


Article

# Regulatory Estimates for Defaulted Exposures: A Case Study of Spanish Mortgages

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**Abstract:** The capital requirements derived from the Basel Accord were issued with the purpose of deploying a transnational regulatory framework. Further regulatory developments on risk measurement is included across several documents published both by the European Banking Authority and the European Central Bank. Among others, the referred additional documentation focused on the models' estimation and calibration for credit risk measurement purposes, especially the Advanced Internal-Ratings Based models, which may be estimated both for non-defaulted and defaulted assets. A concrete proposal of the referred defaulted exposures models, namely the Expected Loss Best Estimate (ELBE) and the Loss Given Default (LGD) in-default, is presented. The proposed methodology is eventually calibrated on the basis of data from the mortgage's portfolios of the six largest financial institutions in Spain. The outcome allows for a comparison of the risk profile particularities attached to each of the referred portfolios. Eventually, the economic sense of the results is analyzed.

**Keywords:** risk management; banking regulation; Basel Accord; defaulted exposures; economic downturn; Expected Loss Best Estimate; Loss Given Default in-default



**Citation:** González, M.R.; Ureña, A.P.; Fernández-Aguado, P.G. Regulatory Estimates for Defaulted Exposures: A Case Study of Spanish Mortgages. *Mathematics* **2021**, *9*, 997. <https://doi.org/10.3390/math9090997>

Academic Editor: Emanuela Rosazza Gianin

Received: 1 March 2021

Accepted: 23 April 2021

Published: 28 April 2021

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## 1. Introduction

An international banking regulation framework has been promoted by means of the Basel Accord [1]. Its implementation regarding the capital calculation, articulated via the Capital Requirements Regulation (CRR, see Reference [2]), has been rolled out across Europe, leading to a more robust and better prepared banking system to face unforeseen challenges like the economic crisis derived from the COVID-19 pandemic circumstances.

Credit risk represents the category that entails greater capital amount in the banking entities, so that an inappropriate capital calculation associated with said category may easily jeopardize the risk management of the banking industry. Certain methods are permitted by law in this regard, namely, the standard approach and the Internal-Ratings Based (IRB) approach, which, in turn, is split between foundation and advanced. Additionally, there are two types of exposures for which the Advanced IRB models may be estimated: defaulted and non-defaulted.

According to CRR, the models for defaulted exposures shall be based on the estimation of the Expected Loss Best Estimate (ELBE) and the Loss Given Default (LGD) in-default. The first one shall account for the expected loss while the second shall reflect both expected and unexpected losses. Nevertheless, the referred regulation did not provide enough details on the specificities that need to be taken into account for the estimation of the mentioned models. Further on, they have been subject to additional guidance issued by the European Banking Authority (EBA) on two documents: Regulatory Technical Standards (RTS) on assessment of methodology for the IRB approach [3] and Guidelines on PD estimation, LGD estimation, and the treatment of defaulted exposures [4]. Lastly, the ECB guide to internal models [5] also includes further guidance in this regard.

Having regard to the increase of depth and complexity of the European banking regulation related to the capital requirements calculation, the financial institutions have devoted significant resources to align their Advanced IRB models to the regulatory expectation. Nevertheless, a high investment in resources may be prone to result in ineffectiveness without having good understanding of the relevant regulation.

There are hardly any academic publications aiming at providing a concrete proposal of Advanced IRB models for defaulted exposures. Kim [6], based on the one factor models from Frye [7] and on the proposal given by Düllmann and Trapp [8], presents several approaches to obtain the LGD in-default and ELBE for small samples with insufficient historical depth. Dermine and de Carvalho [9,10] estimates the consequences of defaulting using two different approaches. The first approach follows the proposal from Altman [11] and Altman and Suggit [12] where the percentage of good and bad loans is analyzed after a number of years after the origination date. An adjustment based on the Kaplan-Meier [13] estimator is presented in order to mitigate the effect of censored data. The second model is based on an empirical estimator from a log-log function. Weissbach et al. [14] focuses their study on the economic capital estimator and proposes the use of Advanced IRB models, especially ELBE and LGD in-default that are calculated as one factor models. The application of survival analysis and a mixture distribution is analyzed by Zhang and C. Thomas [15] for the purpose of the recovery ratio modelling and the subsequent LGD prediction. Altman and Kalotay [16] estimate the defaulted debt recoveries via a mixture of Gaussian distributions. Fenech et al. [17] also uses the Kaplan-Meier estimation to obtain the probability of recovering the complete debt. All the referred publications either provide a non-regulatory proposal or assess certain modelling aspects, which may be related to the models. To date, as already mentioned, there is no specific proposal of IRB defaulted exposure models aimed at being aligned with the most recent regulatory developments.

