# Passivity-Based Tracking Control of a Mobile Manipulator Robot 

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#### Abstract

This work presents a control approach based on the passivity principle, developed to guarantee the performance of the application used to track the trajectory of the mobile manipulator when it is disturbed. By exploiting the particularity of modelling mobile manipulator robots equipped with a nonholonomic mobile base, we present a global control law for the mobile manipulator as a single system. This control allows the whole system to be taken into account and its dynamics to be modified by introducing a highly non-linear regression matrix to take account of uncertainties and modelling constraints, thereby guaranteeing position tracking and ensuring that asymptotic errors tend towards zero and accommodating system operation in the presence of external disorders. The control process is very well implemented via the rules for updating the vector of uncertain parameters, thus ensuring compensation for the effect of disturbances. Next, a control scheme is proposed for the control of a mobile manipulator robot consisting of a manipulator arm with two Degrees of Freedom (2DDL) mounted on a mobile unicycle platform. Simulation tests validate the performance of the proposed approach when external disturbances occur; showing acceptable system stability performance and validated by exploitation of Lyapunov theory.


Keywords: Robotic mobile manipulator, passive control, stability, high accuracy

## 1. Introduction

As technology advances, more and more robots will take the place of human work in many fields. We are now talking about exploration and intervention robots, military robots, medical, assistant or domestic robots, and an infinite number of robotic systems are emerging. These applications require the system to have a high capacity, ease of control, processing, and a sophisticated hierarchical structure to achieve its autonomy. This requires modifying our conventional design techniques, both at the processing and reasoning levels, control and execution levels, to give robots a constantly evolving degree of autonomy. The mobile manipulator includes a robotic manipulator installed on a movable platform with wheels. It combines the agile handling ability provided by the operator's arm with the mobility offered by the mobile platform, allowing it to perform all tasks primarily related to manipulation [1, 2]. The realization of these tasks requires the design of high-performance controllers, allowing them to overcome theoretical and practical challenges, which is the focus of the researchers [2-7]. Indeed, the systems are typical examples of under-actuated mechanical systems with holonomic and nonholonomic constraints [8]. In general, robot control techniques are decoupled into two steps: planning and trajectory tracking. This strategy was developed to reduce the difficulties caused by the complexity of the dynamics of complex manipulators [9]. In the paper [10], the authors presented a control structure based on a preselected configuration (mobility index) for a nonholonomic platform with wheels. The mobile base is controlled in such a way as to place the arm's terminal member in the desired position. Its mobility depends on information about the measured joint position of the robotic arm [11]. The same authors reported that
regard mobile manipulator systems to be two distinct sub-systems facilitates monitoring and predict issues [12]. Moreover, a much better efficient movement control could be obtained through combination of the mobility of the mobile base and the arm manipulation. The paper [13] exposes a trajectory planning strategy based on two completely independent controllers. One of which controls the mobile base and the other the manipulator. The two controllers communicate via an algorithm that ensures cooperative movement between the mobile base and the manipulator's arm. More recently, the whole-body control strategy (manipulator arm-mobile platform) has been the subject of several publications. Silva and Adorno [14] have developed a control that provides trajectory generation for the effector. The input signals for the nonholonomic mobile base and the manipulator's arm are generated using the pseudo-inverse of the Jacobian matrix of the whole system. Wang and Yu [15] developed a position controller for simple rigid bodies based on feedback linearization and dual quaternion algebra. Kussaba et al. [16] presented a hybrid control structure based on unitary double quaternions to develop a control law that guarantees the global asymptotic stability of the closed-loop system. The study presented in this paper aims to develop a control approach based on the passivity principle to guarantee a certain level of performance and, more precisely, during a trajectory tracking application for a mobile manipulator robot in an unknown environment. By exploiting the particularity of the modelling of mobile manipulator robots equipped with a nonholonomic mobile base, we have developed a control law that allows us to consider the whole system by modifying its dynamics through the introduction of a highly nonlinear regressive matrix in order to take in to account several constraints and modelling uncertainties. The Lyapunov theory proves the stability of the mobile manipulator robot. Our study is justified by à comparison between two types of control techniques of a mobile manipulator for trajectory tracking, namely the proportional-integral-derivative (PID) controller and a passivitybased control. The rest of this paper is organized as follows. Section two is devoted to describing the mobile manipulator system and its dynamic modelling. Section three is devoted to presenting the control approach applied to the system. Moreover, we conclude with the presentation of the results of the simulation performed.

