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# Incorporating Perspectival Elements in a Discrete Mathematics Course

Calvin Jongsma

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# Incorporating Perspectival Elements in a Discrete Mathematics Course

## Abstract

Discrete mathematics is a vast field that can be explored along many different paths. Opening with a unit on logic and proof and then taking up some additional core topics (induction, set theory, combinatorics, relations, Boolean algebra, graph theory) allows one to bring in a wealth of relevant material on history, philosophy, axiomatics, and abstraction in very natural ways. This talk looks at how my 2019 textbook on discrete mathematics, focused in this way, came to be, and it highlights the various perspectival elements the book includes.


## Keywords
Christianity, perspective, mathematics, undergraduates

## Disciplines
Christianity | Discrete Mathematics and Combinatorics | Education



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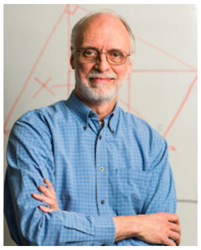
# Incorporating Perspectival Elements in a Discrete Mathematics Course

Calvin Jongsma (Dordt University)



Calvin Jongsma is Professor Emeritus of Mathematics at Dordt University. His interest in history and philosophy of mathematics and logic is evident in the many book reviews he has written for *MAA Reviews* and *Perspectives on Science and Christian Faith*. An essay recapitulating his dissertation on Whately's revival of logic in 19th-century Britain is the opening chapter in the forthcoming book *Aristotle's Syllogism and the Creation of Modern Logic*.

**Abstract**

Discrete mathematics is a vast field that can be explored along many different paths. Opening with a unit on logic and proof and then taking up some additional core topics (induction, set theory, combinatorics, relations, Boolean algebra, graph theory) allows one to bring in a wealth of relevant material on history, philosophy, axiomatics, and abstraction in very natural ways. This talk looks at how my 2019 textbook on discrete mathematics, focused in this way, came to be, and it highlights the various perspectival elements the book includes.

## 1 Introduction

A concern of the Association of Christians in the Mathematical Sciences from the very beginning has been how to connect our Christian faith with our professional interests and work in the mathematical sciences. As one of this year's ACMS pre-conference workshops and a number of speakers demonstrated, an important way to implement this is by attending to historical and philosophical perspectives. What I'd like to do in this essay is show how I incorporated perspectival elements in the discrete mathematics text I wrote for an intermediate-level course that I taught for years at Dordt. I'll share some of my mathography, explain how my text came to be, and briefly outline what the text covers and how it touches on perspectival matters.

## 2 Interest in Mathematics and Perspectival Issues

My love of mathematics goes as far back as I can remember, but in college I discovered that I also enjoyed history of philosophy. While a junior, a philosophy professor helped me deconflict what my major should be: he told me to stick with mathematics! That's not quite the advice you might think it was. He thought there were plenty of Christian academics seeking to work out their faith in philosophy, but fewer of them in mathematics with a historical and philosophical bent. Also, a mathematics professor familiar with my multiple interests counseled me not to philosophize without a solid grounding in mathematics proper; else, he said, no one would listen to me.

Armed with these two sage pieces of advice, I went off to graduate school to study mathematics, alert for ways to enrich my program with history and philosophy of mathematics. After satisfying comprehensive doctoral exams at the State University of New York at Buffalo in algebra and analysis, I was hoping to do a historically weighted dissertation on some topic in algebra or foundations. But a potential supervisor told me that if I did that I would never find a job, and he had no interest in overseeing such an undertaking. This time I ignored my professor's advice. After attending a talk on Gauss by the well-known historian of mathematics Ken May, I decided to transfer to the University of Toronto, where I could do a joint Ph.D. in mathematics and history of mathematics under him, specializing in the history of logic. And, as it turned out, it *was possible* to find a job teaching college mathematics with that kind of professional training.

My association with what came to be called ACMS after 1985 goes back to its inception in the early '70s. Like Bob Brabenec and some others, I thought Christian mathematicians should be more intentional about connecting their faith to their academic work and that this might be helped by engaging with the history, philosophy, and foundations of mathematics, broadly conceived. In addition to attending and speaking at the early conferences, Gene Chase and I collaborated to compile and annotate a *Bibliography of Christianity and Mathematics* (1983).[1] Later, I contributed two historical chapters on mathematization trends in the pre-modern and early modern eras in the book authored by a number of ACMS members, *Mathematics in a Postmodern Age* (2001). And in 2006, I gave the keynote address "Mathematics: Always Important, Never Enough" at a Dordt conference for Christian mathematics educators on the topic of a Christian approach to mathematics and mathematics education. In these ways I was able to draw upon my background interests and professional training in history and philosophy of mathematics to share my ideas with a broader academic audience.

