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A Novel Approximation Approach for the Analytical Solution of the Flow of Micropolar Fluid Through a Permeable Channel

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Abstract:

An attempt is made in this study to investigate the problem of micropolar fluid flow in a porous medium theoretically. Employing the Berman's similarity solution, the model equations governing the flow is transformed into a set of nonlinear ordinary differential equation and solved using Temimi-Ansari method. Expressions for the velocity and micro-rotation profiles are obtained under the impressions of diverse parameters affecting the flow problem. Using symbolic computation software Mathematica, the nondimensional equations are solved numerically using the Keller Box scheme. Comparison between the analytical solution obtained by TAM and the numerical result are compared with results in literature to observe rapid convergence. Findings from the study showed in the presence of N_1, N_3 and Re, the rotation profile plummet, while increase in the parameter, N_2 cause an acceleration in the velocity profile. Similarly, increasing the values of N_1 and Re cause a deceleration in the velocity profile, whereas it increased in the presence of N_2 parameter.

Keywords: *Novel Approximation, Micropolar fluid, permeable channel, Temimi-Ansari Method.*

Introduction

The term micropolar fluid refers to a non-Newtonian fluid with additional microstructural effects, such as rotational motion and couplings between rotation and deformation. This is characterized by the presence of microstructure elements, such as particles, molecules, and other small-scale components, which affect the fluid's behavior (Eringen, 1966). From a physical perspective, micropolar fluids can be thought of as fluids made up of inflexible, spherically, or randomly oriented particles suspended in a viscous medium without regard to fluid particle deformation. The conservation equations for mass, linear momentum, angular momentum, and energy define the fluid's motion in micropolar fluid theory. The paired stress tensor and the microrotation vector, two examples of microstructural components, are incorporated into these equations. The textbook by Eringen, (1971) provides an extensive overview of micropolar fluid theory and some of its applications (Siddiqui, & Mustafa, 2018). As a mathematical extension of classical fluid mechanics, the theory of micropolar fluids, which was developed by Eringen, provides a mathematical model for fluids with microstructure. Unlike Newtonian fluids, which only consider linear stress-strain relationships, micropolar fluids include additional considerations related to the microstructural effects (Sidik, Nazar, & Pop, 2019).

Micropolar fluids are known to exist in nature and have been employed in several disciplines, including biology, geophysics, engineering, and medicine. Flow in porous media, lubricating machine parts, blood flow in biological systems, and microfluidics are a few application areas. Ferrofluid, which is comprised of a stable colloidal suspension of Brownian magnetic particles in a nonmagnetic liquid host, has frequently been modelled as a micropolar fluid Singh et al. (2016). Lubricating fluids in bearings can be studied using the micropolar theory in lubrication theory (Lukaszewicz, 1999; Singh, Kumar Pandey, & Manoj, 2020). Granular flow is also satisfactorily described by a micropolar fluid model (Pandey, & Sharma, 2015; Cimpean, & Iesan, 2005; Ariman, & Turk, 1974). Adomian decomposition approach has been employed for micropolar flow in a porous channel driven by transverse magnetic field by Shakeri Aski et al. (2014). This study discovered that the velocity and micro rotation profiles are influenced by the control conditions in different ways.

Using a novel modification of parameters, Osman and Arslanturk (2020) looked at the problem of micropolar flow in a porous channel. This method is used to solve second order constant differential equations using the Wronskian method. Convergent algorithms for the flow distributions were obtained during the study. Furthermore, the BVP4c subroutine in MATLAB was used to solve the problem numerically. Excellent agreement was found when the generated solutions were compared with the body of existing literature, confirming the dependability and accuracy of the suggested technique. Prakash Meena (2019) has solved the micropolar flow problem in a porous channel using an innovative spectral quasilinearization method that is based on the Newton-Raphson method in numerical analysis. The governing flow equations were simplified to first order ordinary differential equations and solved in terms of the flow distributions using the Berman's similarity approach. The flow variable expressions were plotted and graphically presented to illustrate the properties relevant to the investigation. A finite element solution to mixed convection micropolar flow driven by a porous stretched sheet has been provided by Bhargava et al. (2003). An investigation is conducted into the effects of surface conditions on the temperature function, microrotation, and velocity. The results of this investigation show that micropolar fluids play a major role in the decrease of drag forces in cooling agents and airplanes.

