right|}{\left|[v_i]_{\mathcal{S}_r}\right| + \left|[v_j]_{\mathcal{S}_r}\right|}$ and

$\zeta_{ij} = \rho^{\varphi}(v_i) + \rho^{\varphi}(v_j) + m$

TABLE 6: EDGE LABELLING OF ROUGH GRAPH (FIG.1)

Edges	ζ_{ij}	η	$\frac{\zeta}{\zeta+\eta}$
$Sim(n_1, n_3)$	13.9	1.0	0.932
$Sim(n_1, n_4)$	13.9	1.2	0.920
$Sim(n_1, n_5)$	13.9	1.3	0.914
$Sim(n_1, n_6)$	13.6	1.0	0.931
$Sim(n_2, n_3)$	13.0	0.8	0.944
$Sim(n_2, n_4)$	14.1	1.7	0.892
$Sim(n_2, n_5)$	14.2	2.0	0.876
$Sim(n_2, n_6)$	13.9	1.3	0914
$Sim(n_3, n_4)$	13.8	1.2	0.920
$Sim(n_3, n_6)$	13.6	1.0	0.930
$Sim(n_4, n_5)$	14.1	1.7	0.890
$Sim(n_4, n_6)$	13.8	1.2	0.920
$Sim(n_5, n_6)$	13.9	1.3	0.914

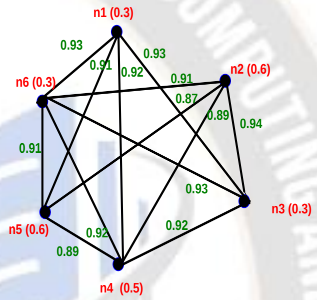

Fig.2. Rough ζ – Labelling graph

I. Representation form of rough ζ -Labeling graphs: (Adjacency matrix):

The adjacency matrix A of a rough graph $G = (V, E, \omega)$ is a $n * n$ matrix defined as $A = [a_{ij}]$ where $a_{ij} = Sim_{\zeta}(v_i, v_j)$.

TABLE 7: ADJACENCY MATRIX FOR FIG 2

	n_1	n_2	n_3	n_4	n_5	n_6
n_1	0	0	0.932	0.920	0.914	0.931
n_2	0	0	0.944	0.892	0.876	0.914
n_3	0	0.932	0	0.920	0	0.930
n_4	0.920	0.892	0.920	0	0.890	0.920
n_5	0.914	0.876	0	0.890	0	0.914
n_6	0.931	0.914	0.930	0.920	0.914	0

III. RESULT AND DISCUSSION

A. Application in Wireless Sensor Network:

- Traditional cluster region

- Based on RMB and Labelling cluster regions are formed.
  Figure 3 shows Cluster Diagram of WSN.