## 2. Description of The Mobile Manipulator Robot

In this section, an atypical nonholonomic mobile manipulator robot is presented. The system consists of a mobile unicycle-type base and a two-degree-of-freedom manipulator mounted on the center of the robot base. The base is a platform with two driving wheels mounted on the same axis, as shown in Fig. 1 [17].


Fig. 1 - Mobile manipulator

### 2.1 Dynamic Modelling of the Mobile Manipulator

The dynamic model is necessary for the simulation; the analysis is of the robot motion and the design of the control algorithm varieties. Several formalisms, such as the Euler-Lagrange formalism, the Newton-Euler formalism, and the D'Alembert principle, allow the dynamic modeling of the mobile manipulator robot. The latter gives the vector of torques. First, the operational coordinates of each joint of the manipulator robot are calculated concerning the fixed reference ( $\mathrm{o}, \mathrm{x}, \mathrm{y}, \mathrm{z}$ ), including the coordinates of the mobile platform. The robot's kinetic and potential energy are calculated to calculate/derive the constrained Lagrange equation, which gives the dynamic model of the system. The coordinates of the two driving wheels are:

$$
\left\{\begin{array}{l}
x_{r}=x+D \sin (\theta)  \tag{1}\\
y_{r}=y-D \cos (\theta) \\
x_{l}=x-D \sin (\theta) \\
y_{l}=y+D \cos (\theta)
\end{array}\right.
$$

By derivation of equation (1), the corresponding speed for each wheel is as follows:

$$
\left\{\begin{array}{l}
\dot{x}_{r}=\dot{x}+D \dot{\theta} \cos (\theta)  \tag{2}\\
\dot{y}_{r}=\dot{y}+D \dot{\theta} \sin (\theta) \\
\dot{x}_{l}=\dot{x}-D \dot{\theta} \cos (\theta) \\
\dot{y}_{l}=\dot{y}-D \dot{\theta} \sin (\theta)
\end{array}\right.
$$

The coordinates of the first segment:

$$
\left\{\begin{array}{l}
x_{1}=x  \tag{3}\\
y_{1}=y \\
z_{1}=0
\end{array}\right.
$$

The coordinates of the second segment:

$$
\left\{\begin{array}{c}
x_{2}=x_{1}-l_{2} \sin \left(\theta_{2}\right) \cos \left(\theta+\theta_{1}\right)  \tag{4}\\
y_{2}=y_{1}-l_{2} \sin \left(\theta_{2}\right) \sin \left(\theta+\theta_{1}\right) \\
z_{2}=2 l_{1}-l_{2} \cos \left(\theta_{2}\right)
\end{array}\right.
$$

By derivation of (3) and (4), the corresponding velocities for each segment of the manipulator arm are as follows:

$$
\left\{\begin{array}{c}
\dot{x}_{1}=\dot{x}  \tag{5}\\
\dot{y}_{1}=\dot{y} \\
\dot{x}_{2}=\dot{x}_{1}-l_{2} \dot{\theta}_{2} \cos \left(\theta_{2}\right) \cos \left(\theta+\theta_{1}\right)+l_{2}\left(\dot{\theta}+\dot{\theta}_{1}\right) \sin \left(\theta_{2}\right) \sin \left(\theta+\theta_{1}\right) \\
\dot{y}_{2}=\dot{y}_{1}-l_{2} \dot{\theta}_{2} \cos \left(\theta_{2}\right) \sin \left(\theta+\theta_{1}\right)-l_{2}\left(\dot{\theta}+\dot{\theta}_{1}\right) \sin \left(\theta_{2}\right) \cos \left(\theta+\theta_{1}\right) \\
\dot{z}_{2}=l_{2} \dot{\theta}_{2} \sin \left(\theta_{2}\right)
\end{array}\right.
$$

The motion of the mobile robot is subject to nonholonomic constraints given by:

$$
\begin{equation*}
A(q) \dot{q}=0 \tag{6}
\end{equation*}
$$

With $A(q)=\left[\begin{array}{lllll}\sin (\theta) & -\cos (\theta) & 0 & 0 & 0\end{array}\right]$ and $q$ : the generalized coordinate vector. Or $S(q)$ which satisfies:

$$
\begin{align*}
& S^{T}(q) A^{T}(q)=0  \tag{7}\\
& S(q)=\left[\begin{array}{cccc}
\frac{r}{2} \cos (\theta) & \frac{r}{2} \cos (\theta) & 0 & 0 \\
\frac{r}{2} \sin (\theta) & \frac{r}{2} \sin (\theta) & 0 & 0 \\
\frac{r}{2 D} & \frac{-r}{2 D} & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right] \tag{8}
\end{align*}
$$

