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Mini-Workshop: Lévy Processes and Related Topics in Modelling

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ABSTRACT. The focus of the meeting was on recent ongoing research and new ideas in the area of infinite divisibility and Lévy processes, with particular view to realistic modelling. As regards more applied aspects, work in mathematical finance, especially concerning modelling and measurement of volatility, figured prominently.

Mathematics Subject Classification (2000): 60Exx, 60Fxx, 60Gxx, 60Kxx, 62Mxx.

Introduction by the Organisers

The theory of infinite divisibility, Lévy processes and Lévy bases and its applications is an area of very active current interest. This trend is of relatively new origin, dating back around 10 years. In this period larger research conferences on “Lévy Processes and their Applications” have been held on a regular basis, about every 2 years and at different locations around Europe. In addition intermingled between these conferences there has been a number of more specialized workshops within the area. One of these was organized at the Isaac Newton Institute, Cambridge, 2 1/2 years ago, the organizers being Professor Neil Shephard and Ole E. Barndorff-Nielsen, on invitation by the Newton Institute. The format of that workshop where the focus was on new ideas and ongoing research, rather than on finished work, and where the presentations were rather short, in some cases even down to about 15 minutes, was found to be very stimulating and productive. The same format was largely followed at the Mini-Workshop under reporting here and was found again to be very productive. Although the workshop was somewhat specialized a rather wide range of topics were treated in the presentations and in informal discussion groups.

Half of the participants were junior researchers, visiting the MFO for the first, or in some cases the second, time. Like the more senior people they were delighted by the warm atmosphere and excellent conditions for research provided by the Institute. And by lucky coincidence, due to the weather the traditional midweek afternoon excursion was shifted to the Thursday afternoon which was the first day of the Fastnacht celebrations in Oberwolfach with the parade of the “coffee aunts”. This made for a rather unique and refreshing break in the otherwise intense programme.

The first talk of the workshop, by Philip Protter, presented an overview of issues in mathematical finance. The talks by Fred Espen Benth, Jean Jacod, Thilo Meyer-Brandis, Mark Podolskij, Robert Stelzer, and Viktor Todorov discussed modelling and inference on volatility and jumps for financial dynamics. A variety of theoretical results on Lévy processes were presented in talks by Søren Asmussen, Friedrich Hubalek, Andreas Kyprianou, Alexander Lindner and Tina Marquardt, while other talks, by Makoto Maejima, Víctor Pérez-Abreu, Jan Rosiński and Steen Thorbjørnsen concerned work on new aspects of infinite divisibility. Finally, the talk by Anna Amirdjanova treated problems in the theory of Gaussian processes.

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Abstracts

Multiple stochastic integrals and their applications to nonlinear filtering with persistent fractional Gaussian observation noise

ANNA AMIRDJANOVA

The first part of the talk is devoted to some recently discovered properties of multiple stochastic integrals defined with respect to general Gaussian random fields. Among them, a stochastic transfer principle, that relates multiple (say, n th order) Itô-type integrals $I_n^X(\cdot)$ with respect to a general d -parameter Gaussian random field X to multiple (n th order) stochastic integrals $I_n^Y(\cdot)$, defined with respect to a Volterra-type random field Y with respect to X , where

$$Y_{t_1, \dots, t_d} = \int_0^{t_1} \cdots \int_0^{t_d} \prod_{i=1}^d K_i(t_i, s_i) dX_{s_1, \dots, s_d},$$

was recently established in [2]. The principle represents an extension of the result, obtained in [5] for multiple integrals with respect to fractional Brownian motion, and asserts that

$$I_n^Y(f) = I_n^X \left(\Gamma_{K_1, \dots, K_d}^{(n)} f \right)$$

for a certain, explicitly identified, operator $\Gamma_{K_1, \dots, K_d}^{(n)}$. As a corollary, the following stochastic independence criterion (in the spirit of [6],[3]) is also obtained: For all f, g (such that $I_n^Y(f)$ and $I_m^X(g)$ are well-defined) the following conditions are equivalent:

- (i) $I_n^Y(f)$ and $I_m^X(g)$ are independent;
- (ii)

$$\int_{\mathbb{T} \times \mathbb{T}} g(\underline{t}_1, \dots, \underline{t}_{m-1}, \underline{u}) (\Gamma_{K_1, \dots, K_d}^{(n)} f)(\underline{s}_1, \dots, \underline{s}_{n-1}, \underline{v}) \gamma(d\underline{u}, d\underline{v}) = 0 \text{ a.e.,}$$

where γ is the covariance function of X ;

- (iii) $\text{Cov}([I_n^Y(f)]^2, [I_m^X(g)]^2) = 0$.

