The Effects of Contract Mechanisms between the Government and Private Hospitals on the Social Utility

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Research Article

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Posted Date: November 16th, 2023

DOI: https://doi.org/10.21203/rs.3.rs-3611472/v1

Additional Declarations: No competing interests reported.

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Abstract

In this work, we deal with a real healthcare system, in which public and private hospitals with different characteristics co-exist. While public hospitals have lower costs, they also suffer from long waiting times, diminishing the perceived quality of care for patients. Conversely, private hospitals, with their higher fees, offer shorter waiting periods, resulting in a more favorable perception of quality. A balanced healthcare system could offer societal benefits. Pricing strategies greatly influence a patient's hospital selection. For instance, reduced fees in private hospitals attract more patients, consequently reducing overcrowding in public facilities and elevating the overall quality of services provided. This study aims to develop pricing models to foster a balanced and socially advantageous healthcare system. Within this system, private hospital pricing is determined through contract mechanisms with the government. Thus, we delve into the ramifications of various contract models between the government and private hospitals on social utility. Our findings underscore the communal advantages of contract mechanisms. Furthermore, we generalize the proposed models to be applicable to similar systems.

Keywords: Healthcare system; Contract mechanisms; Pricing policies; Nash equilibrium; Social utility

1. Introduction

In this study, we model a real healthcare system, in which, public and private hospitals co-exist, nevertheless, they have different features. Service levels in private hospitals are high, waiting times are short, but fees are high. On the other hand, although public hospitals offer much lower or even free service, long waiting times and low service quality decrease patient satisfaction in these hospitals [1]. Patients make a choice between these two types of service providers, depending on their income level and quality sensitivity. Private hospitals commonly serve patients with higher income levels, while those with lower income levels prefer public ones, in general. Many researchers analyze the factors affecting patients' hospital preferences [2]. Qin and Prybutok [3] analyze the factors affecting patients' satisfaction and also their behavior to choose a hospital and identify that the price is one of the most important factors affecting patients' preferences. Andritsos and Tang [4] and also Andritsos and Aflaki [5] report that in addition to public ones, the presence of private service providers in healthcare systems could reduce patient waiting times and government spending. However, Duckett [6], March and Shroyen [7] state that governments should be cautious about supporting the private sector in the healthcare system, owing that improper support may harm the system rather than be beneficial.

Collaboration between governments and private hospitals and likewise price definition takes place based on contract mechanisms. Hence, these mechanisms have an essential role in the design of efficient and fair healthcare systems. Nevertheless, there are considerable studies about the application of contract mechanisms and price competition in various areas of management science [8, 9], but in the health sector, they are not widely researched. As a few examples, Kreis and Schmidt [10] survey the impact of the application and evaluation of health technologies on the public through experiences in France, Germany, and the United Kingdom (UK). You and Kobayashi [11] analyze the effect of mandatory health insurance applications on healthcare expenditures in China. Chick and Mamani [12] discuss the implementation of contract mechanisms in the health sector and indicated that in case of supply uncertainty, the cost-sharing contract could provide good coordination for the flu vaccine supply chain.

In service systems, the scheme of payments affects performance and revenue [13, 14]. Therefore, performance-based contracts are becoming widespread in healthcare systems [15]. For example, to drop off preventable readmissions, some healthcare systems have begun to apply reimbursement schemes such as pay for performance or bundled payment instead of fee for service [16]. So and Tang [17] develop a mathematical model to examine the impact of reimbursement policy for drug usage. Guo et al. [18] examine the impact of these two reimbursement schemes on patient welfare, readmission rate, and waiting time in a public healthcare system.

Zhou et al. [19] denote that the subsidization of private institutions by the governments is an important matter in health reform to access a safe and effective healthcare system. Governments can increase their subsidization rates, allowing more patients to go to private institutions, but this ratio must be consistent with the objectives and resources in the healthcare system. Hoel and Sæther [20] state that in a healthcare system involving public and private institutions, private facilities should be subsidized or public services should be limited to a definite fee if there are capacity constraints. In the case of low subsidy rates, public institutions may face over-crowd, while the capacity of private ones may remain unused. Conversely, private institutions start to become crowded and the health expenditures of the government increase due to the high subsidy payments. For these reasons, finding the best ratio of subsidy is an important research topic. Qian et al. [21] analyze different subsidy mechanisms and report that differentiated price policies are beneficial for healthcare systems. Qian and Zhuang [22] state that subsidy policies can be utilized to direct patients with higher sensitivity to waiting time to private hospitals. Accordingly, the congestion in public hospitals decreases and the system becomes more balanced, and accordingly, the social utility increases [23, 24].

Competition situations in health systems have been thoroughly investigated. Acuna et al. [25] present two novel quantitative frameworks for negotiating with local and regional actors to reduce waiting lists in two-tier health systems. Their game-theoretic model can substantially reduce waiting lists. Acuna et al. [26] describe a two-level Nash-in-Nash method for modelling insurer, hospital, and patient interactions within the healthcare market. In order to account for horizontal hospital mergers and the proliferation of insurance networks, they model eight distinct scenarios. Using a game-theoretic queuing model that characterizes equilibrium points, Li and Zou [27] identify optimal decisions for contract mechanisms. Incorporating mechanism design into actual health insurance scenarios, Sun et al. [28] expand the concept of mechanism design. They present multi-strategy combination plans that align the interests of the healthcare system with patients. Yang et al. [29] examine three categories of hospital relationships using game theory: autonomous decision-making, regional medical information, and government-led collaboration. According to the findings, the proportion of transferred patients is one of the most influential factors on decision-making, cost-sharing, and hospital profit in health information exchange. Moscelli et al. [30] investigate the impact of hospital selection and competition on waiting time disparities. They examine variations in waiting times that are influenced by competition in specific geographic regions. They note that competition in the healthcare industry can lead to undesirable outcomes, such as lengthier, more disorganized waiting times. Niu et al. [31] investigate incentives for the exchange of health information to enhance decision-making coordination among competing institutions. They contemplate the profit and social responsibility objectives of institutions. They demonstrate that the exchange of health information is feasible when hospital profit margins on health examination fees are significantly reduced. Carvalho and Lodi [32] depict a theoretical and computational framework for a multiplayer kidney replacement program. They emphasize the significance of utilizing the concept of Nash equilibria as well as the necessity of additional research to measure the quality of transplants. Alvarado et al. [33] develop a game-theoretic model involving an insurer and a hospital. In an insurer-led Stackelberg game with rational parties, the optimal policy design and the hospital's most suitable response are obtained. Yaya et al. [34] merge the k-means algorithm and data envelopment analysis with a game-theoretic model to evaluate healthcare efficacy. Bisceglia et al. [35] investigate the interdependence of regional regulators using a game-theoretic model. In a simple and realistic framework, they investigate the interactions between price setters in various regions. Han et al. [36] evaluate the character of competition between two participating institutions in an insurer's plan. The insurer seeks to optimize the system's overall achievable quality by selecting a fee-for-service or package payment plan. The findings offer insurers directives for selecting payment models. Panayides et al. [37] apply a game-theoretic model involving three parties, including emergency services. Simulation can also be used to analyze the interaction between multiple hospitals [38, 39].

In this study, we design heuristic-based solution methods. Different heuristics and metaheuristics are employed in the literature for detecting Nash equilibrium points. This is not limited to healthcare system applications. Porter et al. [40] develop two simple search strategies to obtain Nash equilibrium points for two-player games as well as for n-player games. Konak and Kulturel-Konak [41] present a regret-based fitness assignment strategy for evolutionary algorithms. The method can detect Nash equilibria in a non-cooperative simultaneous combinatorial game theoretic model where enumerating all participants' decision alternatives is computationally intractable. Zaman et al. [42] suggest a co-evolutionary method for determining multiple Nash equilibria for n-player in continuous games. The technique is utilized to analyse the competitive electricity market. Lung and Dumitrescu [43] address the problem of determining the Nash equilibria for a multi-player normal-form game. They introduce a Nash-based domination relation that permits evolutionary search operators to converge on multiple solutions of the game. According to Elgers et al. [44], a pure Nash equilibrium is a well-known concept with numerous applications in the field of game theory. They note that it is complex to determine whether a pure Nash equilibrium exists in n-player normal-form games. They emphasize that the current exact methods for computing these quantities are impractical and restricted to small instances. Then, they employ three local search-based metaheuristics to solve the problem. Konak et al. [45] develop a new genetic algorithm, named the Nash equilibrium sorting genetic algorithm, to identify Nash equilibrium points for the competitive maximal covering location problem with two and three competitors. As stated by Wei et al. [46], there may not be an exact Nash equilibrium solution for game models; therefore, a solution method that approximates the Nash equilibrium points may be beneficial. They develop a multi-objective migratory bird optimization algorithm based on game theory, neighborhood operators and path relinking to enhance the search capability. Belabid et al. [47] combine a hybrid greedy algorithm with the concept of Nash equilibrium and genetic operators. In healthcare systems, algorithms similar to those described in this paragraph can have considerable applications.

