

Modified LMS Adaptive Algorithm for the determination of Maximum SNR using Self-Adaptation Technique of Noise Factor for Communication System

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Abstract—The most often used adaptive filter (AF) is the least-mean-square (LMS) filter. This filter has many applications in the area of communication and signal processing. System identification represents a significant use case for adaptive filters. This paper represents a new structure/model of system identification and adaptive noise cancellation (ANC) using state-of-the-art LMS-AF. The paper expressed the algorithm of updated LMS-AF. The major parts of the proposed model are- two adaptive filters, one LMS filter, one correlation function, and one auto-correlation function. The research investigation for the proposed model is based on the self-adaptation or self-learning method to obtain the maximum signal to noise ratio (SNR) based on the real-time inputs. The proposed structure/model converges towards the ideal LMS-ANC system. This paper also includes mathematical analysis, simulation, results, and discussion. The most important comparison parameters are the SNR and mean-square-error (MSE).

Keywords- LMS-AF; ANC; Self-learning/adaptation; SNR; MSE; Correlation; Auto-correlation

I. INTRODUCTION

The filtering is the fundamental method to computation of signals contaminated by additive white Gaussian noise [1–3]. This method is noise cancellation techniques which can be justified after the evaluation and comparison of SNRs. The following digital filters are used as noise cancellation process- FIR [4], IIR [5], Wiener [6], Kalman [7], LMS [8,9] and RLS [10,11], etc. Nowadays, most popular and widely filter is LMS adaptive filters in the application of communication systems due to low computation complexity [12–14]. The major applications of adaptive filters are plant or system identification [15], noise cancellation [16,17], echo cancellation [18], ECG signal analysis [19,20], prediction [21], line enhancer [22], channel equalization [23], inverse modelling [24], channel identification [25], etc. The concepts of LMS-ANC can be expressed with the help of figure 1 [1,26–28]. Figure 1 consists, an adaptive filter, one LMS algorithm block and one subtractor block [2,29]. The adaptive filter has FIR filter structure of filter length L . Adaptive filter generates an output $y(n)$ after the updating of FIR filter weights as a consequence of reference input signal $x(n)$ which is generated from the noise source. This filtered signal $y(n)$ is subtracted in the primary input signal $d(n)$ and received error/system output signal $e(n)$. The signal $d(n)$ is jointly generated from the signal and noise sources. It means signal $d(n)$ is a noisy information/message signal $I(n)$. The FIR filter weights are adjusted and updated by the LMS algorithm. The LMS algorithm depends on the signals $x(n)$ and $e(n)$.

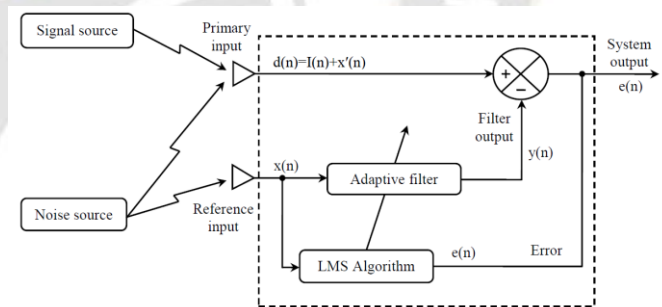


Figure 1. Adaptive noise cancelling concept.

In the present and past researches, the adaptive filter cancels the unwanted noise signal approximately from the noise included information/message signal. The basics of the paper belong to the reference paper [30] but the novelty of the paper is the perfect noise cancellation from the primary input signal using self-learning/adaptation technique. In results, $e(n)$ will be equivalent to $I(n)$ when $x'(n)$ is same as $y(n)$. Signals $I(n)$ and $x(n)$ are uncorrelated but $x(n)$ and $x'(n)$ are correlated.

II. LMS-ANC ALGORITHM

The LMS algorithm is robust and simple [31]. This algorithm for an adaptive noise cancellation had classified into three parts: output signal algorithm, error signal algorithm, and weight updating algorithm [23,25,32–34]. The weight updating algorithm is the backbone of LMS algorithm. The algorithms

are discussed as follows: The output signal algorithm is given in eq. (1),

$$y(n) = W^T(n)X(n) \quad (1)$$

The error signal algorithm is expressed in eq. (2),

$$e(n) = d(n) - y(n) \quad (2)$$

The MSE of error signal is as given in the eq. (3),

$$\xi(n) = E[e^2(n)] \quad (3)$$

Using eq. (1), eq. (2) and eq. (3),

$$\xi(n) = E[d^2(n)] + W^T(n)R_X W(n) - 2W^T(n)P_X \quad (4)$$

Where,

$$R_X = E[X(n)X^T(n)] \text{ and } P_X = E[d(n)X(n)]$$

R_X is denoted as autocorrelation vector of $x(n)$ and P_X is denoted as correlation vector of $d(n)$ and $x(n)$. Evaluating the gradient vector with respect to $W(n)$ using eq. (4),

$$\nabla_W \xi(n) = \frac{\partial}{\partial W(n)} [\xi(n)] = 2R_X W(n) - 2P_X \quad (5)$$

The optimum weight vector will be as follows for the minimum value of MSE,

$$W(n) = W_{opt}(n) \quad (6)$$

The gradient vector will be zero for the optimal result of weight vector. Using eq. (5) and eq. (6); the optimal vector value is expressed as,

$$W_{opt}(n) = R_X^{-1}P_X \quad (7)$$

$$0 < \mu < \frac{2}{\lambda_{max}} \quad (10)$$

Where,

λ_{max} = Maximum Eigen value of autocorrelation vector R_X

μ = Step-size of the LMS-AF

$W(n)$ = Weight matrix of the LMS-AF, dimension $L \times 1$

$X(n)$ = Input signal matrix of the LMS-AF, dimension $L \times 1$

$y(n)$ = Output signal of the LMS-AF, dimension 1×1

$d(n)$ = Primary input signal of the LMS-AF, dimension 1×1

$e(n)$ = System output of the LMS-AF, dimension 1×1

$W(n+1)$ = Updated weight matrix of the LMS-AF, dimension $L \times 1$ and , λ_{max} is defined in the eq. (11) which depends on the energy of $x(n)$ [36,37]. Therefore, the step-size of LMS-AF will depend on energy of the signal.

$$\lambda_{max} = (L+1)E[x(n)^2] \quad (11)$$

$$W(n) = [w_0(n) \ w_1(n) \ \dots \ w_{L-1}(n)]^T \quad (12)$$

$$X(n) = [x(n) \ x(n-1) \ \dots \ x(n-L+1)]^T \quad (13)$$

Where, L = Filter length of 1st LMS-AF

The convergence rate of $e(n)$ is the important analysis parameter. This parameter is calculated [38] using eq. (1), eq. (2) and eq. (9).

$$e(n+1) = e(n)(1 - \mu \lambda_{max}) \quad (14)$$

Where, $e(0) = k$, $W(0) = 0$, $d(n) = k$, $x(n) = x$, and $\mu \ll 1$. The signals $d(n)$ and $x(n)$ are considered as constant for the convergence rate analysis of system output signal $e(n)$.