The second half of the talk is devoted to nonlinear filtering theory in the presence of persistent fractional Brownian motion observation noise. Existing stochastic evolution equations, satisfied by the normalized and unnormalized optimal filters, are discussed. Extensions of these results leading to development of nonlinear filtering theory of random fields, in the case when the observation noise is a fractional Wiener sheet, are suggested (as in [4]) and some open problems are presented. Expansions of the optimal filter, obtained in [1] for the one-parameter case, in terms of multiple fractional Itô and Stratonovich stochastic integrals, are discussed. Suboptimal filters, obtained when the series are truncated and the integrals are properly discretized, are shown to converge to the optimal filter. A numerical example, illustrating the method, is also presented. Finally, some ideas about open problems on nonlinear filtering of random fields in the presence of Volterra sheets are suggested.

REFERENCES

- [1] A. Amirdjanova, S. Chivoret (2006). New method for optimal nonlinear filtering of noisy observations by multiple stochastic fractional integral expansions. *Computers & Mathematics with Applications* **52**, 161–178.
- [2] A. Amirdjanova (2007). Stochastic transfer principle for multiple integrals with respect to Gaussian random fields with applications. Accepted for *Integration: Mathematical Theory and Applications*.
- [3] S. Chivoret, A. Amirdjanova (2006). Product formula and independence criterion for multiple Huang-Cambanis integrals. *Statistics & Probability Letters* **76**, 1255–1260.
- [4] M. Linn, A. Amirdjanova (2006). Nonlinear filtering of random fields in the presence of fractional Wiener sheet. Preprint.
- [5] V. Perez-Abreu, C. Tudor (2002). Multiple stochastic fractional integrals: a transfer principle for multiple stochastic fractional integrals. *Bol. Soc. Mat. Mexicana* **3**, 187–203.
- [6] J. Rosinski, G. Samorodnitsky (1999). Product formula, tails and independence of multiple stable integrals. *Contemp. Math.* **234**, 169–194.

Loss rates for two-sided reflected Lévy processes: representation and asymptotics

SØREN ASMUSSEN

Let $X = \{X(t)\}_{t \geq 0}$ be a Lévy process with $X(0) = 0$ and characteristic triplet μ, σ^2, ν , and let $V^K = \{V^K(t)\}_{t \geq 0}$ be X reflected at the boundaries 0 and $K > 0$ (see below). Because of the compactness of $[0, K]$, it is trivial to infer that a limiting stationary distribution π^K exists. The loss rate ℓ^K is then in loose terms defined as the stationary rate of overflow of X over K . We are concerned with finding expressions for ℓ^K and asymptotic properties as $K \rightarrow \infty$. This question has a long applied history. For example for a finite dam, ℓ^K is the rate of water overflow, and in a finite data buffer measured in units of bits, ℓ^K is the bit loss rate.

More precisely, V^K is defined as the solution of the Skorokhod problem

$$(1) \quad V^K(t) = V^K(0) + X(t) + \ell^0(t) - \ell^K(t),$$

where ℓ^0, ℓ^K are the minimal processes increasing only when V^K is at the boundary 0, resp. K , such that $0 \leq V^K(t) \leq K$ for all t . In [2] XIV.3, a pragmatic approach to existence and uniqueness of a solution is taken and consists in running X until exit of $[0, K]$. If say $(-\infty, 0)$ is hit, one then applies the standard operator giving one-sided (upward) reflection at 0 until (K, ∞) is hit. Then the operator giving one-sided (downward) reflection at K takes over, and so on. However, a number of more formal approaches have been developed and are surveyed in [6], from where we quote the expression

$$(2) \quad V^K(t) = V^K(0) + X(t) - Y_1(t) \vee Y_2(t)$$

where

$$(3) \quad Y_1(t) = \left[(V^K(0) - K)^+ \wedge \inf_{0 \leq u \leq t} (V^K(0) + X(u)) \right],$$

$$(4) \quad Y_2(t) = \sup_{0 \leq s \leq t} \left[(V^K(0) + X(s) - K)^+ \wedge \inf_{s \leq u \leq t} (V^K(0) + X(u)) \right].$$

