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RESEARCH ARTICLE

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A novel approach to legacy donations with long-term benefits supported by mathematical analysis

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Abstract

A novel approach to legacy donations, called the "Master Fund Strategy," is proposed. Potential long-term financial benefits for both donor and nonprofit organizations (NPOs) when compared to a "Traditional Fund Strategy" are established through mathematical analysis and computer simulations, providing nonprofit marketing and fundraising professionals an alternative way to lock in bequest funding. In particular, formulas are developed for computing relevant financial quantities associated with the two strategies. Conditions are presented under which the Master Fund Strategy is better than the Traditional Fund Strategy, in the sense that there is a point in time when the net present value of the distributions to the NPO under the Master Fund Strategy exceeds that of a Traditional Fund Strategy and continues to do so beyond that point. These analytical results are obtained under the assumption that the investment rates of return and the fund payouts rates are known constants; however, formulas for relaxing these restrictions are also developed and the consequences are examined with Monte Carlo simulations.

KEYWORDS
charitable donation, estate planning, legacy donation

Practitioner Points

a. The current methods for making legacy donations to Nonprofit Organizations (NPOs) are (1) to leave funds directly to the NPO, (2) to leave funds in an endowment fund either at the NPO or some other charitable organization that then provides annual distributions to the NPO or (3) establish a private foundation.
b. In this (and a related) paper, a novel approach to making legacy donations with potential long-term financial benefits to the NPO is proposed and analyzed with numerical computations and rigorous mathematics.
c. The proposed approach has these advantages to donors:
   1. Donors can achieve these benefits with a smaller corpus contribution and with lower management costs than establishing a private foundation.
   2. Donors can control their legacy donations in a more convenient and easy-to-implement manner.

3. Donors can implement this approach during their lifetime—with appropriate tax benefits—using Donor Advised Funds and leave written instructions to convert these accounts to permanent funds upon the donor's death.

1 | INTRODUCTION AND LITERATURE REVIEW

Many financial supporters of nonprofit organizations (NPOs) make legacy charitable contributions through estate documents, such as wills and trusts, and many promise such bequests but fail to deliver on them (Wishart & James III, 2021). The challenges facing these donors include matching personal preferences, size of potential donations, desired impact, the time horizon over which they want to support chosen nonprofits (NPOs), and the charitable instruments available for making their contributions. In establishing these gifts, donors are motivated by a desire to provide ongoing financial benefits to their favored NPOs and often, by their desire for continued recognition of their contributions.

Donors who wish to have a long-term impact currently have a limited number of choices for doing so. Wealthy donors can set up their own private foundations, retaining significant control over how funds are distributed to NPOs; however, doing so requires significant overhead to set up and manage. Less wealthy donors have even fewer choices. With a *Traditional Fund Strategy (TFS)*, they can donate to general endowment funds of existing NPOs or, in some cases, such as a university, for example, they can establish an endowment with specific guidelines as to how the funds should be used. Another alternative is to create a fund at an organization—such as the Columbus Foundation in Columbus, Ohio—that manages the investments and distributes earnings to the donor's specified NPOs.

The contribution of the work presented here expands the available choices for making such donations and provides a different fundraising and marketing strategy to NPOs that may result in a greater number and dollar amount of bequests. In particular, a new approach called the *Master Fund Strategy (MFS)* is proposed that may appeal to a broad base of donors due to the following potential benefits that result because the MFS:

- Provides the donor with more control over the rate at which the funds are distributed to the NPO—less so than with a private foundation but also requiring a significantly smaller initial corpus and greatly reduced management costs.
- Is easy to implement, either during one's lifetime using Donor Advised Funds (DAF), which provide tax savings in the United States and other countries (e.g., the United Kingdom, Canada, and Australia) comparable to other planned giving vehicles, or as part of a legacy plan through the donor's estate documents.
- Is amenable to a mathematical analysis that uses a net-present-value (NPV) approach to establish conditions under which the MFS provides greater long-term financial benefits to both the donor and the NPO than using a TFS.

Appropriate background and context within the existing literature is given in the rest of this section, together with an explanation of how the MFS works. In Section 2, mathematical formulas in terms of the investment parameters are presented for comparing the NPV financial benefits of the MFS and the TFS over time. Using these formulas, conditions are given in Section 3 under which the MFS provides greater long-term financial benefits—in terms of the NPV of the annual stream of funds received by the NPO—than those of the TFS. The mathematical conditions in Section 3 assume that investment parameters are constant from year to year. However, the computer simulations in Section 4 use parameters whose values vary each year to show that the mathematical results from Section 3 are robust in the sense that using the MFS with variable parameter values also leads to greater long-term financial benefits. Lastly, we provide a summary and brief comments on the significance of this research for nonprofit research, takeaways for practice, and limitations of the study.

1.1 | Background and context

Phillips et al. (2021) (among others), note that legacy donors may give gifts directly to NPOs, which is the most straightforward way to make a difference. They also note donors' use of intermediary giving vehicles, including donor-advised funds (DAFs), endowments, foundations, and trusts (p. 411). Each of these giving vehicles has different consequences for donors as they attempt to find the best method for themselves in terms of impact to the nonprofit, the timing of donations, and meeting their own preferences for giving. Gifts made during a donor's life can take advantage of taxes in the year when the gift(s) are made and can potentially avoid capital gains taxes (in the United States and some other countries).

The wealthiest of donors can establish their own foundations, customizing their legacy support and controlling their investments and distributions. Doing so requires higher costs of administration and professional staff to manage the foundation's operations, and the need to be personally involved in the foundation's activities. Arden (2013) suggests that the amount of wealth and the right personality and passion are critical factors in whether donors decide to create their own foundations. As this vehicle for legacy giving is limited to those with significant wealth, it is unrealistic for most donors.

An alternative widely available is the DAF, which enables donor involvement in charitable giving decisions during and after their lifetimes (Scaife et al., 2012; see also Stanford PACS, 2020). DAFs can offer (depending on the NPO's stipulations) a method to leave a legacy named after the donor and a way to provide charitable gifts into the future. In the work here, we blend strategies for legacy giving by creating a new strategy with aspects of personal foundations at lower cost and with potentially greater financial returns to NPOs in the future.

To understand why donors give and how our proposed strategy affects their giving choices, we now provide a brief overview of motivations for giving, and in particular, motivations for legacy giving; reasons why charitable organizations set up longer-term giving vehicles; and practical considerations in understanding our strategy.

Hundreds of studies have demonstrated reasons why donors make charitable gifts, for example, they care about issues, religious causes, or have family traditions that affect their choices to give (see, e.g., Sargeant, 1999; Routley & Sargeant, 2014.) They may give based on awareness of need, altruism, reputation, solicitation, and other benefits (Bekkers & Wiepking, 2011), when there are public and private benefits (Konrath & Handy, 2017), or based on their financial situations (Furnham, 1984). They may feel good or receive a "warm glow" from benefiting others (Andreoni, 1998), they may want to make themselves look better in the eyes of others, take advantage of tax benefits, or create a legacy.

Examining those who wish to leave a legacy, Hager (2006) provided two motivations for legacy giving: perpetuity (obtaining gratification from long-lasting impact) and community reputation (garnering respect). Routley and Sargeant (2014, p. 881) found that donors see legacy giving as a means of extending oneself, noting, that a donor's estate can provide "a form of symbolic immortality." Jones and Routley (2022, p.1), in interviewing people who planned to leave a legacy to the Royal Opera House in the United Kingdom found that "the strength of [their connection with the nonprofit] and development of a shared identity can create a sense of symbolic immortality and influence the legacy giving decision."

Jonas et al. (2002) noted that when people think about their own mortality, they become more generous with their assets. Finally, Hansmann (1990) states that legacy giving may exhibit intergenerational equity, with donors providing assets that benefit today's and future generations equally.

Who chooses to leave legacy gifts also affects the vehicles and strategies donors choose for giving, and studies address who plans legacy gifts. James (2008) found that, of the population of U.S. donors over age 50 who donate more than $500 annually, less than 10% had charitable estate plans. Having a family, volunteering, and donating during one's lifetime, and being more religious, more educated, wealthier, and older correlate positively with legacy giving. Further, Wishart and James III (2021), in a study of Australian planned givers, noted that among those who reported having a planned gift in place, "35% generated no actual bequest gift at death" and among their study participants, "58% had never made a gift to the charity during lifetime" (p. 5).

