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Exploring Diverse Rough Neighborhoods Through Graphical Analysis

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Abstract— The world's knowledge is believed to double every ten months, and this vast pool of information often contains incomplete data, imperfections, uncertainties, and vague elements. Converting such data into meaningful patterns is a crucial task for data analysts. In this research, we have explored various mathematical models for this purpose. Among these models, we focused on Pawlak's Rough Set model applied through Rough Graphs. Our work presents a novel form of Rough Neighborhood System, as demonstrated in this paper.

Keywords- Set approximations, Rough neighbourhood, Rough Graph

I. INTRODUCTION

In 1982, Zdzisław Pawlak introduced Rough Set Theory as a mathematical framework for dealing with uncertainty and imperfection in data. He employed equivalence relations to identify patterns within inconsistent or imprecise data. The central idea behind Rough Set Theory is to approximate sets using two fundamental operators, Lower and Upper Approximations. Lower Approximation captures the set of elements that are certainly in the rough set, while Upper Approximation encompasses elements that may or may not belong to the set. The theory has found applications in data analysis, knowledge representation, and decision support systems, particularly in cases where data is incomplete or uncertain.

Abu-Donia conducted research that involved comparing various binary relation-based approximations. This work focused on examining different types of right neighborhoods, which are used to define relationships between elements based on binary relations [1]. By providing comparative results, this research contributed to the understanding of how different approaches to approximations can yield distinct outcomes, thus helping researchers and practitioners choose the most appropriate method for their specific applications.

Kay Zeng introduced a model that leverages kernel functions, particularly the Gaussian Kernel function, to construct Rough Neighborhoods. This model is employed in feature selection, a crucial aspect of machine learning and data analysis. The use of kernel functions allows for more flexible and nuanced representations of neighborhoods, potentially leading to better feature selection results in complex datasets.

Qi Wang and colleagues proposed the Local Neighborhood Rough Set model. This model is designed to provide more optimal results compared to classical Rough Set models, particularly in the context of data analysis [2,3,4]. By introducing the concept of local neighborhoods, they aimed to capture relationships between elements that are not immediately apparent in classical Rough Set models [8-16]. Experimental results supported the idea that the Local Neighborhood Rough Set model can yield improved outcomes in certain scenarios. Furthermore, the combination of the Classical Rough Neighborhood model with the Decision Theoretic Rough Set model [5-7] led to the introduction of new approximations, enhancing the versatility of the Rough Set framework.

Shoubin and collaborators delved into the construction of a remote neighborhood system model [20-28]. This model incorporates the concept of a modular lattice, a mathematical structure that represents the relationships and interactions between various elements. By applying this concept to neighborhood systems, they provided a detailed framework for analyzing data with a focus on remote or indirect relationships. This approach can be valuable in situations where understanding the connections between elements requires a more complex and nuanced representation than traditional neighborhood systems.

In this paper we have introduced δ' and δ''-neighborhood based lower and upper approximation and implemented for rough graph.

II. PRELIMINARIES

A. *Definition 1 [1]*

For a given pair (U, R), where U is a non-empty set and R is any binary relation, the right neighborhood of an element x is defined as the set of all elements y in the universe U for which the relation R holds between x and y. In other words, xRy for each element y in the right neighborhood.

B. *Definition 2 [1]*

Let R be a binary relation on a non-empty set U and let A be a subset of U. The definition of the lower approximation R_A concerning the set A with respect to relation R is defined as the set of all elements x in U such that for each element a in A there exists a pair (x, a) in R represented mathematically as

$[R_A = \{x \in U| \ \forall a \in A, (x,a) \in R\}.]$

The definition of the upper approximation R^A concerning the set A with respect to relation R is defined as the set of all elements x in U for which there exists at least one element a in A such that the pair (x, a) is in R represented mathematically as:

$$R^A = \{x \in U| \ \exists a \in A, (x,a) \in R\}.$$

C. *Definition 3 [3]*

Let (U, A, g, δ) be kernel neighborhood function $\forall a, b \in U$

$$[x]_g^\delta = \begin{cases} 0 & g(a,b) < \delta \\ g(a,b) & g(a,b) \geq 0 \end{cases}$$

where $\delta > 0$, $g-$ Gaussian kernel function and $[x]_g^\delta$ $-$ kernel neighborhood granule of x.

For any fuzzy subset $X \subseteq F(U)$ defining lower and upper approximation of x as

$$P_X = \{k_i \in X \mid [k]_l^\delta \subseteq X\}$$

$$P^X = \{k_i \in X \mid [k]_l^\delta \cap X \neq 0\}$$

D. *Definition 4 [2]*

Let $G = (K(G), L(G))$ be a graph. For each $k \in K(G)$. The j-neighbourhood systems for $x, \forall j \in \{r, l, <r>, <l>, i, u, <i>, <u>\}$ are defined by

i) $N_{r(x)} = \{y \in K(G): xRy\}$
ii) $N_{l(x)} = \{y \in K(G): yRx\}$
iii) $N_{\{<r>\}(x)} = \bigcap_{x \in N_r(y)} N_r(y)$
iv) $N_{\{<l>\}(x)} = \bigcap_{x \in N_r(y)} N_r(y)$
v) $N_{i(x)} = N_{r(x)} \cap N_{l(x)}$
vi) $N_{u(x)} = N_{r(x)} \cup N_{l(x)}$
vii) $N_{\{<i>\}(x)} = N_{\{<r>\}(x)} \cap N_{\{<l>\}(k)}$
viii) $N_{\{<u>\}(x)} = N_{\{<r>\}(x)} \cup N_{\{<l>\}(k)}$

