Mathematical Biosciences and Engineering

http://www.aimspress.com/journal/mbe

MBE, 20(11): 20274–20294.
DOI: 10.3934/mbe.2023897
Received: 06 August 2023
Revised: 17 October 2023
Accepted: 25 October 2023
Published: 07 November 2023

*Research article*

# Design of robust fuzzy iterative learning control for nonlinear batch processes

**Wei Zou**[1], **Yanxia Shen**[1,*] **and Lei Wang**[2]

[1] Engineering Research Center of Internet of Things Technology Applications, Ministry of Education, Jiangnan University, Wuxi 214122, China
[2] School of Automation, Wuxi University, Wuxi 214105, China

* **Correspondence:** Email: shenyx@jiangnan.edu.cn.

**Abstract:** In this paper, a two-dimensional (2D) composite fuzzy iterative learning control (ILC) scheme for nonlinear batch processes is proposed. By employing the local-sector nonlinearity method, the nonlinear batch process is represented by a 2D uncertain T-S fuzzy model with non-repetitive disturbances. Then, the feedback control is integrated with the ILC scheme to be investigated under the constructed model. Sufficient conditions for robust asymptotic stability and 2D $H_\infty$ performance requirements of the resulting closed-loop fuzzy system are established based on Lyapunov functions and some matrix transformation techniques. Furthermore, the corresponding controller gains can be derived from a set of linear matrix inequalities (LMIs). Finally, simulations on the three-tank system and the highly nonlinear continuous stirred tank reactor (CSTR) are carried out to prove the feasibility and efficiency of the proposed approach.

**Keywords:** fuzzy iterative learning control; nonlinear batch processes; uncertain T-S fuzzy model; robust asymptotic stability; 2D $H_\infty$ performance

## 1. Introduction

Due to their high efficiency and flexibility, batch processes play a significant role in modern industries. They are widely applied in specialty chemicals, polymers, pharmaceuticals and biochemicals, advanced alloys, modern agriculture and other fields and have received considerable attention [1, 2]. Batch processes are characterized by strong time-variance, high nonlinearity, an unstable operating point, and a uniquely repetitive nature [3]. These challenges inevitably make the control problem of the batch processes more difficult and complicated. Therefore, it is particularly necessary to develop modeling and control methods for batch processes.

Driven by the feature of repetitive operation in batch processes, iterative learning control (ILC) has been extensively researched by scholars. ILC has proven to be an excellent tool for achieving perfect tracking and control optimization by repeatedly learning from the knowledge of the previous iterations. With this method, the transient response and tracking performance of batch processes can be progressively improved. For constrained batch processes with state-dependent uncertainty, a conic ILC control law was implemented in [4] and a detailed convergence proof was provided. The authors in [5] investigated a point-to-point ILC to deal with the problem of unknown batch-varying initial state in batch processes. Based on system parameter information, Geng et al. [6] put forward the data-based ILC strategy for multiphase batch processes with different dimensions and system uncertainty to address the problem of the robustness conditions. However, these ILC algorithms, when taken as pure ILC schemes, are essentially the open-loop feed-forward control techniques from a separate batch perspective. The pure ILC schemes cannot achieve satisfactory control performance when the batch process is affected by uncertainties and real-time perturbations, since this control law only uses the information of previous batches lacks real-time feedback information [7]. New strategies should be developed to allow for the high precision control of batch processes.

Given this, by taking the inherent two-dimensional (2D) nature of batch processes under ILC into account, a combination of a real-time feedback control mechanism and pure ILC design based on 2D systems theory has been successfully applied. For batch processes with time-varying uncertainties and external disturbances, a generalized extended state observer based indirect-type ILC design was discussed in [8]. In order to achieve an improved control performance of batch processes under uncertainty, the authors in [9] explored a composite ILC strategy where the model's predictive control and ILC are integrated based on the 2D framework. In addition, Li et al. [10] proposed an iterative learning-based predictive algorithm for multiphase batch processes with asynchronous switching and complex characteristics; this 2D integrated control method ensured the system's asymptotic and exponential stability.

Among the aforementioned studies, most of the 2D composite ILC methods are formulated on linear models, which are not sufficient enough to describe a practical nonlinear system. Nevertheless, in modeling practical phenomena, batch processes are frequently in the form of complex nonlinear systems, which pose great difficulties in terms of system analysis and synthesis. Therefore, the Takagi-Sugeno (T-S) fuzzy model is introduced and has attracted considerable attention. In essence, T-S fuzzy models are characterized by a group of fuzzy IF-THEN rules and fuzzy sets to approximate any smooth nonlinear function with arbitrary accuracy [11]. More importantly, it is feasible to apply the 2D systems theory to study the controller design problems for nonlinear batch processes. Yu et al. [12] designed the 2D controller, where the fuzzy iterative learning controller is combined with the reliable guaranteed cost controller for nonlinear batch processes described by the 2D T-S fuzzy model. For nonlinear batch process with an actuator fault and according to the T-S fuzzy model, Luo et al. [13] presented a 2D fuzzy constrained iterative learning predictive fault-tolerant control strategy, which guarantees the steady operation and exponential convergence of the resulting closed-loop system. It is noteworthy that the available results are concerned with 2D fuzzy ILC for nonlinear batch processes without considering the influence of uncertainties. In fact, uncertainties and non-repetitive disturbances are always present in nonlinear batch processes based on T-S fuzzy models. To this end, Li et al. [14] proposed an approach that combines robust predictive fault-tolerant control and an ILC based on a 2D T-S fuzzy Roesser model for batch processes subject to uncertainties, disturbances and partial actuator faults. This scheme applied the robust positive invariant and the terminal constraint set to achieve satisfactory control effects. This paper utilizes the fuzzy Lyapunov theory and 2D $H_\infty$ methodology to ensure robust stability under non-repetitive disturbances and uncertainties.

This paper investigates a robust fuzzy ILC algorithm for nonlinear batch processes based on T-S fuzzy models. With the local-sector nonlinearity method, the nonlinear batch process is transformed into an uncertain 2D T-S fuzzy model with non-repetitive disturbances. Our goal is to develop an effective design strategy that ensures the closed-loop 2D T-S fuzzy system is asymptotically stable and has a $H_\infty$ performance.

The main contributions of this research work are highlighted as follows:

1) T-S fuzzy models have been exploited to represent the nonlinear batch processes.

2) Based on Lyapunov functions and some matrix transformation techniques, the results of asymptotic stability and a 2D $H_\infty$ performance analysis are obtained.

3) A systematic robust fuzzy iterative learning controller design method is derived in terms of a set of linear matrix inequality (LMI) constraints.

The present work is organized as follows. Section 2 gives a problem formulation and some preliminaries. A set of sufficient conditions and a 2D fuzzy ILC law are presented in Section 3. Two practical examples are given in Section 4, and a concise conclusion is drawn in Section 5.

Throughout this paper, all matrices are assumed to be of appropriate dimensions. The superscripts "$-1$", "$T$" and "$\perp$" represent inverse, transpose, and null space of a matrix, respectively. Additionally, the symbol 0 and $I$ refer to the null and identity matrices with compatible dimensions, respectively. For a matrix $X$, $X > 0$ ($X < 0$) means that $X$ is a real symmetric positive definite (negative definite) matrix, $sym(X)$ stands for $X + X^T$ and "$(*)$" indicates the symmetric elements. Finally, the symbol $diag\{X_1, X_2, ..., X_n\}$ indicates a block diagonal matrix with diagonal blocks $X_1, X_2, ..., X_n$. A 2D signal $w(i, j) \in \mathcal{L}_2$ represents that $w(i, j)$ is in the $\mathcal{L}_2$ space, which implies $\|w\|_2 = \sqrt{\sum_{i=0}^{\infty} \sum_{i=0}^{\infty} \|w(i, j)\|^2} < \infty$.

## 2. System description and problem statement

Consider a class of nonlinear continuous-time systems running repeatedly, described by the following state-space model:

$$\begin{cases} \dot{x}(t,k) = f\left[x(t,k), u(t,k)\right] \\ y(t,k) = g\left[x(t,k)\right], \quad 0 \le t \le T_d,\ k \ge 0 \end{cases} \tag{2.1}$$

where on the $k$th batch, $x(t,k) \in R^n$, $u(t,k) \in R^m$, and $y(t,k) \in R^l$ represent the state, input and output vectors, respectively, $T_d$ is the time period of a batch, and $f\left[x(t,k), u(t,k)\right]$ and $g\left[x(t,k)\right]$ denote nonlinear functions with proper dimensions.