We can find the input velocity vector $v=\left[\dot{\theta}_{r} \dot{\theta}_{l} \dot{\theta}_{1} \dot{\theta}_{2}\right]^{T}$, for all given $q$ in the following equation:

$$
\begin{equation*}
\dot{q}=S(q) v(t) \tag{9}
\end{equation*}
$$

The total kinetic energy is the sum of the kinetic energies of each subsystem (manipulator arm and the mobile platform):

$$
\begin{equation*}
E_{C}=\frac{1}{2} m_{p}\left(\dot{x}^{2}+\dot{y}^{2}\right)+\frac{1}{2} I_{p} \dot{\theta}^{2}+\frac{1}{2} m_{1}\left(\dot{x}_{1}^{2}+\dot{y}_{1}^{2}\right)+\frac{1}{2} I_{1} \dot{\theta}_{1}^{2}+\frac{1}{2} m_{2}\left(\dot{x}_{2}^{2}+\dot{y}_{2}^{2}\right)+\frac{1}{2} I_{2}\left(\dot{\theta}_{1}^{2}+\dot{\theta}_{2}^{2}\right) \tag{10}
\end{equation*}
$$

The total potential energy of the system, taking into account that the potential energy of the moving platform is zero, is:

$$
\begin{equation*}
E_{P}=\frac{l_{1}}{2} m_{1} g \sin \left(\theta_{1}\right)+m_{2} g\left(l_{1}+\frac{l_{2}}{2} \sin \left(\theta_{2}\right)\right) \tag{11}
\end{equation*}
$$

The equation of motion of the mobile manipulator is determined using Lagrange's method which is based on the differentiation of energy terms. Taking into consideration the kinematic constraint of the mobile platform (6), we write;

$$
\begin{equation*}
\frac{d}{d t}\left(\frac{d L}{d \dot{q}}\right)-\frac{d L}{d q}=\tau-A(q) \lambda \tag{12}
\end{equation*}
$$

With $L=E_{C}-E_{P}, \tau=\left[\tau_{r} \tau_{l} \tau_{1} \tau_{2}\right]^{T}$ :the torque vector driving the mobile manipulator robot. $\lambda$ : Lagrange multiplier. $q=\left[\theta_{r}, \theta_{l}, \theta_{1}, \theta_{2}\right]:$ Generalized coordinate vector.
The equation of motion of the moving manipulator obtained by the Lagrangian approach as follows:

$$
\begin{equation*}
M(q) \ddot{q}+C(q, \dot{q}) \dot{q}+G(q)=Q \tag{13}
\end{equation*}
$$

Where $Q=B \tau-A^{T}(q) \lambda$
$M(q) \in R^{4 x 4}$ : Inertia matrix, $C(q, \dot{q}) \in R^{4 x 4}$ : Centrifugal and Coriolis force matrix, $G(q) \in R^{1 x 4}$ : The vector of gravitational forces, $\mathrm{Q} \in R^{1 x 4}$ the vector of applied torques and $B=I_{4 x 4}$ is the input matrix.

## 3. Control Design

The time-varying path follows the given control problem and its successive derivatives or describes the desired velocity and acceleration, respectively. To achieve good performance, significant efforts have been devoted to the development of model-based control strategies [23]. Among the control approaches developed, we cite, typical methods include the inverse dynamic control, the feedback linearization technique, and the passivity-based control method.

### 3.1 Linear Control

The PID control consists of three basic actions, namely, proportional, integral and derivative, which represent the current, past and expected future error that covers the whole-time history, the error signal. The appropriate gains are: $k_{p}, k_{i}$ and $k_{d}$. By adjusting these parameters, the system performance and stability (13) can be achieved. The PID controller is given by the continuous time expression:

$$
\begin{equation*}
Q=k_{p} e_{q}+k_{i} \int e_{q} d t+k_{d} \dot{e}_{q} \tag{14}
\end{equation*}
$$

Where: $e_{q}=q_{d}-q$, $\mathrm{e}(\mathrm{t})$, or $e_{q}(t)$ : the position error in radians.
$q_{d}$ : The desired position vector in radian and Q the vector of applied torques.