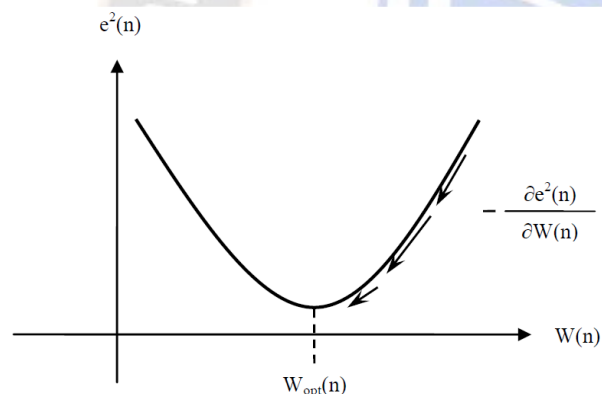


Figure 2. Weight updating using the steepest descent method.

The $W_{opt}(n)$ is the optimum wiener solution of LMS algorithms for noise cancellation. The MSE is minimized at this optimum solution of the weight vector. This weight update algorithm is based on the minimum MSE technique. As per the condition of the minimum MSE, $e(n)$ will be equivalent to $I(n)$ when $x'(n)$ is same as $y(n)$ because $I(n)$ and $x(n)$ are uncorrelated. The LMS algorithm adapts the property of noise and approximately cancels the noise from the primary input signal. Figure 2 shows the weight updating technique using steepest descent method [35]. This method depends on the negative gradient vector of square error signal. The weight update algorithm is described as,

$$W(n+1) = W(n) + \mu \left[- \frac{\partial}{\partial W(n)} \{e^2(n)\} \right] \quad (8)$$

After the simplification of eq. (8) using eq. (2):

$$W(n+1) = W(n) + \mu e(n)X(n) \quad (9)$$

Where, the step-size μ range is expressed as,

III. ANALYSIS OF THE PROPOSED MODEL

The self-learning/adaptation technique of ANC concepts model is revealed in figure 3 with two adaptive filters. The signals $d(n)$ and $x(n)$ are used as inputs of the given model. The input $d(n)$ which is a combination of $I(n)$ and $x_1(n)$ generated from the signal source and noise source respectively. The weights of FIR filter used in the adaptive filter-1 are decided as per LMS algorithm by use of signals $x(n)$ and $e(n)$. The reference input noise signal $x(n)$ is also generated from the noise source. The FIR filter length is L . These weights are also passed to the FIR filter of adaptive filter-2 through digital multiplier with noise factor $\delta(n)$. The proposed parameter noise factor $\delta(n)$ is determined with the help of real-time inputs which are $d(n)$ and $x(n)$. The digital multiplier can be replaced by digital amplifier with gain as noise factor $\delta(n)$. The $y_1(n)$ signal of adaptive filter-2 passes to the subtractor. The subtractor subtracts $y_1(n)$ from $d(n)$ and generates system output $e_1(n)$. This innovative model gives the system output $e_1(n)$ is equivalent to the information/message signal $I(n)$.

A. Algorithms

The algorithms and other mathematical analysis of the proposed model based on self-learning/adaptation technique are described in this sub-section. The LMS algorithms of the proposed model are expressed in the eq. (15) to eq. (27).

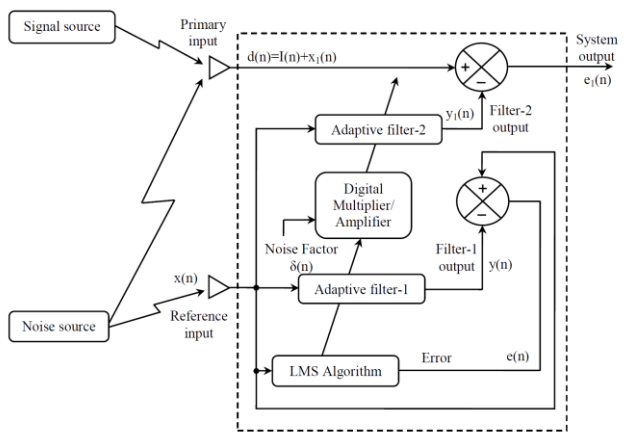


Figure 3. ANC concept using self-learning/adaptation technique.

The filtered signal algorithm of adaptive filter-1 is specified in eq. (15),

$$y(n) = W^T(n)X(n) \quad (15)$$

The error signal algorithm is expressed in eq. (16),

$$e(n) = x(n) - y(n) \quad (16)$$

The weight update LMS algorithm is in eq. (17),

$$W(n+1) = W(n) + \mu X(n)e(n) \quad (17)$$

The digital multiplier or digital amplifier algorithm is expressed in eq. (18),

$$W_1(n) = \delta(n)W(n) \quad (18)$$

The filtered output signal algorithm of adaptive filter-2 is given in eq. (19),

$$y_1(n) = W_1^T(n)X(n) \quad (19)$$

The system output algorithm of proposed model is expressed in eq. (20),

$$e_1(n) = d(n) - y_1(n) \quad (20)$$

B. Error signal convergence rate

The convergence rate of $e_1(n)$ for the proposed model is defined in the eq. (21).

$$e_1(n+1) = e_1(n)(1 - \delta(n)L\mu x^2) \quad (21)$$

Where, $e_1(0) = k$, $W(0) = 0$, $d(n) = k$, $x(n) = x$ and $\mu \ll 1$.