The referred regulation states that LGD in-default shall be estimated by adding a supplement over the estimated ELBE that captures every unexpected loss that may occur during the recovery process of the remaining debt. In this regard, Merton [18] affirms that the recovery ratios volatility disappears with an adequate portfolio diversification. However, Altman et al. [19] confirm that such volatility was observed during different moments of the economic cycle of several countries. Therefore, they conclude that the Merton model's assumption contradicts the empirical evidence presented. The said conclusion gives sense to the regulatory proposal of estimating an add-on that reflects the unexpected loss.

In the methodology for the ELBE and LGD in-default estimation, the time in default variable acquires high significance. Gürtler and Hibbeln [20] conclude that the majority of loans take some time to return to the non-defaulted status. In addition, a positive correlation is found between the LGD and the resolution time. Frontczak and Rostek [21] introduce the time in default as a variable in an LGD model with stochastic collateral and Betz et al. [22] analyze the resolution time, both reaching similar conclusions than Gürtler and Hibbeln.

This article mainly relies on our concrete methodologic proposal for the IRB modelling of defaulted exposures, published recently as a working paper [23], with an additional adjustment that helps to ensure higher conservatism over the LGD in-default. Subsequently, the said proposal is implemented on the basis of the mortgage data from the six largest Spanish institutions. Then, a comparison of the outcome is performed so that various credit risk profiles are analyzed. Eventually, the results are assessed in terms of economic sense, considering the widely-known credit loss distribution defined by Vasicek [24] as a reference.

## 2. Materials and Methods

The initial dataset includes information from recovery processes of defaulted mortgages, which, according to CRR, may be categorized as exposures secured by mortgages on immovable property. Despite the scarcity of data in defaulted exposure databases, the

mortgage portfolios of the six financial entities under study contain sufficient observations for a robust estimation whether the methodology proposed in the paragraphs below is implemented. With the purpose of calibrating the ELBE and LGD in-default parameters, two sets are defined.

- Set of expected and unexpected losses estimation (E): It is built on the basis of sufficiently large sample of exposures that defaulted close to the reference date, which is defined as the date on which the historical data was extracted, as of 31 December 2016. The recovery processes collected shall be sufficiently observed from the default date until the debt repayment is completed. This set is used for estimating both ELBE and LGD in-default models.
- Set of unexpected losses estimation (I): It contains the data associated with exposures in which the default date is closer to the reference date, so that the respective recovery processes are only observed during the first months in default. Therefore, the most recent trend, at the reference date, is captured. In addition, this set contains data collected during observed downturn periods, so that the corresponding downward trend is caught, as requested in the related regulatory documents. This set is also a basis for the LGD in-default estimation.

### 2.1. ELBE Model

The ELBE model is estimated for each date on which recovery flows are collected.

Being  $[0, T] \subset \mathbb{R}$  a time in default interval, being  $0 < t_0 < t_1 < \dots < t_k < \dots < t_n < T$  and  $0 < \tilde{t}_0 < \tilde{t}_1 < \dots < \tilde{t}_k < \dots < \tilde{t}_m < T$ , two partitions of such an interval so that  $t_0 < \tilde{t}_0$  and  $\tilde{t}_m < t_n$ , being also the debt recoveries registered over the defined default interval expressed as  $r_{\tilde{t}_0}, r_{\tilde{t}_1}, \dots, r_{\tilde{t}_k}, \dots, r_{\tilde{t}_m}$  so that  $\tilde{t}_i \in (0, T)$  and, being the EAD the exposure amount at default, the ELBE is defined as follows.

$$ELBE_{t_k} := 1 - \left[ \left( \sum_{\tilde{t}_i > t_k} r_{\tilde{t}_i} \right) / \left( EAD - \sum_{\tilde{t}_i > t_k} r_{\tilde{t}_i} \right) \right] \tag{1}$$

For an adequate estimation of ELBE, the data related to the recovery processes contained in set E are taken into account, i.e., all those  $r_{\tilde{t}_i}$  such that  $\tilde{t}_i \in (0, T)$ . The ELBE evaluated for the values contained in set E on the basis of Equation (1) is indicated as  $ELBE_{t_i}^E$  for all points in time  $t_i \in (0, T)$ .