### 3.2 Passivity-Based Motion Control

While the PID controller is suitable for set point control problems, many tasks require high-precision path following capabilities. Examples of such tasks include plasma welding, laser cutting, or high-speed operations in the presence of obstacles. In these cases, local models require slow traversal through several intermediate set points, which significantly delays task completion. Therefore, to improve the trajectory tracking performance, the controllers must consider the dynamic model of the mobile manipulator via a purely adaptive control law. Assuming the mobile manipulators movements (13) are already saved devoid of the exterior disruptions and that all the velocities of the robot joints are available for feedback, there is room of taking the passivity benefit (properties, which follow) of the rigid robot to derive a command that present many benefits compared to linearization-based algorithms [18-20]. The controller conception planned on the passivity involves two fundamental points. The first point is energy shaping, conceived to make the closed loop system match the required energy function. The second point includes the injection of a damping term, to ensure the stability of the closed-loop system even in the presence of external disturbances. At this phase, the robot system passivity characteristics may be conserved in the closed-circuit system.

Admitting that the equation (13) of the robot's dynamics is known, the passive-based controller of Slotine [21] may be used, where we consider the following dynamic equation:

$$
\begin{equation*}
M\left(q_{r}\right) \ddot{q}_{r}+C\left(q_{r}, \dot{q}_{r}\right) \dot{q}_{r}+G\left(q_{r}\right)+u=Q \tag{15}
\end{equation*}
$$

Considering (7), (8), and replacing (9) and its derivative in (13), the dynamic model can be written as:

$$
\begin{equation*}
D(q) \ddot{v}+F(q, \dot{q}) \dot{v}+N(q)=S^{T} B \tau+u \tag{16}
\end{equation*}
$$

Where $D(q)=S^{T} M S, F(q)=S^{T}(M \dot{S}+C S)$ and $N(q)=S^{T} G, \bar{Q}=S^{T} B \tau$, Knowing that $S^{T} B=I$.
With: $q_{r}=\dot{q}_{d}-\gamma e_{q}, q_{d}$ is the desired trajectory of the robot, $e_{q}=q-q_{d}$, the error vector. $\gamma \in R^{4 x 4} \mathrm{~A}$ diagonal matrix of positive gains; $u=-k_{d 1} . s$ represents a new control input.

Models (13) and (16) have important properties which will be exploited for the calculation of the controller, the most relevant of which are recalled below.

Property 1: The matrix D is symmetric positive definite.
Property 2: The matrix $(\dot{D}-2 F)$, is skew symmetric, i.e. for several vector x , there is:

$$
\begin{equation*}
x^{T}(\dot{D}-2 F) x=0 \tag{17}
\end{equation*}
$$

Property 3 [22]: The system (13) is passive of the simulated monitoring feed $\boldsymbol{\tau}$ to $\boldsymbol{q}$ so:

$$
\begin{equation*}
\langle q ; \quad \tau\rangle=\int_{0}^{\mathrm{t}} \dot{\mathrm{q}}^{\mathrm{T}}(\mathrm{~s}) \tau(\mathrm{s}) \mathrm{ds} \geq-\beta \tag{18}
\end{equation*}
$$

With: $\beta=H_{0}(\boldsymbol{q}(0))>0$, and $H_{0}$ represents all energy of the open-loop system (13),

$$
\begin{equation*}
H_{0}(q)=\frac{1}{2} \dot{q}^{T} M \dot{q} \tag{19}
\end{equation*}
$$

By combining (13) and (16), the dynamic equation error for the closed loop maybe as:

$$
\begin{align*}
& D \dot{s}+\left(F+k_{d 1}\right) s=0  \tag{20}\\
& s=\dot{e_{q}}+\gamma e_{q} \tag{21}
\end{align*}
$$

s: A new variable which is the linear combination of the position error and the speed error. The closed-loop system (20) is strict passive from $\boldsymbol{u}$ to $\boldsymbol{s}$. Thus, the passivity of the robotic system is maintained in the closed- loop structure. Hence the minimum energy of the open-loop system (13), so that: $(\mathrm{q}, \dot{q})=(0,0)$, has been translated to $\left(e_{q}, \dot{e_{q}}\right)=(0,0)$ by the regulator (16). In fact, the controller (16) reforms the functioning energy (19) of the open-loop system (13) into:

$$
\begin{equation*}
H_{1}\left(e_{q}, \dot{e_{q}}\right)=\frac{1}{2} s^{T} M s \tag{22}
\end{equation*}
$$