We won't use expressions of this type, but given (1), we define $\ell^K = \mathbb{E}_{\pi^K} \ell^K(1)$. The form of π^K is known in terms of first passage probabilities of X : $\pi^K[x, K]$ is the probability that X enters $[x, \infty)$ before $(-\infty, x - K)$, cf. [2] IX.4, XIV.3. Thus, π^K is explicitly available basically when the scale function of X is so, for example for X spectrally one-sided, with one-side jumps which are phase-type or have a rational Laplace transform, or for stable processes (but note that even in some of these cases, only Laplace transforms can be found). The expression of [3] for ℓ^K is

$$(5) \quad \ell^K(t) = \frac{\mathbb{E}X(1)}{K} \int_0^K x \pi^K(dx) + \frac{\sigma^2}{2K} + \frac{1}{2K} \int_0^K \pi^K(dx) \int_{-\infty}^{\infty} \varphi^K(x, y) \nu(dy),$$

where

$$(6) \quad \varphi^K(x, y) = \begin{cases} -(x^2 + 2xy) & \text{if } y \leq -x, \\ y^2 & \text{if } -x < y < K - x, \\ 2y(K - x) - (K - x)^2 & \text{if } y > K - x. \end{cases}$$

It is notable that in contrast to the discrete-time case, the proof of (5) is highly non-trivial. The added complication is due largely to the continuous part of ℓ^K . The approach of [3] uses optional stopping of the martingale of [5] together with a limiting argument which is needed to get an expression free of an arbitrary complex number α used in the definition of this martingale. We do not believe it is possible to get rid of the occurrence of π^K in (5).

Asymptotics of ℓ^K as $K \rightarrow \infty$ when $\mathbb{E}X(1) < 0$ is another classical topic of the applied literature. With light tails, it is shown in [3] that $\ell^K \sim De^{-\gamma K}$ where K is the Cramér root, that is, the solution of $\kappa(\gamma) = 0$ where κ is the Lévy exponent. This parallels [7] that treated the discrete time case. With heavy tails, an asymptotic expression is obtained in [4], in discrete time, and its Lévy process analogue, obtained in [1], is

$$(7) \quad \ell^K \sim \int_K^{\infty} \bar{\nu}(x) dx;$$

the precise condition is essentially that $\bar{\nu}(x)$ be a subexponential tail.

A conservation principle easily gives that $\ell^K - \ell^0 = \mathbb{E}X(1)$. An easy argument in [3] gives $\ell^0 \rightarrow 0$, $K \rightarrow \infty$, when $\mathbb{E}X(1) > 0$, so that $\ell^K \sim \mathbb{E}X(1)$; the rate of convergence easily follows from what was just stated for $\mathbb{E}X(1) < 0$. The remaining case $\mathbb{E}X(1) = 0$ is treated in [1] and is somewhat different by involving functional limit theorems. With finite variance, the basic observation is that $\ell^K = \sigma^2/2K$ in the Brownian case $\mu = 0, \nu(dx) \equiv 0$. Since $\{X(tK^2/K)\}_{t \geq 0}$ has a

Brownian limit, with variance constant $\omega^2 = \sigma^2 + \int_{-\infty}^{\infty} x^2 \nu(dx)$, an appropriate scaling argument therefore suggests that $\ell^K \sim \omega^2/2K$. The rigorous verification in [1] involves a continuity property of ℓ^K as function of the characteristic triplet, for which some care in the formulation and proof is needed. With infinite variance, appropriate conditions on regular variation (with index say α) in the left and right tail of ν give a functional stable limit (of course with different normalizing constants), and exact formulas on the stable case from [3] therefore applies to show that $\ell^K \sim c/K^{\alpha-1}$ for some explicit constant.