On the other side of the equation are the actions and practices of NPO leaders. These leaders value longer-term donations because accumulating funds provides security and a sound financial plan for success and survival of the NPO. Setting up endowments provides annual investment income that continues to benefit the NPO (Hansmann, 1990; National Council of Nonprofits, 2022), and NPOs seek to diversify funding sources and mitigate risks (see, e.g., Chang & Tuckman, 1990, 1994). Nonprofit fundraisers value getting commitments in place from potential donors, and as Wishart and James III (2021) conclude, it is important for a nonprofit to move donors to planned gift confirmation.

Taken together, motivations for legacy giving and practices nonprofit leaders use to generate funding to benefit recipients today and into the future leave room for additional strategies for establishing legacy funds. To that end, we note that no research directly addresses the strategy we propose. Klausner (2003) and Afik et al. (2020) come closest by addressing the tradeoff between current and future giving using a discounted cash flow approach and showing how different payout rates and savings and investment decisions affect future nonprofit fund balances. Our proposed strategy follows Klausner's logic and mirrors Afik et al.'s "tailored projection analysis," which provides information on how assets, payout rates, and longevity interact. Our strategy allows legacy donors, under reasonable conditions, to generate superior long-term financial outcomes for NPOs in the future and provides a marketing and fundraising tool that may provide a greater number of bequests and a larger amount of funding over time.

While our strategy provides practical guidance for donors, NPOs, and planned giving professionals, all must concern themselves with regulations, laws, and policies affecting charitable giving. Our strategy, implemented through DAFs, would be created by donors during their lifetimes. DAFs currently allow donors to avoid expensive administrative and management fees, are easy to create, have low or no mandatory payouts, and allow donors to have endowment or foundation personnel manage reporting and other regulatory requirements (Phillips et al., 2021, p. 410).

It is possible, however, that lawmakers will lobby to change how DAFs operate. Some NPOs and beneficiaries of nonprofit work wish to provide/receive goods and services immediately, while other stakeholders place equal or more value on having long-term funds that provide greater stability to the organization and its beneficiaries. Intergenerational equity and efficiency concerns (e.g., Hansmann, 1990), and concerns about the skyrocketing use of DAFs with "limited regulation" result in lawmakers' debates over the benefit of the vehicle to NPOs and stakeholders (Andreoni & Madoff, 2020). Duquette (2017, p. 1161) notes that in the future, policy makers will continue to evaluate the tradeoffs between allowing NPOs to grow their funds and providing funds quickly to beneficiaries. Although having greater long-term funds helps sustain the nonprofit in the long run, some donors and granting bodies may view the funds as "excess" and give less to NPOs that have larger long-term fund balances (Handy & Webb, 2003).

Additional context and literature is given in the companion paper by the authors (see Solow, Symes & Webb 2023). We now provide the details of the proposed strategy, including mathematical analysis and a discussion of how our strategy can generate superior funding to NPOs.

1.2 | The master fund strategy

The novel legacy-donation approach to be proposed is an intermediate strategy that provides some of the same long-lasting benefits that accrue to donors who establish foundations while limiting the amount of the additional costs for creating and managing these funds. In contrast to a Traditional Fund Strategy (TFS)—in which a donor makes an initial contribution to a *Charity Fund* (CF) that then invests the money and makes annual distributions to various NPOs under terms specified in the donor's establishing documents—with the proposed Master Fund Strategy (MFS), the donor divides the initial contribution between two funds: a Charity Fund that functions in the same way as the CF under the TFS, and a new *Master Fund* (MF) that provides annual distributions to the CF (see Figure 1). As some of the initial contribution under the MFS resides in the MF, distributions to the NPOs generated under this strategy are initially lower than under the TFS. However, due to compounding of invested funds, subsequent distributions to NPOs utilizing the MFS can eventually exceed those under the TFS.

In the work here, the performance of the MFS is compared to that of the TFS using an NPV analysis (see Klausner, 2003) on the stream of distributions to the NPOs generated under each of the two strategies. To that end, in Section 2, formulas are given for computing relevant quantities associated with the two strategies. In Section 3, conditions are presented under which the MFS is "better than" the TFS in that there is a time $T > 0$ such that for all time $t > T$, the NPV of all cash flows to the NPOs up to time t under the MFS is greater than that under the TFS. While the financial analysis in Section 3 requires constant investment-return and payout rates, formulas for comparing the two strategies with variable rates are developed in Section 4. Simulations are used to compare fund performances with variable rates to those predicted with fixed rates that also show the long-term benefits of using the MFS.

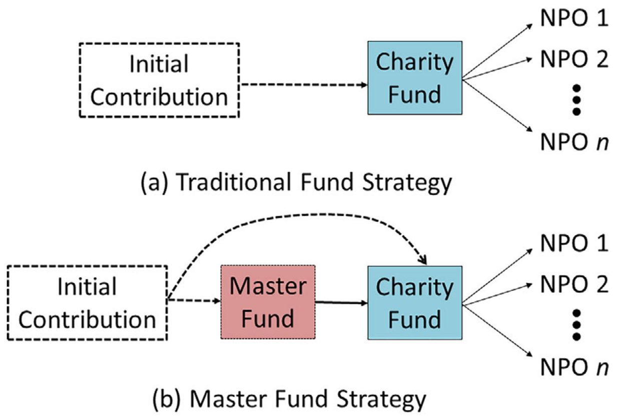

FIGURE 1 Traditional versus master fund strategy.

2 | FORMULAS FOR RELEVANT QUANTITIES FOR THE TWO STRATEGIES

Performing an NPV analysis requires assigning a discount rate to future cash flows to the NPOs. To that end, it is assumed that donors choose a discount rate based on a subjective evaluation of current versus future payouts to the NPOs, and can apply different discount rates to different NPOs. For example, a donor supporting an NPO providing relief to a local community suffering from an environmental disaster might apply a high discount rate resulting from the urgency of the need, while a lower discount rate may be appropriate for a contribution to a university to support student scholarships and faculty research on a long-term basis.

For the analysis here, without loss of generality, assume that only one NPO is to receive distributions. Computing the NPVs of the two strategies requires the following data:

- For the TFS, the amount of the initial donation, p, all of which is used to establish the CF.
- For the MFS, the initial donations, p_c and p_m, to the CF and the MF, respectively. To ensure that the same amount of initial contribution is made under both strategies, it must be that $p = p_c + p_m$.
- The (continuous or annual) investment earnings rates, net of expenses, of the CF and the MF, which are assumed to be constant over time and the same, say i, for both funds. This assumption assures that differences in NPVs arise from the structure of the funding strategies and not from superior investment or fund–management skills for one of the two types of funds.
- The (continuous or annual) distribution rates of the CF and MF, which are assumed to be constant over time and the same, say r, for both funds.
- For convenience, the *fund growth rate* is defined as $g = i - r$ and represents the rate at which a fund increases in value after making distributions from its earnings.
- The donor's specified discount rate, λ.
- The *net discounted growth rate*, $G = g - \lambda$.

The nominal investment earnings rates and the discount rates are both known to contain a "real" and an "inflationary" component and both may rise and fall over time. Since these parameters always appear in tandem in the NPV calculations as a difference between the rates, the inflationary effect is canceled within the calculation, so only the real difference between the earnings rate and the discount rate remains; thus, it is appropriate to use nominal rates for the calculation. Inflation will also necessitate increased periodic distributions to the NPO in order to maintain the same level of services; however, this would be offset by an increase in the inflationary component of the investment earnings rate and increased CF and MF balances over time. In summary, inflationary adjustments are not relevant to the results presented subsequently.

Using the foregoing data, it is possible to obtain formulas for computing fund values and NPVs of NPO distributions at time t for both strategies; however, those formulas, whose derivations are provided in the online Appendix S1, depend on whether all rates are assumed to be compounded annually or continuously.

2.1 | Formulas for annual compounding rates

The formulas for fund balances at the beginning of year t, cumulative distribution amounts through the end of year t, and cumulative NPVs, when rates are compounded once annually, are presented in Table 1, for $t = 1, 2, \ldots$ In that regard, and throughout the rest of this article, any quantity with a bar over it—for example $\overline{NPV}$—refers to that quantity under the MFS. In contrast, a quantity without a bar—for example, $NPV(t)$—refers to that quantity under the TFS.

The last formula in Table 1 enables one to determine the NPV of the stream of funds received by the NPO using each strategy. For specific values of the parameters, it is then possible to use numerical methods to find the *transition year*, if one exists, that is, the first year when the NPV with the MFS is greater than that with the TFS. From the transition year forward, the NPV with the MFS exceeds that with the TFS.