In alignment with the regulation, the conservatism shall be avoided for the ELBE estimation. This condition is met since the proposed approach is based on data reflecting economic circumstances, which were current by the reference date.

### 2.2. LGD In-Default Model

The LGD in-default is obtained as a result of adding a supplement to the ELBE. This add-on is estimated in a manner that, following a conservative approach, the uncertainty of the recovery process is captured, as well as the trend observed during an economic downturn period.

First, it is considered that the last moment registered after default is  $t_s \in (0, T)$  for certain  $s > 0$  and it is defined  $ELBE_{t_i}^{C,p}$  as the percentile p of the ELBE distribution at the moment  $t_i$  evaluated for the values from a set C. Then, the following difference is defined.

$$\Delta_p^{E,I}(t_i) := ELBE_{t_i}^{E,p} - ELBE_{t_i}^{I,50} \forall t_i \in (0, t_s], i \in \{1, \dots, n\}, p \in [50, 100] \tag{2}$$

Thus, the following indicator is also defined.

$$\delta_p := \#\left\{ i \in \{1, \dots, s\} / \Delta_p^{E,I}(t_i) < 0, \forall p \in [50, 100] \right\} \tag{3}$$

Based on the above, two root mean square errors are considered. The first one follows the usual definition while the second one contains an adjustment so that it only accounts for the negative values of  $\Delta_p^{E,I}(t_i)$ .

$$RMSE_p = \sqrt{(1/s) \sum_{i=1}^s \Delta_p^{E,I}(t_i)^2} \quad \forall p \in [50, 100] \tag{4}$$

$$RMSE_p^{neg} := \begin{cases} \sqrt{(1/\delta_p) \sum_{\{i \in \{1, \dots, s\} / \Delta_p^{E,I}(t_i) < 0\}} \Delta_p^{E,I}(t_i)^2} & si \delta_p > 0 \\ 0 & si \delta_p = 0 \end{cases} \tag{5}$$

Subsequently, the Mixed Adjustment Indicator (MAI) is defined as:

$$MAI_p := RMSE_p + RMSE_p^{neg} \quad \forall p \in [50, 100] \tag{6}$$

Hence, the previous concepts are defined as being possible to obtain the percentile that leads to the calculation of a sufficiently conservative supplement, i.e.,  $p^*$  such that  $MAI_{p^*} = \min\{MAI_p / p = 2n \quad \forall n \in \mathbb{N} / 24 < n < 51\}$ .

The additional unexpected loss is, thus, defined as indicated in Equation (2), but evaluating the function only for the values from set E.

$$\Delta_{p^*}^E(t_i) := ELBE_{t_i}^{E,p^*} - ELBE_{t_i}^{E,50} \quad \forall t_i \in (0, T], i \in \{1, \dots, n\} \tag{7}$$

The obtained supplement curve of unexpected loss should have increasing monotonicity in order to make economic sense. Therefore, an adjustment is implemented on the methodology proposed in the already referred working paper [10], thus, ensuring an additional layer of conservatism for the overall approach. The monotonicity is achieved by redefining the add-on, as indicated below.

$$\hat{\Delta}_{p^*}^E(t_i) := \max\{\Delta_{p^*}^E(t_{i-1}), \Delta_{p^*}^E(t_i) / t_i \in (0, T] \wedge i \in \{2, \dots, n\}\} \tag{8}$$

The LGD in-default is eventually determined as:

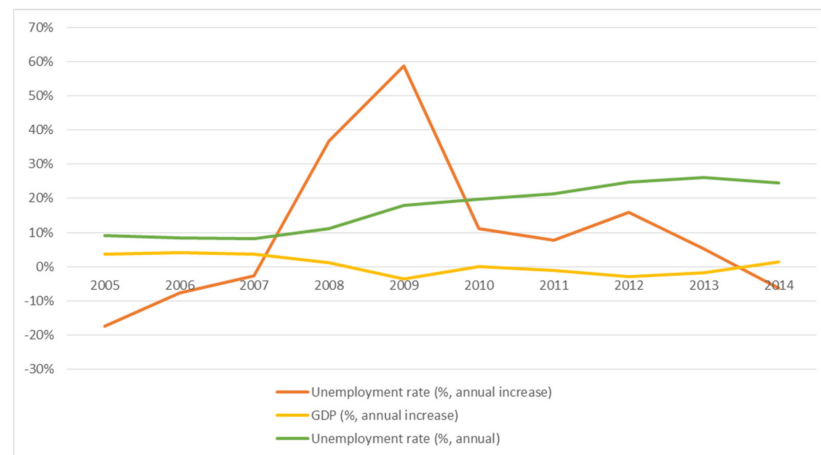
$$LGD_{t_i} := ELBE_{t_i}^E - \hat{\Delta}_{p^*}^E(t_i) \quad \forall t_i \in (0, T], i \in \{1, \dots, n\} \tag{9}$$