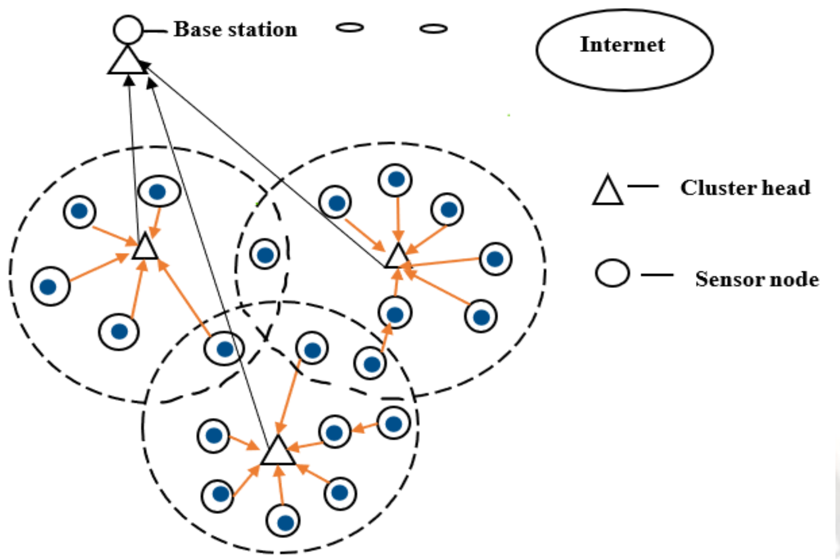

Figure 3. Cluster Diagram of WSN

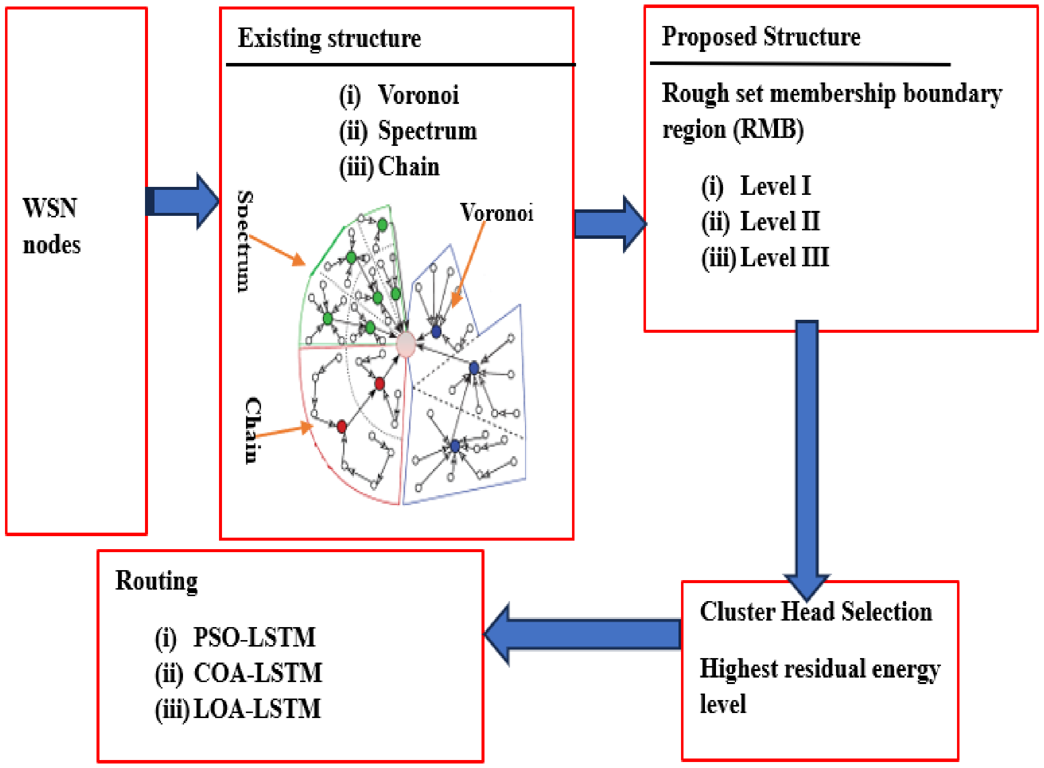

Figure 4. Proposed block diagram for RMB Structure

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B. Methodology of RMB for cluster boundary and routing using LOA-LSTM Algorithm

1) PSO-LSTM

Particle Swarm Optimization (PSO) is a nature-inspired optimization algorithm used to find the optimal solution through iteratively adjusting population of particle. Each particle's movement is based on own best position and the best-known position of neighbouring particles. PSO finds the best parameter values and optimizes objective function. Long Short-Term Memory (LSTM) architecture is effective for modelling the sequences of data, time-series data. LSTMs overcomes the vanishing gradient problem and captures long-range dependencies and patterns of sequential data. The PSO optimize the LSTM model hyper parameters and improve routing performance in WSN. This hybrid approach leverages the optimization capabilities of PSO and fine-tunes the LSTM model for the specific task of routing.

For each particle i:

Update particle's velocity: $v_{i(t+1)} = w * v_{i(t)} + cl * rand() * (pbest_i - x_{i(t)}) + c_2 * rand() * (gbest - x_{i(t)})$Update particle's position:

$x_{i(t+1)} = x_{i(t)} + v_{i(t+1)}$ Where

- $\boldsymbol{v_{i(t)}}$ represents velocity of particle which is at iteration
- $\boldsymbol{x_{i(t)}}$ represents position which meand hyperparameter of particle (i) @iteration (t)
- $pbest_i$ represented position visited through particle i,
- $gbest$ represent global best position,
- w,c_1,c_2 are weight coefficients,
- $rand()$ provides random number.