In this study, a healthcare system that contains public and private hospitals with different characteristics is modeled. In the literature, there are studies on the models that include one public and one private hospital [23, 24]. Different from them, we unify all hospitals in the system under two private hospitals and one public hospital. We assume the hospitals interact with each other and their decisions about pricing affect the preferences of patients to choose them. In general, demand forecasting means predicting future demand based on previous data, which is an important topic in healthcare management [48, 49, 50]. In this study, the probability of selecting a hospital is determined by factors such as waiting times, service quality, and pricing. We show that the models of this study can easily be generalized for the ones with more hospitals. Thus the models in these studies provide the basis for the systems involving multiple hospitals. Heuristic solution methods are used in this study.

Different from many previous studies, we demonstrate that contract mechanisms can boost social utility, in a system in which there is competition between private hospitals. In addition to the state where there is no contract between the government and the private hospitals, the model in which the government proposes a contract to both of the private hospitals is analyzed. In case of rejection of the contract, they make their own pricing decisions. In the model, the Nash equilibrium points for the private hospitals are searched using the game theory techniques. After the introduction of the models, in the experimental results subsection, a comparison is made between the models based on the defined social utility. In this way, we model the behavior of strategic patients in a region to choose one of the nearby hospitals.

The outline of this work can be summarized as follows:

- We model a real regional healthcare system that contains one public hospital and two private hospitals with different characteristics.
- Distinct from many prior studies, we investigate the influence of contract mechanisms on public expenditures, patient decisions, and the profit of private hospitals within the system.
- In addition to the state where there is no contract between the government and the private hospitals, we analyze the model in which the government proposes a contract to both the private hospitals.
- We explore the Nash equilibrium points for the pricing decisions of the private hospitals using the game theory techniques.
- We demonstrate that it is possible to raise both the social utility and the profits of private hospitals by applying suitable contract mechanisms.
- We generalize the model and the solution methods for the case involving multiple public and private hospitals.

The similarities and differences between this study and those found in the literature are outlined in Table 1.

Table 1
A comparison between studies in the literature and our work

	Patient's hospital choice modeling	Contracting/Pricing	Multiple hospitals	Competition in hospitals	Staffing	Capacity sharing health system	Hospital mergers
Our study	√	√	√	√			
Smith et al. [2]	√						
Qin and Prybutok [3]	√						
Jiang et al. [15]		√					
Andritsos and Tang [16]		√					
So and Tang [17]		√					
Guo et al. [18]		√					
Zhou et al. [19]	√	√					
Qian et al. [21]		√					
Kaya et al. [23]	√	√					
Teymourifar et al. [24]	√	√			√		
Acuna et al. [25]	√		√	√		√	
Acuna et al. [26]	√		√	√			√
Li and Zou [27]		√			√		
Sun et al. [28]		√					
Yang et al. [29]			√	√			
Moscelli et al. [30]		√		√	√		
Niu et al. [31]		√		√			
Carvalho and Lodi [32]			√	√			
Alvarado et al. [33]				√			
Yaya et al. [34]			√	√			
Bisceglia et al. [35]		√		√			
Han et al. [36]		√	√	√			
Panayides et al. [37]			√	√			
Kaya et al. [38]	√	√	√		√		
Balan and Brand [39]			√				√

In the next sections of the study, at first, the system and models are explained. Then in the results section, a comparison is made between the models based on social utility, which is defined as a multi-objective function consisting of the utility that the patients obtain and also the spending of the government.

2. Description of the System

The used notations in this section are outlined in Table 2. As mentioned in the introduction, the dealt problem is from a real healthcare system, in which there are one public and two private hospitals in a region i.e. n=2 and m=1. The primary subject of the problem is to investigate the impacts of contract mechanisms to enhance social benefit in this system.

Table 2
Used notations

Notation	Description	Type
n	Number of private hospitals	Parameter
m	Number of public hospitals	Parameter
c_{d_j}	Cost of care for each patient in the j-th public hospital	Parameter
b_{d_j}	Amount paid by the patients to the j-th public hospital	Parameter
c_{o_i}	Cost of care for each patient in the i-th private hospital	Parameter
r_i	Total price of service in the i-th private hospital	Decision variable
b_{o_i}	Amount paid by the patients to the i-th private hospital	Variable
s_{o_i}	Subsidy payment made by the government to the i-th private hospital for each patient	Decision variable
k_{o_i}	Cost of unit capacity in the private hospital	Parameter
k_{d_j}	Cost of unit capacity in the j-th public hospital	Parameter
q_{o_i}	Service quality level in the i-th private hospital	Parameter
q_{d_j}	Service quality level in the j-th public hospital	Parameter
λ	Total arrival rate of all hospitals' patients	Parameter
p_{o_i}	Probability of selecting the i-th private hospital by a patient	Variable
p_{d_j}	Probability of selecting the j-th public hospital by a patient	Variable
ca_{o_i}	Capacity of the i-th private hospital	Parameter
ca_{d_j}	Capacity of the j-th public hospital	Parameter
μ_{o_i}	Service rate per unit capacity in the i-th private hospital	Parameter
μ_{d_j}	Service rate per unit capacity in the j-th public hospital	Parameter
H_d	Amount of expenditure made by the government (public expenditure)	Variable
w_{o_i}	Average waiting time in the i-th private hospital	Variable
w_{d_j}	Average waiting time in the j-th public hospital	Variable
k	Price sensitivity of a patient	Variable
A_n	Thresholds of k	Variable
k^{max}	Upper bound of k	Parameter
$F_k(x)$	Cumulative probability function of k	Definition
I_{min}	Minimum income level of patients	Definition
I_{max}	Maximum income level of patients	Definition
U_1	Total utility received by all patients	Objective function
U_2	Average government expenses per patient	Objective function
U	Total social utility	Objective function
Z_{o_i}	The profit of the i-th private hospital	Objective function
T	Computational time	Output of the solution algorithm

In the current state of the system, there is a contract based on fixed prices between the government and private hospitals, and also the government pays a definite subsidy to the patients that receive service from a private hospital. In this case, as seen in Fig. 1, the service fee, the average waiting time and the perceived quality level in the i-th private hospitals are denoted as b_{o_i}, w_{o_i} and q_{o_i}, $\forall i = 1, 2$, which are b_d, w_d and q_d in the public one. We assume that quality has a favorable impact on the utility while the influence of price and waiting time is adverse. In the literature, a utility function has been defined for similar models [23, 24, 38]. In this study, taking advantage of them we define the utility of the patients in the i-th private hospital as $\frac{q_{o_i}}{w_{o_i}} - k{b_{o_i}}^2$, while it is $\frac{q_d}{w_d} - kb_d^2$ in the public hospital. k is the price sensitivity of the patients and we assume $k \geq 0$, otherwise, the negative effect of price in the utility function is omitted.

We suppose that a strategic patient makes a choice between the hospitals according to the utility she will receive in the chosen hospital.

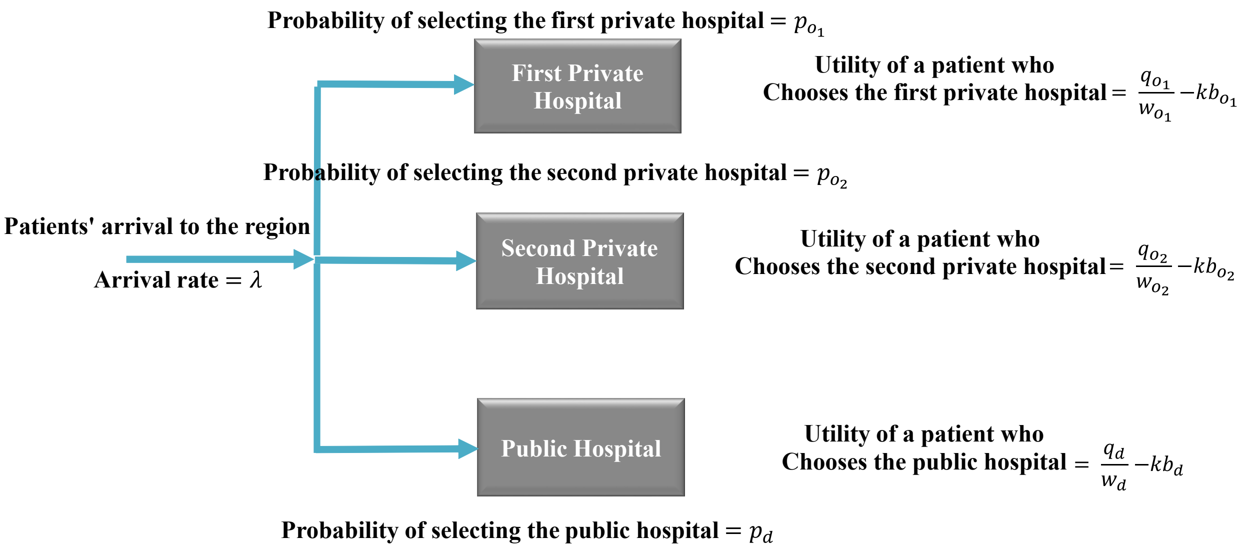

Fig. 1. A strategic patient chooses one of the hospitals based on the utility she gets.