E. *Definition 5 [2]*

Let $G = (K(G), L(G))$ be a graph. For each $x \in K(G)$. The j-adhesion neighbourhood are defined $\forall j \in \{r, l, <r>, <l>, i, u, <i>, <u>\}$ are follows

i) $P_{r(x)} = \{y \in K(G): xR = yR\}$
ii) $P_{l(x)} = \{y \in K(G): Rx = Ry\}$
iii) $P_{<r>}(x) = \{y \in K(G): \bigcap_{x \in yR} yR = \bigcap_{y \in xR} xR\}$
iv) $P_{<l>}(x) = \{y \in K(G): \bigcap_{x \in Ry} Ry = \bigcap_{y \in Rx} Rx\}$
v) $P_{i(k)} = P_{r(x)} \cap P_{l(x)}$
vi) $P_{u(k)} = P_{r(x)} \cup P_{l(x)}$
vii) $P_{\{<i>\}(x)} = P_{\{<r>\}(x)} \cap P_{\{<l>\}(x)}$
viii) $P_{\{<u>\}(x)} = P_{\{<r>\}(x)} \cup P_{\{<l>\}(x)}$

III. MAIN RESULTS

A. *E- neighborhood via set*

1) *Definition 3.1*

Let (U, R, γ_j) be a E-neighbourhood space. A δ' of rough set based on E-neighbourhood ($\delta' E$-neighbourhood rough set) of $A \subseteq U$ in (U, R, γ_j) or with respect to γ_j is a pair $({}^{\delta'}N_{jR}(A), {}^{\delta'}N_j^R(A))$, defined by

$${}^{\delta'}N_{jR}(A) = \cup\{E_j(x) : E_j(x) \subseteq A\}$$

called the δ' lower approximation of A.

$${}^{\delta'}N_j^R(A) = [{}^{\delta'}N_{jR}(A^c)]^c$$

called the δ' upper approximation of A.

The accuracy measure of $\delta' E$-neighbourhood is denoted ${}^{\delta'}\mathrm{A}_j(A)$ by and it is defined as

$${}^{\delta'}\mathrm{A}_j(A) = \frac{|{}^{\delta'}N_{jR}(A)|}{|{}^{\delta'}N_j^R(A)|}, \qquad |{}^{\delta'}N_j^R(A)| \neq 0$$

2) *Preposition 3.1*

Let (U, R, γ_j) be a E-neighbourhood space. The following condition holds for every $A, B \subseteq U$:

i) ${}^{\delta'}N_{jR}(\phi) = \phi$
ii) ${}^{\delta'}N_{jR}(A) \subseteq A$
iii) If $A \subseteq B$ then ${}^{\delta'}N_{jR}(A) \subseteq {}^{\delta'}N_{jR}(B)$
iv) ${}^{\delta'}N_{jR}(A) \cup {}^{\delta'}N_{jR}(B) \subseteq {}^{\delta'}N_{jR}(A \cup B)$
v) ${}^{\delta'}N_{jR}(A) = [{}^{\delta'}N_j^R(A^c)]^c$
vi) ${}^{\delta'}N_{jR}({}^{\delta'}N_{jR}(A)) = {}^{\delta'}N_{jR}(A)$

Proof:-

i) ${}^{\delta'}N_{jR}(\phi) = \cup\{E_j(x): E_j(x) \subseteq \phi\} = \phi$

ii) Since $E_j(X) \subseteq A$, then

$${}^{\delta'}N_{jR}(A) \subseteq A$$

iii) Since $A \subseteq B$, then

$$\cup\{E_j(x): E_j(x) \subseteq A\} \subseteq \cup\{E_j(x): E_j(x) \subseteq B\}$$
$$\Longrightarrow {}^{\delta'}N_{jR}(A) \subseteq {}^{\delta'}N_{jR}(B)$$

iv) Since, ${}^{\delta'}N_{jR}(A) \subseteq A$ then ${}^{\delta'}N_{jR}(B) \subseteq B$, then

$${}^{\delta'}N_{jR}(A) \cup {}^{\delta'}N_{jR}(B) \subseteq {}^{\delta'}N_{jR}(A \cup B)$$

v) If $x \in {}^{\delta'}N_{jR}(A)$ for every $x \in A$, there exists $Ej\ (x) \subseteq A$. Then for every $x \in U - (U - A)$, there exits $E_j(x)$ such that $E_j(x) \cap (U - A) = \phi$. Then $x \notin {}^{\delta'}N_j^R(U - A)$ but

$$x \in U - {}^{\delta'}N_{jR}(U - A)$$
$$\therefore {}^{\delta'}N_j^R(A) = U - ({}^{\delta'}Nj^R(U - A))$$
$$= ({}^{\delta'}N_{jR}(U - A))^c$$
$$= ({}^{\delta'}N_{jR}(A^c))^c$$

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vi) If $x \in {}^{\delta'}N_{jR}(A)$, for every $x \in A$, there exists $E_j(x) \subseteq A$. Since ${}^{\delta'}N_{jR}(A) \subseteq A$, then

$$ {}^{\delta'}N_{jR}\left({}^{\delta'}N_{jR}(A)\right) = {}^{\delta'}N_{jR}(A) $$

3) *Proposition 3.2*

Let (U, R, γ_j) be a E-neighbourhood space. The following condition holds for every $A, B \subseteq U$:

i) ${}^{\delta'}N_j^R(U) = U$
ii) $A \subseteq {}^{\delta'}N_j^R(A)$
iii) ${}^{\delta'}N_{jR}(A) \subseteq {}^{\delta'}N_j^R(B)$
iv) If $A \subseteq B$, then ${}^{\delta'}N_j^R(A) \subseteq {}^{\delta'}N_j^R(B)$
v) ${}^{\delta'}N_j^R(A \cup B) = {}^{\delta'}N_j^R(A) \cup {}^{\delta'}N_j^R(B)$
vi) ${}^{\delta'}N_j^R(A) = \left[{}^{\delta'}N_{jR}(A^c)\right]^c$
vii) ${}^{\delta'}N_j^R\left({}^{\delta'}N_j^R(A)\right) = {}^{\delta'}N_j^R(A)$