2) LSTM Model for WSN Clustering

The LSTM model is designed to capture temporal dependencies in sequences, which could be useful for WSN data clustering. It requires defining the LSTM architecture, hyper parameters, and the clustering evaluation metric. Define LSTM architecture:

LSTM($input_{dim}$, $hidden_{units}$, num_{layers}...).

Train LSTM model: model. Fit(X_train, ...) Predict cluster labels: $cluster_{labels}$ = model.predict(X_{test}). Evaluate clustering performance: $evaluation_{metric}$ =$evaluate_{clusters}$ ($cluster_{labels}$, $true_{labels}$)

Where

- $input_{dim}$ the dimensionality of the input data,
- $hidden_{units}$ is the number of LSTM units in each layer,
- num_{layer} is the number of LSTM layers,
- X_{train} and X_{test} the training and testing data, $true_{labels}$ are the ground-truth cluster labels (if available).

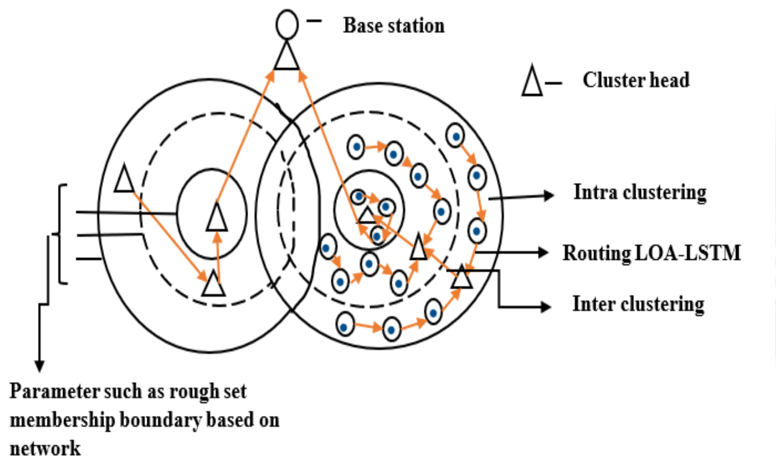

Figure 5. Proposed block diagram rough set membership boundary based on network

3) COA-LSTM

Cat Optimization Algorithm (COA) is an optimization algorithm used in tunning LSTM. The COA algorithm is applied in routing problems. COA-LSTM is referred as hybrid approach that involves using an optimization algorithm and optimize the hyperparameters of an LSTM model for routing in Wireless sensor networks (WSNs). COA-LSTM leverages the strengths and deep learnings achieve improved routing performance.

4) LOA-LSTM

"Loin optimization Algorithm" or LOA optimization - LSTM, incorporates learning or adaptation mechanisms and improves performance. The algorithms often adjust their parameters or strategies based on the observed outcomes or feedback during optimization. LOA and LSTM is applied for WSN routing. This hybrid approach leverages the optimization capabilities of LSTM model, ultimately improves routing performance.

Throughput refers to the amount of data successfully transmitted between nodes in a wireless

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sensor network within a given period. It is a measure of the network data transmission capacity and efficiency. Higher throughput indicates that the network is capable of transmitting more data, which is crucial for applications and requires real-time data. Energy efficiency is a critical concern in wireless sensor networks is referred due to the limited energy resources in nodes. Nodes are battery-powered and energy to be prolonged for network's lifetime. Energy-efficient protocols minimizes energy consumption during various process in WSN such as communication, data processing. The distance between nodes affects the network performance. The Fig.6 shows the performance of proposed algorithm such as PSO-LSTM, COA-LSTM, LOA-LSTM under boundary structures.