The equation (22) is the system's operational energy (20). The phase where the reforming energy is complete. To obtain a stable path-following control, we need to add the damping:

$$
\begin{equation*}
u=-k_{d 1} \cdot s \tag{23}
\end{equation*}
$$

Where, $\boldsymbol{k}_{\boldsymbol{d} \mathbf{1}} \in \mathrm{R}^{4 \times 4}$ : diagonal positive definite matrix. $\boldsymbol{k}_{\boldsymbol{d} \mathbf{1}} . \boldsymbol{S}$ : damping injection. It is the idea of energy reforming and damping of the control approach based on the passivity principle [21].
The Lyapunov candidate function

$$
\begin{equation*}
V=\frac{1}{2} s^{T} D s+e_{q}^{T} \gamma k_{d 1} \dot{e}_{q} \tag{24}
\end{equation*}
$$

Its derivative with respect to time along the trajectory (20) is:

$$
\begin{align*}
& \dot{V}=\frac{1}{2} s^{T} \dot{D} s+s^{T} D \dot{s}+2 e_{q}^{T} \gamma k_{d 1} \dot{e}_{q}  \tag{25}\\
& =\frac{1}{2} s^{T} \dot{D} s-s^{T}(u+F s)+2 e_{q}^{T} \gamma k_{d 1} \dot{e}_{q}  \tag{26}\\
& =\frac{1}{2} s^{T}(\dot{D}-2 F) s-s^{T} k_{d 1} s+2 e_{q}^{T} \gamma k_{d 1} \dot{e}_{q} \tag{27}
\end{align*}
$$

Where $s^{T}(\dot{D}-2 F) s=0$,

$$
\begin{equation*}
\dot{V}=-s^{T} k_{d 1} s+2 e_{q}^{T} \gamma k_{d 1} \dot{e}_{q} \tag{28}
\end{equation*}
$$

By properly choosing $\gamma,(24)$ and (26) establish the closed-loop stability of the system in the Lyapunov sense and the boundedness of the state $q_{r}$ and $\mathrm{e}_{\mathrm{q}}$. Moreover, from (21) and the boundedness of the error $\mathrm{e}_{\mathrm{q}}, \dot{e}_{\mathrm{q}}$ is also bounded. In other words what leads to the conclusion that when $\mathrm{s} \rightarrow 0$ has $\mathrm{t} \rightarrow \infty$, it ends to e and $\dot{e}_{\mathrm{q}} \rightarrow 0$ when $\mathrm{t} \rightarrow \infty$. Except, the operation of the system can be exposed to external disturbances $\left(\tau_{p}\right)$, we introduce adaptive control schemes to reduce these effects. Describing their estimates by $\left(\hat{\tau}_{p}\right)$, the dynamic equation error for the closed loop (20) becomes:

$$
\begin{equation*}
D \dot{s}+\left(F+k_{d 1}\right) s+B \tilde{\tau}_{p}=0 \tag{29}
\end{equation*}
$$

With $\tilde{\tau}_{p}=\hat{\tau}_{p}-\tau_{p}$.
The disturbance estimation law is of the form:

$$
\begin{equation*}
\dot{\hat{\tau}}_{p}=\delta^{-T} B^{T} S \tag{30}
\end{equation*}
$$

To demonstrate closed-loop stability, consider the following Lyapunov function:

$$
\begin{equation*}
V=\frac{1}{2} s^{T} D s+\frac{1}{2} \tilde{\tau}_{p}^{T} \delta \tilde{\tau}_{p}+e_{q}^{T} \gamma k_{d 1} \dot{e}_{q} \tag{31}
\end{equation*}
$$