2.2 | Formulas for continuous compounding rates

In Section 2.1, formulas are presented using annual compounding. In this section, analogous formulas for fund balances, distributions, and NPVs are presented in Table 2 using continuous compounding. In these formulas, t is a nonnegative number indicating the amount of time in years since the initial donation.

Numerical examples of comparing the results in Table 2 for both the TFS and the MFS are presented in the online Appendix S2 and show the potential benefit of using the MFS. In the next section, mathematical conditions are given under which the MFS is better than the TFS.

3 | THE BREAK-EVEN THEOREM

The formulas in Table 2 are interesting in their own right; however, they also provide the ability to determine conditions under which there is—and is not—a *break-even time*, that is, a time $T > 0$ such that $\overline{NPV}(T) = NPV(T)$, after which, $\overline{NPV}(t) > NPV(t)$ for all $t > T$. This means that, after the break-even time T, the MFS provides superior benefits compared to the TFS. These conditions are stated in the following theorem (whose proof is given in Appendix S1):

Theorem 3.1. (**Break-even Theorem**) *The existence of a break-even time T for which $\overline{NPV}(T) = NPV(T)$ depends, as follows, on the sign of the net discounted growth rate $G = g - \lambda$, where $g = i - r$ is the growth rate and λ is the discount rate*:

1. *If $G = g - \lambda = 0$, then $T = \frac{2}{r}$ is the unique break-even time.*
2. *If $G = g - \lambda > 0$, then there is a unique break-even time, T, whose value is the root of the following equation*:

$$rte^{Gt} + \left(1 - e^{Gt}\right)\left(\frac{r}{G} + 1\right) = 0. \quad (1)$$

3. *If $G = g - \lambda < 0$ then,*
   a. *If $i > \lambda$, then there is a break-even time whose value is the root of Equation (1).*
   b. *If $i \le \lambda$, then there is no break-even time.*

Furthermore, whenever there is a break-even time, T, for all $t > T$, $\overline{NPV}(t) > NPV(t)$.

TABLE 1 Fund balances, distributions, and NPVs with annual compounding.

	Traditional fund strategy	Master fund strategy
Charity fund value	$C(t) = (1+g)^{t-1}p$	$\overline{C}(t) = (1+g)^{t-1}p_c + (t-1)(1+g)^{t-2}rp_m$
Master fund value	Not applicable	$\overline{M}(t) = (1+g)^{t-1}p_m$
Cumulative NPO distributions	$D(t) = r\sum_{j=1}^{t} C(j)$	$\overline{D}(t) = r\sum_{j=1}^{t} \overline{C}(j)$
NPV of NPO distributions	$NPV(t) = r\sum_{j=1}^{t} \frac{C(j)}{(1+\lambda)^j}$	$\overline{NPV}(t) = r\sum_{j=1}^{t} \frac{\overline{C}(j)}{(1+\lambda)^j}$

TABLE 2 Fund balances, distributions, and NPVs with continuous compounding.

	Traditional fund strategy	Master fund strategy
Charity fund value	$C(t) = e^{gt}p$	$\overline{C}(t) = e^{gt}(p_c + rp_m t)$
Master fund value	Not applicable	$\overline{M}(t) = e^{gt}p_M$
Cumulative NPO distributions		
($g \neq 0$)	$D(t) = rp\frac{e^{gt}-1}{g}$	$\overline{D}(t) = rp_c\frac{e^{gt}-1}{g} + r^2p_m\left(\frac{e^{gt}}{g}t - \frac{e^{gt}-1}{g^2}\right)$
($g = 0$)	$D(t) = rpt$	$\overline{D}(t) = rp_C t + r^2p_m\frac{t^2}{2}$
NPV of NPO distributions		
($G = g - \lambda \neq 0$)	$NPV(t) = rp\frac{e^{Gt}-1}{G}$	$\overline{NPV}(t) = rp_c\frac{e^{Gt}-1}{G} + r^2p_m\left(\frac{e^{Gt}}{G}t - \frac{e^{Gt}-1}{G^2}\right)$
($G = g - \lambda = 0$)	$NPV(t) = rpt$	$\overline{NPV}(t) = rp_c t + r^2p_m\frac{t^2}{2}$

It is important to note from (1) that the break-even time depends on the growth rate (g), the discount rate (λ), and the distribution rate (r), but is *independent* of the initial allocation of money in the CF and MF. This independence is shown mathematically in Appendix S1.3 but informally is due to the fact that the total initial donation under both strategies is the same. The contribution to the NPV at time T coming from the investment of p_c in the CF is the same for both strategies. Furthermore, because the total NPV at time T is the same for both strategies by definition of break-even, it follows that the remaining NPV under the TFS due to the investment of $p - p_c$ must be the same as the remaining NPV due to the investment of p_m in the MF.

The Break-Even Theorem is divided into multiple cases that depend on the relationships among the growth rate, the distribution rate, and the discount rate. When the earnings rate i is greater than the discount rate λ, there always is a break-even time and so the MFS eventually results in greater NPV benefits than the TFS. When the discounted earnings rate is less than or equal to zero, no break-even time exists and, in this case, the TFS is superior to the MFS. Each of these cases is illustrated graphically using an initial allocation of \$50,000 to the MF, \$50,000 to the CF, an investment rate of $i = 0.08$ and a distribution rate of $r = 0.05$.

For Case 1, a discount rate of $\lambda = 0.03$ is used; the growth rate is equal to the discount rate (that is, $g = \lambda$); and the formulas for computing the NPVs come from the last line of Table 2, where $g - \lambda = 0$. According to Theorem 3.1, for this case the break-even time is $T = \frac{2}{r} = \frac{2}{0.05} = 40$ years as shown in Figure 2a. Figure 2b indicates how the MF and CF balances grow over time for both the MFS and the TFS. As seen in Figure 2b, the Charity Funds for both strategies are equal at $\frac{1}{r} = 20$ years, where the difference between $\overline{NPV}(t)$ and $\text{NPV}(t)$ reaches its maximum point. As t increases beyond this point, under the MFS, the CF and its distributions continue to be greater than those under the TFS.

Unfortunately, for other cases in the Break-even Theorem, there is no closed-form expression for T; however, it is possible to obtain the value using numerical methods by solving equation (1), as is now illustrated.

For Case 2, both $i - \lambda$ and $g - \lambda = i - r - \lambda$ are positive. The numerical example for this case differs from that used in Case 1 only in the distribution rate, specifically, $r = 0.02$ for Case 2 compared to $r = 0.05$ in Case 1. As a result of this lower distribution rate, the CF under the TFS grows faster in Case 2 than in Case 1 and so it takes longer for the CF under the MFS to catch up to that of the TFS. As seen in Figure 3, this occurs at $\frac{1}{r} = 50$ years compared to 20 years in Case 1, delaying the break-even time for Case 2 to 74.4 years compared to 40 years for Case 1.

The third scenario where a break-even time exists arises in Case 3(a) of the Break-even Theorem. In order for Case 3(a) to apply, $i - \lambda > 0$ while $g - \lambda = i - r - \lambda < 0$. Two different numerical examples illustrate this case. In Figure 4a, both the discounted growth rate $i - r - \lambda < 0$ and the undiscounted growth rate $g = i - r < 0$. This means that more funds are being distributed than are being earned on the investments under both strategies and so the funds are depleting over time. This can be seen in Figure 4b, where each of the individual funds eventually begins to diminish. Even though the total of the funds under the MFS is shrinking over time, the Break-even Theorem indicates that the MFS is superior to the TFS, with a break-even time of 25.1 years being shown in Figure 4a.