C. Determination of noise factor $\delta(n)$

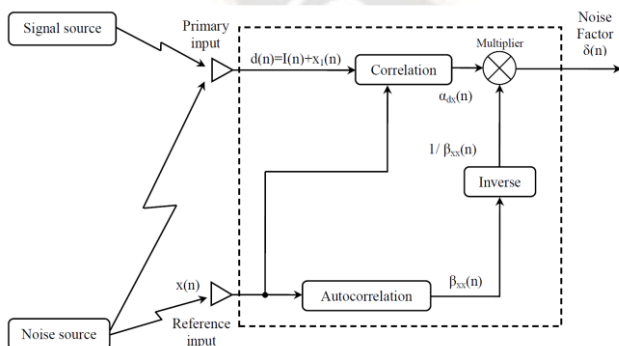


Figure 4. Block diagram for the determination of noise factor $\delta(n)$.

Figure 4 presents the block diagram for the determination of noise factor $\delta(n)$ used in figure 3. The noise factor $\delta(n)$ is used for the perfect ANC. The factor $\delta(n)$ is determined by $d(n)$ and $x(n)$. These input signals are same as inputs given for the

proposed model of figure 3. The determination of noise factor $\delta(n)$ is based on the correlation functions [39]. The correlation function $\alpha_{dx}(n)$ is determined with the help of eq. (22),

$$\alpha_{dx}(n) = \sum_{i=0}^n d(i)x(i-n) \quad (22)$$

The auto-correlation function $\beta_{xx}(n)$ is determined with the help of eq. (23),

$$\beta_{xx}(n) = \sum_{i=0}^n x(i)x(i-n) \quad (23)$$

The inverse function of autocorrelation function of reference input noise signal $x(n)$ is generated i.e. $1/\beta_{xx}(n)$. The noise factor $\delta(n)$ is evaluated with the help of eq. (24),

$$\delta(n) = \frac{\alpha_{dx}(n)}{\beta_{xx}(n)} = \frac{\sum_{i=0}^n d(i)x(i-n)}{\sum_{i=0}^n x(i)x(i-n)} \quad (24)$$

If information/message signal $I(n)$ is sinusoidal and noise is proportional to any function $f(n)$. Therefore, signals $I(n)$, $x'(n)$, $d(n)$ and $x(n)$ can be defined as,

$$I(n) = A \sin(\omega_0 n) \quad (25)$$

$$x'(n) = B(n)f(n) \quad (26)$$

$$d(n) = I(n) + x'(n) = A \sin(\omega_0 n) + B(n)f(n) \quad (27)$$

$$x(n) = C(n)f(n) \quad (28)$$

The correlation function $\alpha_{dx}(n)$ is calculated with the help of eq. (22), (27) and (28),

$$\alpha_{dx}(n) = \text{Corr}(d(n), x(n)) = B(n)C(n) * \text{Corr}(f^2(n)) \quad (29)$$

Where, signals $I(n)$ and $x(n)$ are non-correlated but signals $x'(n)$ and $x(n)$ are correlated. $\text{Corr}(\cdot)$ is the notation of correlation function. The auto-correlation function $\beta_{xx}(n)$ is calculated with the help of eq. (23) as,

$$\beta_{xx}(n) = \text{Corr}(x(n), x(n)) = C^2(n) * \text{Corr}(f^2(n)) \quad (30)$$

The noise factor $\delta(n)$ is calculated with the help of eq. (24), eq. (29) and eq. (30),

$$\delta(n) = \frac{\alpha_{dx}(n)}{\beta_{xx}(n)} = \frac{B(n)C(n) * \text{Corr}(f^2(n))}{C^2(n) * \text{Corr}(f^2(n))} = \frac{B(n)}{C(n)} \quad (31)$$

The noise factor $\delta(n)$ is self-calculated as per the given signals $d(n)$ and $x(n)$ by the model given in figure 4. The calculated value of noise factor $\delta(n)$ passes into the proposed model of noise cancellation as shown in figure 3 for the maximum SNR of system output $e_1(n)$.

TABLE I. PARAMETERS USED IN THE MATLAB SIMULATION FOR THE DETERMINATION OF NOISE FACTOR $\delta(n)$

S. N.	Parameter	Symbol	Value/Range	Reference eq.(s)
1	Magnitude of information/message signal $I(n)$	A	2	25, 27
2	Frequency of information/message signal $I(n)$ in radian/second	ω_0	20	25, 27
3	Noise magnitude present in the signal $d(n)$	B(n)	0.1-5	26, 27
4	Reference noise magnitude	C(n)	1	27
5	Noise function as Gaussian noise	f(n)	Mean = 0, Variance = 0.75	26, 27, 28
6	Sample-time	-	0.001	-
7	Number of Samples/Iterations	N	1001	-

The MATLAB model of figure 4 is simulated using the parameters given in Table 1 and the results are made known in figures 5, 6 & 7. According to the parameter's values and eq. (38), the value of the noise factor $\delta(n)$ will be equal to $B(n)$. Figure 5 shows that the value of $B(n)$ is approximately same as calculated value of noise factor $\delta(n)$. Figure 6 also gives the percentage error in the calculated value of noise factor $\delta(n)$. The figures 6 and 7 present that the percentage error in the calculated value of noise factor $\delta(n)$ is higher if $B(n) \ll 1$ or $B(n) \rightarrow 0$. If $B(n) \ll 1$ or $B(n) \rightarrow 0$ represents the noise factor in signal $d(n)$ is very less, therefore, no need to apply the noise cancellation process or we can ignore this limiting value. As per the simulation results; if $B(n) > 0.5$ approximately, the absolute value of the percentage error for the calculated value of noise factor $\delta(n)$ is less than 4% at the parameter's values given in table 1.

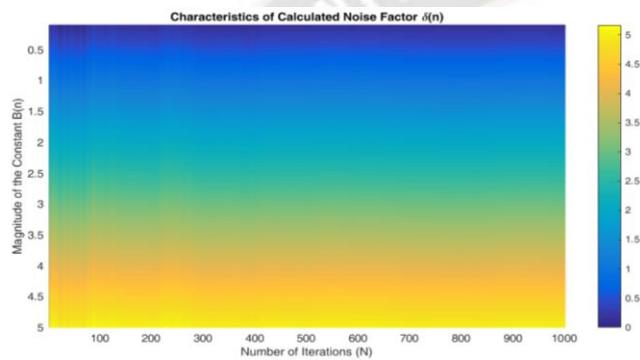


Figure 5. Characteristics of the calculated noise factor $\delta(n)$ w.r.t. constant $B(n)$ and number of iterations N .