Mirgolbabaee et al. (2017) have examined analytically and numerically the semi-analytical investigation of micropolar fluid and heat transfer of micropolar fluid in a permeable channel. Runge-Kutta-Fehlberg method of order four was used to derive numerical expressions for the velocity and microrotation, while the Abkari-Ganji method provided analytical expressions. After tabulating the numerical results, calculating the associated errors, and using Abkari-Ganji, the study was concluded. Steady Magnetohydrodynamic utilizing the semi-analytical homotopy perturbation method, Agrawal et al. (2021) investigated micropolar fluid flow as well as heat and mass transfer in a permeable channel with thermal radiation. The research indicates that the parameters that were entered into the problem had a significant impact on the flow patterns. Tables with Nusselt and Sherwood numbers of coefficients of heat and mass transfer were shown, and they were examined quantitatively. More recently, new methods have been developed to address this problem because of the development of powerful computers. Sheikholeslami et al. (2014), for example, have used the innovative Lattice Boltzmann approach to solve the free convection flow of nanofluid. The same author expanded on this study to examine the combination of heat transfer and micropolar fluid flow in a permeable channel by implementing an analytical method and MHD natural convection and heat transfer employing nanofluid along the micropolar problem. Zheng and Cao (2015) studied the flow of micropolar fluid through an expanding wall using a lie group analysis. The governing model equations were reduced to ordinary differential equations with the use of self-similar solutions that were derived through the application of invariance transformations. The various flow distributions were obtained by solving these equations, which were then displayed graphically. Kelson and Farrell (2001) explored the effects of suction and injection on the micropolar flow through a porous stretched sheet. Similarly, heat transfer via a permeable wall and a micropolar fluid problem have been studied using the differential transform method in conjunction with Laplace transform. The study's findings were determined to be consistent with the body of current literature.

Singh (2018) examined the MHD micropolar fluid layer's thermal convection, which is heated from below. Expressions for the onset of both stationary and oscillatory convection were found by the application of linear stability analysis. The results were verified and found to be consistent with previous research. Singh (2017) has explored the effects of magnetohydrodynamics and the Hall effect on the flow of a visco-elastic micropolar fluid layer heated from below in a porous material. The problem of low and heat transfer of a micro-polar fluid in a porous channel with expanding or contracting walls has been tackled by Zheng et al. (2013). This investigation is predicated on the application of a transverse magnetic field. A parametric study was carried out and the flow fields were substantially affected by the pertinent parameter. The consequences of mass injection on micropolar flow in a porous channel for analytic solution was investigated by Hassan and Rashidi (2014) using the homotopy analysis approach. Similarly, Abdulaziz et al. (2009) used the homotopy analysis method to study a fully developed micropolar fluid flow between vertical plates under the influence of a magnetic field. A homotopy analysis approach solution has been applied by Ziabakhsh and Domairry (2008) to micropolar fluid flow in a porous channel with mass transfer in a permeable channel separated by parallel plates. A comparative study of the acquired results and those found in the literature revealed very good agreement.

Nowadays, a wide range of analytical and semi-analytic iterative techniques have been developed to tackle a wide range of problems in diverse fields of applied sciences such as linear and nonlinear ODEs and PDEs, nonlinear ordinary differential equations, the Korteweg-de Vries Equations, Duffing equations, nonlinear thin film flow problems, and certain chemistry problems, Temimi and Ansari proposed an innovative semi-analytical iterative method known as (TAM). A major advantage of the TAM is that it does not require restrictive assumptions for nonlinear terms, unlike the ADM, which required the so-called Adomian polynomials, reduce the large computational work or the use of other parameters or restrictions associated with other iterative approaches such as VIM and HPM. Furthermore, the TAM procedure has been effectively implemented for solving non-linear PDEs and ODEs. Extensive and successful applications of TAM to several problems can be found in Ebiwareme, (2021); Ebiwareme, (2022); Ehsani et al., (2013); Temimi, H., & Ansari, A. R. (2011); Ebiwareme, L. (2021); Ebiwareme, L., & Bunonyo, K.W. (2023); Ebiwareme, L. & Odok, E.O. (2022); AL-Jawary, M. A., Raham, R. K. (2016); AL-Jawary, M. A. (2017).

None of the previously mentioned studies have examined the Temimi-Ansari semi-analytical method for the investigation of the micropolar fluid flow problem. Thus, the objective of this study is to solve the problem with this new approach and compare it to previous solutions from other approaches to see if there is any convergence. The study is categorized as follows: The model problem formulation and its reduction to nonlinear ordinary differential equations using the Berman similarity solution are outlined in the next two parts. The solution method's foundation and how it applies to the problem are covered in Sections 4 and 5. Section 6 contains tables and a graphic representation of the numerical results of the flow fields under various parameter variations. Section 7 concludes with a summary of the study's key findings that are emphasized.

Mathematical formulations of the Problem

Let us examine a continuous, incompressible, laminar flow of a micropolar fluid which is uniformly injected or removed at a velocity of v_0, as depicted in Figure 1, through a two-dimensional channel with porous walls. Using Cartesian coordinates, the channel walls are taken parallel to the x-axis and are located at a width of $2h$. Given these conditions, the governing equations pertinent to the system are as follows.