In the second numerical example for Case 3(a), $i - \lambda > 0$ while $g - \lambda = i - r - \lambda < 0$, however, the undiscounted growth rate $g = i - r > 0$, so all the funds and the *undiscounted* distributions from them for both strategies continue to grow, even though the *discounted* value of these distributions diminishes over time. These results are shown in Figure 5a. As was seen when comparing Case 1 to Case 2, the first numerical example of Case 3(a) with $r = 0.06$ takes 35.4 years to reach break-even which is longer than the

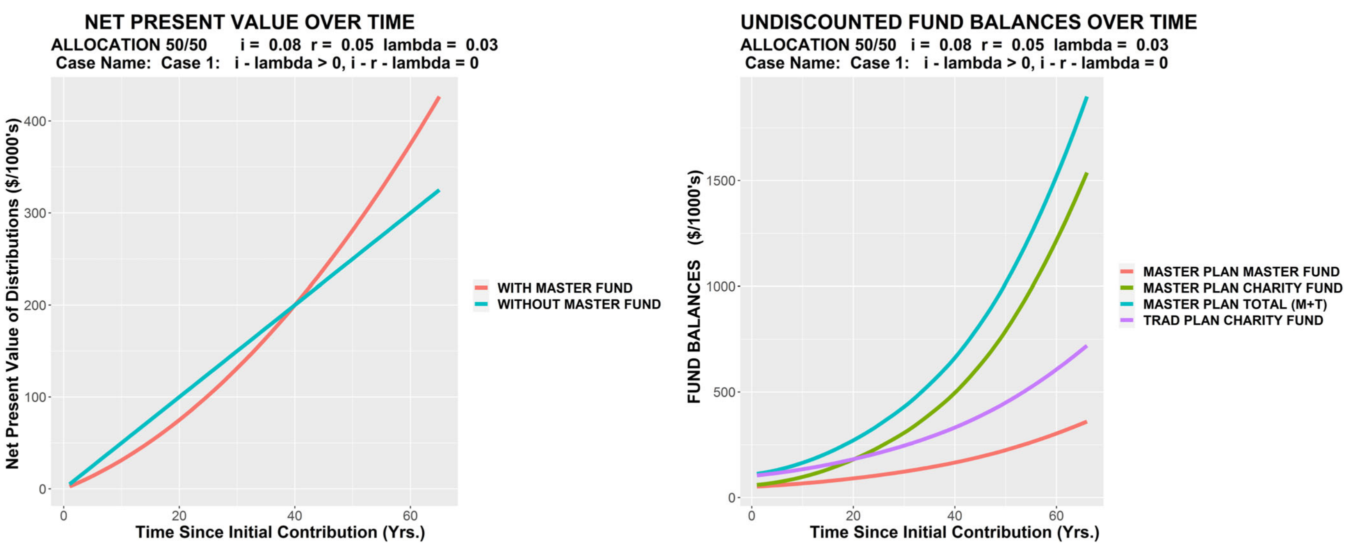

FIGURE 2 Case 1 of the Break-even Theorem.

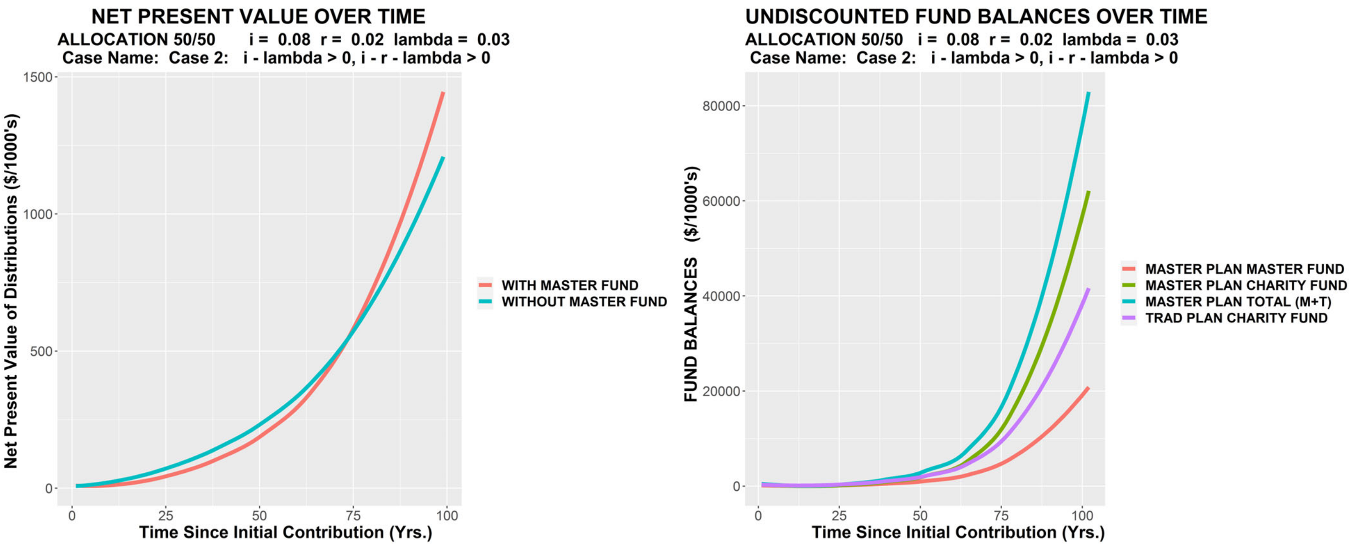

FIGURE 3 Case 2 of Break-even Theorem.

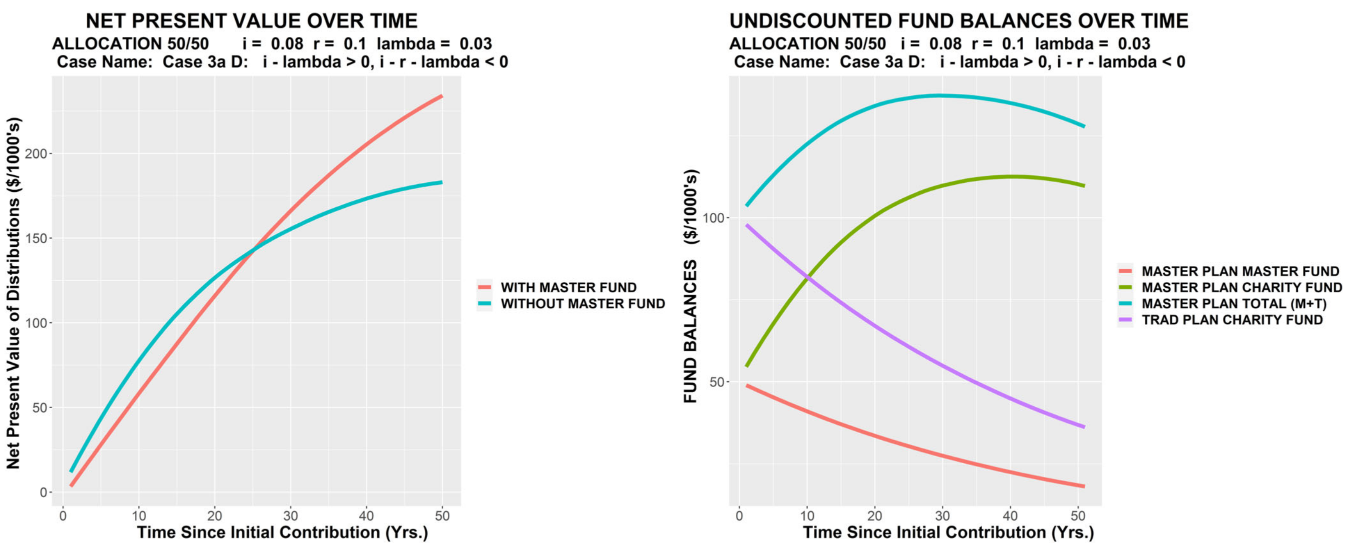

FIGURE 4 Case 3(a) of the Break-even Theorem with $g < 0$.

second numerical example with $r = 0.10$ because the additional contributions from the MF to the CF under the MFS are smaller.

The next numerical example illustrates Case 3(b) and compares the performance of the MFS and the TFS when $i - \lambda < 0$. For this condition, Case 3(b) of the Break-even Theorem indicates that that there is no break-even time so the TFS will be better than the MFS. Case 3(b) arises when $i - \lambda < 0$ or when $i - \lambda = 0$. Also, like Case 3(a), these conditions can occur with $g > r$, with all of the undiscounted fund balances and distributions growing under both strategies, or when $g < r$, with all of the fund balances depleting and distributions diminishing over time. Only the non-depleting case is shown. Figure 6 illustrates the superiority of the TFS for these conditions. Because the CF under the MFS is initially less than that under the TFS, the NPV of distributions to the NPO is initially higher with the TFS, as seen in Figure 6a. In addition, because the funds under either strategy grow at a slower rate than the assumed discount rate, the discounted value of the distributions from the MF, $\overline{NPV}(t)$, never catches up to the NPV of the TFS, even though all of the undiscounted fund balances continue to grow, as seen in Figure 6b.

