Stimulated Raman Scattering of Q-Gaussian Laser Beam in an Unmagnetized Plasma

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Abstract

This study describes the relativistic q-Gaussian laser beam's stimulated Raman scattering (SRS) in an unmagnetized plasma. Moreover, the influence of the pump laser's relativistic self-focusing on the SRS process has been investigated. Using variational theory, we derived analytical solutions to the coupled nonlinear wave equations describing the pump, EPW, and scattered waves. The resulting equations were numerically solved to see the impacts of laser and plasma characteristics on the dynamics of the pump beam and its influence on the power of scattered waves. The power of the scattered wave was observed to be significantly altered via the self-focusing action of the pump beam, where when the effect of self-focus increases, it leads to an increase in the effect of stimulated Raman scattering. The stimulated Raman scattering yield is investigated based on the laser beam's and plasma's intensity. The main finding is that as q increases, the SRS yield increases, and as the intensity of the laser beam and plasma density increase, the SRS yield also increases. The scattering of the self-focused beam takes place at a greater distance than the beam of the pump, due to the relatively diminished level of scattered power. The value of the integrated reflection increases with the increase of q and the growth rate.

1. Introduction

The scattering electromagnetic waves phenomenon from the Eigenmodes of plasma has been well-recognized for several years [1, 2]. The plasma's intrinsic vibrational modes, electron plasma waves, and ion plasma waves may be amplified if a powerful laser beam propagates through the plasma [3-5].

A significant instabilities class includes a big amplitude light wave coupling into a scattered light wave plus either the plasma wave electron (the Raman instability) or an acoustic wave of ion (the Brillouin instability) [6]. And the stimulated Raman forward scattering (SRFS) use is a significant nonlinear phenomenon in the laser-plasma accelerator framework. Its function is to initiate a relativistic plasma wave that facilitates the acceleration of electrons to energies in the multi-MeV range [7, 8]. The phenomenon above holds significance in the context of the fast igniter fusion methodology. Raman instability can generate hot electrons due to the propagation of an ultra-intense laser pulse in the underdense region of expanding fusion target plasma. The plasma wave (ω_p) is

driven by the ponderomotive force between the pump (ω_0) and its Stokes/anti-Stokes sidebands ($\omega_0 \pm \omega_p$) in this particular process. The plasma wave's density fluctuation interacts with the pump, generating nonlinear currents that drive the sidebands upwards [9, 10]. Natural modes in a collisionless plasma are Landau damped due to the absence of a pump wave (laser beam). The electron and ion waves in plasma are stimulated and can cause plasma instabilities if the pump wave power exceeds the damping threshold. The SRS is enhanced if one of the excited modes has a high frequency [11]. Lately, there has been much concern in investigating the nonlinear influence of the laser radiation scattering phenomenon at elevated powers, as they're pertinent to laser-driven fusion. And, an elevated energy laser photon may decline into a plasma and photon or a phonon and photon, corresponding to SRS and SBS. Interaction of this ultra-intense laser pulse with plasmas has been a topic of investigational as well as theoretical study concern owing to its prospective uses in the rapid ignition in the scheme of Inertial Confinement Fusion (ICF), the acceleration of the particle, Laser induced plasma spectroscopy and fresh sources of radiation [12, 13]. SRS and SBS are considered the most important parametric decay instabilities in the ICF. Three waves interact during SRS, one causing forward-scattering (FSRS) and the other producing backward-scattering (BSRS) [14]. Reducing the effects of SRS in the laser-induced plasma is crucial to the success of the laser fusion process. The laser light wave (the pump) decays into two other waves, the scattered light wave (SLW) and the electron plasma wave (EPW), making SRS a three-wave parametric instability [15]. Numerous approaches have been suggested for suppressing the SRS, like presenting a polarization rotation or a laser bandwidth and laser smoothing method (polarization smoothing, smoothing via spectral spreading, and produced spatial incoherence) [16]. The study conducted by Salih et al. [17] examined the SRS in non-magnetized plasma, specifically concerning the relativistic laser beams. The pump laser's self-focusing and the relativistic electron mass's nonlinearity affected the SRS process, which was investigated in this study. Incorporating the nonlinear coupling between the scattered laser beam and the pump laser was observed to have an effect. The transverse static magnetic field impact upon the SRFS of a laser in plasma was investigated by Hasoon et al. [18]. And, an upper hybrid wave and (2) focused Stokes/Anti-Stokes sidebands were excited by the x-mode. As a result, the electrons driving the upper hybrid wave experience a ponderomotive force from the laser and the sideband. Sharma P.[19] studied how relativistic nonlinearity affected the SRS of a laser beam traveling through a collisionless plasma at zero intensity in a center hollow Gaussian beam. And building the equations employed the theory of fluid, which evolved with Maxwell's equations and the partial differential equation. Singh A. et al. [20] used the moment theory to study the SRS of Gaussian laser beam in Relativistic Plasma. They discovered the reduction in the reflectivity of SRS with an increment in the pump wave intensity. In this communication, Walia K. et al. [21] probe the SRS of a high-power beam in Thermal Quantum Plasma. There's a pump beam interaction with the EPW, which causes the backscattered beam creation. Gupta N. et al. [22] theoretically explored the SRS phenomenon of elliptical q-Gaussian laser beams interacting nonlinearly with the underdense plasmas. A variational theory-informed, semi-analytical solution to a system of coupled nonlinear wave equations for three waves (EPW, pump, and scattered). The pump beam's self-focusing effect was an important factor in the scattered waves' strength. The q-Gaussian beam is characterized by a parameter q, distinguishing it from a normal Gaussian beam. When q becomes infinitely large, the q-Gaussian mode converges to a regular Gaussian beam [23]. vAccording to observations by Gupta et al. [24], the self-focusing effect of the pump beam has a major impact on the scattered wave's power. Our research shows that the yield of SRS grows with increasing q, laser beam intensity, and plasma density. Most previous research papers were devoted to studying the Gaussian mode, and recently, there were only two studies in the q-Gaussian modes. These studies considered the ponderomotive force and the power of the scattering wave and its effect due to self-focusing, and they did not study the relativistic and the effect of plasma density on self-focusing of the laser beam and plasma density. This paper studied the relativistic self-focusing of the q-Gaussian beam on Raman scattering.