## 3 Teaching, Textbook Writing, and Perspectival Issues

My teaching career at Dordt College beginning in 1982 encompassed a full range of standard undergraduate mathematics courses. One course I inherited at the outset, though, was a bit of an odd duck. Since Dordt's calculus sequence served engineering and computer science students as well as mathematics majors, our department decided to focus on the theory of calculus in a transition course for our students. To prepare them for navigating rigorous $\epsilon-\delta$ limit arguments and proving the basic theorems of differential and integral calculus, we first studied how to construct mathematical proofs. This gave me an opportunity to tap into my knowledge of developments in logic, though I didn't initially see how to inject very much perspectival material into such a technical course. I did, of course, concentrate on those sorts of issues in my alternate-year capstone course on the history of mathematics, but beyond having students read Judy Grabiner's *Who Gave You the Epsilon: Cauchy and the Origins of Rigorous Calculus*, I didn't pay much attention to perspectival issues in our sophomore-level transition course. Except, a significant undercurrent soon pushed me to do more.

What I had learned about the history of logic and the foundations of mathematics in my doctoral program made it difficult for me to accept some of what was being asserted about logic and mathematical proof in the pamphlets I initially chose to use as a text (first Bittinger's *Logic, Proof, and Sets*; and then Solow's *How to Read and Do Proofs*). I found myself pushing back on some of their key viewpoints, but I discovered that students didn't much appreciate being told that what they were reading was problematic—it's the textbook, after all!

At the same time, I was teaching the introductory logic course for Dordt's philosophy department, using a text (Bergmann, Moor, and Nelson's *The Logic Book*) that presented logic from a twentieth-century natural deduction perspective—the very thing, I soon realized, that my mathematics students needed for learning how to do proofs. The alternative Frege-Hilbert-style approach, better known to mathematicians, focused on tautologies (logical axioms) more than rules of inference

[1] For more information about this project, see my tribute to Gene on page 324 of this *Journal and Proceedings*.

because that had been the basis for implementing the logicists' program of reducing mathematics to logic. Recognizing the divergent goals of these two approaches to logic caused me to revamp the first part of my transition-math course and think about writing my own booklet on mathematical proof. It also made me realize that I could tailor my research interests to enrich my mathematics teaching.

At the time (1990-1991) no publishers among the 25 I contacted were willing to risk adopting my approach to mathematical proof construction. But I was seeing its value in my students' upper-level coursework, so I continued to invest appreciable time and energy in developing a classroom text for the course. This gradually morphed from bare-bones outlines to paragraphs of mathematical prose, using Knuth's recently available typographical system TeX (plain TeX at the time) to produce a publication-quality manuscript.

I also used some of this material on logic in a discrete mathematics course that I taught to computer science majors and engineering students. Thus, when enrollments in our mathematics transition course fell below an acceptable level in the early 2000s, we decided to combine these two courses. I dropped out the baby real analysis from the transition mathematics course and the finite state machines material from the other, redesigning the course to suit our particular clientele. This also gave me an opportunity to try to integrate perspectival elements where there was a natural fit.

Over the years, then, I gradually developed a text in *Discrete Mathematics* for the hybrid course I was teaching on an annual basis. Upon retiring years later in the mid-2010s, I posted chapters of my text to Dordt's Digital Collections to continue to make them available to Dordt students—and, as it happened, to whoever else might be interested in them. To my great surprise, they began to be downloaded all over the world. Within a year, over 9000 chapter-downloads were made by people in some 600 institutions and 90 countries, the most popular chapter being, amazingly enough, the rather technical seventh chapter, *Posets, Lattices, and Boolean Algebra*, geared to students learning about Boolean functions and logic gate circuits—downloads of this chapter proceeded at the rate of around 18 per day for months.

Pleasantly encouraged by this unexpected response, I decided to see whether a publisher might now be interested in my text. Two of the five publishers I sent a new prospectus to (Springer and Wiley) now expressed interest; a third (MAA) declined consideration because they already had two discrete mathematics texts in the pipeline. By the time *Introduction to Discrete Mathematics via Logic and Proof* was published by Springer in late 2019 as part of their *Undergraduate Texts in Mathematics* series, downloads of early chapter-drafts reached almost 80,000 by people connected to 2500 institutions in over 150 countries. Since then, another 20,000 digital chapter-downloads of the published textbook have been tallied by Springer through their licensing program with libraries.