Proof:-
(i),(ii),(iii) follows from the definition 3.1
iv) By definition 2.1 and, if $A \subseteq B$, then

$$ {}^{\delta'}N_j^R(A) = \left[{}^{\delta'}N_{jR}(A^c)\right]^c $$

$= \cap \{[N_j(x)]^c : [N_j(x) \subseteq A^c]^c\}$
$\subseteq \cap \{[N_j(x)]^c : [N_j(x) \subseteq B^c]^c\}$
$= {}^{\delta'}N_j^R(B)$

$$ \therefore {}^{\delta'}N_j^R(A) \subseteq {}^{\delta'}N_j^R(B) $$

v) It follows from (iv) and by definition 3.1
vi) By definition 3.1
vii) It follows from (ii) and by definition 3.1

4) *Example 3.1*

Let $U = \{a, b, c, d\}$, $A = \{a, b\}$ and
$R = \{(a,a), (b,b), (a,c), (a,d), (d,b), (b,d)\}$

	a	b	c	d
E_r	$\{a,b,d\}$	$\{a,b,d\}$	ϕ	$\{a,b,d\}$
E_l	$\{a,c,d\}$	$\{b,d\}$	$\{a,c,d\}$	U
$E_{<r>}$	U	U	U	U
$E_{<l>}$	$\{a,b\}$	$\{a,b,d\}$	ϕ	$\{b,d\}$
E_i	$\{a,d\}$	$\{b,d\}$	ϕ	$\{a,b,d\}$
E_u	U	$\{a,b,d\}$	$\{a,c,d\}$	U
$E_{<i>}$	$\{a,b\}$	$\{a,b,d\}$	ϕ	$\{b,d\}$
$E_{<u>}$	U	U	U	U

TABLE 1: $E -$ neighborhood of a point

	${}^{\delta'}N_{jR}(A)$	${}^{\delta'}N_j^R(A)$
$j = r$	ϕ	U
$j = l$	ϕ	U
$j = < r >$	ϕ	U
$j = < l >$	$\{a,b\}$	U
$j = i$	ϕ	U
$j = u$	ϕ	U
$j = < i >$	$\{a,b\}$	U
$j = < u >$	ϕ	U

TABLE 2: δ' lower and upper approximation of A

5) *Definition 3.2*

Let (U, R, γ_j) be a E-neighbourhood space. A δ'' of rough set based on E-neighbourhood (δ'' $E-$neighbourhood rough set) of $A \subseteq U$ in (U, R, γ_j) or with respect to γ_j is a pair $\left({}^{\delta''}N_{jR}(A), {}^{\delta''}N_j^R(A)\right)$, defined by

$$ {}^{\delta''}N_{jR}(A) = \left[{}^{\delta''}N_j^R(A^c)\right]^c $$

called the δ'' lower approximation of A.

$$ {}^{\delta''}N_j^R(A) = \cup \{E_j(x) : E_j(x) \cap A \neq \phi\} $$

called the δ'' upper approximation of A.

The accuracy measure of δ'' $E-$neighbourhood is denoted by ${}^{\delta''}\mathcal{A}_j(A)$ and it is defined as

$$ {}^{\delta''}\mathcal{A}_j(A) = \frac{\left|{}^{\delta''}N_{jR}(A)\right|}{\left|{}^{\delta''}N_j^R(A)\right|}, \left|{}^{\delta''}N_j^R(A)\right| \neq 0 $$

6) *Preposition 3.3*

Let (U, R, γ_j) be a $E-$neighbourhood space. The following conditions holds for every $A, B \subseteq U$:
i) ${}^{\delta''}N_{jR}(U) = U$
ii) $A \subseteq {}^{\delta''}N_{jR}\left({}^{\delta''}N_j^R(A)\right)$
iii) If $A \subseteq B \Rightarrow {}^{\delta''}N_{jR}(A) \subseteq {}^{\delta''}N_{jR}(B)$
iv) ${}^{\delta''}N_{jR}(A \cap B) = {}^{\delta''}N_{jR}(A) \cap {}^{\delta''}N_{jR}(B)$
v) ${}^{\delta''}N_{jR}(A) \cup {}^{\delta''}N_{jR}(B) \subseteq {}^{\delta''}N_{jR}(A \cup B)$
vi) ${}^{\delta''}N_{jR}(A) = \left({}^{\delta''}N_j^R(A^c)\right)^c$

Proof:
i) It follows from definition 3.2
ii) It also follows from definition 3.2
iii) If $A \subseteq B$, then

$$ {}^{\delta''}N_{jR}(A) = \left[\cup \{P_j(x) : P_j(x) \cap A^c \neq \phi\}\right]^c $$
$$ \subseteq \left[\cup \{P_j(x) : P_j(x) B^c \neq \phi\}\right]^c $$
$$ = {}^{\delta''}N_{jR}(B) $$

iv) By definition 3.2, we have

$$ {}^{\delta''}N_{jR}(A \cap B) = \cap \left[\{P_j(x) : P_j(x) \cap (A \cap B)^c \neq \phi\}\right]^c $$

Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$, then $P_j(x) \subseteq A$ and $P_j(x) \subseteq B$. Then by (iii), we have

$$ {}^{\delta''}N_{jR}(A \cap B) \subseteq {}^{\delta''}N_{jR}(A) \text{ and} $$
$$ {}^{\delta''}N_{jR}(A \cap B) \subseteq {}^{\delta''}N_{jR}(B) $$
$$ \therefore {}^{\delta''}N_{jR}(A) \cap {}^{\delta''}N_{jR}(B) = \cap \left[\{P_j(x) : P_j(x) \cap A^c \neq \phi\}\right]^c $$

and $\cap \left[\{P_j(x) : P_j(x) \cap B^c \neq \phi\}\right]^c = \cap \left[\{P_j(x) : P_j(x) \cap (A \cap B)^c \neq \phi\}\right]^c$

$$ {}^{\delta''}N_{jR}(A \cap B) = {}^{\delta''}N_{jR}(A) \cap {}^{\delta''}N_{jR}(B) $$

v) The proof is similar by (iv)
vi) If $x \in {}^{\delta''}N_{jR}(A)$ for every $x \in A$,there exists $P_j(x) \subseteq A$.Then for every $x \in U-(U-A)$, there exists $P_j(x)$ such that $P_j(x) \cap (U-A) = \phi$. So

$$x \notin {}^{\delta''}N_j^R(U-A), x \in U - \left({}^{\delta''}N_j^R(U-A)\right) \therefore {}^{\delta''}N_{jR}(A) = \left({}^{\delta''}N_j^R(A^c)\right)^c$$

7) *Preposition 3.4*

Let (U, R, γ_j) be a E −neighbourhood space.The following condition holds for every $A, B \subseteq U$

i) ${}^{\delta''}N_j^R(\phi) = \phi$
ii) ${}^{\delta''}N_j^R\left({}^{\delta''}N_{jR}(A)\right) \subseteq A$
iii) If $A \subseteq B \rightarrow {}^{\delta''}N_j^R(A) \subseteq {}^{\delta''}N_j^R(B)$
iv) ${}^{\delta''}N_j^R(A \cap B) \subseteq {}^{\delta''}N_j^R(A) \cap {}^{\delta''}N_j^R(B)$
v) ${}^{\delta''}N_j^R(A \cup B) = {}^{\delta''}N_j^R(A) \cup {}^{\delta''}N_j^R(B)$
vi) ${}^{\delta''}N_j^R(A) = \left({}^{\delta''}N_{jR}(A^c)\right)^c$

Proof:-
Similar to proposition 3.3.

	${}^{\delta''}N_{jR}(A)$	${}^{\delta''}N_j^R(A)$
j=r	{a,b,c}	{a,b}
j=l	{a,b}	{a,b}
$j=<r>$	{a,b}	{a,b}
$j=<l>$	{a,b,c}	{a,b}
j=i	{a,b,c}	{a,b}
j=u	{a,b}	{a,b}
$j=<i>$	{c}	{a,b}
$j=<u>$	{a,b}	{a,b}

TABLE 3: δ'' lower and upper approximation of A

B. *$j-$ neighborhood via graph*

1) *Definition 4.1*

Let $G = (K(G), L(G))$ be a graph and H be a subgraph of G. Define a first type of lower and upper approximation of H which are denoted by $\underline{N}_j(K(H))$ and $\overline{N}_j(K(H))$ respectively.

$$\underline{N}_j(K(H)) = \{x \in K(G): N_j(x) \subseteq K(H)\}$$
$$\overline{N}_j(K(H)) = K(H) \cup \{x \in K(G): N_j(x) \cap K(H) \neq \phi\}$$

2) *Definition 4.2*

Let $G = (K(G), L(G))$ be a graph and H be a subgraph of G. Define the j −boundary, j −positive, j −negative region and j −accuracy measure of H in terms of j −neighborhood is denoted by BND_{N_j}, POS_{N_j}, NEG_{N_j}, and $\mathcal{A}_{N_j}$

i) $BND_{N_j}(K(H)) = \overline{N}_j(K(H)) - \underline{N}_j(K(H))$
ii) $POS_{N_j}(K(H)) = \underline{N}_j(K(H))$
iii) $NEG_{N_j}(K(H)) = K(G) - \overline{N}_j(K(H))$
iv) $\mathcal{A}_{N_j} = \frac{|\underline{N}_j(K(H))|}{|\overline{N}_j(K(H))|} \left|\overline{N}_j(K(H))\right| \neq 0$

3) *Example 4.1*

Let G be a simple graph. j −neighbourhood systems are defined as follows:

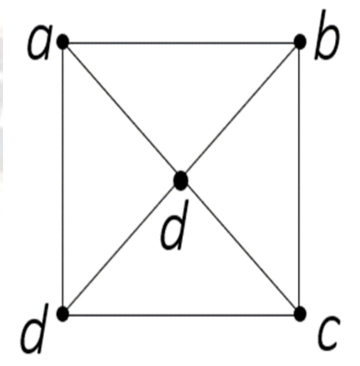

FIGURE 1

If $j = \{r\}$ and $j \in \{l, <r>, <l>, i, u, <i>, <u>\}$, then we have,
i)
$N_j(a) = \{b, d, e\}, N_j(b) = \{c, d\}, N_j(c) = \{e\}, N_j(d) = \{a, b, c\}, N_j(e) = \{d\}$
ii) If j= $\{<r>, <l>, <i>, <u>\}$, then
$N_j(a) = \{a, b, c\}, N_j(b) = \{b\}, N_j(c) = \{c\}, N_j(d) = \{d\}, N_j(e) = \{e\}$
Let $j = r, K(H) = \{a, b, c, d\}$.Then
iii) $\overline{N}_r(K(H))$={a,b,c,d} and $\underline{N}_r(K(H)) = \{b, d, e\}$
iv) $BND_{N_r}(K(H)) = \{a, e\}$
v) $POS_{N_r}(K(H)) = \{b, d, e\}$
vi) $NEG_{N_r}(K(H)) = \{e\}$
vii) $\mathcal{A}_{N_r}(K(H)) = \frac{3}{4}$

4) *Definition 4.3*

Let $G = (K(G), L(G))$ be a graph and H be a subgraph of G. Define the α' type of lower and upper approximation of H which are denoted by ${}^{\alpha'}\underline{N}_j(K(H))$ and ${}^{\alpha'}\overline{N}_j(K(H))$ respectively.