2. Relativistic Self-focusing

Let us contemplate a laser beam characterized by a frequency ω_0 and wave no. k_0, which undergoes propagation in a medium of unmagnetized plasma alongside the z-axis. And the linear polarization of electromagnetic waves, specifically the laser beam, is denoted as $\overrightarrow{E_0} = \hat{y}E_0$, where E_0 represents the wave's electric field. The beam's intensity distribution along the wavefront at a distance of $z = 0$ is expressed as follows [25]:

$$E_0 E_0^* = A_{00}^2 \left[1 + \frac{r^2}{q r_0^2} \right]^{-q} \tag{1}$$

The equation mentioned above involves using a cylindrical coordinate system, where E_0 is the axial amplitude, r denotes the radial coordinate, r_0 is the original width of the beam, and E_0 signifies the axial amplitude. Therefore, the plasma dielectric constant can be expressed as follows [26]:

$$\varepsilon = 1 - \frac{\omega_p^2}{\gamma \omega_0^2} \qquad (2)$$

where ω_p denotes the plasma frequency, defined as $\omega_p (= 4\pi n_e e^2/m)^{1/2}$, k_0 is represented by $k_0 = (\omega_0/c)\varepsilon_0^{1/2}$. The n_e denotes the electron plasma density, while m and e represent the electronic mass and charge, respectively. The relativistic factor, denoted by γ, is expressed as follows [26]:

$$\gamma \approx \left[1 + \frac{e^2 E_0 E_0^*}{m_0^2 \omega_0^2 c^2}\right]^{1/2} \qquad (3)$$

The wave equation is the governing equation for the pump laser beam's electric field in the plasma [17]:

$$\nabla^2 \vec{E}_0 - \nabla(\nabla \cdot \vec{E}_0) + \frac{\omega_0^2}{C^2} \varepsilon_0 \cdot \vec{E}_0 = 0 \qquad (4)$$