A strategic patient selects the first private hospital if Eq. 2.1 is valid.

$$\frac{q_{o_1}}{w_{o_1}} - kb_{o_1}^2 \geq \frac{q_d}{w_d} - kb_d^2 \quad and \quad \frac{q_{o_1}}{w_{o_1}} - kb_{o_1}^2 \geq \frac{q_{o_2}}{w_{o_2}} - kb_{o_2}^2 \tag{2.1}$$

We assume that $b_{o_1} > b_{o_2} > b_d$. Based on our assumption, to select the first one among the private hospitals by a patient, her utility in this hospital would be more than the other, means $\frac{q_{o_1}}{w_{o_1}} - kb_{o_1}^2 \geq \frac{q_{o_2}}{w_{o_2}} - kb_{o_2}^2$ and accordingly $\frac{q_{o_1}}{w_{o_1}} - \frac{q_{o_2}}{w_{o_2}} \geq k(b_{o_1}^2 - b_{o_2}^2)$. Since the right-hand side of the inequality is non-negative, we conclude that $\frac{q_{o_1}}{w_{o_1}} \geq \frac{q_{o_2}}{w_{o_2}}$, which can be generalized as $\frac{q_{o_1}}{w_{o_1}} \geq \frac{q_{o_2}}{w_{o_2}} \geq \frac{q_d}{w_d}$.

We define $\frac{q_{o_i}}{w_{o_i}}$ and $\frac{q_d}{w_d}$ as the satisfaction levels in the i-th private and public hospital. Thus the previous paragraph means that if the price of service in a hospital is higher than the other, it has to provide more satisfaction, otherwise it is not preferred.

k has an important effect on the decision of patients. It is a non-negative random variable with an upper bound of k_{max}. We suppose that if it is low for a patient, she gives more importance to quality than price and vice versa. In addition, we assume that there is an inverse relationship between k and the income level of patients, as seen in Fig. 2.

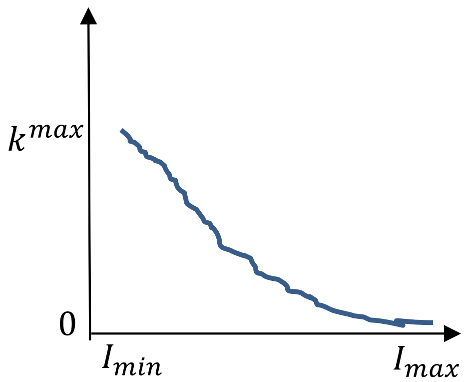

Fig. 2. It is supposed that there is an inverse relationship between k and the income level of patients.

In Fig. 2, I_{min} and I_{max} are the minimum and maximum income levels of patients, while k^{max} is the upper bound of the price sensitivity of patients.

Each patient has a specific value of k. We assume that there is a threshold shown as A in Fig. 3 and if for a patient $k \leq A$ she chooses the private hospitals.

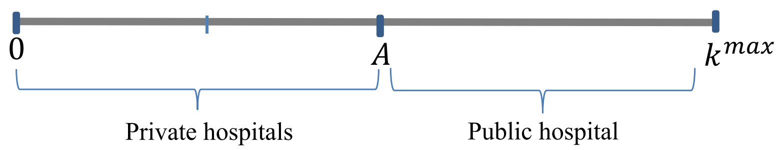

Fig. 3. The price sensitivity threshold of patients influences hospital selection decisions.

In Fig. 3, A is the threshold of the price sensitivity of patients to choose private hospitals. k^{max} is the upper bound of the price sensitivity of patients.

We suppose that if for a patient $k \leq A$, then $\frac{q_{o_i}}{w_{o_i}} - kb_{o_i}^2 \geq \frac{q_d}{w_d} - kb_d^2$, $\forall i = 1, 2$. Therefore, from Eq. 2.1, if k for a patient is as in Eq. 2.2, she chooses the first private hospital.

$$k \leq \frac{\frac{q_{o_1}}{w_{o_1}} - \frac{q_{o_2}}{w_{o_2}}}{b_{o_1}^2 - b_{o_2}^2} \tag{2.2}$$

It is assumed that the average times in the hospitals are according to the M/M/1 queueing model, then they can be calculated as in Eqs. 2.3 and 2.4 [23, 38].

$$w_{o_1} = \frac{1}{ca_{o_1}\mu_{o_1} - \lambda p_{o_1}} \tag{2.3}$$

$$w_{o_2} = \frac{1}{ca_{o_2}\mu_{o_2} - \lambda p_{o_2}} \tag{2.4}$$

where μ_{o_i} and ca_{o_i} are the service rate and the capacity of the i-th private hospital, respectively. We suppose that a value of k is attributed to each patient, which is a random variable with a cumulative distribution function, $F_k(x)$ [23, 38].

So, p_{o_1} is as in Eq. 2.5.

$$p_{o_1} = F_k(\frac{\frac{q_{o_1}}{w_{o_1}} - \frac{q_{o_2}}{w_{o_2}}}{b_{o_1}^2 - b_{o_2}^2}) \tag{2.5}$$

Using Eqs. 2.3 and 2.4, p_{o_1} can be written more clearly as in Eq. 2.6.

$$p_{o_1} = F_k(\frac{q_{o_1}(ca_{o_1}\mu_{o_1} - \lambda p_{o_1}) - q_{o_2}(ca_{o_2}\mu_{o_2} - \lambda p_{o_2})}{b_{o_1}^2 - b_{o_2}^2}) \tag{2.6}$$

In a similar way, p_{o_2} is calculated as in Eq. 2.7.

$$p_{o_2} = F_k(\frac{q_{o_2}(ca_{o_2}\mu_{o_2} - \lambda p_{o_2}) - q_d(ca_d\mu_d - \lambda p_d)}{b_{o_2}^2 - b_d^2}) - p_{o_1} \tag{2.7}$$

U_1, the total utility received by all patients is calculated as in Eq. 2.8.

$$U_1 = \int_0^{A_1} (\frac{q_{o_1}}{w_{o_1}} - kb_{o_1}^2) f(k)dk + \int_{A_1}^{A_2} (\frac{q_{o_2}}{w_{o_2}} - kb_{o_2}^2) f(k)dk + \int_{A_2}^{k^{max}} (\frac{q_d}{w_d} - kb_d^2) f(k)dk \tag{2.8}$$

We supposed k is uniformly distributed between 0 and k^{max}. As seen in Fig. 4, A_1 and A_2 are the critical values of k for the patients to select the hospitals, which are calculated as $A_1 = k^{max} p_{o_1}$ and $A_2 = k^{max}(p_{o_1} + p_{o_2}) = k^{max} p_o$.

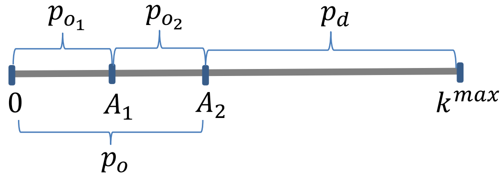

Fig. 4. The threshold of patients' price sensitivity when there are two private hospitals

In Fig. 4, p_{o_1}, p_{o_2} and p_d are the probabilities of selecting the first and second private and public hospitals, respectively. A_1 and A_2 are the critical values of the price sensitivity of patients to select hospitals. k^{max} is the upper bound of the price sensitivity of patients.