$${}^{\alpha'}\underline{N}_j(K(H)) = \cup\{N_j(x): N_j(x) \subseteq K(H)\}$$
$${}^{\alpha'}\overline{N}_j(K(H)) = \left[{}^{\alpha'}\underline{N}_j(K(H))^c\right]^c$$

5) *Definition 4.4*

Let $G = (K(G), L(G))$ be a graph and H be a subgraph of G. Define the $\alpha' j$-boundary,$\alpha' j$-positive,$\alpha' j$-negative region and $\alpha' j$-accuracy measure of H in terms of j −neighbourhood is denoted by $BND_{\alpha' N_j}$, $POS_{\alpha' N_j}$, $NEG_{\alpha' N_j}$, and $\mathcal{A}_{\alpha' N_j}$

i) $BND_{\alpha' N_j}(K(H)) = {}^{\alpha'}\overline{N}_j(K(H)) - {}^{\alpha'}\underline{N}_j(K(H))$
ii) $POS_{\alpha' N_j}(K(H)) = {}^{\alpha'}\underline{N}_j(K(H))$
iii) $NEG_{\alpha' N_j}(K(H)) = K(G) - {}^{\alpha'}\overline{N}_j(K(H))$
iv) $\mathcal{A}_{\alpha' N_j} = \frac{|{}^{\alpha'}\underline{N}_j(K(H))|}{|{}^{\alpha'}\overline{N}_j(K(H))|}$, $\quad |{}^{\alpha'}\overline{N}_j(K(H))| \neq 0$

6) *Example 4.2*

Continued from example 4.1. Let $j = r, K(H) = \{b, d, e\}$.Then

i) ${}^{\alpha'}\overline{N}_r(K(H)) = \{a, b, c, d, e\}$ and ${}^{\alpha'}\underline{N}_r(K(H)) = \{b, d, e\}$

ii) $BND_{\alpha'N_j}(K(H)) = \{a, c\}$

iii) $POS_{\alpha'N_j}(K(H)) = \{b, d, e\}$

iv) $NEG_{\alpha'N_j}(K(H)) = \phi$

v) $\mathcal{A}_{\alpha'N_j}(K(H)) = \frac{3}{5}$

7) Definition 4.5

Let $G = (K(G), L(G))$ be a graph and H be a subgraph of G. Define the α'' type of lower and upper approximation of H which are denoted by ${}^{\alpha''}\underline{N}_j(K(H))$ and ${}^{\alpha''}\overline{N}_j(K(H))$ respectively,

$$ {}^{\alpha''}\underline{N}_j(K(H)) = \left[{}^{\alpha''}\overline{N}_j(K(H))^c\right]^c $$

$$ {}^{\alpha''}\overline{N}_j(K(H)) = \cup\{N_j(x): N_j(x) \cap K(H) \neq \phi\} $$

8) Definition 4.6

Let $G = (K(G), L(G))$ be a graph and H be a subgraph of G. Define the $\alpha''j$-boundary, $\alpha''j$-positive, $\alpha''j$-negative region and $\alpha''j$-accuracy measure of H in terms of j-neighborhood which will be denoted by $BND_{\alpha''N_j}$, $POS_{\alpha''N_j}$, $NEG_{\alpha''N_j}$, and $\mathcal{A}_{\alpha''N_j}$

$BND_{\alpha''N_j}(K(H)) = {}^{\alpha''}\overline{N}_j(K(H)) - {}^{\alpha''}\underline{N}_j(K(H))$

ii) $POS_{\alpha''N_j}(K(H)) = {}^{\alpha''}\underline{N}_j(K(H))$

iii) $NEG_{\alpha''N_j}(K(H)) = K(G) - {}^{\alpha''}\overline{N}_j(K(H))$

iv) $\mathcal{A}_{\alpha''N_j}(K(H)) = \frac{|{}^{\alpha''}\underline{N}_j(K(H))|}{|{}^{\alpha''}\overline{N}_j(K(H))|}, |{}^{\alpha''}\overline{N}_j(K(H))| \neq 0$

9) Example 4.3

Continued from example 4.1. Let $j = \langle r \rangle, K(H) = \{b, d\}$.Then

i) ${}^{\alpha''}\overline{N}_{<r>}(K(H)) = \{a, b, c, d\}$ and ${}^{\alpha''}\underline{N}_r(K(H)) = \{b, d\}$

ii) $BND_{\alpha''N_{<r>}}(K(H)) = \{a, c\}$

iii) $POS_{\alpha''N_{<r>}}(K(H)) = \{b, d\}$

iv) $NEG_{\alpha''N_{<r>}}(K(H)) = \{e\}$

v) $\mathcal{A}_{\alpha'N_{<r>}}(K(H)) = \frac{1}{2}$

C. $j -$ adhesion neighborhood via graph

1) Definition 5.1

Let $G = (K(G), L(G))$ be a graph and H be a subgraph of G. Define the second type of lower and upper approximation of H which are denoted by $\underline{P}_j(K(H))$ and $\overline{P}_j(K(H))$ respectively, and it is defined by,

$\underline{P}_j(K(H)) = \{x \in K(G): P_j(x) \subseteq K(H)\}$ and

$\overline{P}_j(K(H)) = K(H) \cup \{x \in K(G): P_j(x) \cap (K(H)) \neq \phi)\}$

2) Definition 5.2

Let $G = (K(G), L(G))$ be a graph and H be a subgraph of G. Define the $j -$boundary, $j -$positive, $j -$negative regions and $j -$ accuracy measure of H in terms of $j -$ adhesion neighborhood is denoted by $BND_{P_j}, POS_{P_j}, NEG_{P_j}, \mathcal{A}_{P_j}$ respectively

i) $BND_{P_j}(K(H)) = \overline{P}(K(H)) - \underline{P}(K(H))$

ii) $POS_{P_j}(K(H)) = POS_{\underline{P}_j}(K(H))$

iii) $NEG_{P_j} = K(G) - NEG_{\overline{P_j}}(K(H))$

iv) $\mathcal{A}_{P_j}(K(H)) = \frac{|\underline{P}_j(K(H))|}{|\overline{P}_j(K(H))|}, \quad |\overline{P}_j(K(H))| \neq 0$