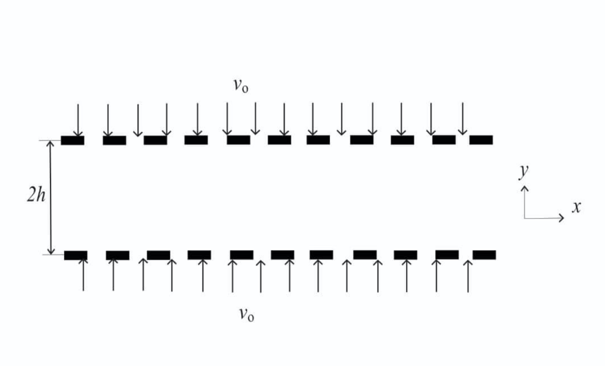


Figure 1. Physical model and coordinate system

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial P}{\partial x}+(\mu+k)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+k\frac{\partial N}{\partial y} \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial P}{\partial y}+(\mu+k)\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-k\frac{\partial N}{\partial y} \tag{3}$$

$$\rho\left(u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}\right)=-\frac{k}{j}+\left(2N+\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)+\left(\frac{\mu_s}{j}\right)\left(\frac{\partial^2 N}{\partial x^2}+\frac{\partial^2 N}{\partial y^2}\right) \tag{4}$$

subject to the appropriate boundary conditions.

$$v=u=0, N=-s\frac{\partial u}{\partial y}\ \text{at}\ y=-h$$

$$v=0, u=\frac{v_0 x}{h}, N=\frac{v_0 x}{h^2}\ \text{at}\ y=+h \tag{5}$$

Nondimensionalization of Model Equations

Utilizing the following non-dimensional variables together with the stream function formulation.

$$\eta=\frac{y}{h}, \psi=-v_0 f(\eta), N=\frac{v_0 x}{h^2}g(\eta), u=\frac{\partial\psi}{\partial y}, v=-\frac{\partial\psi}{\partial x} \tag{6}$$

Using Eq. (6), the governing equations in Eqs. (1-4) reduced to the form.

$$(1+N_1)f^{iv}(\eta)-N_1g(\eta)-Re\big(f(\eta)f''(\eta)-f'(\eta)f''(\eta)\big)=0 \tag{7}$$

$$N_2g''(\eta)+N_1\big(f''(\eta)-2g(\eta)\big)-N_3Re\big(f(\eta)g'(\eta)-f'(\eta)g(\eta)\big)=0 \tag{8}$$

The appropriate boundary conditions are given as

$$\begin{aligned} &f(-1)=f'(-1)=g(-1)=0\\ &f(1)=0, f'(1)=-1, g(1)=1 \end{aligned} \tag{9}$$

where (u,v) are velocities along the x,y axes respectibely, ρ denote the fluid density, μ is the dynamic viscosity, N is the angular or micro rotation velocity, P is the pressure of the fluid, j represent the micro-inertia density, k denote the material parameter and $v_s=\left(\mu+\frac{k}{2}\right)j$ represent the micro rotation viscosity.

Fundamentals of Temimi-Ansari method (TAM)

In accordance with Al-Jawary (2016; 2017) and Ebiwareme (2021; 2022), we consider an operator form representation of a general differential equation as follows:

$$L\big(u(x)\big)+N\big(u(x)\big)+f(x)=0,\quad x\epsilon D \tag{10}$$

$$B\left(u,\frac{du}{dx}\right)=0,\quad x\epsilon\mu \tag{11}$$

where x is the independent variable, $u(x)$ is an unknown function, $f(x)$ is a given known function, L is a linear operator, N is a nonlinear operator and B is a boundary operator.

To implement the standard TAM procedure, we seek an initial guess, $u_0(x)$ which satisfy Eq. (10) subject to the prescribed condition.

$$L\big(u_0(x)\big)+f(x)=0,\quad B\left(u_0,\frac{du_0}{dx}\right)=0 \tag{12}$$

The second iterative problem is given as follows.

$$L\big(u_1(x)\big)+N\big(u_0(x)\big)+f(x)=0,\quad B\left(u_1,\frac{du_1}{dx}\right)=0 \tag{13}$$

We consider the next iteration as follows.

$$L\big(u_2(x)\big) + N\big(u_1(x)\big) + f(x) = 0,\ B\left(u_2, \frac{du_2}{dx}\right) = 0 \tag{14}$$

Continuing the same way, we obtain nth iterative procedure to give the subsequent iterates as

$$L\big(u_{k+1}(x)\big) + N\big(u_k(x)\big) + f(x) = 0,\ B\left(u_{k+1}, \frac{du_{k+1}}{dx}\right) = 0, k \geq 0 \tag{15}$$

In view of Eq. (15), each $u_k(x)$ is considered alone as a solution of Eq. (10). This method is easy to implement, straightforward and direct. The method gives a better approximate solution which converges to the exact solution with only few members.

Computational Procedure using TAM

In this section, we solve the governing dimensionless nonlinear ordinary differential equations (7) and (8) with boundary condition (9), employing the standard Temimi-Ansari technique. We establish an approximate analytical solution for the velocity and micro-rotation profiles in terms of Reynold number, coupling parameter, and spin-gradient viscosity parameter using Mathematica, a symbolic computer program.