REFERENCES

- [1] L. N. Andersen (2007). *Loss rate asymptotics for Lévy processes*. PhD dissertation in progress, Aarhus University.
- [2] S. Asmussen (2003). *Applied Probability and Queues* (2nd ed.). Springer-Verlag.
- [3] S. Asmussen, M. Pihlsgaard (2007). Loss rates for Lévy processes with two reflecting barriers. *Math. Oper. Res.* (in print).
- [4] P. Jelenković (1999). Subexponential loss rates in a GI/G/1 queue with applications. *Queueing Systems* **33**, 91–123.
- [5] O. Kella, W. Whitt (1992). Useful martingales for stochastic storage processes with Lévy input. *J. Appl. Probab.* **29**, 396–403.
- [6] L. Kruk, J. Lehoczky, K. Ramanan, S. Shreve (2006). Double Skorokhod map and reneging real-time queues. *Tom Kurtz Festschrift* (submitted).
- [7] M. Pihlsgaard (2005). Loss rate asymptotics in a GI/G/1 queue with finite buffer. *Stochastic Models* **21**, 913–931.

Options and the stochastic volatility model of Barndorff-Nielsen and Shephard

FRED ESPEN BENTH

In the Barndorff-Nielsen and Shephard (BNS) stochastic volatility model the stock price dynamics is given by

$$\frac{dS(t)}{S(t)} = (\mu + \beta Y(t)) dt + \sqrt{Y(t)} dB(t)$$

where μ and β are constants, $B(t)$ is a Brownian motion and $Y(t)$ is a non-Gaussian Ornstein-Uhlenbeck (OU) process

$$dY(t) = -\lambda Y(t) + dL(\lambda t).$$

The process $L(t)$ is a subordinator and λ a (positive) constant. This model was introduced in [1], where the authors proposed a sum of such OU-processes rather than one. The market is incomplete under the BNS model, and hence a unique price for options are not defined.

We suggest to use the utility indifference approach to single out a price for a plain vanilla call or put option. As is standard, we use an exponential utility function, parameterized by the risk aversion $\gamma > 0$. It is known that by letting $\gamma \downarrow 0$, one recovers the so-called minimal entropy price, which is yielded by the minimal netropy martingale measure (MEMM). In [2] an explicit density process for the

MEMM is derived. Using dynamic programming and a logarithmic transform of the indirect utility functions, we are able to derive an integro-PDE for the option price in the case of a general γ .

Numerical methods for this PDE is developed in [3], and in [4] we use this for finding the implied risk aversion. *I.e.*, given options prices, we use the utility indifference prices to back out the risk aversion $\hat{\gamma}$. In this way we can find out what risk the market assigns to options of different maturities and strikes. The case of Microsoft put options are presented, and a “risk aversion smile” is found. From the empirical investigations, it seems that the market is afraid of market crashes. Furthermore, all the prices in the market were consistent with the seller’s indifference price, and not the buyer.

We investigate the sensitivities of the option prices with respect to the underlying stock price. This is known and the Greeks of the options, and we calculate explicit formulas for the delta and gamma in terms of expectations where the option payoff function is not differentiated. This is possible using the Malliavin Calculus, and via Monte Carlo simulations we show in particular that one can calculate the gamma in a numerically stable way for the BNS model. Results from this analysis will appear in [5].

REFERENCES

- [1] O. E. Barndorff-Nielsen, N. Shephard (2001). Non-Gaussian Ornstein-Uhlenbeck based models and some of their uses in financial economics. *J. R. Stat. Soc. A* **63**, 167–241.
- [2] F. E. Benth, T. Meyer-Brandis (2005). The density process of the minimal entropy martingale measure in a stochastic volatility model with jumps. *Finance Stoch.* **9**, 563–575.
- [3] F. E. Benth, M. Groth (2005). The minimal entropy martingale measure and numerical option pricing for the Barndorff-Nielsen and Shephard stochastic volatility model. *E-print No. 26*, University of Oslo.
- [4] F. E. Benth, M. Groth, M. Lindberg (2007). The implied risk aversion from utility indifference pricing in a stochastic volatility model. *E-print No 1*, University of Oslo.
- [5] F. E. Benth, M. Groth, O. Wallin (2007). Derivative-free greeks for the Barndorff-Nielsen and Shephard stochastic volatility model. *Manuscript*, University of Oslo.