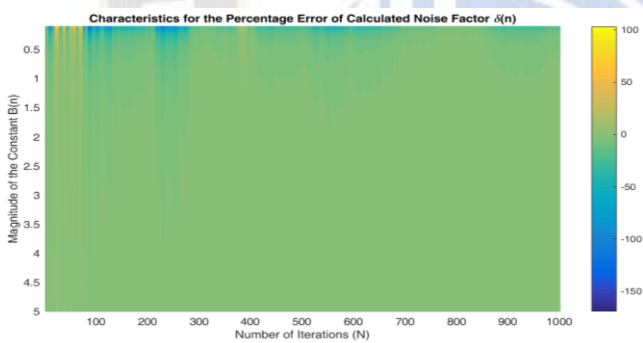


Figure 6. Characteristics for the percentage error value of calculated noise factor $\delta(n)$ with respect to constant $B(n)$ and the number of iterations N .

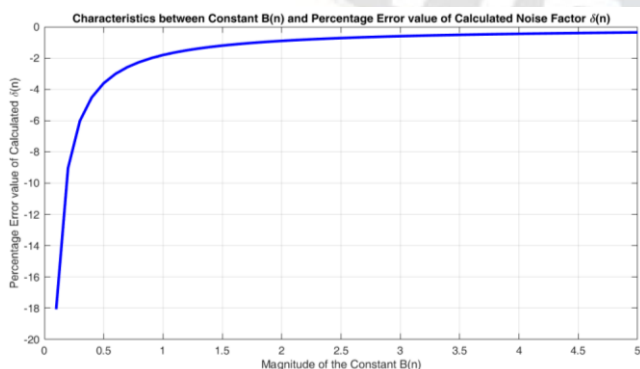


Figure 7. Characteristics of percentage error of calculated noise factor $\delta(n)$ with respect to constant $B(n)$.

D. Optimum value of weight vector

The MSE of the signal $e_1(n)$ is as given in the eq. (32),

$$\xi_1(n) = E[e_1^2(n)] \quad (32)$$

Using eq. (20) and eq. (32),

$$\xi_1(n) = E[d^2(n)] + \delta^2(n)W^T(n)R_X W(n) - 2\delta(n)W^T(n)P_X \quad (33)$$

Where,

$$R_X = E[X(n)X^T(n)] \text{ and } P_X = E[d(n)X(n)]$$

R_X is the autocorrelation vector of signal $x(n)$ and P_X is the correlation vector of signals $d(n)$ and $x(n)$. Evaluating the gradient vector with respect to $W(n)$ using eq. (33),

$$\nabla_W \xi_1(n) = \frac{\partial \xi_1(n)}{\partial W(n)} = 2\delta^2(n)R_X W(n) - 2\delta(n)P_X \quad (34)$$

The optimum weight vector will be as follows for the minimum value of MSE,

$$W(n) = W_{opt,\delta}(n) \quad (35)$$

The gradient vector will be zero for the optimal result. Using eq. (34) and eq. (35); the optimal value of the weight matrix is expressed as,

$$W_{opt,\delta}(n) = \frac{1}{\delta(n)}(R_X^{-1}P_X) \quad (36)$$

The $W_{opt,\delta}(n)$ is the optimum wiener solution of LMS algorithms for noise cancellation. The MSE is minimized at this optimum solution. The relation between optimum weight vector $W_{opt,\delta}(n)$ for the proposed model and the optimum weight vector $W_{opt}(n)$ of traditional LMS using eq. (7) and eq. (36) is defined as,

$$W_{opt,\delta}(n) = \frac{1}{\delta(n)}W_{opt}(n) \quad (37)$$

Therefore, the ratio of optimum weight vector $W_{opt,\delta}(n)$ for the proposed model and the optimum weight vector $W_{opt}(n)$ of traditional LMS is constant because $\delta(n)$ is a constant factor.

E. MSE

The minimum MSE of the projected model using eq. (33) and eq. (35) is,

$$\xi_{1,\min}(n) = E[d^2(n)] + \delta^2(n)W_{opt,\delta}^T(n)R_X W_{opt,\delta}(n) - 2\delta(n)W_{opt,\delta}^T(n)P_X \quad (38)$$

Using eq. (35) and eq. (37),

$$\xi_{1,\min}(n) = E[d^2(n)] + \delta^2(n) \left[\frac{1}{\delta(n)}(R_X^{-1}P_X) \right]^T R_X W_{opt}(n) - 2\delta(n) \left[\frac{1}{\delta(n)}(R_X^{-1}P_X) \right]^T (n)P_X \quad (39)$$

Eq. (39) is simplified as eq. (40) using the identities $(AB)^T = B^T A^T$ and $AA^{-1} = I$,

$$\xi_{1,\min}(n) = E[d^2(n)] - P_X^T R_X^{-1} P_X \quad (40)$$

Where, $(\cdot)^T$ denotes the transpose of the given matrix and $(\cdot)^{-1}$ demotes the inverse of matrix. Using eq. (37) and (40),

$$\xi_{1,\min}(n) = E[d^2(n)] - \delta(n)P_X^T W_{opt,\delta}(n) \quad (41)$$

F. The optimum value of error signal $e_1(n)$

The error signal $e_1(n)$ of the proposed model given in eq. (20). This value is optimized as per the condition given in eq. (35). Therefore,

$$e_{1,opt}(n) = d(n) - \delta(n)W_{opt,\delta}^T(n)X(n) \quad (42)$$

After the simplification of eq. (42) using eq. (36),

$$e_{1,opt}(n) = d(n) - P_X^T R_X^{-1} X(n) \tag{43}$$

The optimal value of signal $e_{1,opt}(n)$ is comparable to $I(n)$ and $d(n)$ is comparable to $I(n) + x_1(n)$. So,

$$e_{1,opt}(n) = d(n) - x_1(n) \tag{44}$$

On comparing the eq. (42), eq. (43) and eq. (44),

$$x_1(n) = P_X^T R_X^{-1} X(n) = \delta(n) W_{opt,\delta}^T X(n) \tag{45}$$

The factor $\delta(n)$ is determined by eq. (46),

$$\delta(n) = \frac{x_1(n)}{W_{opt,\delta}^T X(n)} = \frac{P_X^T R_X^{-1} X(n)}{W_{opt,\delta}^T X(n)} \tag{46}$$

IV. SIMULATION, RESULT AND DISCUSSION

The projected model is a combination of figures 3 and 4. The input signals are $d(n)$ and $x(n)$. The value of factor $\delta(n)$ determines using the model simulated on MATLAB of the block diagram of figure 4. The values of noise factor $\delta(n)$ pass into model of block diagram of figure 3. The combination of adaptive filter-1 and LMS algorithm blocks of figure 3, work as system identification. This combined part identifies the weights of filter according to the signal $x(n)$. The determined values of noise factor $\delta(n)$ and weights are passes into the adaptive filter-2 through digital multiplier/amplifier. The subtractor and adaptive filter-2 work together as a noise canceller.