The symbol ε_0 represents the permittivity of free space. When an electric field is [6]:

$$\vec{E}_0 = \vec{A}_0 \exp\{i(\omega_0 t - k_0 z)\} \qquad (5)$$

The amplitude field of the laser beam is denoted by $\vec{A}_0$. Assuming further the variation of $\vec{A}_0$ as:

$$\vec{A}_0 = \vec{A}_{20} \exp\{-ik_0 S_0(x,y,z)\} \qquad (6)$$

The functions A_{00} and S_0 are defined as real functions of space and giving by:

$$A_{20}^2(r,z) = \frac{A_{00}^2}{f^2(z)}\left[1 + \frac{r^2}{qr_0^2 f^2(z)}\right]^{-q} \qquad (7)$$

$$S_0 = \frac{1}{2} r^2 \frac{1}{f_0} \frac{df_0}{dz} + \Phi_0(z) \qquad (8)$$

The variables f, $\Phi_0(z)$ and r_0 represent the beam width, nonlinear dielectric function and spot size parameter, respectively. The equation for the beamwidth parameter f of the fundamental wave can be derived using the methodology outlined by Salih et al. [27, 28]:

$$\frac{d^2 f}{d\xi^2} = \frac{q^2}{k^2 f^3 (r^2/f^2 + qr_0^2)^2} + \frac{\alpha A_{00}^2 \omega_p^2}{c^2 f^3 k^2 r_0^2} \qquad (9)$$

The Runge-Kutta fourth-order method, Matlab and Mathmatica program [29] has been used to numerically solve equation (9) for the different laser-plasma parameters since an analytical solution is not possible; the laser beam's vacuum wavelength is ($\lambda = 1.053$ μm), as well as the laser original radius is $r_0 = 10$ μm for the intensity of laser beam ($I = 10^{18} W/cm^2$). The graphical representation in Figure 1 displays the fluctuations in the laser beamwidth parameter (f) about the normalized propagation distance, denoted as (ξ), across various q-parameter values. For example, as the parameter q increases from 1 to 10, the efficacy of self-focusing intensifies, but at values greater than 10, the defocusing is obvious.
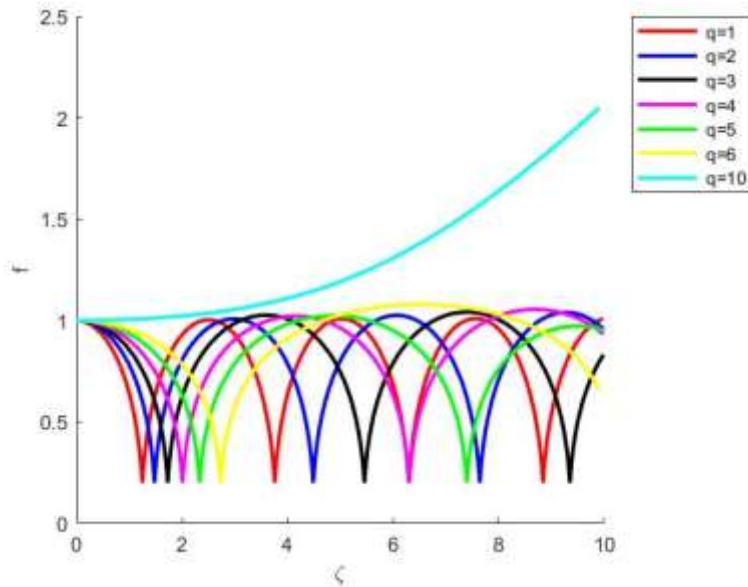
Figure 1: Variation of beamwidth parameter f with propagation distance ξ for parameters: $\alpha E_{00}^2 = 0.5$, $\omega_{p0}^2/\omega^2 = 0.02$, $and\ q = 1, 2, 3, 4, 5, 6, and\ 10$.

The impact of plasma density ω_{p0}^2/ω^2 on the efficacy of self-focusing is illustrated in Figure 2. At the elevated levels of plasma density ω_{p0}^2/ω^2, it has been observed that focusing exhibits greater efficacy than defocusing when the plasma density increases. When the plasma density is less than 0.03, de-focusing occurs.