Its time derivative along the trajectory (29) is:

$$
\begin{align*}
& \dot{V}=\frac{1}{2} s^{T} \dot{D} s+s^{T} D \dot{s}+\dot{\tilde{\tau}}_{p}^{T} \delta \tilde{\tau}_{p}+2 e_{q}^{T} \gamma k_{d 1} \dot{e}_{q}  \tag{32}\\
& =\frac{1}{2} s^{T} \dot{D} s+s^{T}\left(-\left(F+k_{d 1}\right) s-B \tilde{\tau}_{p}\right)+\dot{\tau}_{p}^{T} \delta \tilde{\tau}_{p}+2 e_{q}^{T} \gamma k_{d 1} \dot{e}_{q}  \tag{33}\\
& =\frac{1}{2} s^{T} \dot{D} s+s^{T}\left(-\left(F+k_{d 1}\right) s-B \tilde{\tau}_{p}\right)+\dot{\tilde{\tau}}_{p}^{T} \delta \tilde{\tau}_{p}+2 e_{q}^{T} \gamma k_{d 1} \dot{e}_{q}  \tag{34}\\
& =s^{T}(\dot{D}-2 F) s-s^{T} k_{d} s-s^{T} B \tilde{\tau}_{p}+\dot{\tilde{\tau}}_{p}^{T} \delta \tilde{\tau}_{p}+2 e_{q}^{T} \gamma k_{d 1} \dot{e}_{q} \tag{35}
\end{align*}
$$

Based on equation (17), equation (35) becomes,

$$
\begin{equation*}
\dot{V}=-s^{T} k_{d} s+\left(-s^{T} B+\dot{\tilde{\tau}}_{p}^{T} \delta\right) \tilde{\tau}_{p}+2 e_{q}^{T} \gamma k_{d 1} \dot{e}_{q} \tag{36}
\end{equation*}
$$

Based on (21) and (30), we have:

$$
\begin{equation*}
\dot{V}=-s^{T} k_{d 1} s+2 e_{q}^{T} \gamma k_{d 1} \dot{e}_{q} \tag{37}
\end{equation*}
$$

From equation (37), properly choosing $\gamma, \dot{V}(q)$ is negative, this proves the closed loop stability in the Lyapunov sense. Moreover, from the definition $s=\dot{e_{q}}+\gamma e_{q}$ and the boundedness of the error $e_{q}, \dot{e_{q}}$ is also bounded. In other words what allows us to reach the conclusion that when $\mathrm{s} \rightarrow 0$ has $\mathrm{t} \rightarrow \infty$, it results e and $\dot{\dot{e}_{q}} \rightarrow 0$ when $\mathrm{t} \rightarrow \infty$.

### 3.3 Dealing with Uncertainties

Referring to the work on passivity in [21], [24], we are introducing an adaptive control system based in passivity. Admitting the inertia of the matrix M, Coriolis matrix C, and gravity G contain uncertainties. According to [24], and based on (13), (7), (8), (9) and (16), the dynamic model (13) can be indicated by a following relationship:

$$
\begin{equation*}
\mathrm{D}(q) \ddot{v}+F(q, \dot{q}) \dot{v}+N(q)=Y(\ddot{v}, \dot{v}, \dot{q}, q) \alpha \tag{38}
\end{equation*}
$$

The robot's physical setting is unknown and should be evaluated. Note by $\alpha$ the vector of parametric uncertainties, where $\hat{\alpha}$, is its estimate. The dynamic equation of the system is as follows:

$$
\begin{equation*}
\mathrm{D}(q) \ddot{v}+F(q, \dot{q}) \dot{v}+N(q)=Y(\ddot{v}, \dot{v}, \dot{q}, q) \hat{\alpha} \tag{39}
\end{equation*}
$$

The notation $Y$ known as the regressor is a function known from generalized co-ordinates, and $\alpha$ is a vector of uncertain parameters. In other words, the uncertainties of $M, C$, and $G$ are summarized in those of $\alpha$, and (39) is rewritten as follows:

$$
\begin{equation*}
S^{T} B \tau=Y \hat{\alpha}-k_{d 1} q_{r} \tag{40}
\end{equation*}
$$

Substituting (39) and (40) into (13), we obtain:

$$
\begin{equation*}
D \dot{s}+F s+k_{d} s=Y(\hat{\alpha}-\alpha) \tag{41}
\end{equation*}
$$

The estimation of the vector of uncertainties $\hat{\alpha}$ is as follows:

$$
\begin{equation*}
\dot{\hat{\alpha}}=-\rho^{-1} Y^{T} S \tag{42}
\end{equation*}
$$