The final numerical example arises from Case 3(b) when $i - \lambda = 0$, so no finite break-even time exists. In this case, the donor should opt for the TFS in preference to the MFS, Although this example is not illustrated, it is similar to the example shown in Figure 6 where the NPV for the TFS begins above the NPV for the MFS and remains above it throughout the time period illustrated, except that as t continues to increase, the NPVs under the two strategies converge

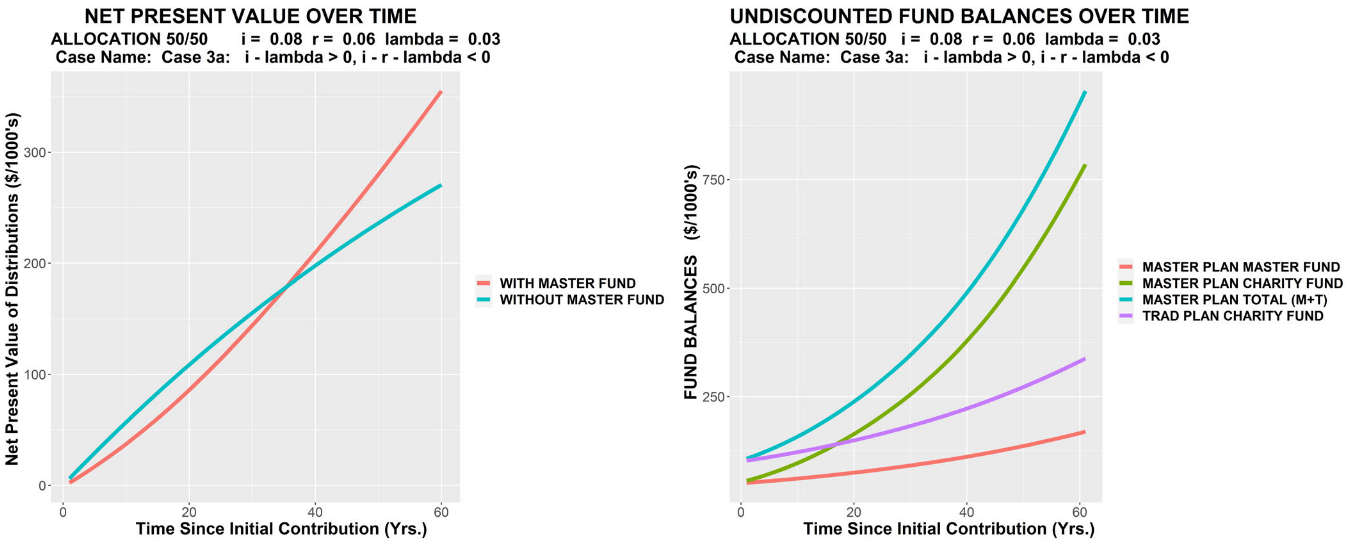

FIGURE 5 Case 3(a) of the Break-even Theorem with $g > 0$.

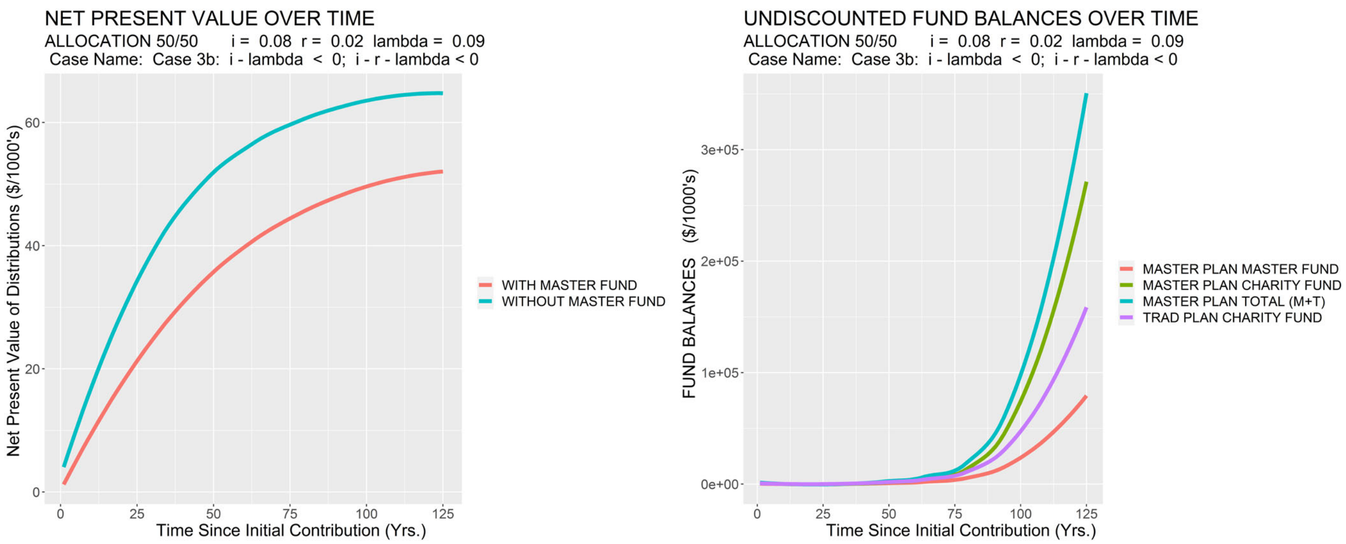

FIGURE 6 Case 3(b) of Break-even Theorem with $i - \lambda < 0$.

to the same value of $100,000, namely, the total initial contribution. This result is confirmed by taking each of the NPV formulas shown in Table 2 for $G \neq 0$, substituting $G = g - \lambda = i - r - \lambda = -r$ (since $i - \lambda = 0$) and taking the limit of each of the two formulas as $t \to \infty$. The result that, when $i = \lambda$, $\lim_{t\to\infty} NPV(t) = \lim_{t\to\infty} \overline{NPV}(t) = p_c + p_m = p$, implies not only that the net present values of the two strategies converge over time, but also that they converge to the original contribution amounts. In other words, when $i = \lambda$, as time increases, the limit of the net present values for both strategies is the same as the value of an immediate, direct contribution to the operating funds of the NPO. As λ increases, the NPVs under both strategies become strictly smaller, so the value of an immediate, direct contribution to the NPO is strictly greater than the value of implementing either the TFS or the MFS.

These results allow for the following optimal Donor Decision Strategy:

Option 1 ($i \leq \lambda$): Choose an immediate, direct contribution to the NPO.

Option 2 ($i > \lambda$): Choose a Master Fund Strategy to support the NPO over time.

4 | VARIABLE INVESTMENT RETURNS AND PAYOUT STRATEGIES

The previous section compared outcomes from the MFS and the TFS assuming that the rates of return on investment, as well as the fund distribution rates, are known and constant over time. In this section, these simplifications are relaxed and formulas are developed to provide for the more realistic case in which investment returns are variable from period to period as well as for a payout strategy that adjusts distributions based on recent investment performance. The performance of the two strategies using these variable rates is compared using a Monte Carlo computer simulation. The results are also compared to those of the previous fixed-rate scenario.

The nominal rate of return on investment for all of the funds in either strategy is now assumed to be a random variable that is normally distributed with a mean of 0.08 and standard deviations of 0.06 and 0.12, as described below. A new rate of return is generated every year over the course of a 100-year simulation. The simulations were created in R programming language (R Core Team, 2020) and returns on investment were drawn from the rnorm function contained in R. Six million random numbers (6 scenarios times 100 years times 10,000 trials) were generated for each simulation, beginning with the initial random seed of 1,650,975,334.

The annual rate of distribution from the funds is adjusted to reflect a widespread practice among NPO fund managers in which payouts increase with better-than-expected investment results and decrease with investment results that are below expectations (see p. 1142 in Duquette (2017) and also Brown et al. (2014)). Specifically, the annual payout rates of the funds are based on a weighted average rate of the return from the current and three most-recent prior periods. In the simulations that follow, the returns for the current period are assigned a weighting of 0.4, while the three prior period returns are weighted, 0.3, 0.2, and 0.1, respectively. The base payout rate is then adjusted higher or lower by the difference between this weighted average and the mean rate of return on investment. In periods where this calculation would result in a negative payout, a zero payout rate is assigned instead. Also, a different diminishing weighting is used for each of the first 3 years of the simulation, since three prior period returns are not available.