The discrete linear model can be obtained by applying the local sector nonlinear method, and using an appropriate average sampling time. Therefore, the nonlinear plant (1) can be converted into a discrete uncertain 2D T-S fuzzy model, which is represented by IF-THEN rules as follows:

Plant Rule $i$: IF $\vartheta_1(t,k)$ is $\mathcal{M}_{i1}$ and $\vartheta_j(t,k)$ is $\mathcal{M}_{ij}, \ldots, \vartheta_p(t,k)$ is $\mathcal{M}_{ip}$, THEN

$$\begin{cases} x(t+1,k) = (A_i + \Delta A_i(t))x(t,k) + (B_i + \Delta B_i(t))u(t,k) + w(t,k) \\ y(t,k) = C_i x(t,k), \qquad i = 1, 2, \ldots, r, \quad j = 1, 2, \ldots, p \end{cases} \tag{2.2}$$

where $\vartheta(t,k) = \left[\vartheta_1(t,k), \vartheta_2(t,k), \ldots, \vartheta_p(t,k)\right]$ stands for a premise variable vector, $\mathcal{M}_{ij}(i = 1, 2, \ldots, r; j = 1, 2, \ldots, p)$ are the fuzzy sets, $r$ is the fuzzy rule number and $p$ is the number of premise variables. It is considered that $x(0,k) = x_0$ is the initial condition for each batch. $w(t,k)$ denotes the unknown disturbance, which is assumed to belong to $\mathcal{L}_2$ space. $\{A_i, B_i, C_i\}$ consists of system matrices with compatible dimensions. Moreover, the output matrices of all fuzzy subsystems are assumed to be common (i.e., $C_1 = C_2 = \cdots = C_r = C$). The common $C$ matrix condition can largely reduce the complexity of the design and computation, though, in theory, it may increase the level of conservativeness [15]. It is assumed that $CB_i \neq 0$. Besides, $\{\Delta A_i(t), \Delta B_i(t)\}$ represents the uncertainty terms in the following form:

$$[\Delta A_i(t) \quad \Delta B_i(t)\ ] = E\Delta(t)\, [F_{Ai} \quad F_{Bi}\ ], \quad \Delta(t)^T \Delta(t) \leq I \tag{2.3}$$

where $E$, $F_{Ai}$ and $F_{Bi}$ are known real constant matrices of appropriate dimensions and $\Delta(t)$ indicates the uncertain perturbation.

Let $\mu_i(\vartheta(t,k))$ represent the normalized fuzzy-basis function of the inferred fuzzy set $\mathcal{M}_i = \prod_{j=1}^{p} \mathcal{M}_{ij}$, which is defined as follows:

$$\mu_i(\vartheta(t,k)) = \frac{\prod_{j=1}^{p} \mathcal{M}_{ij}(\vartheta_j(t,k))}{\sum_{i=1}^{r} \prod_{j=1}^{p} \mathcal{M}_{ij}(\vartheta_j(t,k))} \geq 0, \quad \sum_{i=1}^{r} \mu_i(\vartheta(t,k)) = 1 \tag{2.4}$$

where $\mathcal{M}_{ij}(\vartheta_j(t,k))$ is regarded as the grade of membership of $\vartheta_j(t,k)$ in $\mathcal{M}_{ij}$. In the sequel, $\mu_i(\vartheta(t,k))$ is written by either $\mu_i$ or $\mu_i(t,k)(i = 1, 2, \ldots, r)$ for brevity.

As an application of a standard fuzzy inference method, the final uncertain T-S fuzzy system in the global model can be expressed by the ILC setting as follows:

$$\begin{cases} x(t+1,k) = (A(\mu) + \Delta A(\mu))x(t,k) + (B(\mu) + \Delta B(\mu))u(t,k) + w(t,k) \\ y(t,k) = Cx(t,k) \end{cases} \tag{2.5}$$

where

$$A(\mu) = \sum_{i=1}^{r} \mu_i A_i, \quad \Delta A(\mu) = \sum_{i=1}^{r} \mu_i \Delta A_i, \quad B(\mu) = \sum_{i=1}^{r} \mu_i B_i, \quad \Delta B(\mu) = \sum_{i=1}^{r} \mu_i \Delta B_i \tag{2.6}$$

Define the tracking error on $k$ batch as follows:

$$e(t,k) = y_d(t) - y(t,k) \tag{2.7}$$

where $y_d(t)$ is the reference trajectory vector. This tracking error is used to adjust the input vector such that the actual output $y(t,k)$ gradually approximates the reference trajectory vector.

To formulate the robust ILC design problem in the 2D T-S fuzzy framework, the following classical ILC strategy can be considered, where the current batch input is formed by the combination of the previous batch and a correction term:

$$u(t,k) = u(t,k-1) + r(t,k) \tag{2.8}$$

where $r(t,k)$ is the modification term, and $u(t,0)$ is the initial value of the iterative algorithm, which is commonly reset to zero for implementation. Next, introduce the vector:

$$\delta x(t,k) = x(t,k) - x(t,k-1). \tag{2.9}$$

Without loss of generality, it is assumed that $y_d(0) = y(0,k) = Cx(0,k)$ and hence $\delta x(0,k) = 0$. From (2.2)–(2.9), the following equations can be obtained:

$$\begin{aligned} \delta x(t+1,k) &= (A(\mu) + \Delta A(\mu))\delta x(t,k) + (B(\mu) + \Delta B(\mu))r(t,k) + \bar{w}(t,k) \\ e(t+1,k) &= e(t+1,k-1) - C(A(\mu) + \Delta A(\mu)\delta x(t,k) - C(B(\mu) + \Delta B(\mu))r(t,k) - C\bar{w}(t,k) \end{aligned} \tag{2.10}$$

where $\bar{w}(t,k) = w_1(t,k) + w(t,k) - w(t,k-1)$, $w_1(t,k) = (A(\delta\mu) + \Delta A(\delta\mu))x(t,k-1) + (B(\delta\mu) + \Delta B(\delta\mu))u(t,k-1)$, $A(\delta\mu) = \sum_{i=1}^{r}(\mu_i(\vartheta(t,k)) - \mu_i(\vartheta(t,k-1)))A_i$, $\Delta A(\delta\mu) = \sum_{i=1}^{r}(\mu_i(\vartheta(t,k)) - \mu_i(\vartheta(t,k-1)))\Delta A_i$, $B(\delta\mu) = \sum_{i=1}^{r}(\mu_i(\vartheta(t,k)) - \mu_i(\vartheta(t,k-1)))B_i$, $\Delta B(\delta\mu) = \sum_{i=1}^{r}(\mu_i(\vartheta(t,k)) - \mu_i(\vartheta(t,k-1)))\Delta B_i$.

Motivated by the parallel distributed compensation method, we consider the following fuzzy ILC updating law in this paper:

Rule $i$: IF $\vartheta_1(t,k)$ is $\mathcal{M}_{i1}$ and $\vartheta_j(t,k)$ is $\mathcal{M}_{ij}$, $\ldots$, $\vartheta_p(t,k)$ is $\mathcal{M}_{ip}$, THEN

$$r(t,k) = K_i \begin{bmatrix} \delta x(t,k) \\ e(t+1,k-1) \end{bmatrix} \tag{2.11}$$

where $K_i$ is the controller gain to be solved. By fuzzy blending, the overall fuzzy correction law can be rewritten in a compact form:

$$r(t,k) = K(\mu) \begin{bmatrix} \delta x(t,k) \\ e(t+1,k-1) \end{bmatrix} \tag{2.12}$$

where $K(\mu) = \sum_{i=1}^{r} \mu_i K_i$.

Let $\begin{bmatrix} x^h(t,k) \\ x^v(t,k) \end{bmatrix} = \begin{bmatrix} \delta x(t,k) \\ e(t+1,k-1) \end{bmatrix}$ and then $\begin{bmatrix} x^h(t+1,k) \\ x^v(t,k+1) \end{bmatrix} = \begin{bmatrix} \delta x(t+1,k) \\ e(t+1,k) \end{bmatrix}$. From (2.10) and (2.12), the closed-loop 2D fuzzy system is

$$\begin{cases} \begin{bmatrix} x^h(t+1,k) \\ x^v(t,k+1) \end{bmatrix} = \hat{A}(\mu) \begin{bmatrix} x^h(t,k) \\ x^v(t,k) \end{bmatrix} + \hat{B}\bar{w}(t,k) \\ z(t,k) = \hat{C} \begin{bmatrix} x^h(t,k) \\ x^v(t,k) \end{bmatrix} \end{cases} \tag{2.13}$$

where $\hat{A}(\mu) = (\bar{A}(\mu) + \Delta\bar{A}(\mu)) + (\bar{B}(\mu) + \Delta\bar{B}(\mu))K(\mu)$, $\hat{B} = \begin{bmatrix} I \\ -C \end{bmatrix}$, $\hat{C} = \begin{bmatrix} 0 & I \end{bmatrix}$, $\bar{A}(\mu) = \begin{bmatrix} A(\mu) & 0 \\ -CA(\mu) & I \end{bmatrix}$, $\Delta\bar{A}(\mu) = \begin{bmatrix} \Delta A(\mu) & 0 \\ -C\Delta A(\mu) & 0 \end{bmatrix} = \hat{E}\Delta(t)\hat{F}_A(\mu)$, $\bar{B}(\mu) = \begin{bmatrix} B(\mu) \\ -CB(\mu) \end{bmatrix}$, $\Delta\bar{B}(\mu) = \begin{bmatrix} \Delta B(\mu) \\ -C\Delta B(\mu) \end{bmatrix} = \hat{E}\Delta(t)F_B(\mu)$, $\hat{E} = \begin{bmatrix} E \\ -CE \end{bmatrix}$, $\hat{F}_A(\mu) = \begin{bmatrix} F_A(\mu) & 0 \end{bmatrix}$. The boundary conditions are supposed to satisfy the following:

$$\lim_{N\to\infty} \sum_{n=0}^{N} (|x^h(0,n)|^2 + |x^v(n,0)|^2) < \infty \tag{2.14}$$

In order to show the main purpose of this study, several lemmas and definitions are introduced as follows.