3) Example 5.1

Let G be a simple graph. $j -$adhesion neighborhood system is defined as follows:

FIGURE 2

If j={r} and $j \in \{l, \langle r \rangle, \langle l \rangle, i, u, \langle i \rangle, \langle u \rangle\}$, then we have,

i) If $j = \{r, l, i, u\}$, then
$P_j(a) = \{a, d\}, P_j(b) = \{b, c\}, P_r(c) = \{b, c\}, P_j(d) = \{a, d\}$

ii) If $j = \{\langle r \rangle, \langle l \rangle, \langle i \rangle, \langle u \rangle\}$, then
$P_j(a) = \{a, d\}, P_j(b) = \{b, c\}$,
$P_j(c) = \{b, c\}, P_j(d) = \{a, d\}$

Let $j = r, K(H) = \{a, b\}$. Then

iii) $\overline{P_j}(K(H)) = \{a, b, c, d\}$ and $\underline{P}_j(K(H)) = \phi$

iv) $BND_{P_j}(K(H)) = \{a, b, c, d\}$

v) $POS_{P_j}(K(H)) = \phi$

vi) $NEG_{P_j}(K(H)) = \phi$

vii) $\mathcal{A}_{P_j}(K(H)) = 0$

4) Definition 5.3

Let $G = (K(G), L(G))$ be a graph and H be a subgraph of G. Define the β' lower and upper approximation of H which are denoted by ${}^{\beta'}\underline{P}_j(K(H))$ and ${}^{\beta'}\overline{P}_j(K(H))$ respectively,

$$ {}^{\beta'}\underline{P}_j(K(H)) = \cup\{P_j(x): P_j(x) \subseteq K(H)\} \; {}^{\beta'}\overline{P}_j(K(H)) = \left[{}^{\beta'}\underline{P}_j(K(H))^c\right]^c $$

5) Definition 5.4

Let $G = (K(G), L(G))$ be a graph and H be a subgraph of G. Define the $\beta'j$-boundary, $\beta'j$-positive, $\beta'j$-negative region and $\beta'j$-accuracy measure of H in terms of j-neighborhood is denoted by $BND_{\beta'P_j}, POS_{\beta'P_j}, NEG_{\beta'P_j}$, and $\mathcal{A}_{\beta'P_j}$

i) $BND_{\beta'P_j}(K(H)) = {}^{\beta'}\overline{P}_j(K(H)) - {}^{\beta'}\underline{P}_j(K(H))$

ii) $POS_{\beta'P_j}(K(H)) = {}^{\beta'}\underline{P}_j(K(H))$

iii) $NEG_{\beta' P_j}\big(K(H)\big) = \big(K(G)\big) - {}^{\beta'}\overline{P}_j\big(K(H)\big)$

iv) $\mathcal{A}_{\beta' P_j} = \frac{\left|{}^{\beta'}\underline{P}_j(K(H))\right|}{\left|{}^{\beta'}\overline{P}_j(K(H))\right|}, \quad \left|{}^{\beta'}\overline{P}_j\big(K(H)\big)\right| \neq 0$

6) Example 5.2

Let $j = r, K(H) = \{b, c\}$.Then

i) ${}^{\beta'}\underline{P}_j\big(K(H)\big)$={b,c} and ${}^{\beta'}\overline{P}_j\big(K(H)\big) = \{a, d\}$

ii) $BND_{\beta' P_j}\big(K(H)\big) = \{a, d\}$

iii) $POS_{\beta' P_j}\big(K(H)\big) = \{b, c\}$

iv) $NEG_{\beta' P_j}\big(K(H)\big) = \{b, c\}$

v) $\mathcal{A}_{\beta' P_j} = 1$

7) Definition 5.5

Let $G = (K(G), L(G))$ be a graph and H be a subgraph of G. Define the β'' lower and upper approximation of H which are denoted by ${}^{\beta''}\underline{P}_j\big(K(H)\big)$ and ${}^{\beta''}\overline{P}_j\big(K(H)\big)$ respectively. It is defined as follows

$${}^{\beta''}\underline{P}_j\big(K(H)\big) = \left[{}^{\beta''}\overline{P}_j\big(K(H)\big)^c\right]^c \quad {}^{\beta''}\overline{P}_j\big(K(H)\big) = \cup\,\{P_j(x) : P_j(x) \cap K(H) \neq \phi\}$$

8) Definition 5.6

Let $G = (K(G), L(G))$ be a graph and H be a subgraph of G. Define the $\beta'' j$-boundary,$\beta'' j$-positive,$\beta'' j$-negative region and $\beta'' j$-accuracy measure of H in terms of j −neighbourhood is denoted by $BND_{\beta'' P_j}$, $POS_{\beta'' P_j}$, $NEG_{\beta'' P_j}$, and $\mathcal{A}_{\beta'' P_j}$

i) $BND_{\beta'' P_j}\big(K(H)\big) = {}^{\beta''}\overline{P}_j\big(K(H)\big) - {}^{\beta''}\underline{P}_j\big(K(H)\big)$

ii) $POS_{\beta'' P_j}\big(K(H)\big) = {}^{\beta''}\underline{P}_j\big(K(H)\big)$

iii) $NEG_{\beta'' P_j}\big(K(H)\big) = K(G) - {}^{\beta''}\overline{P}_j\big(K(H)\big)$

iv) $\mathcal{A}_{\beta'' P_j}\big(K(H)\big) = \frac{\left|{}^{\beta''}\underline{P}_j(K(H))\right|}{\left|{}^{\beta''}\overline{P}_j(K(H))\right|}, \quad \left|{}^{\beta''}\overline{P}_j\big(K(H)\big)\right| \neq 0$