To implement the simulation with investment returns and payout rates varying with each period, the continuous compounding formulas in Table 2 are replaced by the recursive formulas in Table 3. In these formulas, the constants g and G from Table 2 are replaced by g_t and G_t, indicating that these values are different in each time period, t. Specifically, $g_t = i_t - r_t$, where i_t is the rate of return on investments in period t as determined by random values from a normal distribution and r_t is the payout rate adjusted for the weighted average rates of return for the periods $t-3$ through t, as described above. Also, $G_t = g_t - \lambda$ is the discounted growth rate in period t. The boundary conditions for these formulas are obtained by substituting $t = 1$ in the corresponding formulas from Table 2. The recursive nature of these formulas, with fund balances, cumulative distributions, and net present values in period t all dependent on the level of these funds in the prior period, $t-1$, provides for the previous investment results within each simulation trial to be carried forward to the next time period. The same periodic investment results and payout strategy adjustments are applied to all of the funds under both strategies, so differences in performance depend only on the structure of the strategy and not on random or systematic differences in investment performance.

The histograms in Figures 7 and 8 display the results of simulations in which parameter values conform to each of the cases in the Break-even Theorem. For Cases 3(a) and 3(b), two sets of histograms are presented, one where all the funds are expected to grow over time, and one where the expected payout rates exceed the expected growth rates, and the funds are expected to deplete over time. Ten thousand trials are performed for each of the simulations with the initial donation being allocated equally to the MF and the CF under the MFS, while 100% of the initial donation is allocated to the CF under the TFS.

Rates of return on investment for each period and for each trial are chosen randomly from a normal distribution with a mean of 0.08 and a standard deviation of 0.06. These values reflect returns that might be anticipated from a portfolio consisting of a combination of common stocks and fixed income securities. Donor discount rates and target payout rates vary so that each of the simulations represents a different break-even case. The specific rates used to implement each simulation appear in the captions on the histogram for that case. Variable-rate simulations are considered as belonging to a specific break-even case based on the expected rate of investment return and the target payout rate, without regard to the standard deviation of

TABLE 3 Fund balances, distributions, and NPVs with variable rates.

	Traditional fund strategy	Master fund strategy
Charity fund value	$C(t) = C(t-1)e^{g_t}$	$\overline{C}(t) = e^{g_t}\left(\overline{C}(t-1) + r_t\overline{M}(t-1)\right)$
Master fund value	Not applicable	$\overline{M}(t) = e^{g_t}\overline{M}(t-1)$
Cumulative NPO distributions		
$(g_t \neq 0)$	$D(t) = D(t-1) + r_t C(t-1)\frac{e^{g_t}-1}{g_t}$	$\overline{D}(t) = \overline{D}(t-1) + r_t\overline{C}(t-1)\frac{e^{g_t}-1}{g_t} + r_t^2\overline{M}(t-1)\left(\frac{e^{g_t}}{g_t} - \frac{e^{g_t}-1}{g_t^2}\right)$
$(g_t = 0)$	$D(t) = D(t-1) + r_t C(t-1)$	$\overline{D}(t) = \overline{D}(t-1) + r_t\overline{C}(t-1) + \frac{1}{2}r_t^2\overline{M}(t-1)$
NPV of NPO distributions		
$(G_t = g_t - \lambda \neq 0)$	$NPV(t) = NPV(t-1) + r_t C(t-1)\frac{e^{G_t}-1}{G_t}e^{-\lambda(t-1)}$	$\overline{NPV}(t) = \overline{NPV}(t-1) + r_t\overline{C}(t-1)\frac{e^{G_t}-1}{G_t}e^{-\lambda(t-1)} + r_t^2\overline{M}(t-1)\left[\frac{e^{G_t}}{G_t} - \frac{e^{G_t}-1}{G_t^2}\right]e^{-\lambda(t-1)}$
$(G_t = g_t - \lambda = 0)$	$NPV(t) = NPV(t-1) + r_t C(t-1)\,e^{-\lambda(t-1)}$	$\overline{NPV}(t) = \overline{NPV}(t-1) + \left[r_t\overline{C}(t-1) + \frac{1}{2}r_t^2\overline{M}(t-1)\right]e^{-\lambda(t-1)}$

the investment returns. The results, including mean years to break-even, are compared to the calculated number of years to break-even under the fixed-rate scenario, where the fixed rate is equal to the expected rate of return and the payout rate is equal to the target payout rate for the variable rate scenario in each of these panels.

In the fixed-rate scenario, Break-even Theorem Cases 1, 2, and 3(a), all have a finite break-even time. For variable rates, the four simulations illustrated in Figure 7 corresponding to these cases, achieve a break-even time in all 10,000 out of 10,000 trials. The case parameters for these four simulations are all identical except for the target payout rates. The mean break-even time of the fixed-rate scenario and the corresponding variable-rate simulation is shown in Table 4.

Comparing the results of the variable-rate simulation to the results calculated for the comparable fixed-rate scenarios shows differences in the time to break-even of less than 1 year for all four cases. For three of the four cases, the years to break-even for the variable-rate cases are greater than those of the corresponding fixed-rate cases. The shorter break-even time for the variable-rate simulation of Case 2 results from the lower payout rate in this scenario. While all of the mean break-even times for the variable-rate scenarios

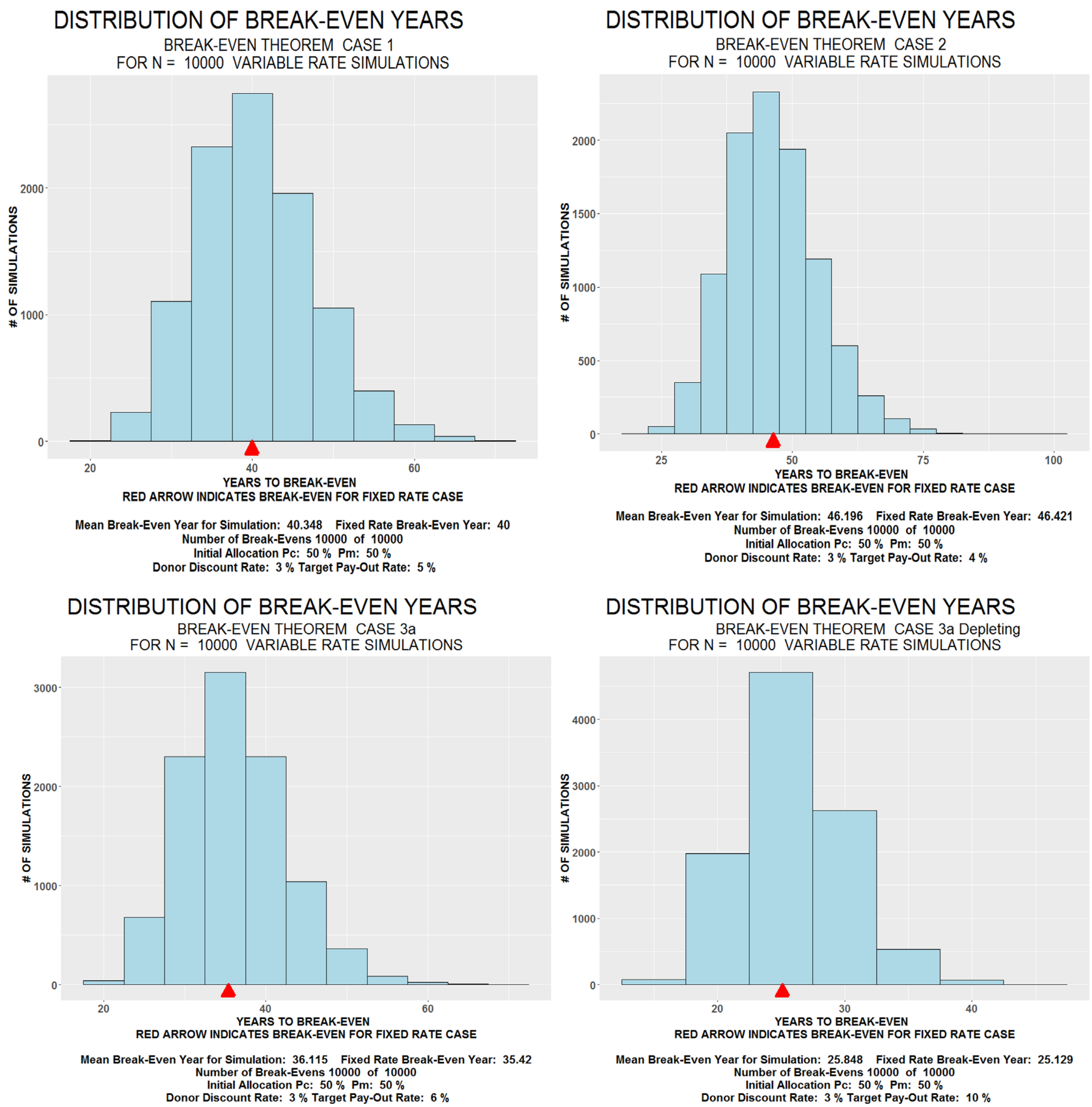

FIGURE 7 Histograms for variable rate cases of Break-even theorem with $i - \lambda > 0$.

are within 1 year of their corresponding fixed-rate cases, these differences are statistically significant at a 99% confidence level. Also shown in Table 4, in addition to breaking even in similar time-frames, the NPV of distributions at break-even for the variable-rate simulations is also similar to the calculated NPV for the comparable fixed-rate scenarios. The differences between the fixed-rate NPVs and the variable-rate NPVs are also statistically significant at a 99% confidence level for all four cases. While the variable-rate trials might take a longer time to break-even in three of the four comparisons, all four scenarios produced higher NPVs at break-even for the variable-rate simulations compared to the fixed-rate break-even NPVs. The optimal decision rules presented in Section 3 support a donor decision to choose a MFS over a TFS under all four simulated scenarios.