H_d, the public expenditure is as in Eq. 2.9.

$$H_d = \lambda p_d(c_d - b_d) + \lambda(p_{o_1} s_{o_1} + p_{o_2} s_{o_2}) + k_d ca_d^2 \tag{2.9}$$

Also, U_2, the average public expenditure is defined as in Eq. 2.10.

$$U_2 = \frac{H_d}{\lambda} \tag{2.10}$$

The social utility, which is defined as in Eq. 2.11 consists of U_1 and U_2.

$$U = \alpha_1 U_1 - \alpha_2 U_2 \tag{2.11}$$

The profit functions of the private hospitals are as in Eqs. 2.12 and 2.13.

$$Z_{o_1} = (r_1 - c_{o_1})p_{o_1}\lambda - k_{o_1} ca_{o_1}^2 \tag{2.12}$$

$$Z_{o_2} = (r_2 - c_{o_2})p_{o_2}\lambda - k_{o_2} ca_{o_2}^2 \tag{2.13}$$

Generalization of the model for *n* private and *m* public hospitals

The described model can be easily generalized to the case where the system contains n private and m public hospitals. We suppose that $b_{d_j} = b_d$, $w_{d_j} = w_d$ and $q_{d_j} = q_d$, $\forall j = 1, 2, ..., m$. Therefore, there is no competition among public hospitals and it does not matter which one is chosen. Thus, all public hospitals can be unified under one and as seen in Fig. 5, the system can be modeled as n private hospitals and one public hospital. We also assume that $b_{o_1} > b_{o_2} > ... > b_{o_n} > b_{d_j} = b_d$, then it can be concluded that $\frac{q_{o_1}}{w_{o_1}} \geq \frac{q_{o_2}}{w_{o_2}} \geq ... \geq \frac{q_{o_n}}{w_{o_n}} \geq \frac{q_{d_j}}{w_{d_j}} = \frac{q_d}{w_d}$, $\forall j = 1, 2, ..., m$.

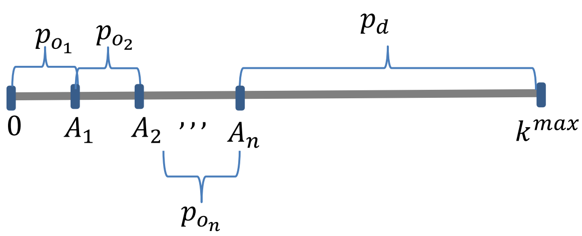

Fig. 5. A_i, $\forall i = 1, 2, ..., n$ are the thresholds of patients' price sensitivity to select the i-th private hospitals.

In Fig. 5, p_{o_i} and p_d are the probabilities of selecting the i-th private hospital and all public hospitals. A_n and k^{max} are the thresholds and the upper bound of the price sensitivity of patients.

p_{o_1} is as in Equation 2.6. p_{o_i}, $\forall i = 2, ..., n-1$, and p_{o_n} are as in Equations 2.14 and 2.15, respectively.

$$p_{o_i} = F_k\left(\frac{q_{o_i}(ca_{o_i}\mu_{o_i} - \lambda p_{o_i}) - q_{i-1}(ca_{i-1}\mu_{i-1} - \lambda p_{i-1})}{b_{o_i}^2 - b_{i-1}^2}\right) - \sum_{ii=1}^{i-1} p_{o_{ii}}, \ \forall i = 2, ..., n-1 \tag{2.14}$$

$$p_{o_n} = F_k\left(\frac{q_{o_n}(ca_{o_n}\mu_{o_n} - \lambda p_{o_n}) - q_d(ca_d\mu_d - \lambda p_d)}{b_{o_n}^2 - b_d^2}\right) - \sum_{i=1}^{n-1} p_{o_i} \tag{2.15}$$

U_1, H_d, and Z_{o_i} are as in Equations 2.16, 2.17, and 2.18, respectively.

$$U_1 = \int_0^{A_1} (\frac{q_{o_1}}{w_{o_1}} - kb_{o_1}^2) f(k)dk + \int_{A_1}^{A_2} (\frac{q_{o_2}}{w_{o_2}} - kb_{o_2}^2) f(k)dk + ... + \int_{A_{n-1}}^{A_n} (\frac{q_{o_{n-1}}}{w_{o_{n-1}}} - kb_{o_{n-1}}^2) f(k)dk + \int_{A_n}^{k^{max}} (\frac{q_d}{w_d} - kb_d^2) f(k)dk \tag{2.16}$$

$$H_d = \lambda p_d(c_d - b_d) + \lambda(\sum_{i=1}^{n} p_{o_i} s_{o_i}) + k_d ca_d^2 \tag{2.17}$$

$$Z_{o_i} = (r_i - c_{o_i}) p_{o_i} \lambda - k_{o_i} ca_{o_i}^2 \tag{2.18}$$

3. Model NC: No Contract Between the Government and the Private Hospital

In this model, which is summarized in Fig. 6, as in the base case problem, there are two private hospitals and one public hospital. Each private hospital determines its own examination fees to maximize its profit. The examination fee, the average waiting time and the perceived quality level are r_i, w_{o_i} and q_{o_i} in the i-th private hospital, while in the public hospital they are b_d, w_d and q_d. Thus, the utility of the patients is $\frac{q_{o_i}}{w_{o_i}} - kr_i^2$ in the i-th private hospital and $\frac{q_d}{w_d} - kb_d^2$ in the public hospital. As it is described in Eq. 3.1, a strategic patient selects the first private hospital if she earns more utility there.

$$\frac{q_{o_1}}{w_{o_1}} - kr_1^2 \geq \frac{q_d}{w_d} - kb_d^2 \quad and \quad \frac{q_{o_1}}{w_{o_1}} - kr_1^2 \geq \frac{q_{o_2}}{w_{o_2}} - kr_2^2 \tag{3.1}$$

As illustrated in Fig. 3, we presume that if for a patient $k \leq A$, then $\frac{q_{o_i}}{w_{o_i}} - kb_{o_i}^2 \geq \frac{q_d}{w_d} - kb_d^2$, $\forall i = 1, 2$. Therefore, for a strategic patient to choose the first private hospital, Equation 3.2 is sufficient.

$$\frac{q_{o_1}}{w_{o_1}} - kr_1^2 \geq \frac{q_{o_2}}{w_{o_2}} - kr_2^2 \tag{3.2}$$

No contract between the government and the private hospital

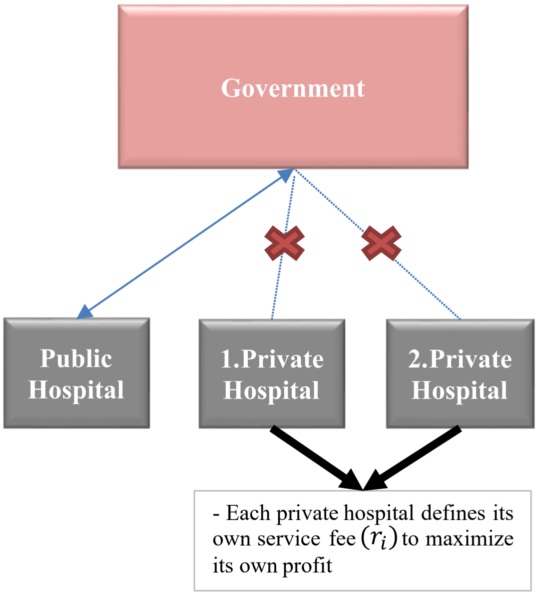

Fig. 6. Model NC

So, the patient chooses the first private hospital, if the value of k is as in Eq. 3.3.

$$k \leq \frac{\frac{q_{o_1}}{w_{o_1}} - \frac{q_{o_2}}{w_{o_2}}}{r_1^2 - r_2^2} \tag{3.3}$$

p_{o_1} is as in Eq. 3.4, where $F_k(x)$ is the cumulative probability function of k.

$$p_{o_1} = F_k(\frac{q_{o_1}(ca_{o_1}\mu_{o_1} - \lambda p_{o_1}) - q_{o_2}(ca_{o_2}\mu_{o_2} - \lambda p_{o_2})}{r_1^2 - r_2^2}) \tag{3.4}$$

In a similar way, p_{o_2} is written as in Eq. 3.5.

$$p_{o_2} = F_k(\frac{q_{o_2}(ca_{o_2}\mu_{o_2} - \lambda p_{o_2}) - q_d(ca_d\mu_d - \lambda p_d)}{r_2^2 - b_d^2}) - p_{o_1} \tag{3.5}$$

It is clear that the probability of choosing the public hospital by a patient is $p_d = 1 - p_{o_1} - p_{o_2}$.

$$H_d = \lambda p_d(c_d - b_d) + k_d c a_d^2 \tag{3.6}$$

In this model, U_1 is defined in Eq. 3.7.

$$U_1 = \int_0^{A_1} (\frac{q_{o_1}}{w_{o_1}} - kr_1^2) f(k)dk + \int_{A_1}^{A_2} (\frac{q_{o_2}}{w_{o_2}} - kr_2^2) f(k)dk + \int_{A_2}^{k^{max}} (\frac{q_d}{w_d} - kb_d^2) f(k)dk \tag{3.7}$$

U_2 and U are as in Eqs. 2.10 and 2.11 and also we assume that k is uniformly distributed between 0 and k^{max}.