As already mentioned, the methodology conservativeness shall exclusively come out of the unexpected losses estimation, according to the regulatory developments.

### 2.3. Models Calibration

As already mentioned, the historical data available for the study was retrieved as of 31 December 2016, which is set as the reference date. It comprises mortgages data from 2005 to 2014, containing both buoyant economic years as well as downturn periods from the six largest Spanish financial institutions.

First, the sets E and I of reference are defined for model calibration purposes. It is, thus, convenient to perform in advance a macroeconomic analysis in order to identify the downturn years for each institution. With this aim, the Spanish macroeconomic variables presented in Figure 1 are assessed in combination with the mortgages' default behaviour.



**Figure 1.** Macroeconomic variables’ evolution in Spain (2005–2014). Source: World Bank Open Data (2021).

The annual unemployment rate together with its annual increase and the Gross Domestic Product (GDP) annual increase are presented above. A turning point is observed in the annual increase of the unemployment rate in 2008, but the change is more pronounced in 2009. Right after that year, the said annual increase is significantly reduced. On the other hand, it is observed that the GDP annual growth rate is generally negative from 2009 until 2013 while the unemployment rate shows a clear upward trend across the period of 2007–2013.

The bad economic conditions do not have an immediate impact in the creditworthiness of the mortgage portfolios. In fact, the most significant increase in the share of defaults is not only observed in 2008 but also in 2009 for some institutions (Table 1). A decrease in defaults is observed in 2010, due to the refinancing processes that were triggered in order to temporarily mitigate the impact of the crisis, so that the natural evolution of the defaulted portfolios affected by the global economic crisis cannot be completely tracked. However, attending to the refinancing measures applied in 2010 and the increase in the defaulted share registered in 2011 figures, it may be considered that a significant deterioration of the financial situation was experienced during 2010.

For the above reasons, 2009–2010 is selected as the downturn period.

**Table 1.** Percentages of mortgages’ defaulted exposures per year over the period of 2005–2014.

	Institution 1	Institution 2	Institution 3	Institution 4	Institution 5	Institution 6
2005	2.72%	4.28%	3.76%	1.97%	3.63%	7.60%
2006	2.93%	5.30%	3.12%	2.25%	4.07%	6.80%
2007	3.49%	8.58%	3.98%	3.72%	5.05%	7.40%
2008	9.78%	20.94%	7.56%	10.81%	6.18%	12.62%
2009	10.29%	14.79%	13.17%	13.76%	11.23%	13.27%
2010	9.83%	4.53%	11.03	6.71%	8.61%	8.91%
2011	12.45%	8.53%	12.42%	9.22%	9.70%	10.34%
2012	12.31%	10.40%	16.28%	13.75%	19.94%	9.62%
2013	22.45%	10.08%	14.71%	19.42%	19.05%	8.29%
2014	13.75%	12.56%	13.97%	18.39%	12.54%	15.15%

On the other hand, the data containing debt recovery processes, which reflect current economic circumstances as of the reference date is derived from the sample of mortgages that defaulted from 2012 until 2014.

Both above referred time periods, 2009–2010 and 2012–2014, capturing downturn economic conditions and recent economic circumstances, respectively, are used for the construction of set I.

Eventually, the set E is built in order to collect exposures that defaulted quite recently and that reflects debt recovery processes, which are sufficiently observed from the default until the complete repayment. For this reason, the information retrieved from mortgages that defaulted from 2009–2012 is considered for the construction of set E, since it ensures at least four years of observation of the recovery processes.