The last two histograms shown in Figure 8 represent growing and depleting scenarios, respectively, for Case 3(b) of the Break-even Theorem in which there is no break-even time; consequently, a donor prefers the TFS to the MFS. However, as shown in these exhibits, 82 of the 10,000 simulations for Case 3(b) and 349 of the 10,000 simulations for Case 3(b) with depleting payouts resulted in break-even times within the 100 year simulation period, with the mean number of years to break-even of 64.8 and 45.2, respectively, for these two simulations.

In the scenarios presented in Figures 7 and 8, the standard deviation of investment returns for all of the funds under either strategy is 0.06. In contrast, the scenarios in Table 5 include simulations using a standard deviation of 0.12, which is twice that used in the scenarios in Figures 7 and 8. This higher level of investment risk might be consistent with risk levels of an all-stock portfolio, or a stock/bond portfolio in an investment environment where returns are more volatile than historical levels. Although higher risk investments generally are associated with higher expected returns, the rate of return for the simulations in Table 5 is the same as those for the corresponding

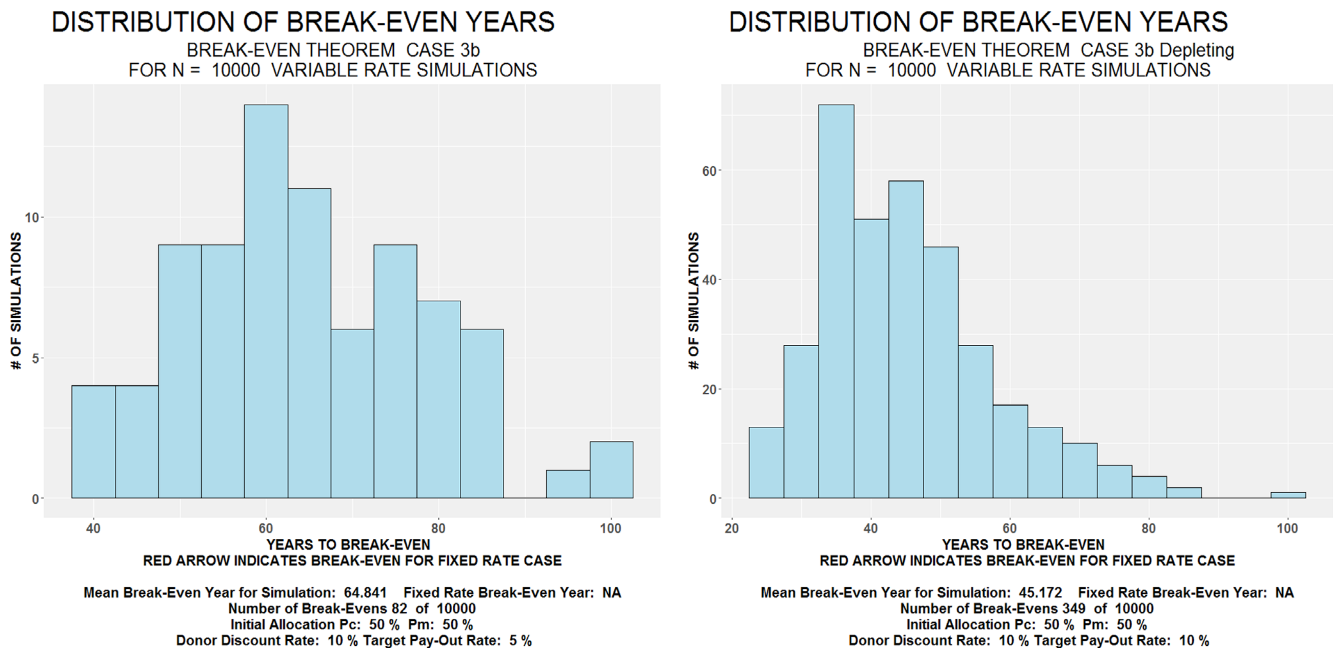

FIGURE 8 Histograms for variable rate cases of Break-even theorem with $i - \lambda \leq 0$.

TABLE 4 Summary of simulation results for cases expected to break even.

	Case 1	Case 2	Case 3(a)	Case 3(a) depleting
Computed years to break-even				
Fixed-rate	40.0	46.4	35.4	25.1
Mean years to break-even				
Variable-rate simulation	40.3	46.2	36.1	25.8
	Case 1	**Case 2**	**Case 3(a)**	**Case 3(a) depleting**
Computed NPV at break-even				
Fixed-rate ($ per $1 invested)	2.0	2.36	1.80	1.43
Mean NPV at break-even				
Variable-rate simulation ($ per $1 invested)	2.05	2.38	1.84	1.46

TABLE 5 Summary of high standard deviation simulation results.

	Case 1	Case 2	Case 3(a)	Case 3(a) depleting	Case 3(b)	Case 3(b) depleting
Variable Sim. St. Dev. = 0.06						
# of Break-evens out of 10,000	10,000	10,000	10,000	10,000	82	349
Mean years to break-even	40.3	46.2	36.1	25.8	64.8	45.2
Variable Sim. St. Dev. = 0.12						
# of Break-evens out of 10,000	9999	9992	9995	9998	1597	2243
Mean years to break-even	37.3	41.1	34.4	26.1	48.1	32.7

scenarios in Figures 7 and 8, namely 0.08, to preserve comparability to these lower risk scenarios.

Histograms are not presented for the higher-risk scenarios, but the results for six of these simulations are summarized in Table 5. As an example, the mean break-even time of 41.1 years for the high-risk Case 2 simulation of Table 5, is substantially earlier than that of 46.2 years for the corresponding lower risk scenario in Figure 7 and is also earlier than the break-even time under the fixed-rate scenario. Both of these comparisons are significant at a 99% confidence level. The high-risk simulation for Case 2 also produced 8 out of 10,000 trials that failed to achieve break-even within the 100-year simulation period. As a further example, for the Case 3(b) simulation, the higher-risk scenario produced 1597 out of 10,000 trials where a break-even time was achieved, even though there is no break-even time for the corresponding fixed-rate case.

5 | SUMMARY

The novel legacy-donation Master Fund Strategy (MFS) of using a Master Fund whose annual distributions support a Charity Fund that, in turn, makes distributions to NPOs is analyzed mathematically and compared to a Traditional Fund Strategy (TFS) of creating a Charity Fund that donates directly to NPOs. The strategies consist of parameters for the amount of the initial donation, p (all of which goes to the Charity Fund under the TFS while p_c and p_m go to the Charity Fund and the Master Fund, respectively, under the MFS); the investment earnings rates of all funds; the distribution rates of the Charity Fund and the Master Fund; and the donor's specified discount rate. A Break-even Theorem is developed that establishes conditions on the parameters under which there is a break-even time, that is, a time T when the NPVs of the two strategies up to time T are equal and, for all $t > T$, the NPV of the MFS is greater than that of the TFS. Although break-even times can be fairly long, in any scenario where the investment earning rate is greater than the discount rate, the donor and the NPOs achieve a higher level of benefit with the MFS.

Based on the results in Sections 2.2 and Section 3, a Donor Decision Strategy is presented indicating that under high discount-rate scenarios it is optimal to make contributions directly to NPO operations while in lower discount-rate scenarios, a MFS is superior to both direct contributions or contributions to a TFS.