The following relation gives the analytical solution to this equation:

$$f(\zeta) = \sqrt{1 + q^2\zeta^2 + R\zeta^2} \tag{10}$$

Where:

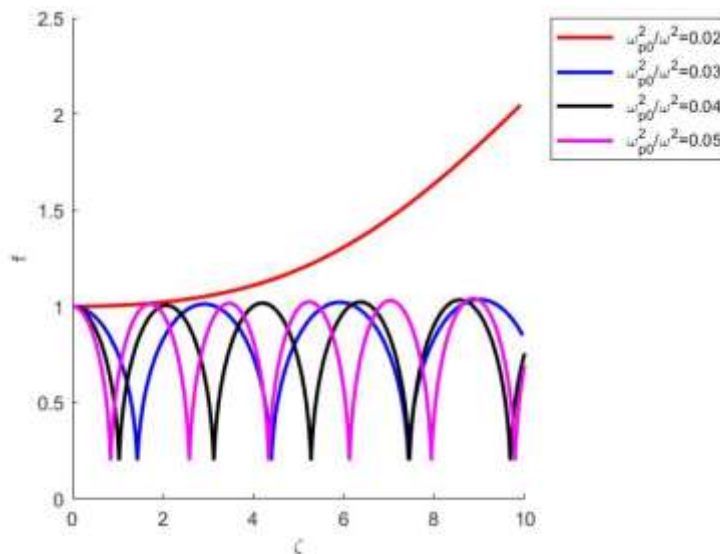$$R = \frac{\alpha A_{00}^2 \omega_p^2}{c^2 k^2 r_0^2} \tag{11}$$

Figure 2: Variation of beamwidth parameters f with propagation distance ξ for parameters: $E_{00}^2 = 0.5$, $\omega_{p0}^2/\omega^2 = 0.02, 0.03, 0.04, 0.05, and\ q = 10$.

3. Stimulated Raman Scattering

Electron plasma waves have frequencies ω and wave numbers k, and when they interact with a powerful laser beam, they produce SRS waves with frequencies ω_s and wave numbers k_s, where $\omega_s = \omega - \omega_0$ and $k_s = k - k_0$. The nonlinear coupling between the SRBS wave and the pump wave reduces the strength of the pump wave. The wave equations for the dispersed laser beam and the depleted laser beam with frequencies ω_d and wave numbers k_d are:

$$\nabla^2 \vec{E}_d + \frac{\omega_d^2}{c^2}\left(1 - \frac{\omega_p^2}{\gamma\omega_d^2}\right)\vec{E}_d + \frac{ie^2\omega_p^2 k |\vec{E}_s\vec{E}_s^*|\vec{E}_d}{16\Gamma_e m_0^2\gamma^3 c^2\omega_d\omega_s^2} = 0 \qquad (12)$$

And,

$$\nabla^2 \vec{E}_s + \frac{\omega_s^2}{c^2}\left(1 - \frac{\omega_p^2}{\gamma\omega_s^2}\right)\vec{E}_s + \frac{ie^2\omega_p^2 k^2 |\vec{E}_d\vec{E}_d^*|\vec{E}_s}{16\Gamma_e m_0^2\gamma^3 c^2\omega_s\omega_d^2} = 0 \qquad (13)$$

Where:

$$\vec{E}_d = \vec{E}_{d0}\exp(i\{\omega_d t - k_d[z + S_d]\}) \qquad (14)$$
$$\vec{E}_s = \vec{E}_{s0}\exp(i\{\omega_s t - k_s[z' + S_s]\}) \qquad (15)$$

The symbol Γ_e represents the damping factor arising from Landau damping, collisional damping, and other relevant factors. The remaining symbols in the equation retain their conventional meanings.

By substituting the expressions for $\vec{E}_d$ and $\vec{E}_s$ obtained from Equations (14) and (15) into Equation (12) and subsequently separating the real and imaginary components, one arrives at a set of interdependent equations governing the behavior of the depleted pump laser beam. For example, the equation for the real part is as follows:

$$2\frac{\partial S_d}{\partial z} + \left(\frac{\partial S_d}{\partial r}\right)^2 = \frac{\omega_d^2\varepsilon_d}{c^2 k_d^2} + \frac{1}{k_d^2 E_{d0}}\left(\frac{\partial^2 E_{d0}}{\partial r^2} + \frac{1}{r}\frac{\partial E_{d0}}{\partial r}\right) \qquad (16)$$

Where:

$$E_{d00}^2(r,z) = \frac{E_{d20}^2}{f_d^2(z)}\left[1 + \frac{r^2}{qr_0^2 f_d^2(z)}\right]^{-q} e^{-2l_d z} \qquad (17)$$

$$S_{d0} = \frac{1}{2}r^2\frac{1}{f_d}\frac{df_d}{dz} + \Phi_d(z) \qquad (18)$$

$$E_{s00}^2(r,z) = \frac{E_{s20}^2}{f_s^2(z)}\left[1 + \frac{r^2}{qr_0^2 f_s^2(z)}\right]^{-q} e^{-2l_s z'} \qquad (19)$$