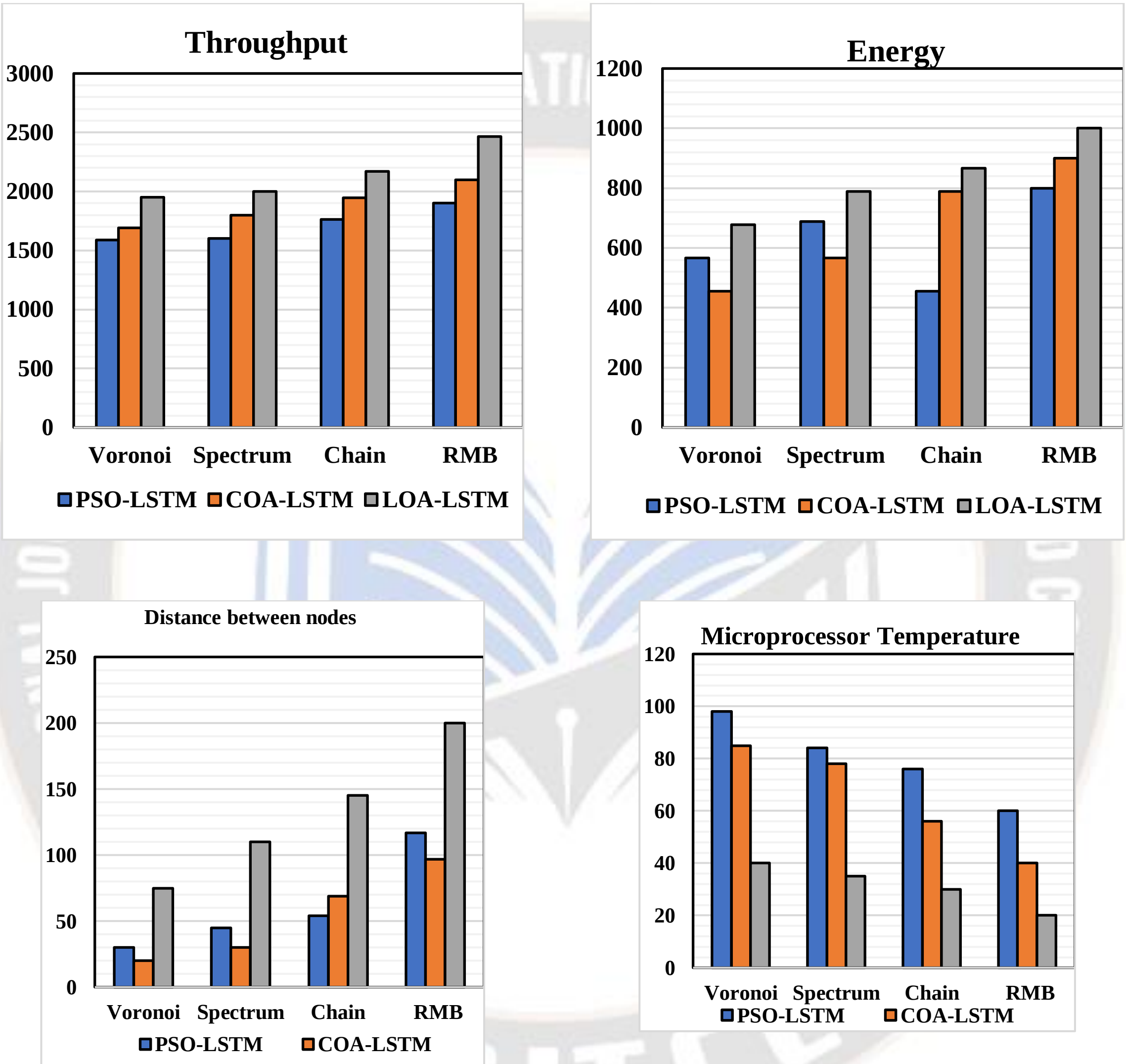

Figure 6. Output of proposed algorithms PSO-LSTM, COA-LSTM and LOA-LSTM for different cluster boundaries

Communication range determines whether nodes directly communicate with each other or require intermediate nodes for relaying. The physical distance can impact signal strength, quality, and latency. Properly managing node placement and communication range is achieved. optimal network coverage and connectivity. The temperature of microprocessors (or processing units) in sensor nodes is a crucial factor in determining the node's performance and lifespan. Elevated temperatures increase energy consumption, reduces processing speed, and even hardware degradation. Efficient thermal management is essential for maintenance of node performance and prolong the operational lifespan of the sensor devices.