Consider the following Lyapunov function to demonstrate the stability of the system with uncertainties:

$$
\begin{equation*}
V=\frac{1}{2} s^{T} D s+e_{q}^{T} \gamma k_{d} \dot{e}_{q}+\frac{1}{2} \tilde{\alpha}^{T} \rho \tilde{\alpha} \tag{43}
\end{equation*}
$$

Where: $\tilde{\alpha}=\hat{\alpha}-\alpha$. Its time derivative along the trajectory of (41) is given by:

$$
\begin{gather*}
\dot{V}=\frac{1}{2} s^{T} \dot{D} s+s^{T} D \dot{s}+2 e_{q}^{T} \gamma k_{d 1} \dot{e}_{q}+s^{T} Y \tilde{\alpha}  \tag{44}\\
\dot{V}=s^{T}(\dot{D}-2 F) s-s^{T} k_{d} s-s^{T} Y \tilde{\alpha}+2 e_{q}^{T} \gamma k_{d} \dot{e}_{q} \tag{45}
\end{gather*}
$$

Meanwhile, the time derivative of $\frac{1}{2} \tilde{\alpha}^{T} \rho \tilde{\alpha}$ is:

$$
\begin{equation*}
\frac{d}{d t}\left(\frac{1}{2} \tilde{\alpha}^{T} \rho \tilde{\alpha}\right)=-\tilde{\alpha}^{T} Y^{T} S \tag{46}
\end{equation*}
$$

Where the equation (45) holds because $\alpha$ is invariable. Considering (17), and combining (45) and (46), we get:

$$
\begin{equation*}
\dot{V}=-s^{T} k_{d 1} s+2 e_{q}^{T} \gamma k_{d 1} \dot{e}_{q} \tag{47}
\end{equation*}
$$

From equation (47) and by properly choosing $\gamma, \dot{V}(q)$ is negative, we conclude the stability of the closed loop system in the Lyapunov sense. Moreover, from equation (21) and the boundedness of the error $e_{q}, \dot{e_{q}}$ is also bounded. In other words what allows us to obtain: when $\mathrm{s} \rightarrow 0$ has $\mathrm{t} \rightarrow \infty$, it allows e and $\dot{e_{q}} \rightarrow 0$ when $\mathrm{t} \rightarrow \infty$. The applied controller diagram is shown in figure bellow.


Fig. 2 - Diagram of applied control

## 4. Results of The Simulation

To test the performance of the presented study, simulations were performed under Matlab/Simulink software for a desired trajectory of the platform of the following form:

$$
\begin{equation*}
\theta(t)=\left(\frac{2 p i}{5}\right) t+5 \tag{48}
\end{equation*}
$$

And reference trajectories for the two segments of the manipulator arm as follows:

$$
\begin{align*}
& \theta_{1}(t)=1.3 \sin \left(t+\frac{p i}{6}\right)  \tag{49}\\
& \theta_{2}(\mathrm{t})=2\left(1-\cos \left(t+\frac{p i}{6}\right)\right) \tag{50}
\end{align*}
$$

The simulations were conducted in uncertain conditions to evaluate the ability of the proposed scheme. The uncertain conditions considered here consist of the effect of perturbations in the parameters and external perturbations applied (dist $\tau(\mathrm{t}))$ to the different components of the torque vector $\mathrm{Q}=\mathrm{f}\left(\tau+d i s t_{-} \tau\right)$ (torque of the platform wheels and arm joints), of the form:

$$
\text { dist_}_{-} \tau=\left[\begin{array}{l}
d_{1 s t} \tau_{1}  \tag{51}\\
\text { dist_ }_{1} \\
\text { dist_ }_{2} \tau_{l} \\
\text { dist_}_{-} \tau_{r}
\end{array}\right]=\left[\begin{array}{c}
2 \sin (3 t) \\
2 \cos (3 t) \\
5 \sin (1.5 t) \\
5 \cos (1.5 t)
\end{array}\right]
$$

The perturbations in the parameters are chosen as follows:

$$
\begin{align*}
& m_{p}=12+2 \sin \left(\frac{2 p i}{5} t\right)[k g], m_{1}=12+2 \sin \left(\frac{2 p i}{5} t\right)[k g], m_{2}=2+\sin \left(\frac{2 p i}{5} t\right)[k g]  \tag{52}\\
& I_{0}=1.2+0.2 \sin \left(\frac{2 p i}{5} t\right)[k g], I_{1}=I_{2}=0.12+0.02 \sin \left(\frac{2 p i}{5} t\right)[k g] \tag{53}
\end{align*}
$$