In view of the TAM procedure, we firstly express the given equations in operator form as

$$f^{iv}(\eta) = \frac{1}{(1+N_1)}\left[N_1 g(\eta) + Re\big(f(\eta)f''(\eta) - f'(\eta)f''(\eta)\big)\right] \tag{16}$$

$$g''(\eta) = \frac{1}{N_2}\left[N_3 Re\big(f(\eta)g'(\eta) - f'(\eta)g(\eta)\big) - N_1\big(f''(\eta) - 2g(\eta)\big)\right] \tag{17}$$

where $L\big(f(\eta)\big) = f^{iv}(\eta), N\big(f(\eta)\big) = \frac{1}{(1+N_1)}\left[N_1 g(\eta) + Re\big(f(\eta)f''(\eta) - f'(\eta)f''(\eta)\big)\right],$

$$f(\eta) = 0,\ L\big(f_0(\eta)\big) = 0, f_0(-1) = 0,\ f_0'(-1) = 0, f_0'(1) = -1, f_0(1) = 0 \tag{18}$$

Similarly, $L\big(g(\eta)\big) = g''(\eta), N\big(g(\eta)\big) = \frac{1}{N_2}\left[N_3 Re\big(f(\eta)g'(\eta) - f'(\eta)g(\eta)\big) - N_1\big(f''(\eta) - 2g(\eta)\big)\right]$ $g(\eta) = 0,\ L\big(g_0(\eta)\big) = 0, g_0(1) = 1,\ g_0(-1) = 0$ (19)

The first problems to be solved are given as follows.

$$L\big(f_0(\eta)\big) = 0, L\big(g_0(\eta)\big) = 0 \tag{20}$$

Solving Eq. (20) gives the first iterative solution given as

$$f_0(\eta) = \delta_1 + \delta_2\eta + \delta_3\frac{\eta^2}{2} + \delta_4\frac{\eta^3}{6}$$

$$g_0(\eta) = \alpha_1 + \alpha_1\eta \tag{21}$$

The second problem to be solved is given by.

$$f_1^{iv}(\eta) = N\big(f_0(\eta)\big), f_1(-1) = 0,\ f_1'(-1) = 0, f_1'(1) = -1, f_1(1) = 0$$

$$g_1''(\eta) = N\big(g_0(\eta)\big), g_1(1) = 1,\ g_1(-1) = 0 \tag{22}$$

Integrating both sides of Eq. (22) from 0 to η four times for $f_1(\eta)$ and $g_1(\eta)$, we have the expressions.

$$f_1(\eta) = \frac{1}{1+N_1}\int_0^\eta\int_0^\eta\int_0^\eta\int_0^\eta\left[N_1 g_0(\eta) + Re\big(f_0(\eta)f_0''(\eta) - f_0'(\eta)f_0''(\eta)\big)\right] d\eta d\eta d\eta d\eta \tag{23}$$

$$g_1(\eta) = \frac{1}{N_2}\int_0^\eta\int_0^\eta\left[N_3 Re\big(f_0(\eta)g_0'(\eta) - f_0'(\eta)g_0(\eta)\big) - N_1\big(f_0''(\eta) - 2g_0(\eta)\big)\right] d\eta d\eta \tag{24}$$

Following the same procedure, the third problem to be solved is given as

$$f_2^{iv}(\eta) = N\big(f_1(\eta)\big), f_2(-1) = 0,\ f_2'(-1) = 0, f_2'(1) = -1, f_2(1) = 0$$

$$g_2''(\eta) = N\big(g_1(\eta)\big), g_2(1) = 1,\ g_2(-1) = 0 \tag{25}$$

Integrating both sides of Eq. (25) from 0 to η four times for $f_1(\eta)$ and $g_1(\eta)$, we have the expressions.

$$f_2(\eta) = \frac{1}{1+N_1}\int_0^\eta\int_0^\eta\int_0^\eta\int_0^\eta\left[N_1 g_1(\eta) + Re\big(f_1(\eta)f_1''(\eta) - f_1'(\eta)f_1''(\eta)\big)\right] d\eta d\eta d\eta d\eta \tag{26}$$

$$g_2(\eta) = \frac{1}{N_2}\int_0^\eta\int_0^\eta\left[N_3 Re\big(f_1(\eta)g_1'(\eta) - f_1'(\eta)g_1(\eta)\big) - N_1\big(f_1''(\eta) - 2g_1(\eta)\big)\right] d\eta d\eta \tag{27}$$

The approximation solution for the velocity and micro-rotation profiles are given by the expressions.

$$f(\eta) = \sum_{n=0}^{\infty} f_n(\eta) = f_0(\eta) + f_1(\eta) + f_2(\eta) + \cdots g(\eta) = \sum_{n=0}^{\infty} g_n(\eta) = g_0(\eta) + g_1(\eta) + g_2(\eta) + \cdots \tag{28}$$

Results and Discussions

In this section, we present the approximate analytical results obtained in solving the nonlinear ordinary differential equations resulting from the nondimensionalization of the model equations. To demonstrate the accuracy, dependability, and convergence, we compare the results obtained using Temimi-Ansari method with the numerically obtained solution using Keller Box Scheme. The characteristics of the diversified parameters on the velocity and micro-rotation profiles are presented graphically and in tables. The results obtained in this study have excellent agreement with results in literature which proves the validity and efficiency of the semi-analytical technique.