In this model, private hospitals attempt to maximize their profits by defining appropriate examination fees. So this model consists of two problems; in the first problem, the profit of the first hospital, and in the second one the profit of the second hospital are to be maximized. The problem for the first hospital is defined as in Eq. 3.8.

$$Max_{r_1} \; Z_{o_{1nc}} = \lambda p_{o_1}(r_1 - c_{o_1}) - k_{o_1} ca_{o_1}^2 \tag{3.8}$$

The problem of the second hospital is defined in Eq. 3.9.

$$Max_{r_2} \; Z_{o_{2nc}} = \lambda p_{o_2}(r_2 - c_{o_2}) - k_{o_2} ca_{o_2}^2 \tag{3.9}$$

The constraints of the model are defined in Eqs. 3.10 to 3.19.

$$p_{o_1} \leq 1 \tag{3.10}$$
$$p_{o_1} \geq 0 \tag{3.11}$$
$$\lambda p_{o_1} \leq ca_{o_1}\mu_{o_1} \tag{3.12}$$

$$p_{o_2} \leq 1 \tag{3.13}$$
$$p_{o_2} \geq 0 \tag{3.14}$$
$$\lambda p_{o_2} \leq ca_{o_2}\mu_{o_2} \tag{3.15}$$

$$\lambda p_d \leq ca_d\mu_d \tag{3.16}$$
$$r_1, r_2 \geq 0 \tag{3.17}$$
$$r_1 \geq r_2 \tag{3.18}$$
$$\frac{q_{o_1}}{w_{o_1}} \geq \frac{q_{o_2}}{w_{o_2}} \tag{3.19}$$

As seen in Fig. 7, the service fees in the private hospitals begin from their minimum points, which are the cost of care for each patient and when one of the private hospitals grows the fee, the other one also raises its own fee. The equilibrium occurs at the point where the fees intersect after they become stable.

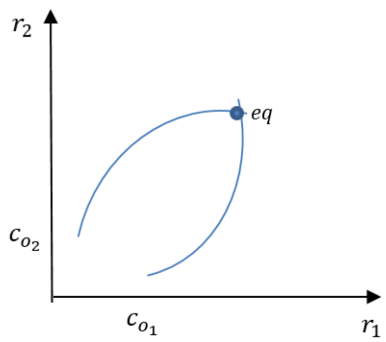

Fig. 7. Equilibrium point of r_1 and r_2 .

In Fig. 7, r_i, c_{o_i}, and eq are respectively the total price of service, cost of care for each patient in the i-th private hospital, and the equilibrium point.

Generalization of the Model NC for n private and m public hospitals

In this generalization, we assume that $r_{o_1} > r_{o_2} > ... > r_{o_n} > b_{d_j} = b_d$, then it can be concluded that $\frac{q_{o_1}}{w_{o_1}} \geq \frac{q_{o_2}}{w_{o_2}} \geq ... \geq \frac{q_{o_n}}{w_{o_n}} \geq \frac{q_{d_j}}{w_{d_j}} = \frac{q_d}{w_d}$, $\forall j = 1, 2, ..., m$. Hence, as shown in Fig. 5 all public hospitals can be unified under one. Thus, the system can be modeled as n private hospitals and one public hospital.

p_{o_1} is as in Equation 2.6. p_{o_i}, $\forall i = 2, ..., n-1$, and p_{o_n} are as in Equations 3.20 and 3.21, respectively.

$$p_{o_i} = F_k(\frac{q_{o_i}(ca_{o_i}\mu_{o_i} - \lambda p_{o_i}) - q_{i-1}(ca_{i-1}\mu_{i-1} - \lambda p_{i-1})}{r_{o_i}^2 - r_{i-1}^2}) - \sum_{ii=1}^{i-1} p_{o_{ii}}, \ \forall i = 2, ..., n-1 \tag{3.20}$$

$$p_{o_n} = F_k(\frac{q_{o_n}(ca_{o_n}\mu_{o_n} - \lambda p_{o_n}) - q_d(ca_d\mu_d - \lambda p_d)}{r_{o_n}^2 - b_d^2}) - \sum_{i=1}^{n-1} p_{o_i} \tag{3.21}$$

U_1 and H_d are respectively as in Equations 3.22 and 3.23.

$$\begin{aligned} U_1 = &\int_0^{A_1} (\frac{q_{o_1}}{w_{o_1}} - kr_{o_1}^2)f(k)dk + \int_{A_1}^{A_2} (\frac{q_{o_2}}{w_{o_2}} - kr_{o_2}^2)f(k)dk + ... + \int_{A_{n-1}}^{A_n} (\frac{q_{o_{n-1}}}{w_{o_{n-1}}} - kr_{o_{n-1}}^2)f(k)dk + \\ &\int_{A_n}^{k^{max}} (\frac{q_d}{w_d} - kb_d^2)f(k)dk \end{aligned} \tag{3.22}$$

$$H_d = \lambda p_d(c_d - b_d) + \lambda(\sum_{i=1}^{n} p_{o_i}s_{o_i}) + k_d ca_d^2 \tag{3.23}$$

As defined in Equation 3.24, the goal of private hospitals is to maximize their own profits and the decision variable is r_i.

$$Max_{r_i} \ \ Z_{o_i} = (r_i - c_{o_i})p_{o_i}\lambda - k_{o_i} ca_{o_i}^2 \tag{3.24}$$

The constraints of the model are defined in Eqs. 3.25 to 3.30.

$$p_{o_i} \leq 1 \tag{3.25}$$

$$p_{o_i} \geq 0 \tag{3.26}$$

$$\lambda p_{o_i} \leq ca_{o_i}\mu_{o_i} \tag{3.27}$$

$$r_i \geq 0 \tag{3.28}$$

$$r_1 \geq r_2 \geq ... \geq r_n \tag{3.29}$$

$$\frac{q_{o_1}}{w_{o_1}} \geq \frac{q_{o_2}}{w_{o_2}} \geq ... \geq \frac{q_{o_n}}{w_{o_n}} \tag{3.30}$$

Solution method

The designed solution method is summarized in Algorithm 1.

Algorithm 1: Solution method for Model NC

1 **Initialize** Best r_2 and r_2
2 **while** $Best\ r_2 \neq r_2$ **do**
3     $r_2 \leftarrow$ Best r_2;
4   **Search** on r_1;
5   **Find Best** r_1 that maximizes $Z_{o_1}(r_1, r_2)$;
6   **Search** on r_2;
7   **Find Best** r_2 that maximizes $Z_{o_2}(Best\ r_1, r_2)$;
8 **Calculate** $Z_{o_1}(Best\ r_1, Best\ r_2)$;
9 **Calculate** $Z_{o_2}(Best\ r_1, Best\ r_2)$;

While this study does not account for capacity decisions, they can be easily incorporated into Algorithm 1 by adding constraints to the searches at lines 4 and 6.

As seen in Algorithm 1, the search initializes at line 1. Then, each hospital defines a price to maximize its own profit after taking into consideration the other one's decision. This process continues until the equilibrium point, in which there is no gain for any changes in price for both of the hospitals. This stable point reaches when the condition in line 2 is realized. It should be noted that the search in rows 4 and 6 is accomplished between 0 and 500 with an increment value equal to one. These values are determined based on the advice of experts. Nonetheless, the same outcomes are acquired when the upper limit of the search is higher. Therefore this is a heuristic method and it yields near-optimal solutions instead of optimal.

We consider a region with one public hospital and two private hospitals. However, it should be noted that the system is not isolated, and patients may seek services in neighbouring regions. Therefore, the solution methods should be applicable to a greater number of hospitals. Algorithm 1 is easily generalizable to n private hospitals. Algorithm 2 is a generalization for three private hospitals.