**Lemma 1.** [16] Let $\Gamma = \Gamma^T$, $\Lambda$ and $\Sigma$ be given matrices of appropriate dimensions. Then, the following two statements are equivalent:

1) There exists a matrix variable $W$ such that satisfies:

$$\Gamma + sym\{\Lambda^T W \Sigma\} < 0.$$

2) The following conditions hold:

$$\begin{cases} \Lambda^{\perp T}\Gamma\Lambda^{\perp} < 0, & if \Sigma^{\perp} = 0, \Lambda^{\perp} \neq 0; \\ \Sigma^{\perp T}\Gamma\Sigma^{\perp} < 0, & if \Lambda^{\perp} = 0, \Sigma^{\perp} \neq 0; \\ \Lambda^{\perp T}\Gamma\Lambda^{\perp} < 0,\ \Sigma^{\perp T}\Gamma\Sigma^{\perp} < 0, & if \Lambda^{\perp} \neq 0, \Sigma^{\perp} \neq 0. \end{cases}$$

where $\Lambda^{\perp}$ and $\Sigma^{\perp}$ are arbitrary matrices whose columns form a basis of the null spaces of $\Lambda$ and $\Sigma$, respectively.

**Lemma 2.** [17] Let $R = R^T$, $X$, $Y$ and $\Delta(t)$ be appropriately dimensioned matrices or vectors, and $\Delta(t)^T \Delta(t) \leq I$. Then,

$$R + sym\{X\Delta(t)Y\} < 0$$

holds if and only if there exists $\varepsilon > 0$ such that

$$R + \varepsilon XX^T + \varepsilon^{-1} Y^T Y < 0.$$

**Lemma 3.** [18] For matrices $\Phi_{ij}(i, j = 1, 2, \ldots, r)$, $\sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i(\vartheta(t,k))\mu_j(\vartheta(t,k))\Phi_{ij} < 0$ is fulfilled if the following inequalities hold:

$$\Phi_{ii} < 0, \quad i = 1, 2, \ldots, r,$$
$$\frac{1}{r-1}\Phi_{ii} + \frac{1}{2}(\Phi_{ij} + \Phi_{ji}) < 0, \quad i \neq j,\ i, j = 1, 2, \ldots, r.$$

**Definition 1.** [19] The resulting uncertain 2D fuzzy system (2.13) with $\bar{w}(t,k) = 0$ is is called 2D robustly asymptotically stable if the system meets the following:

$$\lim_{t+k\to\infty} \left\| \begin{bmatrix} x^h(t,k) \\ x^v(t,k) \end{bmatrix} \right\| = 0 \tag{2.15}$$

for any boundary condition satisfying Eq (2.14).

**Definition 2.** [12] For any scalar $\gamma > 0$, the resulting closed-loop system (2.13) is said to have $H_\infty$ performance level $\gamma$ if under zero boundary conditions, $\|z\|_2 < \gamma^2 \|\bar{w}\|_2$ holds for all non-zero $\bar{w} \in \mathcal{L}_2$.

The main purpose of this article is to derive the sufficient conditions for the design of the fuzzy ILC scheme in the form of (2.8) and (2.12) such that the resulting closed-loop fuzzy system (2.13) achieves asymptotic stability with a robust $H_\infty$ performance index.

## 3. Main results

### 3.1. Performance analysis of 2D fuzzy ILC system

In this section, sufficient conditions are presented to guarantee both the asymptotic stability and the $H_\infty$ performance of the resulting 2D closed-loop fuzzy system (2.13).

**Theorem 1.** Given a scalar $\gamma > 0$, a robust ILC scheme described by the 2D closed-loop system (2.13) is asymptotically stable and has $H_\infty$ performance $\gamma$ if there exists a symmetric positive matrix $P = diag\left\{P^h, P^v\right\}$ satisfying the following condition for all $\mu$:

$$\begin{bmatrix} \hat{A}(\mu)^T P\hat{A}(\mu) - P + \hat{C}^T\hat{C} & \hat{A}(\mu)^T P\hat{B} \\ * & \hat{B}^T P\hat{B} - \gamma^2 I \end{bmatrix} < 0 \tag{3.1}$$

**Proof.** First, we prove the asymptotic stability of the closed-loop system (2.13) under the condition $\bar{w}(t,k) = 0$. The following fuzzy Lyapunov function candidate is considered:

$$\begin{aligned} V(t,k) =& V^h(t,k) + V^v(t,k) \\ V_h(t,k) =& x^h(t,k)^T P^h x^h(t,k) \\ V_v(t,k) =& x^v(t,k)^T P^v x^v(t,k) \end{aligned} \tag{3.2}$$

Then, along the trajectory of system (2.13), one has the following:

$$\begin{aligned} \nabla V(t,k) =& V^h(t+1,k) - V^h(t,k) + V^v(t,k+1) - V^v(t,k) = \\ & \begin{bmatrix} x^h(t,k) \\ x^v(t,k) \end{bmatrix}^T (\hat{A}(\mu)^T P\hat{A}(\mu) - P) \begin{bmatrix} x^h(t,k) \\ x^v(t,k) \end{bmatrix} \end{aligned} \tag{3.3}$$

Recalling (3.1), it can be obtained that $\hat{A}(\mu)^T P\hat{A}(\mu) - P < 0$. Hence, for arbitrary $\begin{bmatrix} x^h(t,k) \\ x^v(t,k) \end{bmatrix} \neq 0$, the following condition is valid,

$$V^h(t+1,k) + V^v(t,k+1) \leq V^h(t,k) + V^v(t,k) \tag{3.4}$$

Similar to [20], one can take the summation on both sides of (3.4) from 0 to $n$ with respect to $t$ and from $n$ to 0 with respect to $k$; then,

$$\begin{aligned} & V^h(1,n) + V^v(0,n+1) + V^h(2,n-1) + V^v(1,n) + \cdots + V^h(n+1,0) + V^v(n,1) \\ &= \sum_{t+k=n+1} V^h(t,k) + \sum_{t+k=n+1} V^v(t,k) \\ &= \sum_{t+k=n+1} V(t,k) \\ &\leq V^h(0,n) + V^v(0,n) + \cdots + V^h(n,0) + V^v(n,0) \\ &= \sum_{t+k=n} V(t,k). \end{aligned} \tag{3.5}$$

Obviously, the function is decreasing along the state trajectories, and it is easy to obtain that $\lim_{t+k\to\infty} V(t,k) = 0$, namely,

$$\lim_{t+k\to\infty} \left\| \begin{bmatrix} x^h(t,k) \\ x^v(t,k) \end{bmatrix} \right\| = 0 \tag{3.6}$$

From Definition 1, the asymptotic stability of the resulting 2D dynamics (2.13) is guaranteed.

Next, we shall establish the 2D $H_\infty$ performance of the system (2.13) under zero boundary conditions for any nonzero $\bar{w}(t,k) \in \mathcal{L}_2$. One can obtain the following:

$$\begin{aligned}&\nabla V(t,k) + z(t,k)^T z(t,k) - \gamma^2 \bar{w}(t,k)^T \bar{w}(t,k)\\ &= \begin{bmatrix} x^h(t,k) \\ x^v(t,k) \\ \bar{w}(t,k) \end{bmatrix}^T \left\{ \begin{bmatrix} \hat{A}(\mu)^T P \hat{A}(\mu) - P & \hat{A}(\mu)^T P \hat{B} \\ * & \hat{B}^T P \hat{B} \end{bmatrix} + \begin{bmatrix} \hat{C}^T \hat{C} & 0 \\ * & -\gamma^2 I \end{bmatrix} \right\} \begin{bmatrix} x^h(t,k) \\ x^v(t,k) \\ \bar{w}(t,k) \end{bmatrix}\end{aligned} \tag{3.7}$$

Thus, if condition (3.1) holds, it is easy to have the following:

$$z(t,k)^T z(t,k) - \gamma^2 \bar{w}(t,k)^T \bar{w}(t,k) < -\nabla V(t,k) \tag{3.8}$$

Summing up both sides of (3.8), one can obtain the following:

$$\begin{aligned}&\sum_{t=0}^{T_0} \sum_{k=0}^{K_0} \left\{ z(t,k)^T z(t,k) - \gamma^2 \bar{w}(t,k)^T \bar{w}(t,k) \right\} < -\sum_{t=0}^{T_0} \sum_{k=0}^{K_0} \{\nabla V(t,k)\}\\ &= -\sum_{k=0}^{K_0} \left\{ V^h(T_0+1,k) - V^h(0,k) \right\} - \sum_{t=0}^{T_0} \{ V^v(t,K_0+1) - V^v(t,0) \}\end{aligned} \tag{3.9}$$

It is clear that $\sum_{k=0}^{K_0} V^h(T_0+1,k) \geq 0$ and $\sum_{t=0}^{T_0} V^v(t,K_0+1) \geq 0$. Combining the zero boundary condition, one has the following:

$$\|z\|_2 < \|\bar{w}\|_2 \tag{3.10}$$

Therefore, the 2D $H_\infty$ performance of the resulting closed-loop system (2.13) is satisfied and the proof is complete.

Theorem 1 is not usable. By defining new variables and using matrix inequality transformation techniques, the following Theorem is presented.