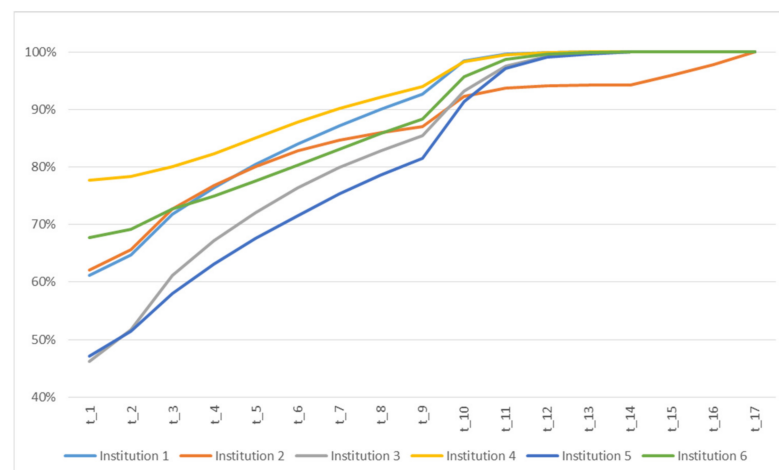
In order to calculate both ELBE and LGD in-default, as indicated in Equations (1) and (9), respectively, the partitioning of the time in default is defined as indicated in Table 2.

**Table 2.** Correspondence between ELBE and LGD in-default estimates and time in default.

Variables	Time in Default
ELBE <sub>1</sub> and LGD <sub>1</sub>	1 month
ELBE <sub>2</sub> and LGD <sub>2</sub>	2 months
ELBE <sub>3</sub> and LGD <sub>3</sub>	2 quarters
ELBE <sub>4</sub> and LGD <sub>4</sub>	3 quarters
ELBE <sub>5</sub> and LGD <sub>5</sub>	4 quarters
ELBE <sub>6</sub> and LGD <sub>6</sub>	5 quarters
ELBE <sub>7</sub> and LGD <sub>7</sub>	6 quarters
ELBE <sub>8</sub> and LGD <sub>8</sub>	7 quarters
ELBE <sub>9</sub> and LGD <sub>9</sub>	8 quarters
ELBE <sub>10</sub> and LGD <sub>10</sub>	3 years
ELBE <sub>11</sub> and LGD <sub>11</sub>	4 years
ELBE <sub>12</sub> and LGD <sub>12</sub>	5 years
ELBE <sub>13</sub> and LGD <sub>13</sub>	6 years
ELBE <sub>14</sub> and LGD <sub>14</sub>	7 years
ELBE <sub>15</sub> and LGD <sub>15</sub>	8 years
ELBE <sub>16</sub> and LGD <sub>16</sub>	9 years
ELBE <sub>17</sub> and LGD <sub>17</sub>	10 years

### 3. Results

The calibration, carried out on the basis of the ELBE methodology, which is reflected in Equation (1), is based on the already defined sets of reference and leads to the following results split by the institution and across time in default (Figure 2).



**Figure 2.** Estimated ELBE of the Spanish institutions by time in default.

Then, the supplement of unexpected loss is calculated as the volatility observed in the debt recovery processes’ evolution per institution over time in default, following Equation (8). As it may be observed below, it has increasing monotonicity by construction.

The resulting unexpected loss add-on is presented in Figure 3, reflecting a highly conservative outcome. The LGD in-default curves (Figure 4) are eventually obtained after

summing up the corresponding estimation of ELBE and the referred add-on, per institution and by time in default, as indicated in Equation (9).

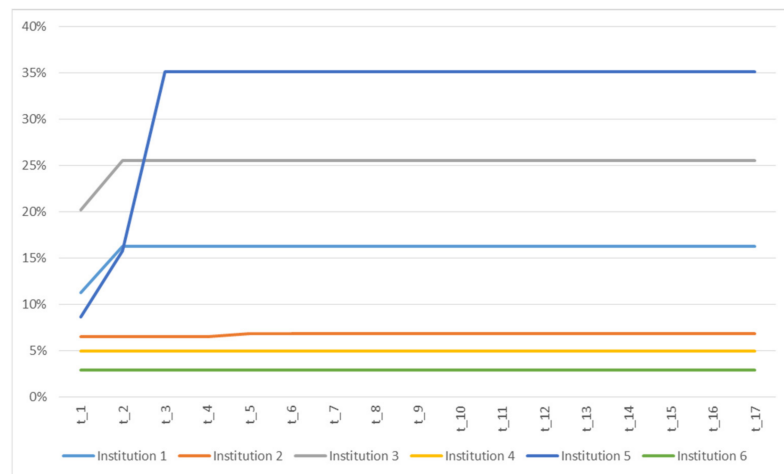


Figure 3. Unexpected loss add-on of the Spanish institutions by time in default.