TABLE II. PARAMETERS USED IN THE MATLAB SIMULATION FOR THE DETERMINATION OF NOISE FACTOR $\delta(n)$ AT FIXED $B(n)$

S. N.	Parameter	Symbol	Value/Range	Eq.(s) used
1	Magnitude of information/message signal $I(n)$	A	2	25, 27
2	Frequency of information/message signal $I(n)$ in radian/second	ω_0	2	25, 27
3	Noise magnitude in signal $d(n)$	$B(n)$	1.05	26, 27
4	Reference noise magnitude	$C(n)$	1	28
5	Noise function as random noise	$f(n)$	-	26, 27, 28
6	Number of samples	-	1-2000	-

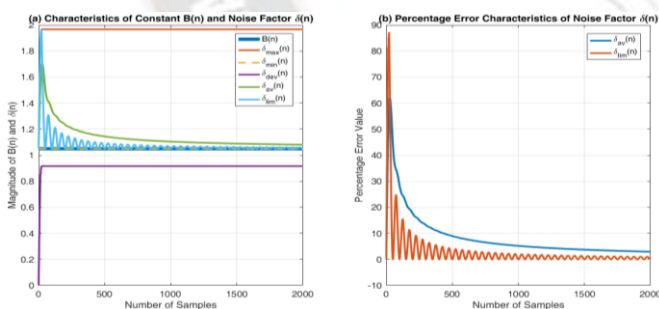


Figure 8. Characteristics of (a) constant $B(n)$, $\delta_{min}(n)$ - minimum value of constant $\delta(n)$, $\delta_{max}(n)$ - maximum value of constant $\delta(n)$, $\delta_{dev}(n)$ - deviation in the constant $\delta(n)$, $\delta_{av}(n)$ - average value of constant $\delta(n)$, $\delta_{lim}(n)$ - limiting value of constant $\delta(n)$ (b) percentage error in the average value of noise factor $\delta(n)$, percentage error in the limiting value of noise factor $\delta(n)$ w.r.t. number of samples for the sinusoidal information/message signal.

The proposed model for the determination of noise factor $\delta(n)$ at fixed $B(n)$ is simulated on the parameters given in table 2. According to the eq. (31), the value of $B(n)$ will be the same as noise factor $\delta(n)$ if $C(n)$ is unity. The $C(n)$ is taken as unity for the reference here. The two types of values of $B(n)$ can be used for the simulation – fixed and random. The value of $B(n)$ and $C(n)$ will be constant because these are the magnitudes of

input signals $x_1(n)$ and $x(n)$ respectively. The factor $\delta(n)$ will be fixed only because signals $x_1(n)$ and $x(n)$ generated from the common source. The simulation results are described here for the fixed value of constant $B(n)$. The figure 8 present different forms determined noise factor $\delta(n)$ when $B(n)$ is fixed i.e. assumed as 1.05 for 2500 samples of sinusoidal information/message signal using MATLAB model of figure 4 and sample-time is 0.01 sec. Figures 8(a) to 8(h) shows the characteristics of constant $B(n)$, $\delta_{min}(n)$ is the minimum value of noise factor $\delta(n)$, $\delta_{max}(n)$ is the maximum value of noise factor $\delta(n)$, $\delta_{dev}(n)$ is the deviation in noise factor $\delta(n)$, $\delta_{av}(n)$ is the average value of noise factor $\delta(n)$, $\delta_{lim}(n)$ is the limiting value of noise factor $\delta(n)$, percentage error in the average value of noise factor $\delta(n)$ and percentage error in the limiting value of noise factor $\delta(n)$ respectively and dependent on samples of $I(n)$. All the results of sub-figures of noise factor $\delta(n)$ describe that the value of noise factor $\delta(n)$ converges to constant values of $B(n)$.

The more than a few parameters of noise factor are classified from eq. (47) to eq. (53).

$$\delta_{min}(n) = \min[\delta(0), \delta(1), \dots, \delta(n)] \tag{47}$$

$$\delta_{max}(n) = \max[\delta(0), \delta(1), \dots, \delta(n)] \tag{48}$$

$$\delta_{dev}(n) = \delta_{max}(n) - \delta_{min}(n) \tag{49}$$

$$\delta_{av}(n) = \frac{1}{n} \sum_{i=0}^n \delta(i) \tag{50}$$

$$\delta_{lim}(n) = \delta(n) \tag{51}$$

$$\text{Percentag error of } \delta_{av}(n) = (\delta_{av}(n) - B(n)) * 100 / B(n) \tag{52}$$

$$\text{Percentag error of } \delta_{lim}(n) = (\delta_{lim}(n) - B(n)) * 100 / B(n) \tag{53}$$

Where, $\min(\cdot)$ and $\max(\cdot)$ demote the minimum value and maximum value of given elements.

Figure 9 presents the simulation results of traditional and proposed LMS-ANC models using the manual control of noise factor $\delta(n)$ based on the block diagram of figure 3. The figure also presents the SNR and MSE characteristics for the parameters given in table 3. The traditional LMS-ANC is independent of noise factor $\delta(n)$ therefore the characteristics of SNR and MSE of figures 9(a) and 9(b) respectively only depend on the other parameters of table 3. The simulation results of proposed LMS-ANC using the manual control of noise factor $\delta(n)$ gives the better SNR and MSE with respect to tradition LMS-ANC for the given range of noise factor $\delta(n)$.