**Theorem 2.** Given a scalar $\gamma > 0$, an ILC scheme described as the 2D fuzzy system (2.13) is asymptotically stable with $H_\infty$ performance $\gamma$ if there exists a symmetric positive matrix $P = diag\left\{P^h, P^v\right\}$, and matrices $W_1$, $W_2$, $W_3$, $W_4$, $W_5$, $W_6$, $W_7$, $W_8$, $W_9$, for all $\mu$, satisfying the following condition:

$$\begin{bmatrix} I - sym\{W_7\} & -W_8 + (\hat{B}W_2)^T & -W_9 & 0 & W_7^T - W_2^T \\ * & -P + sym\{\hat{B}W_3\} & \hat{B}W_4 & \hat{A}(\mu)W_1 + \hat{B}W_5 & W_8^T - W_3^T + \hat{B}W_6 \\ * & * & -\gamma^2 I & \hat{C}W_1 & W_9^T - W_4^T \\ * & * & * & P - sym\{W_1\} & -W_5^T \\ * & * & * & * & -sym\{W_6\} \end{bmatrix} < 0 \tag{3.11}$$

**Proof.** The inequality (3.11) can be equivalently transformed to the following:

$$\Gamma_1 + sym\{\Lambda_1^T \tilde{W}_1 \Sigma_1\} < 0 \tag{3.12}$$

where $\Gamma_1 = \begin{bmatrix} I - sym\{W_7\} & -W_8 & -W_9 & 0 & W_7^T \\ * & -P & 0 & 0 & W_8^T \\ * & * & -\gamma^2 I & 0 & W_9^T \\ * & * & * & P & 0 \\ * & * & * & * & 0 \end{bmatrix}$, $\Lambda_1^T = \begin{bmatrix} 0 & 0 \\ \hat{A} & \hat{B} \\ \hat{C} & 0 \\ -I & 0 \\ 0 & -I \end{bmatrix}$, $\Sigma_1 = I$, $\tilde{W}_1 = \begin{bmatrix} 0 & 0 & 0 & W_1 & 0 \\ W_2 & W_3 & W_4 & W_5 & W_6 \end{bmatrix}$, and hence $\Lambda_1^{\perp} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ 0 & \hat{A}^T & \hat{C}^T \\ 0 & \hat{B}^T & 0 \end{bmatrix}$. Based on Lemma 1, the following condition can be established:

$$\Lambda_1^{\perp T}\Gamma_1\Lambda_1^{\perp} < 0 \tag{3.13}$$

Then, inequality (3.13) can be expressed as follows:

$$\Gamma_2 + sym\{\begin{bmatrix} -I \\ \hat{B} \\ 0 \end{bmatrix}\begin{bmatrix} W_7 & W_8 & W_9 \end{bmatrix}\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}\} < 0 \tag{3.14}$$

where $\Gamma_2 = \begin{bmatrix} I & 0 & 0 \\ * & \hat{A}(\mu)P\hat{A}(\mu)^T - P & \hat{A}(\mu)P\hat{C}^T \\ * & * & \hat{C}P\hat{C}^T - \gamma^2 I \end{bmatrix}$. Introducing the matrices $\Lambda_2 = \begin{bmatrix} -I & \hat{B}^T & 0 \end{bmatrix}$, $\tilde{W}_2 = \begin{bmatrix} W_7 & W_8 & W_9 \end{bmatrix}$, $\Sigma_2 = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$, we get $\Lambda_2^{\perp} = \begin{bmatrix} \hat{B}^T & 0 \\ I & 0 \\ 0 & I \end{bmatrix}$, $\Sigma_2^{\perp} = 0$. Another application of Lemma 1 to (3.14) yields the following:

$$\Lambda_2^{\perp T}\Gamma_2\Lambda_2^{\perp} < 0 \tag{3.15}$$

Furthermore, after some routine matrix manipulations, the inequality (3.15) can be reformulated as follows:

$$\begin{bmatrix} \hat{A}(\mu) & I \\ \hat{C} & 0 \end{bmatrix}\begin{bmatrix} P & 0 \\ * & -P \end{bmatrix}\begin{bmatrix} \hat{A}(\mu) & I \\ \hat{C} & 0 \end{bmatrix}^T + \begin{bmatrix} \hat{B} & 0 \\ * & I \end{bmatrix}\begin{bmatrix} I & 0 \\ * & -\gamma^2 I \end{bmatrix}\begin{bmatrix} \hat{B} & 0 \\ * & I \end{bmatrix}^T < 0 \tag{3.16}$$

This last inequality is a dual version of (3.1) in Theorem 1, which completes the proof.

### *3.2. Controller design for the nominal model case*

Based on the analysis result of Theorems 1 and 2, the problem of designing the corresponding matrices in the updating law (2.12) is investigated, such that the resulting ILC scheme is asymptotically stable and meets the 2D $H_\infty$ performance specification.

The plant (2.2) without time-varying parametric uncertainties is considered, that is, $\Delta(t) = 0$, $\Delta A_i(t) = 0$, $\Delta B_i(t) = 0$, $\Delta\bar{A}(\mu) = 0$, $\Delta\bar{B}(\mu) = 0$, then $\hat{A}(\mu) = \bar{A}(\mu) + \bar{B}(\mu)K(\mu)$, and $w_1(t, k)$ in plant (2.10) is rewritten as $w_1(t, k) = A(\delta\mu)x(t, k-1) + B(\delta\mu)u(t, k-1)$. The following theorem provides sufficient conditions for the solvability of the considered problem, where a finite set of LMI constraints makes the design conditions.

**Theorem 3.** Given a scalar $\gamma > 0$, a robust ILC scheme described as the 2D fuzzy system (2.13) in the absence of uncertainty is asymptotically stable with $H_\infty$ performance $\gamma$ if there exists a symmetric positive matrix $P = diag\left\{P^h, P^v\right\}$, and matrices $W_1$, $W_2$, $W_3$, $W_4$, $W_5$, $W_6$, $W_7$, $W_8$, $W_9$ and $Y_i$ exist, such that the following LMIs hold:

$$\Upsilon_{ii} < 0, \quad i = 1, 2, \ldots, r \tag{3.17}$$

$$\frac{1}{r-1}\Upsilon_{ii} + \frac{1}{2}(\Upsilon_{ij} + \Upsilon_{ji}) < 0, \quad i \neq j,\ i, j = 1, 2, \ldots, r \tag{3.18}$$

where $\Upsilon_{ij} = \begin{bmatrix} I - sym\{W_7\} & -W_8 + (\hat{B}W_2)^T & -W_9 & 0 & W_7^T - W_2^T \\ * & -P + sym\{\hat{B}W_3\} & \hat{B}W_4 & \Omega_{ij} + \hat{B}W_5 & W_8^T - W_3^T + \hat{B}W_6 \\ * & * & -\gamma^2 I & \hat{C}W_1 & W_9^T - W_4^T \\ * & * & * & P - sym\{W_1\} & -W_5^T \\ * & * & * & * & -sym\{W_6\} \end{bmatrix}$,
$\Omega_{ij} = \bar{A}_i W_1 + \bar{B}_i Y_j$.

In this case, the required ILC law matrices of (2.11) can be computed by the following:

$$K_i = Y_i W_1^{-1} \tag{3.19}$$

**Proof.** According to the fuzzy system parameters, the inequality (3.11) in Theorem 2 can be reorganized as follows:

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\vartheta(t,k))\mu_j(\vartheta(t,k))\Upsilon_{ij} < 0 \tag{3.20}$$

Using Lemma 3, if the conditions (3.17) and (3.18) hold, then (3.20) is fulfilled. Therefore, the proof is complete.

### *3.3. Controller design for the uncertain model case*

In this section, the design of robust ILC schemes for an uncertain fuzzy plant model is developed by making extensive use of the previously developed results when there are uncertainties in the model structure (i.e., the matrix $\Delta A_i(t)$ and $\Delta B_i(t)$ exist in (2.2) and are of the form (2.3)). The results of Theorems 1–3 can be extended to design ILC schemes for the uncertain fuzzy system. The next result gives new LMI-based conditions for the solvability of the considered problem.

**Theorem 4.** Given a scalar $\gamma > 0$, a robust ILC scheme described as the 2D fuzzy system (2.13) with uncertainty structure modeled by (2.2) is robustly asymptotically stable with $H_\infty$ performance $\gamma$, if there exists a symmetric positive matrix $P = diag\left\{P^h, P^v\right\}$, matrices $W_1$, $W_2$, $W_3$, $W_4$, $W_5$, $W_6$, $W_7$, $W_8$, $W_9$, $Y_i$, and a scalar $\varepsilon > 0$ exist, such that the following LMIs hold:

$$\Psi_{ii} < 0, \quad i = 1, 2, \ldots, r \tag{3.21}$$

$$\frac{1}{r-1}\Psi_{ii} + \frac{1}{2}(\Psi_{ij} + \Psi_{ji}) < 0, \quad i \neq j,\ i,j = 1,2,\ldots,r \tag{3.22}$$

where $\Psi_{ij} = \begin{bmatrix} I - sym\{W_7\} & -W_8 + (\hat{B}W_2)^T & -W_9 & 0 & W_7^T - W_2^T & 0 & 0 \\ * & -P + sym\{\hat{B}W_3\} & \hat{B}W_4 & \Omega_{ij} + \hat{B}W_5 & W_8^T - W_3^T + \hat{B}W_6 & \varepsilon\hat{E} & 0 \\ * & * & -\gamma^2 I & \hat{C}W_1 & W_9^T - W_4^T & 0 & 0 \\ * & * & * & P - sym\{W_1\} & -W_5^T & 0 & M_{ij}^T \\ * & * & * & * & -sym\{W_6\} & 0 & 0 \\ * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & 0 & -\varepsilon I \end{bmatrix}$,

$\Omega_{ij} = \bar{A}_i W_1 + \bar{B}_i Y_j$, $M_{ij} = \hat{F}_{Ai} W_1 + F_{Bi} Y_j$.