9) Example 5.3

Continued from example 5.1. Let $j = r, K(H) = \{a, b\}$. Then

i) ${}^{\beta''}\underline{P}_j\big(K(H)\big) = \{a, d\}$ and ${}^{\beta''}\overline{P}_j\big(K(H)\big) = \{a, b, c, d\}$

ii) $BND_{\beta'' P_j}\big(K(H)\big) = \{b, c\}$

iii) $POS_{\beta'' P_j}\big(K(H)\big) = \{a, d\}$

iv) $NEG_{\beta'' P_j}\big(K(H)\big) = \phi$

v) $\mathcal{A}_{\beta'' P_j}\big(K(H)\big) = \frac{1}{2}$

D. E − neighborhood via graph

1) Definition 6.1

Let $G = (K(G), L(G))$ be a graph, for each $x \in K(G)$.The E −neighbourhood system for x,$\forall j \in \{r, l, i, u, <r>, <l>, <i>, <u>\}$ are defined by

i) $E_r(x)$={$y \in K(G): N_r(y) \cap N_r(x) \neq \phi$}

ii) $E_l(x)$={$y \in K(G): N_l(y) \cap N_l(x) \neq \phi$}

iii) $E_{<r>}(x)$={$y \in K(G): N_{<r>}(y) \cap N_{<r>}(x) \neq \phi$}

iv) $E_{<l>}(x)$={$y \in K(G): N_{<l>}(y) \cap N_{<l>}(x) \neq \phi$}

v) $E_i(x)$={$E_r(x) \cap E_l(x)$}

vi) $E_u(x)$)={$E_r(x) \cup E_l(x)$}

vii) $E_{<i>}(x)$={$E_{<r>}(x) \cap E_{<l>}(x)$}

viii) $E_{<u>}(x)$={$E_{<r>}(x) \cup E_{<l>}(x)$}

2) Example 4.1

Let G be a simple graph. E − neighborhood system is defined as follows:

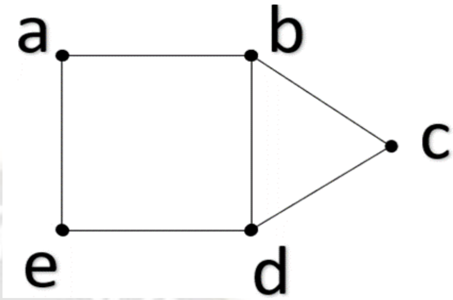

FIGURE 3

If $j = \{r\}$ and $j \in \{l, <r>, <l>, i, u, <i>, <u>\}$, then we have,

i) If $j = \{r, l, i, u\}$, then
$E_j(a) = \{a, c, d\}, E_j(b) = \{b, c, d, e\}, E_j(c) = \{a, b, c, d, e\}$
$E_j(d) = \{a, b, c, d\}, E_j(e) = \{b, c, e\}$

ii) If j= $\{<r>, <l>, <i>, <u>\}$, then
$E_j(a) = \{a, d\}, E_j(b) = \{b, e\}, E_j(c) = \{c\}$
$E_j(d) = \{a, d\}, E_j(e) = \{b, e\}$

3) Definition 6.2

Let $G = (K(G), L(G))$ be a graph and H be a subgraph of G. Define the third type of lower and upper approximation of H which are denoted by $\underline{E}_j\big(K(H)\big)$ and $\overline{E}_j\big(K(H)\big)$ respectively. It is defined as follow,

$$\underline{E}_j\big(K(H)\big) = \{x \in K(G): E_j(x) \subseteq K(H)\} \overline{E}_j\big(K(H)\big) = K(H) \cup \{x \in K(G): E_j(x) \cap \big(K(H)\big) \neq \phi\}$$

4) Definition 6.3

Let $G = (K(G), L(G))$ be a graph and H be a subgraph of G. Define the j −boundary, j −positive, j −negative region and j −accuracy measure of H in terms of E −neighbourhood is denoted by BND_{E_j}, POS_{E_j}, NEG_{E_j}, and $\mathcal{A}_{E_j}$

i) $BND_{E_j}\big(K(H)\big) = \overline{E}_j\big(K(H)\big) - \underline{E}_j\big(K(H)\big)$

ii) $POS_{E_j}\big(K(H)\big) = \underline{E}_j\big(K(H)\big)$

iii) $NEG_{E_j}\big(K(H)\big) = K(G) - \overline{E}_j\big(K(H)\big)$

iv) $\mathcal{A}_{E_j}\big(K(H)\big) = \frac{\left|\underline{E}_j(K(H))\right|}{\left|\overline{E}_j(K(H))\right|}, \left|\overline{E}_j\big(K(H)\big)\right| \neq 0$

5) Example 6.2

Continued from example 6.1 Let $j = r, K(H) = \{a, b, c\}$.Then

i) $\underline{E}_j\big(K(H)\big) = \{c, d\}$ and $\overline{E}_j\big(K(H)\big) = \{a, b, c, d, e\}$

ii) $BND_{E_j}\big(K(H)\big) = \{a, b, e\}$

iii) $POS_{E_j}\big(K(H)\big) = \{c, d\}$

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iv) $NEG_{E_j}\big(K(H)\big) = \phi$
v) $\mathcal{A}_{E_j}\big(K(H)\big) = \frac{2}{5}$

6) Definition 6.4

Let $G = (K(G), L(G))$ be a graph and H be a subgraph of G. Define the δ' lower and upper approximation of H which are denoted by ${}^{\delta'}\underline{E}_j\big(K(H)\big)$ and ${}^{\delta'}\overline{E}_j\big(K(H)\big)$ respectively,

$${}^{\delta'}\underline{E}_j\big(K(H)\big) = \cup\,\{E_j(x): E_j(x) \subseteq K(H)\}\ {}^{\delta'}\overline{E}_j\big(K(H)\big) = \left[{}^{\delta'}\underline{E}_j\big(K(H)\big)^c\right]^c$$