TABLE III. PARAMETERS USED IN THE MATLAB SIMULATION FOR THE PROPOSED MODEL AT FIXED $B(n)$

S.N.	Parameter	Symbol	Value/Range	Eq.(s) used
1	Magnitude of information/message signal $I(n)$	A	2	25, 27
2	Frequency of information/message signal $I(n)$ in radian/second	ω_0	2	25, 27
3	Noise magnitude in signal $d(n)$	$B(n)$	1.05	26, 27
4	Reference noise magnitude	$C(n)$	1	28
5	Noise function as random noise	$f(n)$	-	26, 27, 28
6	Number of samples	-	1-2000	-
7	Filter Length	L	2	12, 13
8	Initial filter weights	$w_0(0)$, $w_1(0)$	0	9
9	Step-size	μ	0.01-0.25	9, 17
10	Noise factor	$\delta(n)$	0.1-5	18, 19, 20

TABLE IV. COMPARATIVE STUDY OF SNR RANGE FOR BOTH TRADITION AND PROPOSED LMS-ANC FOR THE MANUAL CONTROL OF NOISE FACTOR $\delta(n) = 0.1$ TO 5

S.N.	Particular	SNR for Tradition LMS (in dB)	SNR for Proposed LMS (in dB)
1	Minimum	22.0730	31.0901
2	Maximum	53.4488	73.5309
3	Deviation = Maximum-Minimum	31.3758	42.4408

TABLE V. COMPARATIVE STUDY OF SNR RANGE FOR BOTH TRADITION AND PROPOSED LMS-ANC FOR THE SELF-DETERMINED OF NOISE FACTOR $\delta(n)$

S.N.	Particular	SNR for Tradition LMS (in dB)	SNR for Proposed LMS (in dB)
1	Minimum	-4.6337	33.5373
2	Maximum	53.5900	71.5344
3	Deviation = Maximum-Minimum	58.2237	37.9971

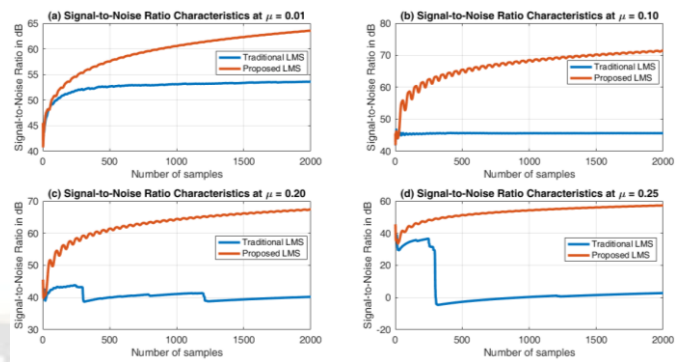


Figure 12. Characteristics of SNR respect to sample-time/time at (a) step-size $\mu = 0.01$, (b) step-size $\mu = 0.1$, (c) step-size $\mu = 0.2$ and (d) step-size $\mu = 0.25$.

The comparative study of the SNR range is also given in table 4. Figure 9 and Table 4 present that the tradition LMS-ANC has a wide range of SNR with respect to proposed LMS-ANC for the given parameters. Figure 9 and Table 4 also present that the proposed LMS-ANC gives the improved SNR at any value of noise factor $\delta(n)$. The value of noise factor $\delta(n)$ is equal to constant $B(n)$ using eq. (31) if $C(n)$ is unity. Therefore, proposed LMS-ANC gives the maximum SNR at noise factor $\delta(n) = \text{constant } B(n) = 1.05$.

Figure 10 presents the simulation results of traditional and proposed LMS-ANC models using the self-learning/adapting of noise factor $\delta(n)$ based on the block diagram of figures 3 and 4. The figure also presents the SNR and MSE characteristics for the parameters given in table 3 but noise factor $\delta(n)$ is self-determined. The traditional LMS-ANC is independent of noise factor $\delta(n)$ therefore the characteristics of SNR and MSE of figures 10(a) and 10(b) respectively only depend on the other parameters of table 3. The simulation results of proposed LMS-ANC using the self-control of noise factor $\delta(n)$ gives the better SNR characteristics and MSE characteristics with respect to tradition LMS-ANC and proposed LMS-ANC for manual control for the given range of noise factor $\delta(n)$. The comparative study of SNR range is also given in table 5. Figure 10 and Table 5 present that the tradition LMS-ANC has a wide range of SNR with respect to proposed LMS-ANC for the given parameters. The figure 10 and table 5 also present that the proposed LMS-ANC gives the improved SNR at any value of noise factor $\delta(n)$ but the value of noise factor $\delta(n)$ is self-determined.

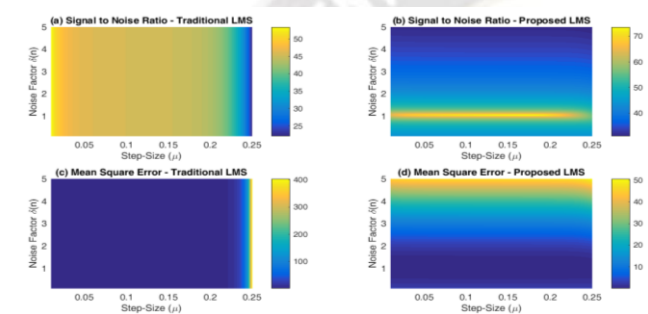


Figure 9. Contour plot of SNR and MSE w.r.t. step-size (μ) and noise factor $\delta(n)$ for the manual control of noise factor $\delta(n) = 0.1$ to 5 (a) SNR - Traditional LMS (b) SNR - Proposed LMS (c) MSE - Traditional LMS (d) MSE - Proposed LMS.

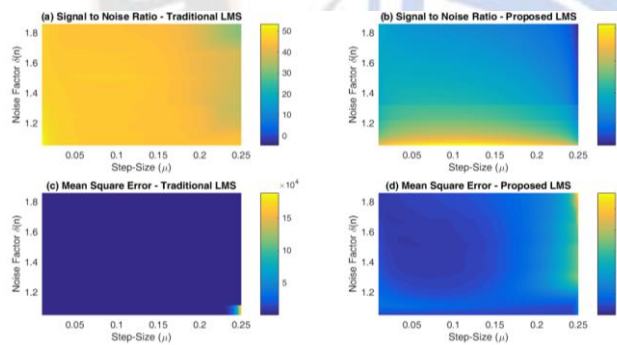


Figure 10. Contour plot of SNR and MSE w.r.t. step-size (μ) and noise factor $\delta(n)$ for self-determined of noise factor $\delta(n)$ (a) SNR - Traditional LMS (b) SNR - Proposed LMS (c) MSE - Traditional LMS (d) MSE - Proposed LMS.