In the case of the RMB structure, the high accuracy of LOA-LSTM improved data routing decisions. Accurate classifications and predictions by LOA-LSTM allow nodes for optimal paths selection for data transmission, reduces data collisions and retransmissions. As a result, throughput in the RMB structure increases with LOA-LSTM compared to PSO-LSTM and COA-LSTM. LOA-LSTM's precise decision-making minimizes delays and improves overall data delivery rates. The enhanced accuracy of LOA-LSTM reduces energy consumption in RMB cluster through accurate decisions and transmit data, LOA-LSTM minimizes unnecessary communication and reduces energy-intensive retransmissions. Consequently, nodes utilizing LOA-LSTM exhibit lower energy consumption compared to the other LSTM variants in the RMB structure. This energy efficiency prolongs network lifetime and improves operational sustainability. The RMB structure's randomly distributed nodes might exhibit varying distances between them, LOA-LSTM's has high accuracy, directly influences node distances, indirectly contributes for uniform data distribution across the network. Accurate decision-making in effective sent to nodes regardless of their location and balances the data coverage and potentially mitigate

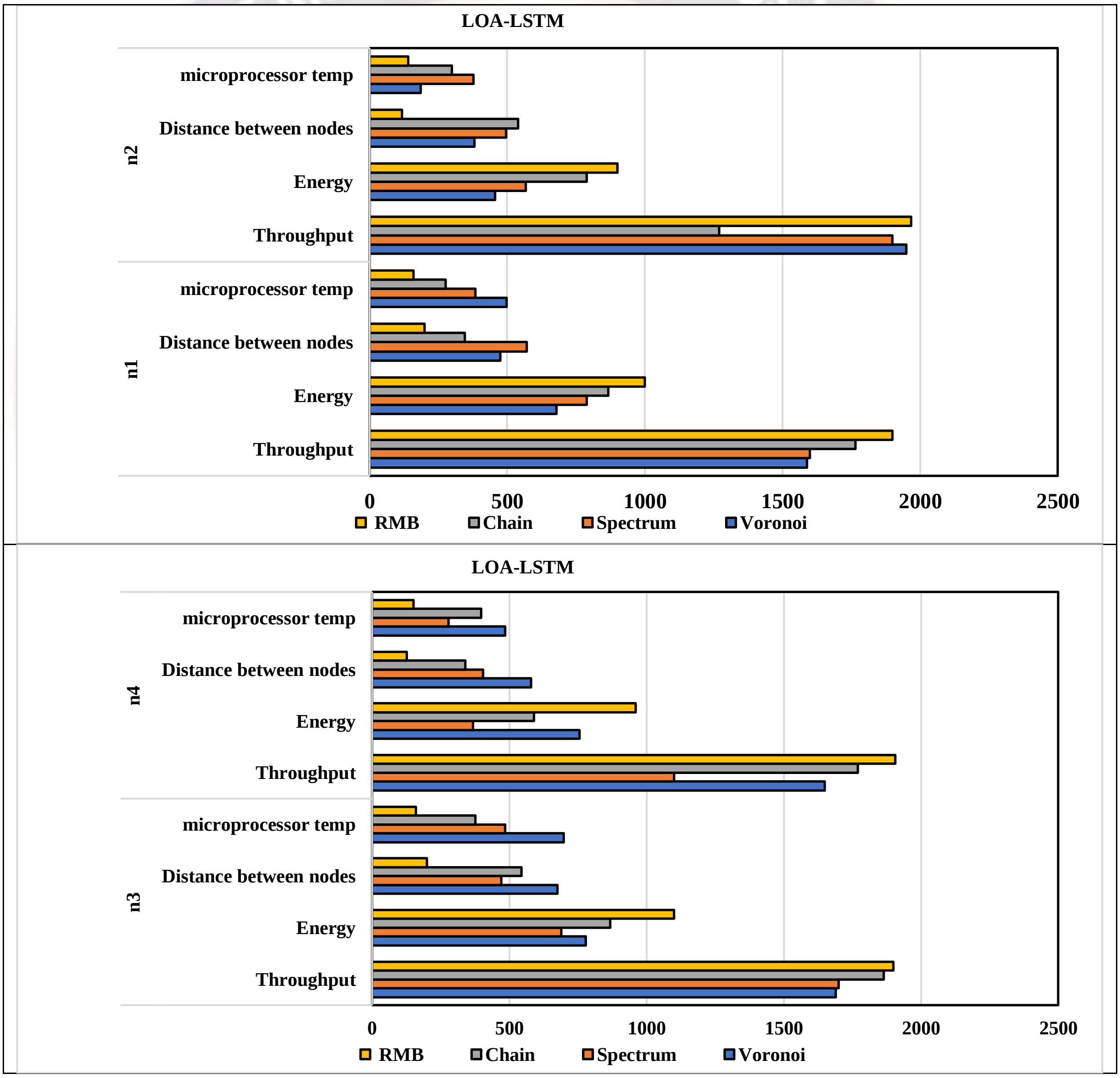

Figure 7. Proposed LOA-LSTM WSN different parameter under different cluster boundary

communication gaps. The RMB structure's node arrangement could result in varied processing loads, impact microprocessor temperature. LOA-LSTM's accuracy optimizes processing requirements. Accurate predictions mean less unnecessary processing, contributes to lower microprocessor temperatures.

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This is beneficial in scenarios where certain nodes are handling more intensive com- putations due to their position in the RMB structure. Fig.7 shows the Proposed LOA-LSTM WSN different parameter under different cluster boundary.

Fig. 8 shows the performance of proposed routing algorithms such as PSO-LSTM, COA-LSTM and LOA-LSTM for different parameters. Precision predicts positive instances. In RMB structure,

LOA-LSTM's has high accuracy and which indicates a large proportion of the positive predictions are correct. This precision improvement with LOA-LSTM is robust for decision-making, which reduces false positive predictions. In comparison, PSO-LSTM and COA-LSTM, the LOA-LSTM performs well, LOA-LSTM's precision is notably higher due to accuracy. Accuracy is for overall correctness of predictions, considering both true positives and true negatives. In the RMB structure, LOA-LSTM's high accuracy indicates that it predicts both positive and negative instances correctly. The RMB's random dis- tribution might pose challenges for accurate pre- diction but LOA-LSTM's ability to handle complex relationships contributes to its high overall accuracy.