Table 1. Comparison between numerical result, VPM, ADM and TAM Solutions for $f(\eta)$ and $g(\eta)$ profiles when Re=0.25, N_1=N_2=N_3=0.5

η	$f(\eta)$				$g(\eta)$			
	Keller Box Scheme	VPM	ADM	TAM	Keller Box Scheme	VPM	ADM	TAM
0.00	0.000000	0.000000	0.000000	0.0000000	0.000000	0.000000	0.000000	0.000000
0.10	0.150798	0.150798	0.150798	0.150798	-0.040795	-0.040795	-0.040795	-0.040795
0.20	0.298451	0.298451	0.298451	0.298451	-0.079260	-0.079260	-0.079260	-0.079260
0.30	0.439825	0.439825	0.439825	0.439825	-0.113030	-0.113030	-0.113030	-0.113030
0.40	0.571811	0.571811	0.571811	0.571811	-0.139680	-0.139680	-0.139680	-0.139680
0.50	0.691341	0.691341	0.691341	0.691341	-0.156700	-0.156700	-0.156700	-0.156700
0.60	0.795401	0.7915401	0.7915401	0.7915401	-0.161460	-0.161460	-0.161460	-0.161460
0.70	0.881059	0.881059	0.881059	0.881059	-0.151210	-0.151210	-0.151210	-0.151210
0.80	0.945482	0.945482	0.945482	0.945482	-0.122990	-0.122990	-0.122990	-0.122990
0.90	0.985973	0.985973	0.985973	0.985973	-0.073700	-0.073700	-0.073700	-0.073700
1.00	1.000000	1.000000	1.00000	1.00000	0.000000	0.000000	0.000000	0.000000

Table 2. Comparison between numerical result, OHAM, DTM and TAM Solutions for $f(\eta)$ and $g(\eta)$ profiles when Re=0.25, N_1=N_2=N_3=0.5

η	$f(\eta)$				$g(\eta)$			
	Keller Box Scheme	OHAM	DTM	TAM	Keller Box Scheme	OHAM	DTM	TAM
0.00	0.000000	0.000000	0.0000000	0.0000000	0.000000	0.000000	0.000000	0.000000
0.10	0.150798	0.149991	0.149991	0.149991	-0.040795	-0.040103	-0.040103	-0.040103
0.20	0.298451	0.296953	0.296953	0.296953	-0.079260	-0.077978	-0.077978	-0.077978
0.30	0.439825	0.437849	0.437849	0.437849	-0.113030	-0.111348	-0.111348	-0.111348
0.40	0.571811	0.569633	0.569633	0.569633	-0.139680	-0.137845	-0.137845	-0.137845
0.50	0.691341	0.689251	0.689251	0.689251	-0.156700	-0.154969	-0.154969	-0.154969
0.60	0.795401	0.793657	0.793657	0.793657	-0.161460	-0.160059	-0.160059	-0.160059
0.70	0.881059	0.879835	0.879835	0.879835	-0.151210	-0.150267	-0.150267	-0.150267
0.80	0.945482	0.944829	0.944829	0.944829	-0.122990	-0.122540	-0.122540	-0.122540
0.90	0.985973	0.985783	0.985783	0.9857831	-0.073700	-0.073611	-0.073611	-0.073611
1.00	1.000000	1.000000	1.000000	1.000000	0.000000	0.000000	0.000000	0.000000

Table 3. Comparison between numerical result, AGM, HAM, and TAM Solutions for $f(\eta)$ and $g(\eta)$ profiles when Re=0.25, N_1=N_2=N_3=0.5

η	$f(\eta)$				$g(\eta)$			
	Keller Box Scheme	AGM	HAM	TAM	Keller Box Scheme	AGM	HAM	TAM
0.00	0.0924003	0.0924003	0.0924003	0.0924003	0.0924004	0.0924004	0.0924004	0.0924004
0.10	0.1046310	0.1046310	0.1046310	0.1046310	0.1120970	0.1120970	0.1120970	0.1120970
0.20	0.1080870	0.1080870	0.1080870	0.1080870	0.1405680	0.1405680	0.1405680	0.1405680
0.30	0.1006590	0.1006590	0.1006590	0.1006590	0.1799230	0.1799230	0.1799230	0.1799230