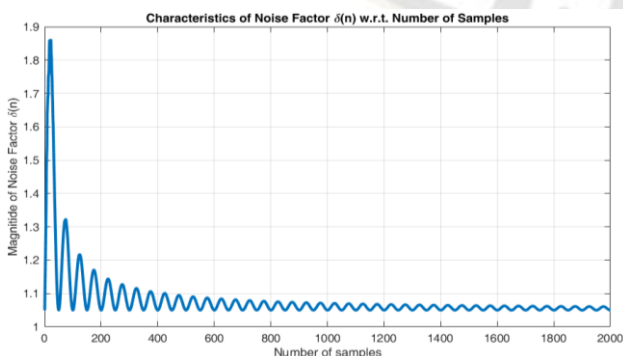


Figure 11. Characteristics of constant $\delta(n)$ with respect to sample-time/time.

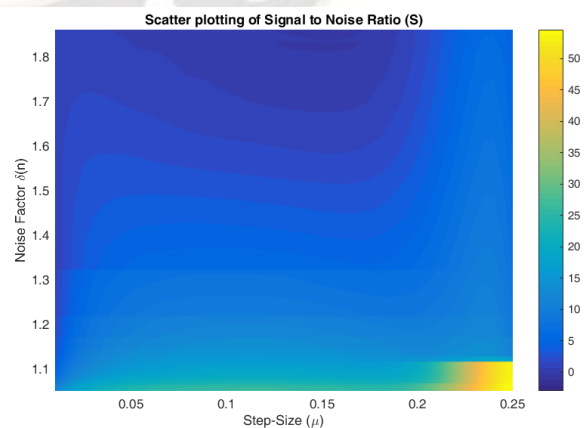


Figure 13. Scatter plotting of SNR w.r.t. step-size (μ) and noise factor $\delta(n)$ for self-determined of noise factor $\delta(n)$.

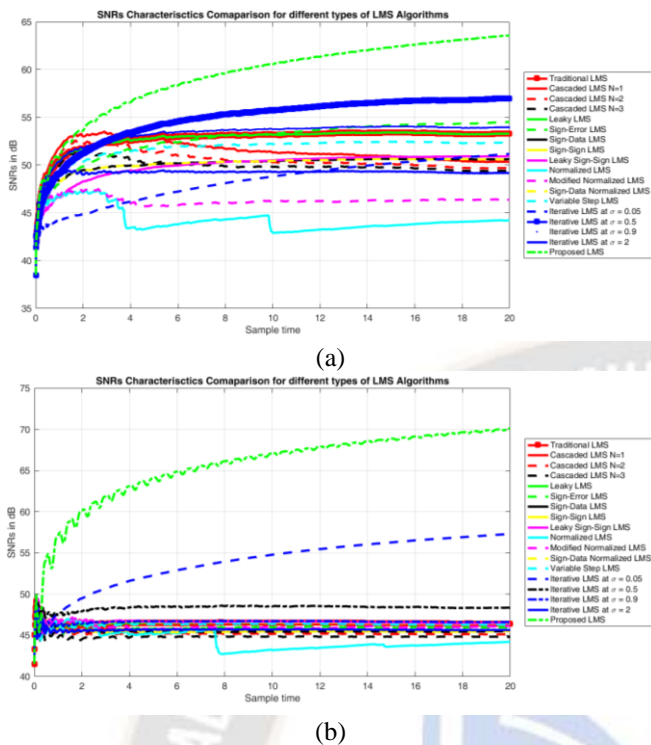


Figure 14. Characteristics of multiple LMS-ANC algorithms for the comparison of SNRs at (a) step-size (μ) = 0.01 (b) step-size (μ) = 0.05.