Algorithm 2: Solution method for Model NC with three private hospitals

1 **Initialize** Best r_3, r_3 and r_2
2 **while** *Best* $r_3 \neq r_3$ **do**
3 $\quad r_3 \leftarrow$ Best r_3;
4 $\quad$ **Search** on r_1;
5 $\quad$ **Find Best** r_1 that maximizes $Z_{o_1}(r_1, r_2, r_3)$;
6 $\quad$ **Search** on r_2;
7 $\quad$ **Find Best** r_2 that maximizes $Z_{o_2}(Best\ r_1, r_2, r_3)$;
8 $\quad$ **Search** on r_3;
9 $\quad$ **Find Best** r_3 that maximizes $Z_{o_3}(Best\ r_1, Best\ r_2, r_3)$;
10 **Calculate** $Z_{o_1}(Best\ r_1, Best\ r_2, Best\ r_3)$;
11 **Calculate** $Z_{o_2}(Best\ r_1, Best\ r_2, Best\ r_3)$;
12 **Calculate** $Z_{o_3}(Best\ r_1, Best\ r_2, Best\ r_3)$;

Within Algorithm 2, constraints related to capacity decisions can be incorporated into the searches at lines 4, 6, and 8.

4. Model SC: Contract Mechanism Based on Subsidy Payments

In this model, which is summarized in Fig. 8, the private hospitals decide to accept or reject the contract proposed by the government according to their own profits. The contract includes the price of the examination and a subsidy. In this model, the choice of hospital for a strategic patient is made based on Eqs. 2.1, 2.2, 2.5 and 2.7. Therefore, H_d, U_1, U_2, U, Z_{o1} and Z_{o2} are defined as in Eqs. 2.8 – 2.13. The generalization of p_{o_i} for the case where there are n private and m public hospitals in the system is as in Equations 2.14 and 2.15. U_1, H_d, and Z_{o_i} are as in Equations 2.16, 2.17, and 2.18, respectively.

In this model, first of all, the profits of the private hospitals are calculated for the case that none of them accepts the contract, which is indicated as Reject- Reject (RR) in Table 3. The profits of the private hospitals are calculated according to the price and subsidy proposed by the government. Each private hospital compares its own profit with RR state; if the contract provides more profit, the hospital accepts it. Otherwise, by rejecting the contract, the hospital defines its own service fee.

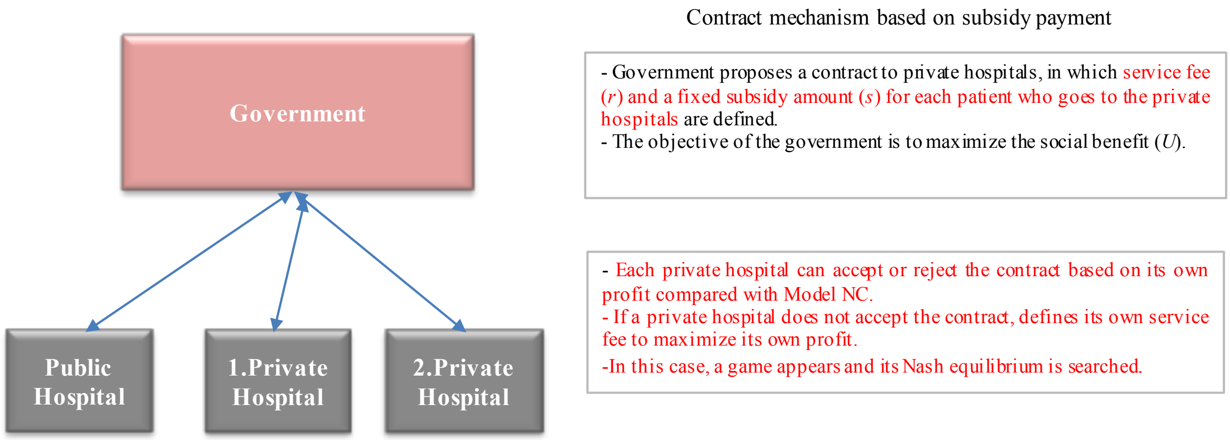

Fig. 8. Model SC

Table 3
Profit of private hospitals and social utility in different regions

		Second private hospital	
		Reject	Accept
First private hospital	Reject	$Z_{o_{1_{RR}}}, Z_{o_{2_{RR}}}, U_{RR}$	$Z_{o_{1_{RA}}}, Z_{o_{2_{RA}}}, U_{RA}$
	Accept	$Z_{o_{1_{AR}}}, Z_{o_{2_{AR}}}, U_{AR}$	$Z_{o_{1_{AA}}}, Z_{o_{2_{AA}}}, U_{AA}$

Solution method

In the algorithm implemented in the MATLAB software, $Z_{o_{1_{RR}}}$ and $Z_{o_{2_{RR}}}$, which are the profits that the two private hospitals obtain when they reject the contract are calculated. Then, the profits are computed for the cases summarized below, and subsequently, the Nash equilibrium point is searched.

Reject-Reject (RR) state: if $Z_{o_{1_{RR}}} > Z_{o_{1_{RA}}}$ and $Z_{o_{2_{RR}}} > Z_{o_{2_{AR}}}$ then both hospitals reject the contract.

Accept-Accept (AA) state: if $Z_{o_{1_{RR}}} > Z_{o_{1_{AR}}}$ and $Z_{o_{2_{RR}}} > Z_{o_{2_{RA}}}$ then both hospitals accept the contract.

Reject-Accept (RA) state: if $Z_{o_{1_{RA}}} > Z_{o_{1_{AA}}}$ and $Z_{o_{2_{RA}}} > Z_{o_{2_{RR}}}$ then the first hospital rejects and the second one accepts the contract.

Accept-Reject (AR) state: if $Z_{o_{1_{AR}}} > Z_{o_{1_{RR}}}$ and $Z_{o_{2_{AR}}} > Z_{o_{2_{AA}}}$ then the second hospital rejects and the first one accepts the contract.

After determining the Nash equilibrium point, the relevant social utility is also calculated. The solution method is summarized in Algorithm 3.

Algorithm 3: Solution method for Model SC

1 **Search** on r and s, and **Calculate** U and Z_{o_i} in states of RA, AR, AA and RR as:
2 Run Algorithm 1;
3 **foreach** *r and s* **do**
4 RA (1. private hospital rejects, while 2. private hospital accepts);
5 **Find** Best r_1 that maximizes $Z_{o1}(r_1, r, s)$;
6 **Calculate** $Z_{o1_{RA}}(Best\ r_1, r, s)$;
7 **Calculate** $Z_{o2_{RA}}(Best\ r_1, r, s)$;
8 **Calculate** $U_{RA}(Best\ r_1, r, s)$;
9 AR (1. private hospital accepts, while 2. private hospital rejects);
10 **Find** Best r_2 that maximizes $Z_{o2}(r, s, r_2)$;
11 **Calculate** $Z_{o1_{AR}}(Best\ r, s, r_2)$;
12 **Calculate** $Z_{o2_{AR}}(Best\ r, s, r_2)$;
13 **Calculate** $U_{AR}(Best\ r, s, r_2)$;
14 AA (both private hospitals accept);
15 **Calculate** $Z_{o1_{AA}}(Best\ r, s)$;
16 **Calculate** $Z_{o2_{AA}}(Best\ r, s)$;
17 **Calculate** $U_{AA}(Best\ r, s)$;
18 **if** $Z_{o1_{AA}} > Z_{o1_{RA}}$ *and* $Z_{o2_{AA}} > Z_{o2_{AR}}$ **then**
19 Equilibrium point is AA;
20 **Calculate** U_{AA};
21 **else if** $Z_{o1_{RR}} > Z_{o1_{AR}}$ *and* $Z_{o2_{RR}} > Z_{o2_{RA}}$ **then**
22 Equilibrium point is RR;
23 **Calculate** U_{RR};
24 **else if** $Z_{o1_{RA}} > Z_{o1_{AA}}$ *and* $Z_{o2_{RA}} > Z_{o2_{RR}}$ **then**
25 Equilibrium point is RA;
26 **Calculate** U_{RA};
27 **else if** $Z_{o1_{AR}} > Z_{o1_{RR}}$ *and* $Z_{o2_{AR}} > Z_{o2_{AA}}$ **then**
28 Equilibrium point is AR;
29 **Calculate** U_{AR};
30 **Find** the Best U, the corresponding fees and related Equilibrium point;
31 **Calculated** Z_{o_1} and Z_{o_2};

In Algorithm 3, constraints tied to capacity decisions can be integrated into the searches on lines 5 and 10.

Similar to the procedure outlined in Algorithm 1, the solution approach summarized in Algorithm 3 is a heuristic method, that yields near-optimal solutions instead of optimal ones. The lower and upper bounds, as well as the increment of the search in lines 2, 4, and 9 are the same as the description after Algorithm 1.

Algorithm 3 can also be easily generalized for a system with n private and m public hospitals, as in Algorithm 4. We assume that there is no competition between public hospitals.