0.40	0.0800048	0.0800048	0.0800048	0.0800048	0.2325040	0.2325040	0.2325040	0.2325040
0.50	0.0435011	0.0435011	0.0435011	0.0435011	0.3009360	0.3009360	0.3009360	0.3009360
0.60	-0.0118118	-0.0118118	-0.0118118	-0.0118118	0.3881780	0.3881780	0.3881780	0.3881780
0.70	-0.0892912	-0.0892912	-0.0892912	-0.0892912	0.4975910	0.4975910	0.4975910	0.4975910
0.80	-0.1927610	-0.1927610	-0.1927610	-0.1927610	0.6330060	0.6330060	0.6330060	0.6330060
0.90	-0.3265860	-0.3265860	-0.3265860	-0.3265860	0.7988030	0.7988030	0.7988030	0.7988030
1.00	-0.4957600	-0.4957600	-0.4957600	-0.4957600	0.9999990	0.9999990	0.9999990	0.9999990

Table 4. Comparison between numerical result, OHAM, DTM and TAM Solutions for $f(\eta)$ and $g(\eta)$ profiles when Re=0.25, N_1=N_2=N_3=0.5

η	$f(\eta)$				$g(\eta)$			
	Keller Box Scheme	SQLM	FEM	TAM	Keller Box Scheme	SQLM	FEM	TAM
0.00	0.000000	0.000000	0.0000000	0.0000000	0.000000	0.000000	0.000000	0.000000
0.10	0.150798	0.149991	0.149991	0.149991	-0.040795	-0.040103	-0.040103	-0.040103
0.20	0.298451	0.296953	0.296953	0.296953	-0.079260	-0.077978	-0.077978	-0.077978
0.30	0.439825	0.437849	0.437849	0.437849	-0.113030	-0.111348	-0.111348	-0.111348
0.40	0.571811	0.569633	0.569633	0.569633	-0.139680	-0.137845	-0.137845	-0.137845
0.50	0.691341	0.689251	0.689251	0.689251	-0.156700	-0.154969	-0.154969	-0.154969
0.60	0.795401	0.793657	0.793657	0.793657	-0.161460	-0.160059	-0.160059	-0.160059
0.70	0.881059	0.879835	0.879835	0.879835	-0.151210	-0.150267	-0.150267	-0.150267
0.80	0.945482	0.944829	0.944829	0.944829	-0.122990	-0.122540	-0.122540	-0.122540
0.90	0.985973	0.985783	0.985783	0.9857831	-0.073700	-0.073611	-0.073611	-0.073611
1.00	1.000000	1.000000	1.000000	1.000000	0.000000	0.000000	0.000000	0.000000

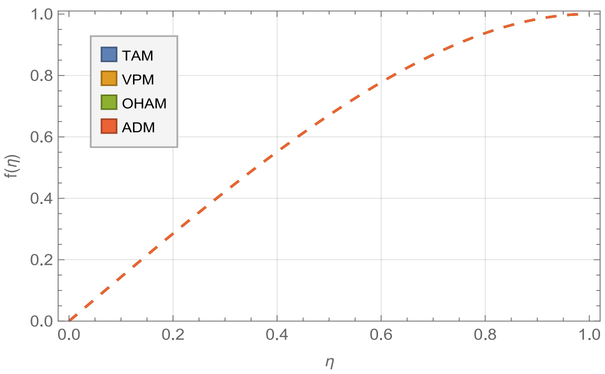


Figure 2. Variation in velocity profile when $N_1 = N_2 = 1, N_3 = 0.1, Re = 1$

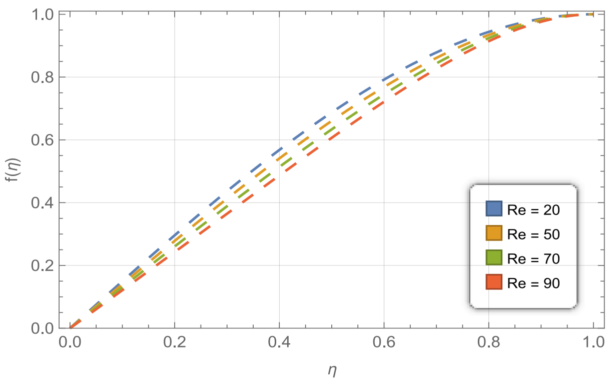
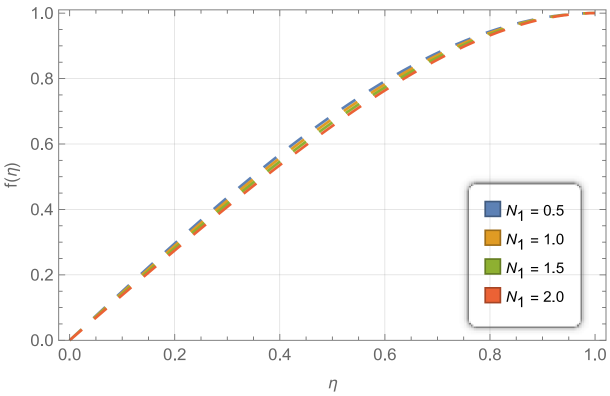