By substituting Equations 8 and 9 into Equation 10, one gets the following equation:

$$\frac{d^2 f_d}{d\xi^2} = \frac{q^2}{k^2 f_d^3(r^2/f_d^2 + qr_0^2)^2} + \frac{\alpha E_{d20}^2\omega_p^2}{c^2 f_d^3 k^2 r_0^2} e^{-2l_d z} \qquad (20)$$

Additionally, the variation of E_{s0} is presumed to be:

$$E_{s0} = E_{s00}\exp\{-ik_s S_s\} \qquad (21)$$

In a similar way to get Equation 21 and assuming that:

$$S_{s0} = \frac{1}{2}r^2\frac{1}{g}\frac{dg}{dz} + \Phi_s(z') \qquad (22)$$

And, by separating into the real part and imaginary part, one gets the real part as:

$$2\frac{\partial S_s}{\partial z'} + \left(\frac{\partial S_s}{\partial r}\right)^2 = \frac{\omega_s^2 \varepsilon_s}{c^2 k_s^2} + \frac{1}{k_s^2 E_{s0}}\left(\frac{\partial^2 E_{s0}}{\partial r^2} + \frac{1}{r}\frac{\partial E_{s0}}{\partial r}\right) \qquad (23)$$

Substituting Equations 19, 21, and 22 into Equation 23 and equating the coefficients of the variable r^2 on the two equation sides, one gets the subsequent equation:

$$\frac{d^2 g}{d\xi'^2} = \frac{q^2}{k^2 f_s^3 (r^2/f_s^2 + qr_0^2)^2} + \frac{\alpha E_{s20}^2 \omega_p^2}{c^2 f_s^3 k^2 r_0^2}e^{-2l_s z'} \qquad (24)$$

l_s is the power of scatter wave, where [18]:

$$l_s = \frac{-e^2 \omega_p^2 k^2 |E_{d0}|^2_{r=0}}{32m_0^2 \gamma_0^3 \Gamma_e c \omega_s^2 \omega_d^2 \varepsilon_s^{\frac{1}{2}}} \qquad (25)$$

E_{d0} is a complex function, where,

$$E_{d0} = E_{d00}exp\{-ik_d S_d\}. \qquad (26)$$

Substituting Equation (26) into Equation (25) gives:

$$l_s = -G_{\text{srs}}\frac{\alpha_d' E_{d0}^2}{f_0^2 \gamma_0^3}e^{-2g_s z} \qquad (27)$$

G_{srs} is the growth rate of stimulated Raman scattering, where [17]:

$$G_{\text{srs}} = \frac{\omega_p^2 k^2 c}{32\Gamma_e \omega_s \varepsilon_s^{1/2}} \qquad (28)$$

And, $z' = L - z$ is the interaction length. A seed beam intensity of 2×10^{-5} times the ultimate pump intensity has been selected. And the findings are in graphical format, as depicted in Figures 3 and 4. Figure 3 illustrates the changes in the beamwidth parameter of the scattered wave g as a function of the normalized propagation distance ξ for varying plasma densities ω_{p0}^2/ω^2 (0.02, 0.03, 0.04, and 0.05). The results indicate that a rise in plasma density is associated with a decrease in the scatter wave's beamwidth parameter, as evidenced by Figure 3. This phenomenon is because the self-focusing of the laser beam is directly proportional to the plasma density.
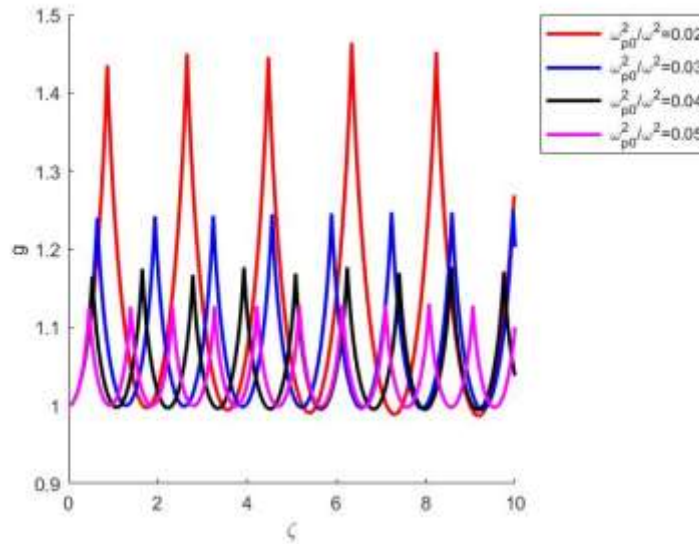
Figure 3: Variation of stimulated Raman scattering pump beam backscattered laser beamwidth parameters g with propagation distance ξ for parameters: $\alpha A_{00}^2 = 0.5$, $\omega_{p0}^2/\omega^2 = 0.02, 0.03, 0.04, 0.05$, $and\ q = 10$.