Moreover, the ILC law matrices of (2.11) can be selected as in (3.19).

**Proof.** If there are uncertainties in the state-space model, the inequality (3.11) in Theorem 2 can be rewritten as follows:

$$\mathbb{R} + sym\{\mathbb{X} \triangle (t)\mathbb{Y}\} < 0 \tag{3.23}$$

where $\mathbb{R} = \begin{bmatrix} I - sym\{W_7\} & -W_8 + (\hat{B}W_2)^T & -W_9 & 0 & W_7^T - W_2^T \\ * & -P + sym\{\hat{B}W_3\} & \hat{B}W_4 & \bar{A}(\mu)W_1 + \bar{B}(\mu)Y(\mu) + \hat{B}W_5 & W_8^T - W_3^T + \hat{B}W_6 \\ * & * & -\gamma^2 I & \hat{C}W_1 & W_9^T - W_4^T \\ * & * & * & P - sym\{W_1\} & -W_5^T \\ * & * & * & * & -sym\{W_6\} \end{bmatrix}$,

$\mathbb{X} = \begin{bmatrix} 0 & \hat{E}^T & 0 & 0 & 0 \end{bmatrix}^T$, $\mathbb{Y} = \begin{bmatrix} 0 & 0 & 0 & \hat{F}_A(\mu)W_1 + F_B(\mu)Y(\mu) & 0 \end{bmatrix}$. Employing Lemma 2 to deal with the uncertainties in inequality (3.23), it produces the following:

$$\mathbb{R} + \begin{bmatrix} \mathbb{X} & \mathbb{Y}^T \end{bmatrix} \begin{bmatrix} \varepsilon I & 0 \\ * & \varepsilon^{-1} I \end{bmatrix} \begin{bmatrix} \mathbb{X}^T \\ \mathbb{Y} \end{bmatrix} < 0 \tag{3.24}$$

where $\varepsilon > 0$. Applying the Schur complement, the following is obtained:

$$\begin{bmatrix} \mathbb{R} & \mathbb{X} & \mathbb{Y}^T \\ * & -\varepsilon^{-1} I & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0 \tag{3.25}$$

pre- and post-multiplying (3.25) by $diag\{I, I, I, I, I, \varepsilon I, I\}$ and its transpose, respectively, we have the following:

$$\sum_{i=1}^{r} \sum_{j=1}^{r} \Psi_{ij} < 0 \tag{3.26}$$

Thus, on the basis of Lemma 3, (3.26) holds if the LMIs (3.21) and (3.22) are satisfied, which completes the proof.

**Remark 1.** When the uncertainty is present in the 2D fuzzy system (2.13), the robust asymptotic stability of the uncertain closed-loop system should be considered first. This stability analysis follows from a direct application of identical steps within in proof of Theorem 1; hence, the details are omitted. We can use the result in Theorem 2 to obtain the robust asymptotic stability condition for the uncertain closed-loop system (2.13), so that the LMIs (3.21) and (3.22) ensure that the uncertain plant (2.13) is robustly asymptotically stable.

## 4. Simulation case study

To illustrate the effectiveness and feasibility of the formulated results on fuzzy ILC scheme designs, two simulation examples are provided in this section.

**Example 1.** As a typical nonlinear batch process, the three-tank system, regarded as a model of many controlled objects in industrial processes, has been extensively researched. The three-tank hydraulic system in [21] was studied. As shown in Figure 1, the system consists of three identical tanks with two pumps pumping water into Tanks 1 and 2. Tank 3 is connected to Tanks 1 and 2 by pipes of an identical circular cross section. The physical model of the three-tank system is given by the following [21]:

$$\begin{cases} \dot{h}_1 = \frac{Q_1}{\mathcal{A}} - \frac{a_1 s_{13} sgn(h_1 - h_3)\sqrt{2g|h_1 - h_3|}}{\mathcal{A}} \\ \dot{h}_2 = \frac{Q_2}{\mathcal{A}} + \frac{a_3 s_{23} sgn(h_3 - h_2)\sqrt{2g|h_3 - h_2|} - a_2 s_0 \sqrt{2gh_2}}{\mathcal{A}} \\ \dot{h}_3 = \frac{a_1 s_{13} sgn(h_1 - h_3)\sqrt{2g|h_1 - h_3|}}{\mathcal{A}} - \frac{a_3 s_{23} sgn(h_3 - h_2)\sqrt{2g|h_3 - h_2|}}{\mathcal{A}} \end{cases} \tag{4.1}$$

where $h_1$, $h_2$ and $h_3$ denote the level in the three tanks, respectively. $Q_1$ and $Q_2$ are the incoming mass flow. $a_1$, $a_2$ and $a_3$ represent the coefficients of flow for pipe 1, pipe 2, and pipe 3, respectively. $\mathcal{A}$ is the cross-sectional area of tanks, and $s_{13} = s_{23} = s_0 = s_n$ is the cross-sectional area of pipes.

Defining $x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T = \begin{bmatrix} h_1 & h_2 & h_3 \end{bmatrix}^T$, $u = \begin{bmatrix} Q_1 & Q_2 \end{bmatrix}^T$, the water levels $h_1$ and $h_2$ are measurable output variables. According to the continuous-time T-S fuzzy model, we can obtain a discrete-time fuzzy model with a sampling period of 0.5 *sec*. Taking into account the non-repetitive disturbance in the discrete-time fuzzy system, the following state-space model is given:

$$\begin{cases} x(t+1,k) = \sum_{i=1}^{3} \mu_i \left[A_i x(t,k) + B_i u(t,k))\right] + w(t,k) \\ y(t,k) = Cx(t,k) \end{cases} \tag{4.2}$$

where $A_1 = \begin{bmatrix} 0.9885 & -0.0005 & 0.0099 \\ -0.0012 & 0.9818 & 0.0095 \\ 0.0106 & 0.0106 & 0.9787 \end{bmatrix}$, $A_2 = \begin{bmatrix} 0.9935 & -0.0003 & 0.0046 \\ -0.0003 & 0.9878 & 0.0051 \\ 0.0054 & 0.0053 & 0.9893 \end{bmatrix}$, $A_3 = \begin{bmatrix} 0.9949 & -0.0002 & 0.0034 \\ 0.0001 & 0.9892 & 0.0041 \\ 0.0040 & 0.0040 & 0.9919 \end{bmatrix}$, $B_1 = B_2 = B_3 = \begin{bmatrix} 0.0032 & 0 \\ 0 & 0.0032 \\ 0 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$.

The membership functions for the premise variable $x_1(t,k)$ are depicted in Figure 2. The non-repetitive disturbance is set up as $w(t,k) = \begin{bmatrix} 0.3e^{(-0.2\delta_1 k)} \\ 0.2sin(0.25t\delta_2 + 0.12k\delta_1) \\ 0.25e^{(-0.2\delta_1 k)} \end{bmatrix}$, where $\delta_1$ and $\delta_2$ are randomly varying in the interval [0, 1].

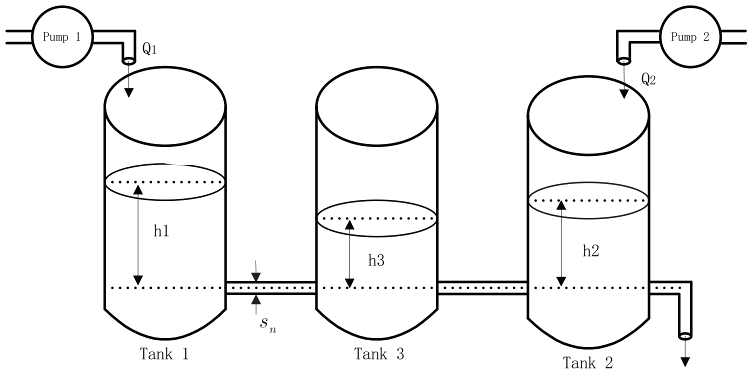


**Figure 1.** Schematic diagram of three-tank system [21].

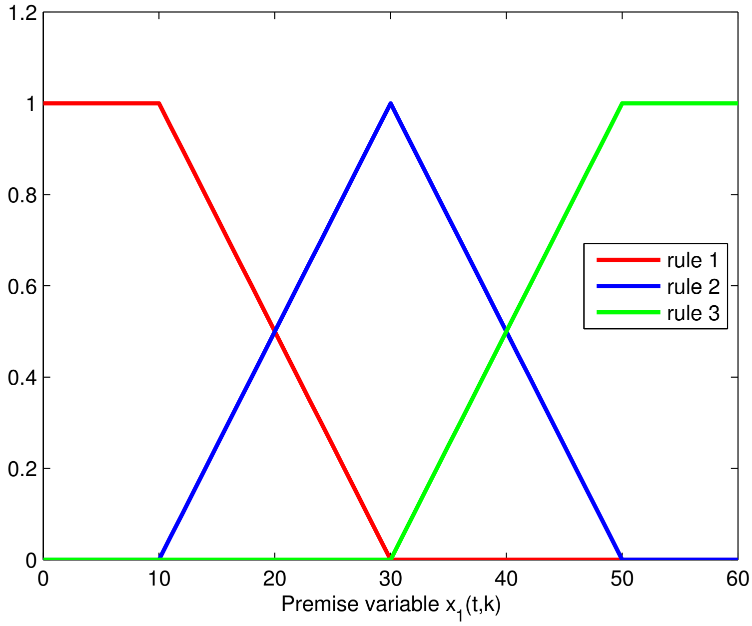


**Figure 2.** Membership functions for the three-tank system.

The initial state $x(0, k)$ and the input vector $u(0, k)$ are supposed to be zero $\forall k \geq 1$. The the reference trajectories are as follows:

$$y_{1d}(t) = \begin{cases} 0.04t\,sin(t/1000), & 0 \leq t \leq 500\,(sec) \\ 20, & 500 < t \leq 1000\,(sec) \\ 20 + 0.02(t - 1000), & 1000 < t \leq 1500\,(sec) \\ 30, & 1500 < t \leq 2000\,(sec) \\ 30 + 0.06(t - 2000), & 2000 < t \leq 2500\,(sec) \\ 60, & 2500 < t \leq 3000\,(sec) \end{cases} \tag{4.3}$$

$$y_{2d}(t) = \begin{cases} 0.1t, & 0 \le t \le 500\,(sec) \\ 50, & 500 < t \le 1000\,(sec) \\ 50 - 0.02(t - 1000), & 1000 < t \le 1500\,(sec) \\ 40, & 1500 < t \le 2000\,(sec) \\ 40 - 0.04(t - 2000), & 2000 < t \le 2500\,(sec) \\ 20, & 2500 < t \le 2800\,(sec) \\ 20 + 0.8(t - 2800), & 2800 < t \le 3000\,(sec) \end{cases} \tag{4.4}$$