## 4 Content and Perspectival Elements

I'll now briefly describe the material contained in *Introduction to Discrete Mathematics via Logic and Proof* (see the Appendix for the Table of Contents), and explain how perspectival elements are embedded within it.

The book opens with a somewhat leisurely study of logic, which (as the book's title indicates) is a coherent integrative thread running through the text. *Logic* is a field with its own content and concerns, but it clearly also has important links to mathematics and computer science. Logic forms the proper basis, I believe, for a genuine understanding of how proofs work—under-the-hood, as it were—in mathematics and elsewhere. I know this is disregarded by some mathematics practitioners who believe that guided practice at constructing proofs is sufficient and who think that logic introduces extraneous and distracting content. Having suffered under this benign-neglect approach as a student, however, I beg to differ, even though I agree that practice is absolutely necessary. I do find this negative attitude understandable, however, given mathematicians' experience with logic as a specialized subfield of mathematics dealing with truth tables and tautologies as well as foundational matters such as consistency and decidability. My different purpose for including logic is methodological, which requires an alternative approach to deduction. I employ the natural deduction approach due to Jaśkowski and Fitch, an approach that emphasizes suppositional inference rules such as *Conditional Proof*, *Proof by Contradiction*, and *Cases*. These are the life-blood of mathematical proof. When haven't you seen a mathematical proof declare, "suppose such and such"?

In addition to supplying a good basis for learning how to construct proofs, the replacement rules of *Propositional Logic* have close connections to results in elementary *Set Theory*, the topic of the fourth chapter. And this system of logic is, of course, also the theoretical basis for discussing Boolean functions and logic circuits, a main focus of the seventh chapter.

The interconnected roles played by logic are pointed out as the text proceeds, but logic is also the focus of a few historical and philosophical discussions scattered throughout. I survey the historically evolving role of logic at the outset, noting that logic's initial link to mathematics was as the deductive instrument (Aristotle's *organon*) for organizing and developing Euclidean geometry and Pythagorean number theory. Logic was completely unconnected to computation and algebra until the nineteenth and twentieth centuries when the work of Boole and Shannon became foundational for logic and computer science. Around the same time, the relation between logic and mathematics was tipped upside down by Frege, Dedekind, and Russell, who reshaped logic to be a content foundation for (parts of) mathematics. These logicist developments are elaborated in remarks integral to the text where germane.

I also make explicit connections to the other main foundational and philosophical developments in the early twentieth century. Brouwer's intuitionistic position on mathematical argumentation along with some push-back by Hilbert and others is mentioned, and differences between intuitionist logic and classical logic on issues involving negation are pointed out, particularly with respect to *Proof by Contradiction*.

With regard to Hilbert's formalist philosophy of mathematics, students are pressed to think through the meaning and value of formalizing mathematics. A formal approach is important for allowing multiple interpretations or models of mathematical theories, something germane to fields such as *Abstract Algebra*, but this is also important for parsing out the logical properties of and connections between a theory's axioms, as was seen in the rise of non-Euclidean geometry. Knowing these things encourages students to distinguish between taking a formal approach for technical mathematical and metalogical considerations and adopting a formalistic viewpoint on mathematics as a whole. Formalization is also relevant for computer-aided or computer-generated proofs, such as those developed for the *Four-Color Theorem*, a topic in Chapter 8's *Topics in Graph Theory*.

Closely connected to these philosophical schools of thought and various nineteenth- and twentieth-century developments is the praxis of axiomatization with its attendant methodologies of formalization and abstraction.

Axiomatization first comes up in the text for a theory having a familiar intended interpretation:

*Peano Arithmetic*. Computational axioms were first isolated and stressed in mid-nineteenth century British algebra (De Morgan, Hamilton), being later systematized by Dedekind and Peano in the late 1880s. This brought ordinary arithmetic into the axiomatic fold. Students are often surprised that properties of the computational apparatus they've used all their lives can be deduced (with some work) from a few definitions and five axioms characterizing the counting process, including the seminal *Axiom of Induction*. As a student once observed to me about this whole process, "calculus is easy; it's arithmetic that's hard!"