Algorithm 4: Solution method for Model SC with three private hospitals

1 **Search** on r and s, and **Calculate** U and Z_{o_i} in states of AAA, RRR, ARR, RAR, RRA, AAR, ARA, RAA:
2 Run Algorithm 2;
3 **foreach** r *and* s **do**
4   AAA (all private hospitals accept);
5   **Calculate** $Z_{o_{1_{AA}}}(Best\ r, s)$, $Z_{o_{2_{AA}}}(Best\ r, s)$, $Z_{o_{3_{AA}}}(Best\ r, s)$, $U_{AAA}(Best\ r, s)$;
6   ARR (1. private hospital accepts, while 2. and 3. private hospitals reject);
7   Run Algorithm 1 for 2. and 3. private hospitals;
8   **Find** Best r_2 and r_3 that maximize $Z_{o_2}(r, s, r_2, r_3)$ and $Z_{o_3}(r, s, r_2, r_3)$;
9   **Calculate** $Z_{o_{1_{ARR}}}(Best\ r, s, r_2, r_3)$, $Z_{o_{2_{ARR}}}(Best\ r, s, r_2, r_3)$, $Z_{o_{3_{ARR}}}(Best\ r, s, r_2, r_3)$, $U_{ARR}(Best\ r, s, r_2, r_3)$;
10   **Do** similar steps between lines 6 and 9 for the RAR and RRA states;
11   AAR (1. and 2. private hospitals accept, while 3. private hospital rejects);
12   **Find** Best r_3 that maximizes $Z_{o_3}(r, s, r_3)$;
13   **Calculate** $Z_{o_{1_{AAR}}}(Best\ r, s, r_3)$, $Z_{o_{2_{AAR}}}(Best\ r, s, r_3)$, $Z_{o_{3_{AAR}}}(Best\ r, s, r_3)$, $U_{AAR}(Best\ r, s, r_3)$;
14   **Do** similar steps between lines 11 and 13 for the ARA and RAA;
15   **Find** the state in which equilibrium occurs;
16 **Find** the Best U, the corresponding fees and related Equilibrium point;
17 **Calculated** Z_{o_1}, Z_{o_2} and Z_{o_3};

Capacity decisions can be added to Algorithm 4 as constraints on the searches in lines 8 and 12.

Algorithms 1-4 primarily consist of basic 'if' conditions, 'for' loops, and 'while' loops. Thus, it can be inferred that these algorithms are not complex and they can be implemented easily.

5. Experimental Results

In this section, the numerical results of the models are presented. The used parameters are summarized in Table 4, which are taken from a district of Eskişehir province in Turkey. As mentioned earlier, there are two private hospitals and one public hospital in the district. The data was collected between 2015-2019 from the system.

Service quality levels were obtained through a questionnaire filled out by 250 respondents in the hospitals. In the questionnaire, which is designed by the Turkish Ministry of Health, there are questions about the staff (nurses, doctors, etc.), facilities, and cleanliness in hospitals. More details about the procedure of data collection and the English translation of the questionnaire are available in [38].

Table 4
Values of base case parameters

$\lambda = 70,000$	$\alpha_1 = 0.01$	$\alpha_2 = 1$	$b_d = 5$	$k^{max} = 1$
$c_{o_1} = 20, c_{o_2} = 15, c_d = 10$	$k_{o_1} = 50,000, k_{o_2} = 40,000, k_d = 10,000$	$q_{o_1} = 0.7, q_{o_2} = 0.6, c_d = 0.5$	$t_{o_1} = 8, t_{o_2} = 8, t_d = 3$	$ca_{o_1} = 3, ca_{o_2} = 3, ca_d = 4$

The procedures specified in Algorithms are implemented in MATLAB. A system with an Intel Core i5 processor, 2.4 GHz and 12 GB of RAM is utilized. The obtained results are presented in Table 5, in which T is in seconds.

Table 5
Results obtained by the models with the base case parameters

Model		Hospital	r	s_o	p_o	w_o	Z	w_d	H_d	U_1	U_2	U	T
	Model NC	First private hospital	230	0	0.081	0.07	739281.16	5.69	447367.13	516.82	6.39	-1.22	< 1
		Second private hospital	176	0	0.098	0.08	744996.25						
Model SC	RR region	First private hospital	230	0	0.081	0.07	739281.16	5.69	447367.13	516.82	6.39	-1.22	6.57
		Second private hospital	176	0	0.098	0.08	744996.25						
	RA region	First private hospital	**272**	**0**	**0.070**	**0.06**	**784625.27**	**0.22**	**799935.97**	**1979.49**	**11.43**	**8.37**	
		Second private hospital	**117**	**34**	**0.155**	**0.13**	**745935.01**						
	AR region	First private hospital	117	34	0.141	0.11	508820.00	0.19	765520.00	2029.50	10.94	9.36	
		Second private hospital	134	0	0.089	0.07	381080.00						
	AA region	First private hospital	117	34	0.137	0.11	480970.00	0.13	1034700.00	2999.70	14.78	15.21	
		Second private hospital	117	34	0.121	0.09	506710.00						

Fig. 9 also summarizes the acquired results.

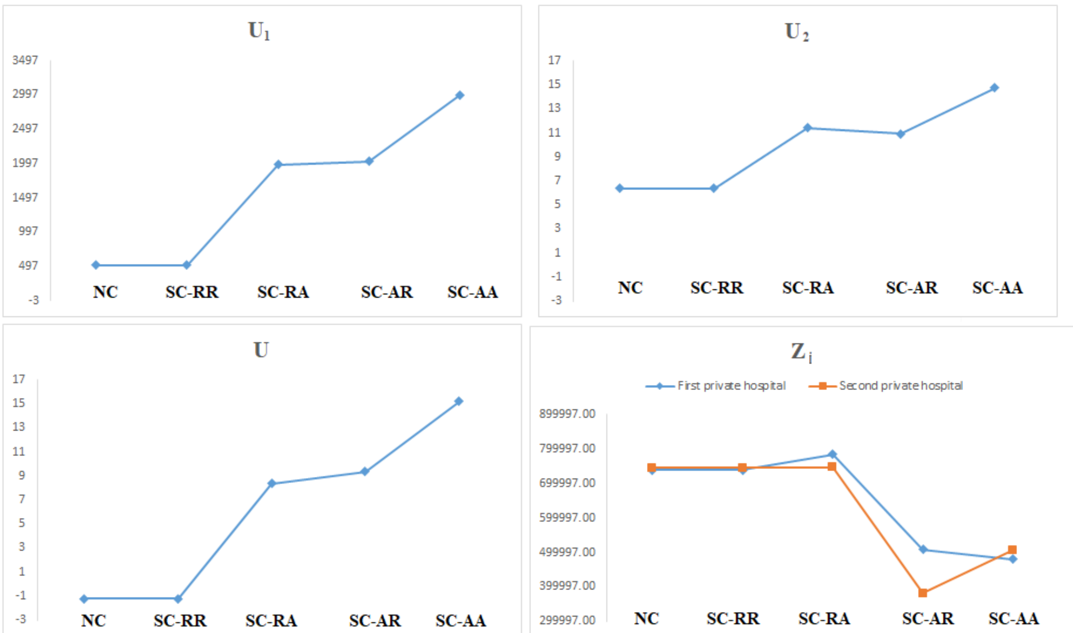

Fig. 9. Values of U_1, U_2, U and Z_i in the models

In Model NC, where there is no contract between the government and the private hospitals, the examination prices in the hospitals are defined as 230 TL and 176 TL. When the government proposes a contract to private hospitals under Model SC, the first private hospital rejects it, while the second one accepts it, that is to say, the equilibrium state occurs in the RA region. The corresponding result is bold in Table 5. Compared to Model NC, the social utility increases by 786% by Model SC, in which the second private hospital made a discount of approximately 33%, while the payment of patients decreases by 47% in the second hospital, and as a result the number of patients who select the second private hospital increases by about 6%. Model SC decreases the average waiting time in the public hospital, and as a result, U_1 improves 383%. Although the expenditure of the government increases, Model SC provides beneficial results for society, and even the profits of both private hospitals increase. As it is clear in Fig. 9, in the RA region where the equilibrium point is formed, the profits of both private patients are higher than in other regions.

Overall, it can be concluded that it is possible to raise both the social utility and the profits of private hospitals by applying suitable contract mechanisms.

To demonstrate the generalizability of the models and solution methods, we assume that there is a third private hospital in the region. We presume the following parameters for the third hospital: $c_{o_3} = 17$, $k_{o_3} = 45000$, $q_{o_3} = 0.6$, $t_{o_3} = 8$ and $ca_{o_3} = 3$. Other parameters are the same as in Table 4. The results of applying Algorithm 2 to solve this problem based on Model NC are shown in Table 6.