Meanwhile, let $e_m(t, k)$, $m = 1, 2$ represent the errors on $k$ batch. In order to evaluate the tracking performance along the batch, the root mean square (RMS) is introduced:

$$RMS\,(mk) = \sqrt{\frac{1}{6001}\sum_{t=0}^{6000} e_m(t, k)^2} \tag{4.5}$$

According to Theorem 3 with $\gamma = 10$, the controller gain matrices in (2.11) are given by the following:

$$K_1 = \begin{bmatrix} -308.9075 & 0.1550 & -3.2102 & 291.0808 & -0.0036 \\ 0.3738 & -306.8137 & -3.0844 & -0.0036 & 291.0808 \end{bmatrix}$$
$$K_2 = \begin{bmatrix} -310.4694 & 0.0931 & -1.5564 & 291.0809 & -0.0036 \\ 0.0931 & -308.6882 & -1.7119 & -0.0036 & 291.0808 \end{bmatrix}$$
$$K_3 = \begin{bmatrix} -310.9067 & 0.0621 & -1.1822 & 291.0809 & -0.0036 \\ -0.0317 & -309.1254 & -1.4001 & -0.0036 & 291.0809 \end{bmatrix}.$$

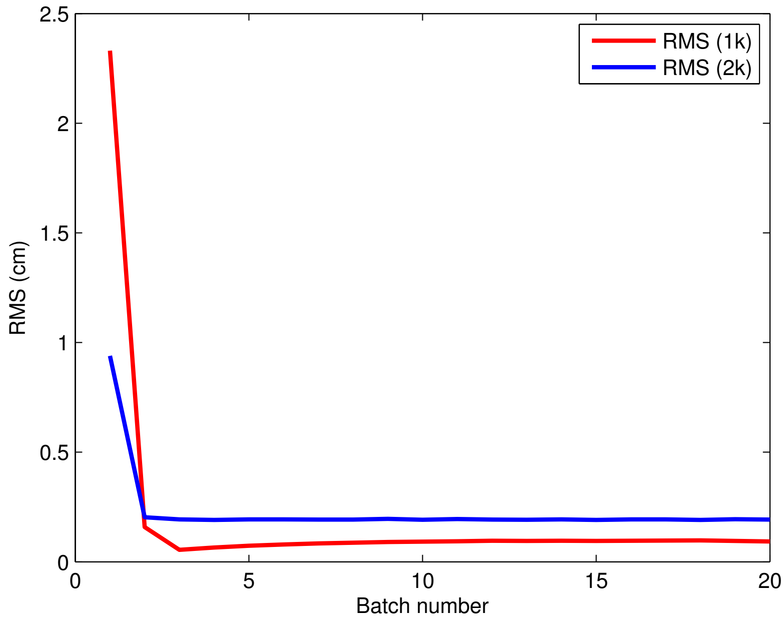


**Figure 3.** RMS performance against batch number for example 1.

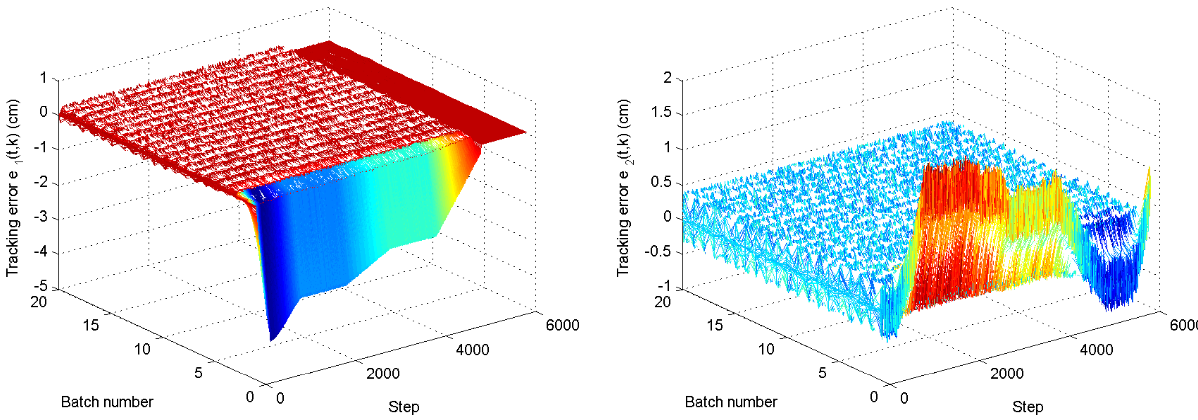


**Figure 4.** The tracking errors for example 1.

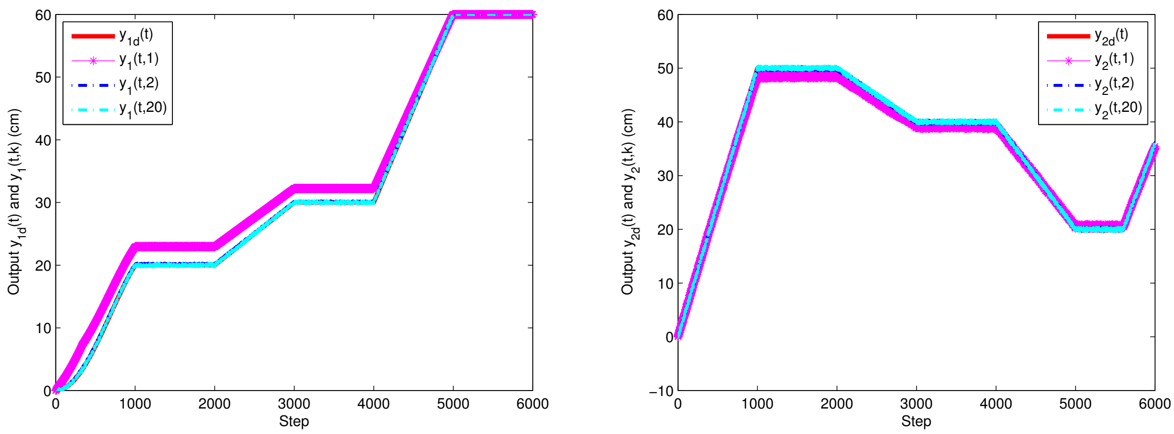


**Figure 5.** Output response of several batches for example 1.

The simulation results of the resulting system are given in Figures 3–5. As the batch increases, the actual outputs gradually track the reference trajectory. However, because the system is affected by non-repetitive disturbances, the root mean squared (RMS) values of the tracking error converge to an acceptable range. Although the designed ILC scheme cannot completely eliminate the effect of non-repetitive uncertainties, it can maintain the asymptotic stability and 2D $H_\infty$ performance of the fuzzy system.

**Example 2.** In this example, the uncertain T-S fuzzy system with non-repetitive interference is considered. We will demonstrate the validity of the studied method by applying it to a highly nonlinear model of the CSTR, which has been investigated in [22, 23]. The dynamics of the CSTR model is described as follows:

$$\begin{cases} \dot{C}_A = \frac{q}{V}(C_{Af} - C_A) - k_0 \exp(-\frac{E_0/R_0}{T})C_A \\ \dot{T} = \frac{q}{V}(T_f - T) + \frac{(-\Delta H)}{\rho C_p}k_0 \exp(-\frac{E_0/R_0}{T})C_A + \frac{UA}{V\rho C_p}(T_c - T) \end{cases} \tag{4.6}$$

where $C_A$ is the concentration of $\mathcal{A}$ in the irreversible reaction ($\mathcal{A} \to \mathcal{B}$), $T$ represents the reactor temperature, and the controlled variable is coolant stream temperature $T_c$; other notations are suitably referred to [22].