The continuously compounded formulas for fixed investment and payout rates are modified to permit variable investment returns and payout rates. These formulas are implemented through a simulation whose results are consistent with supporting the optimal funding strategy determined by the Break-even Theorem under a fixed-rate assumption.

The significance of this work for nonprofit research is that it provides the foundation for researchers to examine other vehicles, models, and methods of raising funds for nonprofits over time. In practice, it benefits nonprofit marketing and fundraising by offering a method to secure more bequests, addressing an asymmetry in fundraising behavior and bequest income. As most fundraisers report metrics on commitments made for legacy giving rather than bequest income received at a much later date in the future, the strategy we propose aligns incentives for fundraisers to spend their time on cultivating bequest donations today as it provides fundraisers an alternative method to secure bequest donations much earlier, during the donors' lifetimes. Clearly, a direction for future research is to collect data to determine the degree to which donors are interested in our MFS. To that end, we have created a website (see http://faculty.weatherhead.case.edu/dxs8/master-fund/) that explains the idea of the MFS and are planning to include a survey to collect the appropriate data. Finally, while the benefits of our model depend somewhat on current permitted activities of DAFs under the law in a handful of countries, and indeed, the examination of practices in only a handful of countries, researchers and practitioners in other countries may choose to explore how our strategy might work in their own contexts.

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CONFLICT OF INTEREST STATEMENT

My co-authors and I have recently published a paper entitled "A novel approach to legacy donations with long-term benefits supported by numerical illustrations (see Solow, Symes & Webb 2023)." In that paper, we propose a novel approach, called the "Master Fund Strategy" for making legacy donations and show, *through numerical examples*, that this approach is potentially "better" than a traditional approach in the sense that the net-present-value of the stream of distributions to the charities under the new approach eventually exceeds that of the traditional approach. In this article, we are submitting here to the *Journal of Philanthropy and Marketing*, however, a mathematical model is used to obtain both analytical and computer simulation results supporting the previous work. In particular, formulas are developed for computing relevant financial quantities associated with the two strategies. Conditions are presented under which the Master Fund Strategy is better than the Traditional Fund Strategy in the sense that there is a point in time when the net-present-value of the distributions to the NPO under the Master Fund Strategy exceeds that of a Traditional Fund Strategy and continues to do so beyond that point. These analytical results are obtained under the assumption that the investment rates of return and the payouts rates are known constants. Formulas for relaxing these restrictions are developed and the consequences are examined with Monte Carlo simulations.

DATA AVAILABILITY STATEMENT

The data in this manuscript is either given explicitly in the paper or is generated randomly for a computer simulation.

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REFERENCES

Afik, Z., Benninga, S., & Katz, H. (2020). Grantmaking foundations' asset management, payout rates, and longevity under changing market conditions: Results from a Monte Carlo simulation study. *Nonprofit and Voluntary Sector Quarterly*, *49*(2), 424–447. https://doi.org/10.1177/0899764019873972

Andreoni, J. (1998). Toward a theory of charitable fund-raising. *Journal of Political Economy*, *106*(6), 1186–1213. https://doi.org/10.1086/250044

Andreoni, J., & Madoff, R. (2020). *Calculating DAF payout and what we can learn when we do it correctly*. National Bureau of economic research working paper 27888. NBER.

Arden, D. (2013, June 24). When clients want their own foundations. *Wall Street Journal*.

Bekkers, R., & Wiepking, P. (2011). A literature review of empirical studies of philanthropy. *Nonprofit and Voluntary Sector Quarterly*, *40*(5), 924–973. https://doi.org/10.1177/0899764010380927

Brown, J. R., Dimmock, S. G., Kang, J. K., & Weisbenner, S. J. (2014). How university endowments respond to financial market shocks: Evidence and implications. *American Economic Review*, *104*(3), 931–962. https://doi.org/10.1257/aer.104.3.9

Chang, C. F., & Tuckman, H. P. (1990). Why do nonprofit managers accumulate surpluses, and how much do they accumulate? *Nonprofit Management and Leadership*, *1*(2), 117–135.

Chang, C. F., & Tuckman, H. P. (1994). Revenue diversification among non-profits. *VOLUNTAS: International Journal of Voluntary and Nonprofit Organizations*, *5*(3), 273–290. https://doi.org/10.1007/bf02354036

Duquette, N. J. (2017). Spend or save? nonprofits' use of donations and other revenues. *Nonprofit and Voluntary Sector Quarterly*, *46*(6), 1142–1165. https://doi.org/10.1177/0899764017728368

Furnham, A. (1984). Many sides of the coin: The psychology of money usage. *Personality and Individual Differences*, *5*, 501–509. https://doi.org/10.1016/0191-8869(84)90025-4

Hager, M. A. (2006). Should your nonprofit build an endowment. *Nonprofit Quarterly*, Summer, June 21, 57–60. https://nonprofitquarterly.org/should-your-nonprofit-build-an-endowment

Handy, F., & Webb, N. J. (2003). A theoretical model of the effects of public funding on saving decisions by charitable nonprofit service providers. *Annals of Public and Cooperative Economics*, *74*(2), 261–282. https://doi.org/10.1111/1467-8292.00224

Hansmann, H. (1990). Why do universities have endowments? *The Journal of Legal Studies*, *19*(1), 3–42. https://doi.org/10.1086/467841

James, R. N. (2008). Health, wealth, and charitable estate planning. *Nonprofit and Voluntary Sector Quarterly*, *38*(6), 1026–1043. https://doi.org/10.1177/0899764008323860

Jonas, E., Schimel, J., Greenberg, J., & Pyszczynski, T. (2002). The scrooge effect: Evidence that mortality salience increases prosocial attitudes and behavior. *Personality and Social Psychology Bulletin*, *28*(10), 1342–1353. https://doi.org/10.1177/014616702236834

Jones, M., & Routley, C. (2022). When I go my family will see my life in programmes: Legacy giving and identity at the Royal Opera House. *Journal of Philanthropy and Marketing*, *27*(2), e1728. https://doi.org/10.1002/nvsm.1728

Klausner, M. (2003). When time Isn't money. *Stanford Social Innovation Review*, *1*(1), 51–59. https://doi.org/10.48558/HF5H-2E25

Konrath, S., & Handy, F. (2017). The development and validation of the motives to donate scale. *Nonprofit and Voluntary Sector Quarterly*, *47*(2), 347–375. https://doi.org/10.1177/0899764017744894

National Council of Nonprofits. (2022, April 24). *Endowments*. National Council of Nonprofits Retrieved June 29, 2022, from https://www.councilofnonprofits.org/tools-resources/endowments

Phillips, S. D., Dalziel, K., & Sjogren, K. (2021). Donor advised funds in Canada, Australia and the US: Differing regulatory regimes, differing streams of policy drift. *Nonprofit Policy Forum*, *12*(3), 409–441. https://doi.org/10.1515/npf-2020-0061

R Core Team. (2020). *R: A language and environment for statistical computing*. R Foundation for Statistical Computing https://www.R-project.org/

Routley, C., & Sargeant, A. (2014). Leaving a bequest. *Nonprofit and Voluntary Sector Quarterly*, *44*(5), 869–885. https://doi.org/10.1177/0899764014542687

Sargeant, A. (1999). Charitable giving: Towards a model of donor behaviour. *Journal of Marketing Management*, *15*(4), 215–238. https://doi.org/10.1362/026725799784870351

Scaife, W., Williamson, A., McDonald, K., & Smyllie, S. (2012). *Foundations for giving: Why and how Australians structure their philanthropy*. Australian Centre for Philanthropy and Studies, Queensland University of Technology.

Solow, D., Symes, R., & Webb, N. (2023), A novel approach to legacy donations with long-term benefits supported by numerical illustrations. *Journal of Philanthropy and Marketing*, https://onlinelibrary.wiley.com/doi/10.1002/nvsm.1803

Stanford PACS – Center on Philanthropy and Civil Society. (2020). *The Stanford PACS guide to effective philanthropy*. Board of Trustees of the Leland Stanford Junior University.

Wishart, R., & James, R. N., III. (2021). The final outcome of charitable bequest gift intentions: Findings and implications for legacy fundraising. *Journal of Philanthropy and Marketing*, *26*, e1703. https://doi.org/10.1002/nvsm.1703

SUPPORTING INFORMATION

Additional supporting information can be found online in the Supporting Information section at the end of this article.

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