Table 6
Results for Model NC with three private hospitals

Model	Hospital	r	s_o	p_o	w_o	Z	w_d	H_d	U_1	U_2	U	T
	First private hospital	229	0	0.061	0.06	445642.39						
Model NC	Second private hospital	150	0	0.076	0.07	362865.26	0.20	430432.54	2132.97	6.15	15.18	< 1
	Third private hospital	136	0	0.090	0.07	341554.11						

As seen, the outputs can be interpreted similarly to the results of Model NC in Table 5. However, since the total probability of choosing private hospitals is higher than the case with two private hospitals, U_1 and, accordingly, U are higher. Nevertheless, it should be noted that the computation time is under one second. Therefore, it can be inferred that the solution methods are not complicated and can be applied to scenarios involving more hospitals.

5.1. *Sensitivity Analysis*

To show that similar consequences are obtained according to different parameters, the results of sensitivity analysis are provided in Table 7.

Table 7
Results obtained by the models according to different parameters

	Model NC	Model SC	
	U	U	Equilibrium Region
$\lambda = 75000$	-1.44	-0.79	RA
$\lambda = 65000$	13.18	21.12	RA
$c_{o_1} = 25$	-1.64	7.97	RA
$c_{o_2} = 20$	-1.58	8.12	RA
$c_{o_2} = 10$	-0.87	8.63	RA
$c_d = 15$	-5.33	4.49	RA
$c_d = 5$	2.88	12.24	RA
$k_d = 15000$	-2.37	7.22	RA
$k_d = 5000$	-0.08	9.51	RA
$q_{o_1} = 0.8$	0.01	9.72	RA
$q_{o_1} = 0.6$	-2.07	7.32	AA
$q_{o_2} = 0.7$	-1.34	8.97	AA
$q_{o_2} = 0.5$	-1.38	7.85	RA
$q_d = 0.6$	-1.65	9.50	RA
$t_{o_1} = 9$	-2.05	5.31	RA
$t_{o_1} = 7$	3.05	12.01	RA
$t_{o_2} = 9$	-2.21	5.45	RA
$t_{o_2} = 7$	0.74	11.93	RA
$t_d = 4$	-2.50	-3.40	RA
$ca_{o_1} = 3.5$	3.85	12.62	RA
$ca_{o_1} = 2.5$	-2.20	3.96	RA
$ca_{o_2} = 2.5$	-2.24	4.00	RA
$ca_d = 4.5$	18.90	26.44	RA
$ca_d = 3.5$	-1.08	-1.08	RR
$b_{td} = 10$	2.56	4.58	RA
$b_{td} = 0$	-5.22	4.59	RA
$k_{max} = 5$	-26.69	10.68	RA
$\alpha_1 = 0.02$	3.95	41.33	AA
$\alpha_1 = 0.005$	-3.81	-0.56	RA

If $\lambda = 65000$, in the equilibrium point, the social utility increases but the profit of private hospitals decreases. When $\lambda = 75000$, the social utility decreases, and in this case, while the profit of the first private hospital increases, the profit of the second private hospital reduces. In both states of $\lambda = 65000$ and 75000, the equilibrium point for Model SC is in the RA area, i.e. the first hospital rejects the contract and the second accepts it. The same equilibrium point (RA) is also obtained with parameters $c_{o_1} = 25$, $c_{o_2} = 10$ and $ca_{o_1} = 2.5$. However, if $ca_d = 3.5$ and $q_{o_1} = 0.6$, the equilibrium point is in the RR and AA areas, respectively.

The results of Table 7 are summarized in Fig. 10, where the green up arrow and red down arrow respectively indicate the boost and decline in the corresponding parameter. Equilibrium points are noted in blue.

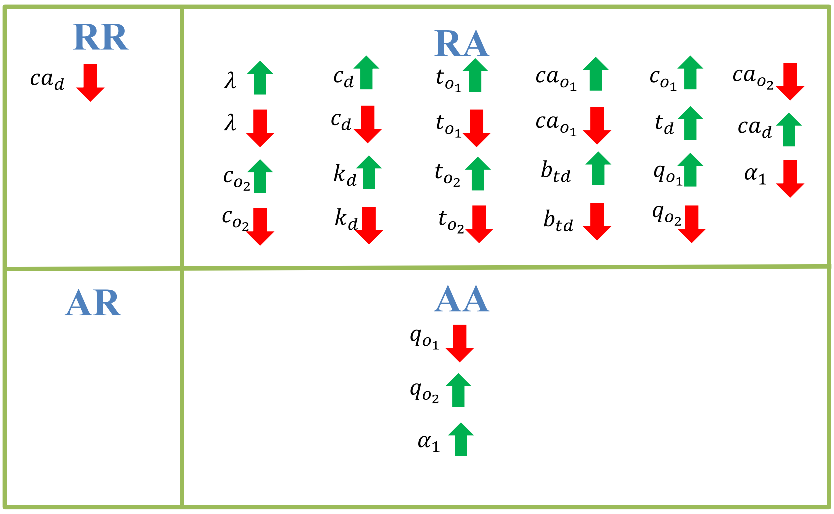

Fig. 10. Graphical summary of the sensitivity analysis.

In general, the highest social utility with the parameters summarized in Table 4 is provided by Model SC. The profit of both private hospitals in this model is higher than in Model NC.

6. Conclusion and Future Works

In this study, we cope with a problem from a real healthcare system and we develop models to solve it. The main topic of the problem is to analyze the effects of contract mechanisms to improve social utility. Although the impacts of many contract mechanisms are analyzed in the literature on supply chain management, there is not enough work on this topic in healthcare management studies. In the system, the pricing decisions are based on the contract mechanisms between the government and private hospitals, which affect the general state of the healthcare system owing to that they have significant effects on the preference of patients to choose a hospital.

According to the results obtained, contract mechanisms between the government and private hospitals are beneficial in terms of society. In the model where the government offers a contract to the private hospitals, in most of the obtained results according to different parameters, the first private hospital, i.e., the hospital providing higher quality service, does not accept the contract and determines its own price. In this case, even though the number of patients in the first private hospital, which defines a higher price than in the absence of a contract, decreases slightly, its profit increases. In this state, the second private hospital, which provides a quality of service between the public and the first private hospital, significantly reduces the price, supplies service for more patients, and increases its own profit.

The problem is from a regional healthcare system in the Eskişehir Province of Turkey, which contains two private hospitals and one public hospital. But the proposed models are generalized for multiple hospitals. In addition, the presented sensitivity analysis shows that the results are also valid for different parameters. The described healthcare system may exist in different countries. From managerial insight, the results can be summarized like this: governments can provide a more balanced healthcare system with appropriate contract mechanisms. In this case, with the enhancement in social utility, the profit of private hospitals that accept the contract also increases. Contract acceptance may require certain managerial decisions. Capacity levels may need to be redefined. In the solution methods, we explain where capacity decisions can be made. However, private hospitals may not always have adequate personnel or equipment to raise capacity, in which case capacity-sharing policies can be considered.

This study has some limitations. The capacity decisions of the hospitals have not been taken into account. Quality is defined as an exogenous variable. In real life, generally, there is a correlation between patients' perception of quality and waiting times. k, the price sensitivity of patients, is assumed to be uniformly distributed. Also, the sensitivity coefficient is defined only for the price. It should also be taken into account that country-specific situations, especially public policies, affect the competitive situation between hospitals. Public hospitals may receive financial incentives depending on the number of patients served; thus, service quality and waiting times influence demand even if competition does not determine prices. Instead of analytical methods, heuristic methods are used to find equilibrium points. In this case, it is not known how many equilibrium points exist. In future studies, we plan to design more general models by overcoming these limits.

Simulation can be a suitable tool for analyzing models involving more hospitals. In future studies, hybrid models using simulation and game theory will be designed. Besides, as seen in Table 1, potential research directions can be depicted as the models that analyze the competition between multiple hospitals by taking into account capacity, staffing, and districting factors.

Supporting Information

All implemented codes and obtained results are accessible via the corresponding author's email address.

Ethics Declarations

Conflict of interest

The authors declare no conflict of interest (financial or non-financial).

Human and animal rights

The research does not involve any data collected from human participants or experiments with animals.

Acknowledgments

This work is supported by the Scientific and Technological Research Council of Turkey (TUBITAK) under the grant number 115M185.

This work is supported by the Portuguese Foundation for Science and Technology (FCT) in the framework of the project CEECINST/00137/2018.

Financial support from Fundação para a Ciência e Tecnologia (through project UIDB/00731/2020) is gratefully acknowledged.

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