Figure 4 manifests the scattered wave's beamwidth parameter fluctuation about the normalized propagation distance ξ, with the varying values of q (1, 2, 3, 4, 5, 6, and 10). This phenomenon can be attributed to the high sensitivity of the scatter wave to the degree of self-focusing exhibited by the laser beam.
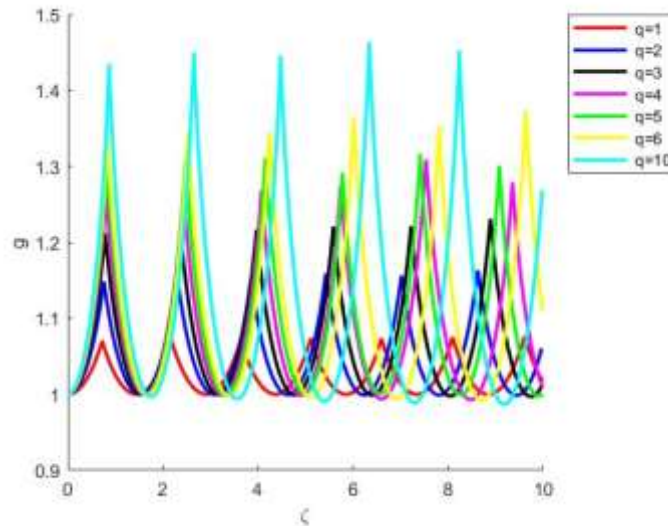

Figure 4: Variation of stimulated Raman scattering pump beam backscattered laser beamwidth parameters g with propagation distance ξ for parameters: $\alpha A_{00}^2 = 0.5$, $\omega_{p0}^2/\omega^2 = 0.02$, $and\ q = 1, 2, 3, 4, 5, 6\ and\ 10$.

The SRS yield is influenced by the peak intensity of the pump laser beam, as depicted in Figures. 5. This effect can be attributed to the laser beam's increased self-focusing at higher intensities. This figure shows that an increase in the laser beam's intensity and plasma density results in a corresponding increase in the SRS yield. At higher intensities, the laser beam exhibits greater self-focusing, which can be credited to increased SRS yield.
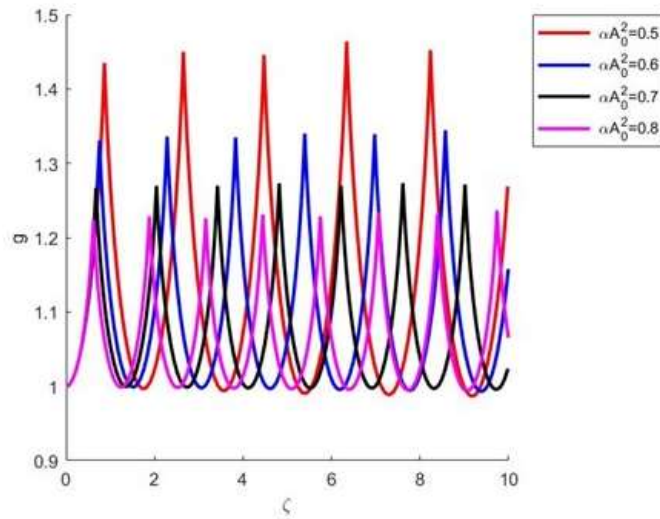
Figure 5: Variation of stimulated Raman scattering pump beam backscattered laser beamwidth parameters g with propagation distance ξ for parameters: $\alpha A_{00}^2 = 0.5, 0.6, 0.7, 0.8$, $\omega_{p0}^2/\omega^2 = 0.02$, and $q = 10$.