7) Definition 6.5

Let $G = (K(G), L(G))$ be a graph and H be a subgraph of G. Define the $\delta' j$-boundary, $\delta' j$-positive, $\delta' j$-negative region and $\delta' j$-accuracy measure of H in terms of $E-$neighborhood is denoted by $BND_{\delta'_{E_j}}$, $POS_{\delta'_{E_j}}$, $NEG_{\delta'_{E_j}}$, and $\mathcal{A}_{\delta'_{E_j}}$

i) $BND_{\delta'_{E_j}}\big(K(H)\big) = {}^{\delta'}\overline{E}_j\big(K(H)\big) - {}^{\delta'}\underline{E}_j K(H)$
ii) $POS_{\delta'_{E_j}}\big(K(H)\big) = {}^{\delta'}\underline{E}_j\big(K(H)\big)$
iii) $NEG_{\delta'_{E_j}}\big(K(G)\big) = \big(\overline{K}(H)\big) - {}^{\delta'}\overline{E}_j\big(K(H)\big)$
iv) $\mathcal{A}_{\delta'_{E_j}}\big(K(H)\big) = \frac{\left|{}^{\delta'}\underline{E}_j(K(H))\right|}{\left|{}^{\delta'}\overline{E}_j(K(H))\right|}, \left|{}^{\delta'}\overline{E}_j\big(K(H)\big)\right| \neq 0$

8) Example 6.3

Continued from example 6.1. Let $j = r\ K(H) = \{b, d, e\}$.Then

i) ${}^{\delta'}\underline{E}_j\big(K(H)\big)$={b,e} and ${}^{\delta'}\overline{E}_j\big(K(H)\big) = \{a, b, d, e\}$
ii) $\overline{BND}_{\delta'_{E_j}}\big(K(H)\big) = \{a, d\}$
iii) $POS_{\delta'_{E_j}}\big(K(H)\big) = \{b, e\}$
iv) $NEG_{\delta'_{E_j}}\big(K(H)\big) = \{c\}$
v) $\mathcal{A}_{\delta'_{E_j}}\big(K(H)\big) = 1$

9) Definition 6.6

Let $G = (K(G), L(G))$ be a graph and H be a subgraph of G. The δ'' lower and upper approximation of H which are denoted by ${}^{\delta''}\underline{E}_j\big(K(H)\big)$ and ${}^{\delta''}\overline{E}_j\big(K(H)\big)$ respectively and it is defined by

$${}^{\delta''}\underline{E}_j\big(K(H)\big) = \left[{}^{\delta''}\overline{E}_j\big(K(H)\big)^c\right]^c\ {}^{\delta''}\overline{E}_j\big(K(H)\big) = \cup\,\{E_j(x); E_j(x) \cap K(H) \neq \phi\}$$

10) Definition 6.7

Let $G = (K(G), L(G))$ be a graph and H be a subgraph of G. Define the $\delta'' j$-boundary, $\delta'' j$-positive, $\delta'' j$-negative region and $\delta'' j$-accuracy measure of H in terms of $E-$neighborhood is denoted by $BND_{\delta''_{E_j}}$, $POS_{\delta''_{E_j}}$, $NEG_{\delta''_{E_j}}$, and $\mathcal{A}_{\delta''_{E_j}}$

i) $BND_{\delta''_{E_j}}\big(K(H)\big) = {}^{\delta''}\overline{E}_j\big(K(H)\big) - {}^{\delta''}\underline{E}_j\big(K(H)\big)$
ii) $POS_{\delta''_{E_j}}\big(K(H)\big) = {}^{\delta''}\underline{E}_j\big(K(H)\big)$
iii) $NEG_{\delta''_{E_j}}\big(K(H)\big) = \overline{K}(G) - {}^{\delta''}\overline{E}_j\big(K(H)\big)$
iv) $\mathcal{A}_{\delta''_{E_j}}\big(K(H)\big) = \frac{\left|{}^{\delta''}\underline{E}_j(K(H))\right|}{\left|{}^{\delta''}\overline{E}_j(K(H))\right|}, \left|{}^{\delta''}\overline{E}_j\big(K(H)\big)\right| \neq 0$

11) Example 6.7

Continued from example 6.1. Let $j =< r >, K(H) = \{a, b, c, d\}$.Then

i) ${}^{\delta''}\underline{E}_j\big(K(H)\big) = \{a, c, d\}$ and ${}^{\delta''}\overline{E}_j\big(K(H)\big) = \{a, b, c, d, e\}$
ii) $\overline{BND}_{\delta''_{E_j}}\big(K(H)\big) = \{d, e\}$
iii) $POS_{\delta''_{E_j}}\big(K(H)\big) = \{a, c, d\}$
iv) $NEG_{\delta''_{E_j}}\big(K(H)\big) = \phi$
v) $\mathcal{A}_{\delta''_{E_j}}\big(K(H)\big) = \frac{3}{5}$

IV. CONCLUSION

The paper presents an extended exploration of rough neighborhoods at various levels, thereby contributing to an enriched understanding of Rough Sets in the context of Knowledge Discovery. This research delves into the concept of rough neighborhoods across different levels and provides valuable insights and findings that have the potential to enhance and broaden the application of Rough Sets in the field of knowledge discovery. The future work introduces a novel approach by applying different types of neighborhoods, represented through graphs, to the core concept of rough set theory, particularly in the context of data mining, which focuses on the reduction of information systems. This innovative approach demonstrates that visualizing reducts using graphs is significantly more intuitive and accessible compared to the traditional Pawlak's rough set theory. By employing these graphical representations, the paper not only enhances the clarity and interpretability of the reduction process but also opens up new possibilities for advancing the field of data mining in conjunction with rough set theory.

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