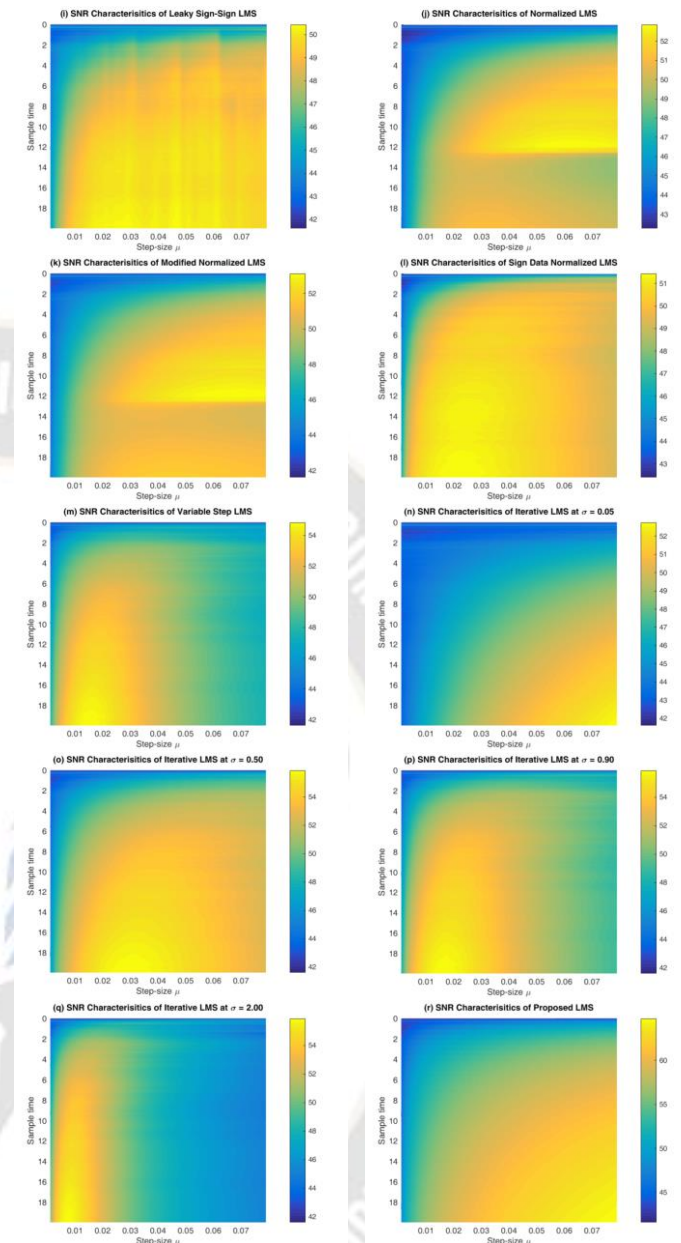
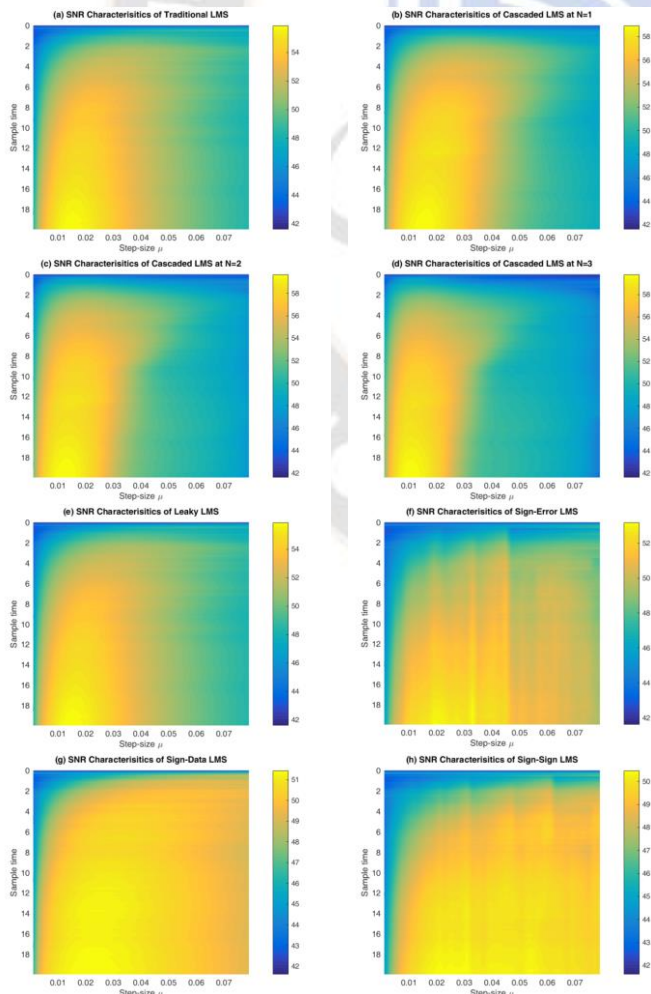


Figure 15. Characteristics of multiple LMS-ANC algorithms for the comparison of SNRs w.r.t. sample time and the step-sizes for (a) Traditional (b) Cascaded at $N = 1$ (c) Cascaded at $N = 2$ (d) Cascaded at $N = 3$ (e) Leaky (f) Signum-Error (g) Signum-Data (h) Signum-Signum (i) Leaky Signum-Signum (j) Normalised (k) Modified-Normalized (l) Signum-Data Normalized (m) Variable-Step (n) Iterative at $\sigma = 0.05$ (o) Iterative at $\sigma = 0.5$ (p) Iterative at $\sigma = 0.9$ (q) Iterative at $\sigma = 2$ (r) Proposed.

The value of noise factor $\delta(n)$ is equal to constant $B(n)$ using eq. (31) if $C(n)$ is unity. Therefore, the ideal value of noise factor $\delta(n) = B(n) = 1.05$. So, the proposed LMS-ANC gives the maximum SNR at $\delta(n) = B(n) = 1.05$. Figure 11 presents the characteristics of noise factor $\delta(n)$ with respect to sample-time or time and the values of the parameter are determined using the model of figure 4. The characteristics determined using the parameters given in table 3 and adopting the behaviour of noise factor $\delta(n)$ with the help of eq. (31) and related previous equations. The minimum, maximum, deviated, average, limiting values and percentage error of average values of noise factor $\delta(n)$ are 1.05, 1.8602, 0.8102, 1.08, 1.05 and

2.8614% respectively. The noise factor $\delta(n)$ converges to its ideal value on time-axis. Figure 12 is a part of figure 10 on some values of step sizes. The step-sizes are 0.01, 0.1, 0.2 and 0.25 for the figures 12(a) to 12(d) respectively. All the sub-figures of figure 12 show that the SNRs for proposed LMS is better than the traditional LMS and increases with respect to sample-time or time.

The scatter plotting of the SNR presents incremental value of the SNR for the proposed LMS-ANC with respect to traditional LMS-ANC as shown in figure 13. The characteristics show that maximum region gives that higher SNR. The percentage area covered in the characteristics for the incremental values, no change values and decremented values of SNR for the given range of parameters are 99.4449%, 0.0500%, and 0.5051% respectively. Therefore, proposed model gives the higher SNR when the noise factor $\delta(n)$ adapted and converges to its ideal value.

Figure 14(a) and 14(b) present the comparative study of the SNRs for multiple LMS-ANC algorithms [30] including the projected model of ANC at step-sizes 0.01 and 0.05 respectively. The proposed LMS gives the behaviour of increasing SNR with respect to sample time and also have maximum SNRs on comparison of other given types of LMS algorithms. The figures 15(a) to 15(r) present the SNRs characteristics of the above given LMS filters for step-sizes 0.001 to 0.08. All the sub-figures of figure 15 is showing that the SNR of proposed LMS is higher than, the other given LMS adaptive filters or algorithms.

V. CONCLUSION

This paper investigates and implemented newly developed structure of LMS-AF for ANC using the self-adaptation technique. This technique is used for the adaptation of used noise factor $\delta(n)$ with respect to time and input signals. The projected model is analyzed and simulated. The input signals are $d(n)$ and $x(n)$ for the projected model of LMS-ANC. The noise factor $\delta(n)$ converges to its ideal value with respect to time for the proposed model. Therefore, the SNR found better for the proposed model and increases with respect to time also. Due to the self-adaptive behaviour of noise factor $\delta(n)$ the proposed model of LMS-ANC is better than the tradition LMS-ANC algorithms in comparison of SNRs. The SNR of proposes LMS-ANC algorithm having increasing behaviour of SNR with respect to sample-time or time because it's self-adopted noise factor converges towards their ideal value in time.

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