The integrated reflectivity R is defined as the scattering power P_s divided by the pump power P_0 and giving by:

$$R_{z=0} = \frac{E_{s20}^2}{A_{00}^2} \frac{r_{s0}}{r_0} e^{-l_s z'} \qquad (29)$$

Figure 6 shows the relationship between the integrated reflection and the growth rate in the SRS, where the value of the reflectivity increases with the increase in the value of q and also with the increase in the value of the growth rate for the SRS since the value of the growth rate depends on the parameter of the bandwidth of the pump wave and the scattering wave for these cases.
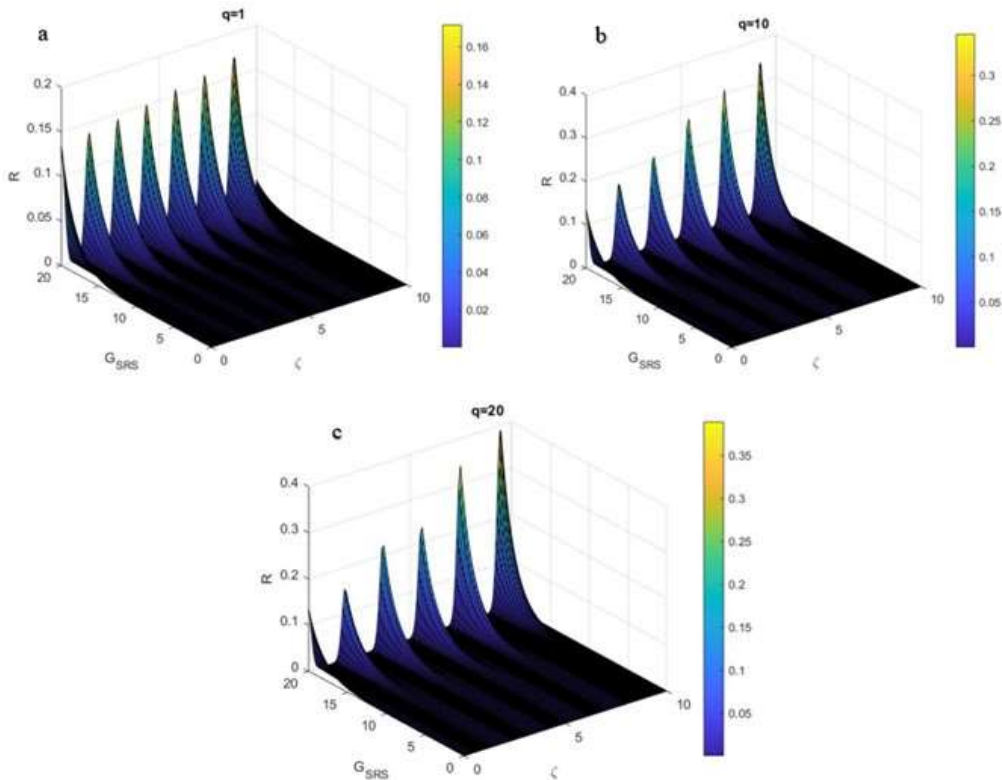

Figure 6: Evolution of integrated reflectivity with growth rate for a) q=1 , b) q=10 and c) q=20 for parameter $\alpha A_{00}^2 = 0.5$, $\omega_{p0}^2/\omega^2 = 0.02$.

4. Conclusions

In summary, the present study has examined the occurrence of SRS when a q-Gaussian laser beam propagates through an unmagnetized plasma. And it can be inferred that the scattered beam's relativistic self-focusing decreases substantially as the laser beam's amplitude structure approaches the optimal Gaussian profile. Also, the self-focusing of the q-Gaussian laser beam portrays a decreasing trend, while its nonlinear absorption demonstrates an increasing trend as q is increased. Additionally, the scattering of the self-focused beam takes place at a greater distance than the beam of the pump, due to the relatively diminished level of scattered power. As q increases, the SRS yield increases, and as the intensity of the laser beam and plasma density increase, the SRS yield also increases. Also, the value of the integrated reflection is affected by increasing the value of q, as its value increases with the increase of q and the growth rate.

Conflict of Interest

The authors declare that they have no conflict of interest.

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