To validate the presented technique by comparing its performers to the known methods, such as, the Proportional Integral Derivative (PID). All the gain matrices of the controllers are defined so that all the monitors performing satisfactorily below conditions of no external disturbances (dist $\tau=0$ ). The parameters of the PID controller and the proposed controller are: $k_{p}=\operatorname{diag}\left(35 I_{4 x 4}\right), k_{i}=\operatorname{diag}\left(25 I_{4 x 4}\right), k_{d}=$ $\operatorname{diag}\left(5 I_{4 x 4}\right)$ and for the proposed controller $; \gamma=6 I_{1 \times 4}$ and $k_{d 1}=\operatorname{diag}\left(20 I_{4 x 4}\right)$.The simulation results are presented in the figures fig. 3 to fig. 6.Figures 3 and 4 illustrate the system's behavior in the absence of disturbances, respectively, in the case of the application of the classical PID controller and the case of the application of the proposed technique (adaptive passive). Figures (fig. 5 and fig. 6) illustrate the behavior of the system under disturbances at time $t$ $=10 \mathrm{~s}$, respectively, including the case of applying the conventional PID controller and the case of applying the proposed (passive adaptive) technique.


Fig. 3 - System behavior without disturbances, PID case


Fig. 4 - Behavior of the system without disturbances, passivity case


Fig. 5 - System behavior with disturbances ( $\mathbf{t}=10 \mathrm{sec}$ ), PID case


Fig. 6 - Behavior of the system with disturbances ( $\mathbf{t}=10 \mathrm{sec}$ ), passivity case


Fig. 7 - Torque curve at the beginning of the disturbances at $t=10 \mathrm{sec}$, PID case


Fig. 8 - Torque behavior at the beginning of the disturbance at $t=10 \mathrm{sec}$, passivity case
The results confirm the advantages of the presented control scheme. The curves show that the proposed controller provides higher accuracy in position tracking control than those provided by PID controllers. In addition, the proposed control technique can achieve a better dynamic performance of the systems. By observing figures (Fig. 3 and Fig.4), it can be verified that the proposed control law can provide a faster convergence rate of position tracking errors along the desired trajectories compared to PID controllers. By observing figures (Fig. 5and Fig. 6), it is quickly noticed that the proposed control method is more efficient for operating the mobile manipulator robot under external disturbances. Not only does it ensure the tracking of the position provide asymptotic errors tend to zero, but it also allows accommodation of the system's operation in the presence of external disturbances. In other words, we conclude that the tracking results of PID based control shows its limits and the mobile manipulator unsuccessfully tends to the referenced trajectory. This shows that PID controller is not robust against external disturbances (Fig. 7). However, the passivity-based adaptive control law, which can correct the effects of disturbances (Fig. 8), provides good outputs and achieves good trajectory tracking performance. Which, Fig. 6 clearly shows that the adaptive control process based on passivity in the presence of external disturbances and parametric uncertainties is very well realized via the rules of updating the vector of uncertain parameters, thus ensuring the compensation of the effect of disturbances (external and parametric uncertainties) allowing to reach a superior performance of tracking the trajectory. Fig. 8 shows the torques generated by the various actuators of the mobile manipulator.

## 5. Conclusion

This work presents a control approach based on the passivity principle, developed to guarantee performance of the application used to track the trajectory of mobile manipulator when disturbed. Although already known by several researchers, the passivity-based control used in our work has been adapted according to the context of the system. Since the mobile robot manipulator is a non-linear system and the internal parameters of the system are not precisely known, we had to use the adaptive passivity-based control that considers these perturbations. The added value of our work regarding the control of mobile manipulator robots by an adaptive control strategy based on passivity is that we have introduced a highly nonlinear regressive matrix to consider uncertainties and disturbances. The results presented show that the control law developed allows to drive the motion of the mobile manipulator robot to converge to the desired operation although in the presence of modeling uncertainties and disturbances on the torques. The stability of the system and the asymptotic convergence of the tracking errors are established using Lyapunov's theory.
As perspectives, we will consider applying neural intelligent methods to control the manipulator mobile. Neural methods establish significant analytical relationships for the control phase and have great flexibility because there is no limit to the number of input and output system parameters.

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