Denote the nonzero equilibrium as $\{C_A^{eq}, T^{eq}, T_c^{eq}\}$ and introduce $x = \begin{bmatrix} C_A - C_A^{eq} & T - T^{eq} \end{bmatrix}^T$, $u = T_c - T_c^{eq}$, and $y = x_2$ as the system state, input and output variables. The discrete-time T–S fuzzy model can be obtained by discretizing the continuous-time T-S fuzzy model in [22] with a 0.01 min sampling time. Furthermore, we can obtain the following fuzzy system by considering the uncertainties and non-repetitive disturbances:

$$\begin{cases} x(t+1,k) = \sum_{i=1}^{4} \mu_i \left[(A_i + \Delta A_i)x(t,k) + (B_i + \Delta B_i)u(t,k))\right] + w(t,k) \\ y(t,k) = Cx(t,k) \end{cases} \tag{4.7}$$

One can choose $x_2$ as the premise variable, the membership functions are defined as follows:

$$\begin{cases} \mu_1(x_2(t,k)) = \frac{1}{2}\frac{g_1(x_2(t,k)) - g_1(-\beta)}{g_1(\beta) - g_1(-\beta)} \\ \mu_2(x_2(t,k)) = \frac{1}{2}\frac{g_1(\beta) - g_1(x_2(t,k))}{g_1(\beta) - g_1(-\beta)} \\ \mu_3(x_2(t,k)) = \frac{1}{2}\frac{g_2(x_2(t,k)) - g_2(-\beta)}{g_2(\beta) - g_2(-\beta)} \\ \mu_4(x_2(t,k)) = \frac{1}{2}\frac{g_2(\beta) - g_2(x_2(t,k))}{g_2(\beta) - g_2(-\beta)} \end{cases}$$

where

$$\begin{cases} g_1(x_2(t,k)) = \varphi_1(x_2(t,k)) - \varphi_1^0 \\ g_2(x_2(t,k)) = \varphi_2(x_2(t,k)) - \varphi_2^0 \end{cases}$$

and

$$\begin{cases} \varphi_1(x_2(t,k)) = k_0 \exp(-\frac{E_0/R_0}{x_2(t,k)+T^{eq}}) \\ \varphi_2(x_2(t,k)) = k_0 \left[\exp(-\frac{E_0/R_0}{x_2(t,k)+T^{eq}}) - \exp(-\frac{E_0/R_0}{T^{eq}})\right] C_A^{eq} \\ \varphi_1^0 = \frac{1}{2}\left[\varphi_1(-\beta) + \varphi_1(\beta)\right] \\ \varphi_2^0 = \frac{1}{2}\left[\varphi_2(-\beta) + \varphi_2(\beta)\right] \end{cases}$$

Impose the constraint on $x_2$ as $|x_2(t,k)| \le \beta$ ($\beta = 10$). The model parameters are given as $A_1 = \begin{bmatrix} 0.9628 & -0.0004 \\ 1.3538 & 0.9880 \end{bmatrix}$, $A_2 = \begin{bmatrix} 0.9929 & -0.0004 \\ -0.1404 & 0.9883 \end{bmatrix}$, $A_3 = \begin{bmatrix} 0.9776 & -0.0006 \\ 0.6162 & 1.0001 \end{bmatrix}$, $A_4 = \begin{bmatrix} 0.9778 & -0.0001 \\ 0.6088 & 0.9763 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ 0.0208 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 \\ 0.0208 \end{bmatrix}$, $B_3 = \begin{bmatrix} 0 \\ 0.0209 \end{bmatrix}$, $B_4 = \begin{bmatrix} 0 \\ 0.0207 \end{bmatrix}$, $E = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$, $F_{A1} = \begin{bmatrix} 0.08 & 0 \\ 0.05 & 0.14 \end{bmatrix}$, $F_{A2} = \begin{bmatrix} 0.06 & 0 \\ 0 & 0.15 \end{bmatrix}$, $F_{A3} = \begin{bmatrix} 0.05 & 0 \\ 0.02 & 0.25 \end{bmatrix}$, $F_{A4} = \begin{bmatrix} 0.08 & 0 \\ 0.03 & 0.12 \end{bmatrix}$, $F_{B1} = \begin{bmatrix} 0 \\ 0.0018 \end{bmatrix}$, $F_{B2} = \begin{bmatrix} 0 \\ 0.0018 \end{bmatrix}$, $F_{B3} = \begin{bmatrix} 0 \\ 0.0021 \end{bmatrix}$, $F_{B4} = \begin{bmatrix} 0 \\ 0.002 \end{bmatrix}$, $\Delta(t) = \begin{bmatrix} 0.5sin(0.25t) & 0 \\ 0 & 0.85cos(0.3t) \end{bmatrix}$, $C = \begin{bmatrix} 0 & 1 \end{bmatrix}$. The non-repetitive disturbances are assumed to be $w(t,k) = \begin{bmatrix} 0.01 & 0.02 \end{bmatrix}^T sin(0.1t\delta_3 + 0.15k\delta_3)$, and $\delta_3$ is randomly varying in the interval [0, 1]. For $k \ge 1$, the initial state is $x(0,k) = \begin{bmatrix} 0.25 & 1 \end{bmatrix}^T$ and the input vector $u(0,k)$ is set to zero. The control objective is to regulate the reactor temperature to the equilibrium point, so the reference trajectory is chosen as follows:

$$y_d(t) = \begin{cases} 1 + 0.045t, & 0 \le t \le 10\,(min) \\ 4 + 3cos(\pi(t-20)/40)^2, & 10 < t \le 30\,(min) \\ 5.5 - 10sin(\pi(t-30)/120), & 30 < t \le 50\,(min) \\ 0.5 - 0.05(t-50), & 50 < t \le 60\,(min) \end{cases} \tag{4.8}$$

By applying Theorem 4 with the $H_\infty$ performance bound $\gamma = 8$, we can obtain the following fuzzy iterative learning controller parameters:

$$\begin{aligned} K_1 &= \begin{bmatrix} -65.0722 & -47.4586 & 47.4359 \end{bmatrix} \\ K_2 &= \begin{bmatrix} 6.7480 & -47.6611 & 47.4476 \end{bmatrix} \\ K_3 &= \begin{bmatrix} -29.4971 & -47.8198 & 47.1977 \end{bmatrix} \\ K_4 &= \begin{bmatrix} -29.4711 & -47.3230 & 47.6602 \end{bmatrix} \end{aligned}$$

In this case, the RMS is $RMS(k) = \sqrt{\frac{1}{6001}\sum_{t=0}^{6000} e(t,k)^2}$.