Figure 3. Effect of velocity profile for different values of Re when $N_1 = N_2 = 1, N_3 = 0.1, Re = 1$



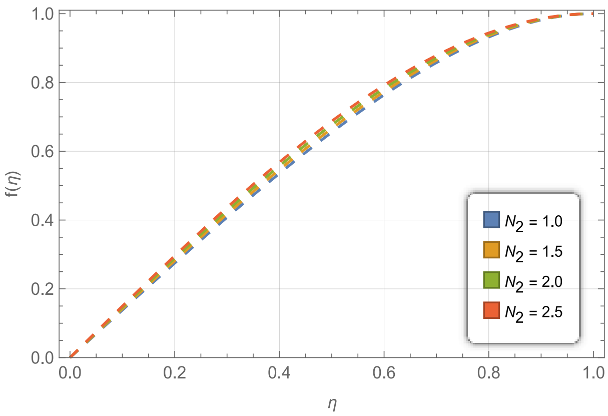
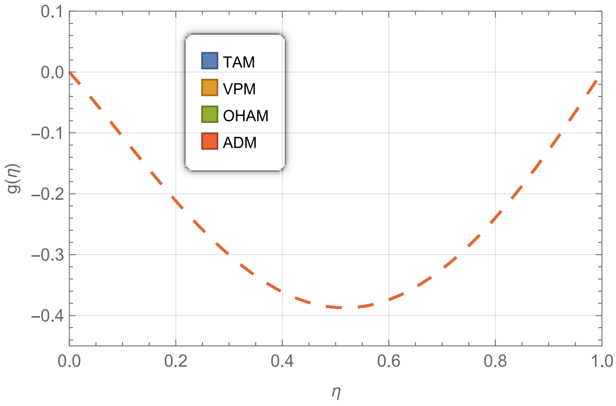


Figure 4. Influence of N_1 on velocity profile when $N_1 = N_2 = 1, N_3 = 0.1, Re = 1$

Figure 5. Effect of N_2 on velocity profile when $N_1 = N_2 = 1, N_3 = 0.1, Re = 1$



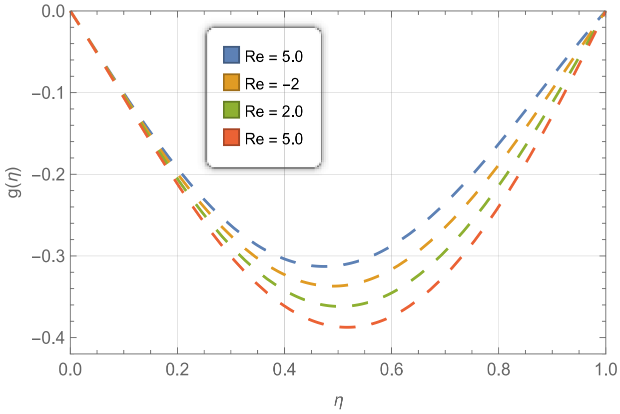
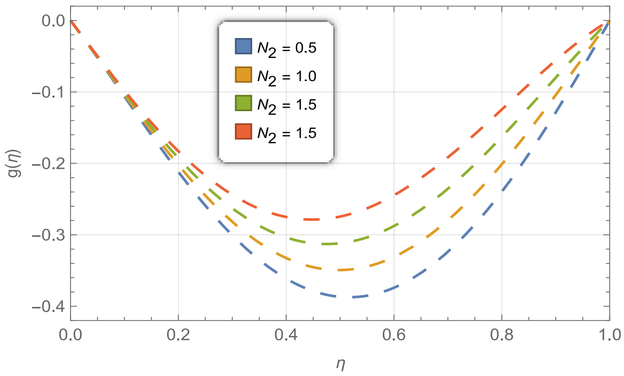


Figure 6. Variation of rotation profile for $N_1 = N_2 = 1, N_3 = 0.1, Re = 1$

Figure 7. Influence of Reynold number on micro-rotation when $N_1 = N_2 = 1, N_3 = 0.1, Re = 1$