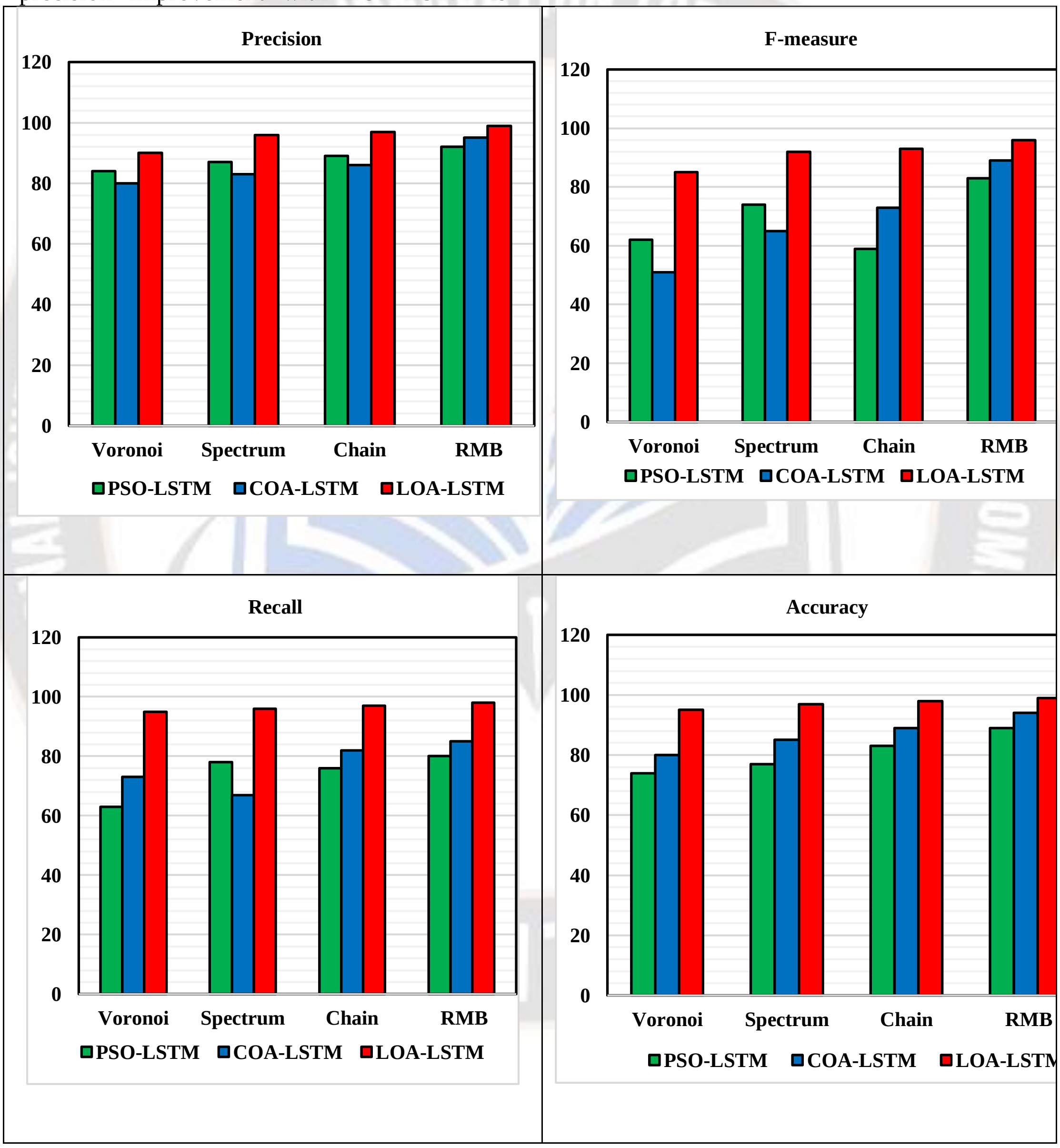

Figure 8. Performance of PSO-LSTM, COA-LSTM and LOA-LSTM

PSO-LSTM and COA-LSTM algorithms in routing also has accuracy. LOA-LSTM's accuracy is significantly. Recall is for proportion of actual positive instances correctly predicted as positive. In RMB structure, LOA-LSTM's high accuracy contributes to a higher recall. Its accurate predictions ensure that more true positive instances are correctly identified, reduce instances of missed positive predictions. PSO-LSTM and COA-LSTM perform less than LOA-LSTM's in RMB structure and improve recall. F-measure is for harmonic mean of precision and recall balanced evaluation for model performance. In RMB structure, LOA-LSTM has high accuracy, precision, and recall with strong F-measure. LOA-LSTM achieve both accurate positive predictions and minimization of false positives and negatives, suits for handling the complexities data in the RMB structure. PSO-LSTM and COA-LSTM has competitive F-measures, LOA-LSTM's exceptional accuracy lead to better F-measure.

IV. CONCLUSION

In this paper, rough graph constructed from information tables of each node in WSN accompanied with rough labelling technique and membership for cluster boundary structure is proposed and termed as RMB algorithm. The RMB minimizes boundary regions and demonstrates the synergistic potential of wireless sensor networks (WSNs) through rough labelling graphs. PSO-LSTM, COA-LSTM and LOA-LSTM is proposed for routing in WSNs. The robust rough set theory with complementary techniques and elevates data categorization for cluster boundary structure and reduces the energy consumption in nodes and cluster head. At its core, the integration of membership functions magnifies the capacity to capture the subtleties inherent in rough labelling graphs in boundary structure for each cluster. This RMB structure reduces the distance between nodes and routing with LOA-LSTM algorithm achieves high throughput and energy saving. Further, RMB can be applied in the underwater acoustic network within WSN. Our research, therefore, does not merely refine existing practices but propels the frontiers of effective decision-making to new horizons.

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