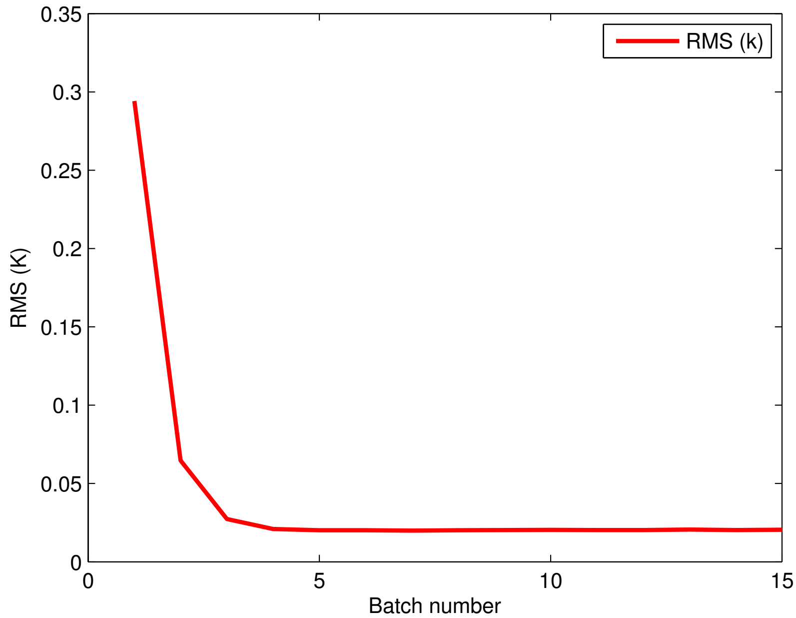


**Figure 6.** RMS performance against batch number for example 2.

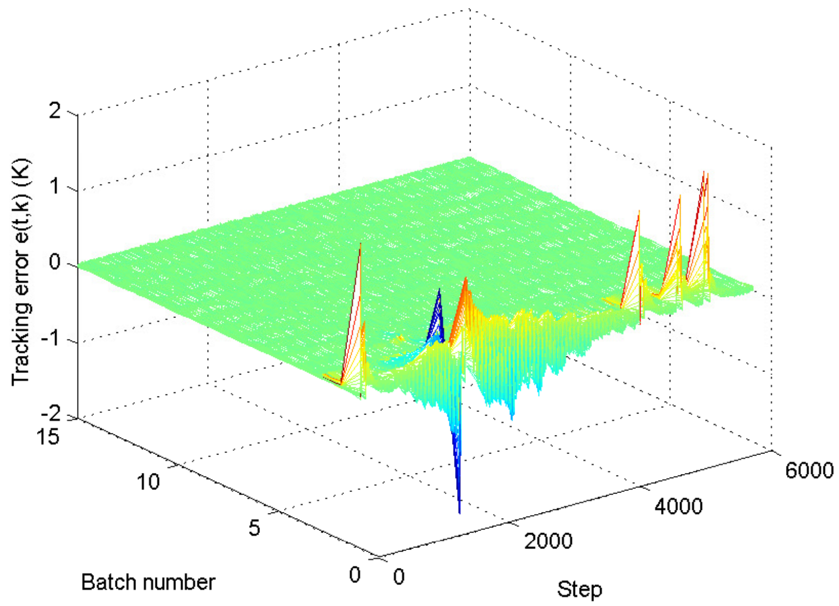


**Figure 7.** The tracking errors for example 2.

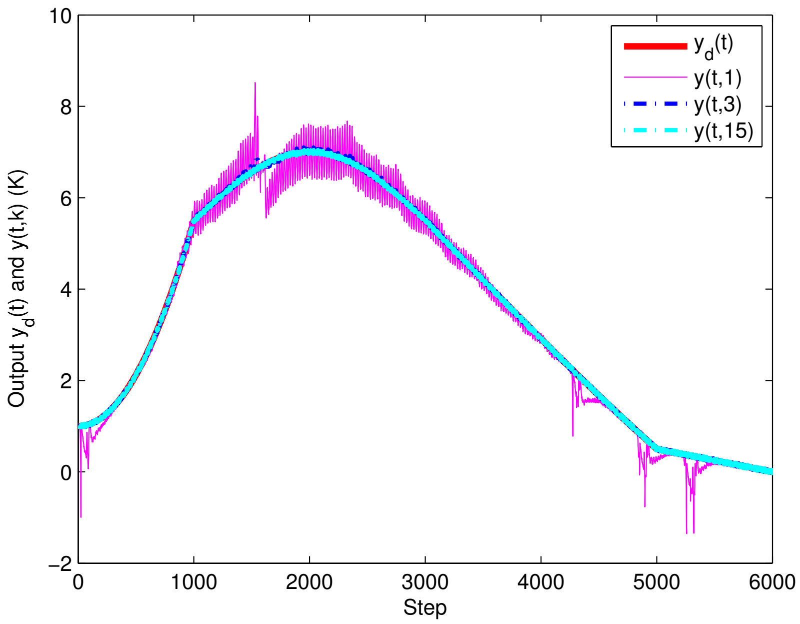


**Figure 8.** Output response of several batches for example 2.

For the uncertain T-S fuzzy system with non-repetitive interference, the test results for this design are portrayed in Figures 6–8. Figure 6 depicts the RMS response of the tracking error reduction to a bounded value with respect to the batches. The evolution of the tracking error e(t, k) is depicted in Figure 7. Figure 8 shows that as the batches progress, the actual outputs of the T-S fuzzy plant gradually approach the desired trajectory. From these figures, it can be immediately observed that the controlled system exhibits robust stability and convergence. Finally, the results confirm that the method proposed in this article has a good attenuation effect on the non-repetitive disturbance of the fuzzy system.

## 5. Conclusions

This paper proposes a 2D fuzzy ILC strategy for nonlinear batch processes. The nonlinear batch process model was represented by the uncertain T-S model with non-repetitive disturbances by using the sector nonlinear method; a 2D compound ILC scheme was designed by exploiting the 2D and repetitive characteristics of batch processes. Based on the 2D Lyapunov theory, sufficient conditions are derived to guarantee that the resulting closed-loop system is asymptotically stable and has a prescribed $H_\infty$ attenuation level. The design problem is formulated as the solution of a set of LMIs. Finally, simulation results show the validity and practical value of the developed method.

## Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

## Acknowledgments

This work is supported by National Natural Science Foundation of China (61573167, 61572237), Postgraduate Research & Practice Innovation Program of Jiangsu Province (KYCX20_1771) and the Start-up Fund for Introducing Talent of Wuxi University (2021r045).

## Conflict of interest

The authors declare there is no conflict of interest.

## References

1. H. Yoo, H. E. Byun, D. H. Han, J. H Lee, Reinforcement learning for batch process control: Review and perspectives, *Annu. Rev. Control*, **52** (2021), 108–119. https://doi.org/10.1016/j.arcontrol.2021.10.006
2. X. C. Sheng, W. L. Xiong, Soft sensor design based on phase partition ensemble of LSSVR models for nonlinear batch processes, *Math. Biosci. Eng.*, **17** (2020), 1901–1921. https://doi.org/10.3934/mbe.2020100
3. L. M. Zhou, L. Jia, Y. L. Wang, D. G. Peng, W. D. Tan, An integrated robust iterative learning control strategy for batch processes based on 2D system, *J. Process Control*, **85** (2020), 136–148. https://doi.org/10.1016/j.jprocont.2019.11.011

4. Y. Q. Zhou, D. W. Li, F. R. Gao, Conic iterative learning control using distinct data for constrained systems with state-dependent uncertainty, *IEEE Trans. Ind. Inf.*, **18** (2022), 3095–3104. https://doi.org/10.1109/TII.2021.3107522
5. X. D. Zhao, Y. Q. Wang, Improved point-to-point iterative learning control for batch processes with unknown batch-varying initial state, *ISA Trans.*, **125** (2022), 290–299. https://doi.org/10.1016/j.isatra.2021.07.007
6. Y. Geng, X. E. Ruan, Y. Yang, Q. H. Zhou, Data-based iterative learning control for multiphase batch processes, *Asian J. Control*, **25** (2022), 1392–1406. https://doi.org/10.1002/asjc.2936
7. C. Y. Zhou, L. Jia, Y. Zhou, A two-stage robust iterative learning model predictive control for batch processes, *ISA Trans.*, **135** (2023), 309–324. https://doi.org/10.1016/j.isatra.2022.09.034
8. S. L. Hao, T. Liu, E. Rogers, Extended state observer based indirect-type ILC for single-input single-output batch processes with time- and batch-varying uncertainties, *Automatica*, **112** (2020), 108673. https://doi.org/10.1016/j.automatica.2019.108673
9. R. D. Zhang, F. R. Gao, Two-dimensional iterative learning model predictive control for batch processes: A new state space model compensation approach, *IEEE Trans. Syst. Man Cybern. Syst.*, **51** (2021), 833–841. https://doi.org/10.1109/TSMC.2018.2883754
10. H. Li, S. Q. Wang, H. Y. Shi, C. L. Su, P. Li, Two-dimensional iterative learning robust asynchronous switching predictive control for multiphase batch processes with time-varying delays, *IEEE Trans. Syst. Man Cybern. Syst.*, **53** (2023), 6488–6502. https://doi.org/10.1109/TSMC.2023.3284078
11. Z. You, H. C. Yan, H. Zhang, H. Zhang, S. M. Chen, M. Wang, Fuzzy-dependent-switching control of nonlinear systems with aperiodic sampling, *IEEE Trans. Fuzzy Syst.*, **29** (2021), 3349–3359. https://doi.org/10.1109/TFUZZ.2020.3018552
12. J. X. Yu, H. Dang, L. M. Wang, Fuzzy iterative learning control-based design of fault tolerant guaranteed cost controller for nonlinear batch processes, *Int. J. Control Autom. Syst.*, **16** (2018), 2518–2527. https://doi.org/10.1007/s12555-017-0614-0
13. W. P. Luo, L. M. Wang, 2D fuzzy constrained fault-tolerant predictive control of nonlinear batch processes, *IEEE Access*, **7** (2019), 119259–119271. https://doi.org/10.1109/ACCESS.2019.2936214
14. H. Li, S. Q. Wang, H. Y. Shi, C. L. Su, P. Li, Iterative learning hybrid robust predictive fault-tolerant control for nonlinear batch processes with partial actuator faults, *J. Process Control*, **129** (2023), 103054. https://doi.org/10.1016/j.jprocont.2023.103054
15. Y. C. Wang, L. F. Zheng, H. G. Zhang, X. W. Zheng, Fuzzy observer-based repetitive tracking control for nonlinear systems, *IEEE Trans. Fuzzy Syst.*, **28** (2020), 2401–2415. https://doi.org/10.1109/TFUZZ.2019.2936808
16. S. Boyd, E. L. Ghaoui, E. Feron, V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, SIAM, Philadelphia, USA, 1994. https://doi.org/10.1137/1.9781611970777
17. I. R. Petersen, A stabilization algorithm for a class of uncertain linear systems, *Syst. Control Lett.*, **8** (1987), 351–357. https://doi.org/10.1016/0167-6911(87)90102-2

18. H. D. Tuan, P. Apkarian, T. Narikiyo, Y. Yamamoto, Parameterized linear matrix inequality techniques in fuzzy control system design, *IEEE Trans. Fuzzy Syst.*, **9** (2001), 324–332. https://doi.org/10.1109/91.919253

19. X. P. Xie, H. G. Zhang, Stabilization of discrete-time 2-D T-S fuzzy systems based on new relaxed conditions, *Acta Autom. Sinica*, **36** (2010), 267–273. https://doi.org/10.1016/S1874-1029(09)60012-4

20. L. M. Wang, C. J. Zhu, J. X. Yu, L. Ping, R. D. Zhang, F. R. Gao, Fuzzy iterative learning control for batch processes with interval time-varying delays, *Ind. Eng. Chem. Res.*, **56** (2017), 3994–4001. https://doi.org/10.1021/acs.iecr.6b04637

21. H. Y. Han, Y. Yang, L. L. Li, S. X. Ding, Performance-based fault detection and fault-tolerant control for nonlinear systems with T-S fuzzy implementation, *IEEE Trans. Cybern.*, **51** (2021), 801–814. https://doi.org/10.1109/TCYB.2019.2951534

22. B. C. Ding, X. B. Ping, Output feedback predictive control with one free control move for nonlinear systems represented by a Takagi–Sugeno model, *IEEE Trans. Fuzzy Syst.*, **22** (2014), 249–263. https://doi.org/10.1109/TFUZZ.2013.2251637

23. H. F. Xie, J. Wang, X. M. Tang, Robust constrained model predictive control for discrete-time uncertain system in Takagi-Sugeno's form, *Asian J. Control*, **20** (2018), 1566–1581. https://doi.org/10.1002/asjc.1603

AIMS Press