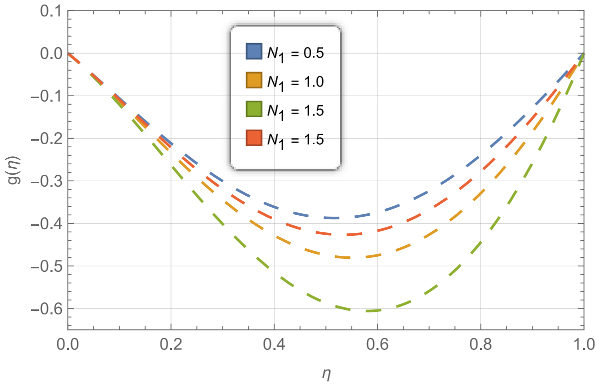


Figure 8. Influence of N_2 on micro-rotation when $N_1 = N_3 = 1, Re = 1$

Figure 9. Influence of N_1 on micro-rotation when $N_2 = N_3 = 1, Re = 1$

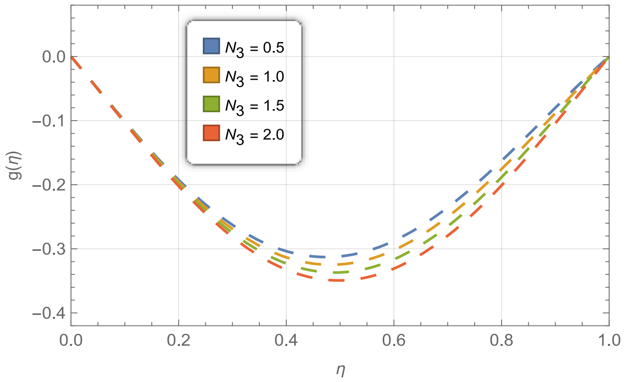


Figure 10. Influence of N_3 on micro-rotation when $N_1 = N_2 = 1, Re = 1$

The convergence of the Temimi-Ansari technique with the numerical scheme (Keller Box method) and other semi-analytical methods for the velocity profile is displayed in Figure 2. It is clear that the result agrees with various other methods, including the Abkari-Ganji method (AGM), the Spectral Quasilinearization method (SQLM), the Finite Element method (FEM), the

Adomian decomposition method (ADM), the Differential transform method (DTM), the Homotopy analysis method (HAM), and the optimal homotopy asymptotic method (OHAM).

In figure 3, the impact of Reynold number on the velocity profile is presented. We observe that increasing the value of the Reynold number lead to deceleration in the velocity profile. The influence of N_1 on the velocity distribution is given in figure 4. The finding reveals that, the velocity profile plumets by increase in the value of the N_1 parameter.

The varying effect of the N_2 parameter on the velocity profile is shown in Figure 5. The results show that raising the N_1 value causes the velocity profile to increase

Figure 6 shows the convergence of the Adomian decomposition approach, Temimi-Ansari method (TAM), variation of parameter method (VPM), and optimal homotopy asymptotic method (OHAM) on the micro-rotation profile with numerical scheme (Keller Box scheme). The outcome made it abundantly clear that the solution obtained by applying the suggested technique agreed with the findings of other approaches.

The micro-rotation profile's characteristics are shown in Figures 7–10 under different conditions, including Reynold number, N_1, N_2, and N_3. We observed that the micro-rotation profile decreased with rising values of Reynold number (Re), N_1 and N_2, while the micro-rotation profile increased with increasing N_2 parameter.

Conclusion

Approximate analytical Solution of the micropolar fluid problem passing through a porous permeable channel is examined in this study. A novel semi-analytical technique known as Temimi-Ansari method is implemented to solve the nonlinear equations upon similarity transformation. Using graphical representation, the characterises of the flow distributions are illustrated as they are affected by the parameters entering the flow model. Based on our findings, the conclusions of the study are summarized as follows.

- The result obtained using TAM agree with results of other studies as see in figures 1 and 6.
- Increasing values of Reynold number cause a decrement in the velocity profile.
- The effect of increased in the values of N_1 cause a decrease in the velocity profile.
- The velocity profile increased by increase in the values of N_2.
- It is observed that increasing values of the Reynold lead to a decrease in the micro-rotation profile of the flow.
- Increase in N_2 lead to an increase in the micro- rotation profile.
- Increase in the parameter N_1 lead to a decrease in the micro-rotation profile.
- Micro-rotation profile decreased with an increase in the N_3 parameter.

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Appendix 1

Nomenclature

Symbols	Definition
C	Specie concentration
D^*	thermal conductivity and molecular diffusivity
f	Dimensionless stream functi
g	Dimensionless microrotation
h	Half width of the channel
j	Micro-inertia density
N	microrotation/angular velocity
$N_{1,2,3}$	Dimensionless parameters
$\delta_1, \delta_2, \delta_3, \delta_4$	Undetermined constants
α_1, α_2	Undetermined constants
N_{ux}	Local Nusselt number
S_{hx}	Local Sherwood number
P	pressure
Pr	Prandtl number
q	mass transfer parameter
Re	Reynold number
T	Fluid temperature
s	microrotation boundary condition
(u, v)	Cartesian velocity components
(x, y)	Cartesian coordinate components parallel and normal to channel axis.
Greek Symbols	
η	Similarity variable
μ	dynamic viscosity
ρ	Fluid density
ψ	stream function
σ	electric conductivity
θ	dimensionless temperature
ϕ	dimensionless mass transfer parameter
k	coupling coefficient
ν_s	microrotation/spin-gradient viscosity

Abbreviation

VPM	Variation of parameter method
OHAM	Optimal Homotopy asymptotic method
DTM	Differential transformation method
ADM	Adomian decomposition method
TAM	Temimi-Ansari method
HAM	Homotopy analysis method
FEM	Finite Element method
AGM	Abkari-Ganji method
SQLM	Spectral Quasilinearisation method