Axiomatization next appears in the context of studying *Set Theory*. After learning about Cantor's and Dedekind's work in elucidating the nature of infinite collections, students are informally introduced to axiomatic *Set Theory*. This was first proposed by Zermelo primarily as a way to organize and deductively justify some key set-theoretic results (such as the *Well-Ordering Theorem*) but also in order to avoid *Set Theory*'s paradoxes (such as *Russell's Paradox*). Its use as a foundation for mathematics is illustrated by noting how *Peano Arithmetic* and other mathematical ideas and results can be modeled (or coded) inside *Set Theory* (e.g., by von Neumann in 1923), and its impact on later developments in mathematics education such as the *New Math Revolution* of the '60s is pointed out. Metalogical concerns such as Gödel's and Cohen's independence results for the *Continuum Hypothesis* are also cited, though obviously not argued.

Finally, axiomatization is introduced with respect to exploring Boolean lattices, generalizing and systematizing its results as *Boolean Algebra*. This leads students deeper into contemporary mathematics, stretching them to think more abstractly about mathematical axioms and models, something they will experience in spades in certain upper-level mathematics courses.

As I'm more of a historian than a philosopher, my default is to include historical material wherever relevant. Thus, the text highlights the significance of scores of people—for *Logic* (Aristotle, Boole, Leibniz, De Morgan, Frege, Russell, Tarski, Shannon, Jaśkowski); for *Set Theory* (Cantor, Dedekind, Venn, Zermelo, Gödel); and for *Graph Theory* (Euler, Hamilton, Kuratowski, Kempe, Heawood)—about 70 mathematicians in all. Naturally, I can't do much besides mention most of these people, but doing this in context is more than name-dropping: it gives students and professors a hook for further historical exploration.

So, the text raises issues both of philosophical and historical interest. What about connections to religious outlooks? Are there even any, and if so, can such things be broached in a commercial textbook? This question can be answered along several different lines. Obviously, Christian doxological reflections on God as the Source and Sustainer of mathematical and logical realities would be deemed out of bounds by a secular publisher like Springer. Such comments can be integrated into an instructor's use of my textbook, however: certain parts of the text provide a stage for adding such sentiments in a rather natural way. And raising various foundational issues provides an information platform for evaluating prominent philosophical perspectives from a Christian viewpoint. I think aspects of all three or four major early twentieth-century foundational philosophies can be challenged from the vantage point of a Christian worldview, even while recognizing legitimate advances in logic and mathematics due to their outlook. Rejecting the reductionist tendency of the logicist program (or of the related set-theoretic foundation), for instance, comports well with a Christian philosophy that emphasizes the rich multidimensional character of creation. This non-reductionist outlook is partly what makes me choose a natural deduction approach to logic over that of Frege and Russell—another reason being that a stress on inference rules as governing valid argumentation fits better with an emphasis on God's sovereignty over truth and validity. I believe that similar sorts of reflections can be made about intuitionist and formalist philosophies of mathematics.

There is one point in the book where a connection to Christian theology is explicitly made: Cantor's view of infinity was formed through both mathematical and theological reflection on this notion. I cite this fact because of its historical influence, not because I believe Christian mathematicians should anoint Cantor as God's prophet of the infinite. Yet noting this connection provides an opportunity to ponder mathematics' appropriation of infinity from theology and philosophy, and it also explains why some mathematicians at the time thought Cantor had lost his way and wandered off into metaphysics.

## 5 Concluding Remarks

I hope it's clear how I've used perspectival elements to integrate, contextualize, and enrich some fairly standard discrete mathematics material. Still, I know that my particular emphasis and choice of topics isn't for everyone: *Discrete Mathematics* is a bit of an amorphous grab-bag that can be taken in many different directions. I've chosen topics that connect with one another in natural ways. There is some flexibility in the book to tailor topics to fit one's interests or needs, though, and instructors can always proceed at their own pace. Some sections can be combined by omitting certain subsections, and extensive exercise sets encourage exploring topics in more depth if that is wanted; an instructor's *Solution Manual* provides complete solutions to half of the book's 1500 problems.

In conclusion, while I have emphasized the historical and philosophical features of my book, treating it as a case study in how one can incorporate perspectival elements in a technical mathematics text, I believe that anyone who examines the text will recognize that it adheres to the advice I was given as an undergraduate: do your philosophizing and historicizing in the context of substantial mathematical considerations.

## 6 Appendix

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