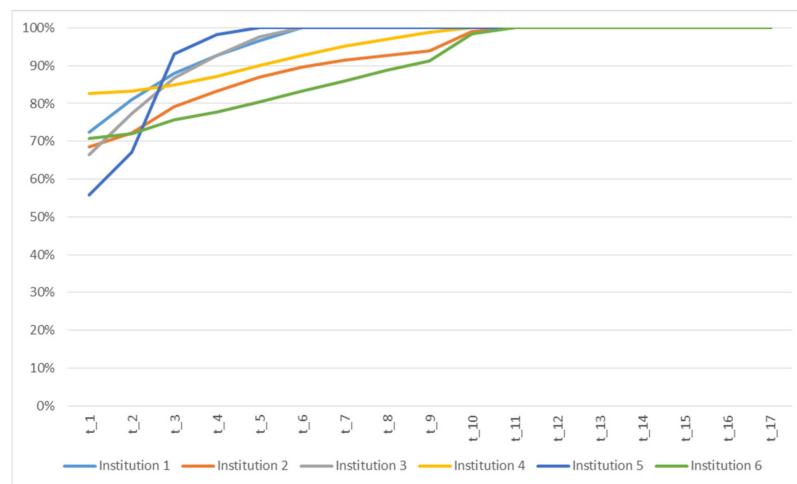


Figure 4. Estimated LGD in-default of the Spanish institutions by time in default.

It is worth highlighting that, as expected by construction, both defaulted exposure estimates do not exceed 100%, which represents the complete loss of the remaining debt.

#### 4. Discussion

The proposed methodologies are deemed as regulatory-compliant for the following reasons.

- The ELBE reflects economic conditions, which were current by the reference date because set E contains mortgages that defaulted close to said date of reference.
- The LGD in-default considers potential adverse changes in the economic conditions during the expected length of the recovery process as well as downturn conditions. The use of reference set I allows us to capture the economic trend of data that is even closer to the reference date, covering possible untoward effects, which might be observed as of the date of reference. In addition, downturn data is included in set I.
- The LGD in-default also contains an additional layer of conservatism because an increasing monotonicity was imposed, leading to a conservative outcome as required by the regulation.
- As a consequence, the estimated unexpected loss is greater than zero, which is in alignment with the regulatory expectation.



The resulting ELBE, presented in Figure 2, allow us to perform a good comparison, in terms of quality, of the debt recovery monitoring politics and the mortgage admission process across banks. First, the convergence-to-100% speed is basically influenced by the effectiveness of the politics and techniques implemented on each entity to face the recovery of the debt. Second, the higher starting ELBE value ( $t_1$ ) is, the less strict policies are applied in the associated mortgage admission process.

The economic sense of the results is also shown since the institutions in scope of the present study may be represented through the widespread distributions used for credit loss modelling first proposed by Vasicek [24]. It is worth noting that, according to Figures 2 and 3, institution 5 started with a lower level of ELBE, but it is the one that reflects a higher unexpected loss. In turn, institution 4 has a high estimated ELBE, but its level of unexpected loss is rather low. Said trade-off between both types of losses behaves as they were intended, i.e., in accordance with the Vasicek formula.

This article, thus, provides a concrete proposal of defaulted exposures models, aimed to be regulatory compliant. Although both ELBE and LGD in-default have been implemented on the basis of the mortgages' portfolios of the financial entities under study, the proposed methodology may be extended to another scope because the model driver considered (time in default) is usually available across portfolios of a different nature and characteristics. Therefore, it is easily applicable on the basis of the credit institutions' historical data.

**Author Contributions:** Conceptualization, methodology, and software, M.R.G. Validation, A.P.U. Formal analysis, A.P.U. and P.G.F.-A. Data curation, P.G.F.-A. Writing—original draft preparation, M.R.G. Writing—review and editing, A.P.U. and P.G.F.-A. Supervision, A.P.U. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research received no external funding.

**Institutional Review Board Statement:** Not applicable.

**Informed Consent Statement:** Not applicable.

**Data Availability Statement:** Data not available due to privacy issues.

**Acknowledgments:** We acknowledge the financial support of the Regional Government of Andalusia, Spain (Research Group SEJ-555).

**Conflicts of Interest:** The authors declare no conflict of interest.

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