On small- and large-time expansions for Lévy semigroups on the real line

FRIEDRICH HUBALEK

We review our results from joint work with Ole Barndorff-Nielsen on the semigroup associated with an infinite activity pure jump subordinator with Levy density [1], [5], [2]. We present our three methods to compute a series expansion

$$p(t, x) = \sum_{n \geq 1} u_n(x) \frac{t^n}{n!}$$

for the semigroup density from the Levy density: the compound Poisson approximation, the method based on derivatives of convolution powers of the upper tail integral, and the method using analytic continuation and contour integration.

We then give several examples of series expansions for the semigroup density for a general Levy process on the real line: The variance-gamma, the Meixner, the normal inverse Gauss, the Cauchy distribution.

In the finite-variation case, or in the "almost-finite variation" case, when the Blumenthal-Gettoor index is one, we obtain quite analogous results as in the subordinator case. We also observe that for all examples of subordinators $p(t, x)$ is for fixed $x > 0$ an entire function in $t \in \mathbb{C}$. For all of our examples on \mathbb{R} we observe, that $p(x, t)$ is for fixed $x \neq 0$ an analytic function in $|t| < |x|$.

In the case when the Blumenthal-Gettoor index is in the interval (1,2) our methods yield typically divergent asymptotic series. The key example is again provided by the stable laws, see, for example, [7, Remark 14.18].

In the final part of the talk we indicate, which and how convergent expansions can be obtained, using the Mellin transform [5], [3], contour integration and the residue theorem. At present it is not clear, whether the approach is limited to general tempered stable distributions in Rosinski's sense [6] or can be extended, for examples to distribution with regularly varying Levy measures.

REFERENCES

- [1] O. E. Barndorff-Nielsen (2000). Probability densities and Lévy densities. *Research Report 18*, Aarhus University, Centre for Mathematical Physics and Stochastics (Ma-PhySto).
- [2] O. E. Barndorff-Nielsen, F. Hubalek (2006). Probability measures, Lévy measures, and analyticity in time. *Thiele Research Report 2006–12*, University of Aarhus, Denmark, The Thiele Centre.
- [3] P. Flajolet, X. Gourdon, P. Dumas (1995). Mellin transforms and asymptotics: harmonic sums. *Theoretical Computer Science*, **144**, 3–58.
- [4] R. A. Handelsman, J. S. Lew (1969). Asymptotic expansion of a class of integral transforms via Mellin transforms. *Archive for Rational Mechanics and Analysis*, **35**, 382–396.
- [5] F. Hubalek (2002). On a conjecture of Barndorff-Nielsen relating probability densities and Lévy densities. In Ole E. Barndorff-Nielsen, editor, *Proceedings of the 2nd MaPhySto Conference on Lévy Processes: Theory and Applications*, **22** in Ma-PhySto Miscellanea.
- [6] J. Rosinski. Tempering stable processes (2007). *Stochastic Processes and Their Applications*. (to appear).
- [7] K.-I. Sato (1999). *Lévy processes and infinitely divisible distributions*. Cambridge University Press, Cambridge.

Some recent developments about estimation of jumps and stochastic volatility

JEAN JACOD

The general setting is as follows: we have an underlying Itô semimartingale X , that is

$$X_t = \int_0^t b_s ds + \int_0^t \sigma_s dW_s + \int_0^t \int_E \delta(s, x) 1_{\{|\delta(s, x)| \leq 1\}} (\mu - \nu)(ds, dx) + \int_0^t \int_E \delta(s, x) 1_{\{|\delta(s, x)| > 1\}} \mu(ds, dx).$$

Here W is a Wiener process and μ is a Poisson random measure on $[0, \infty) \times E$ (E is an auxiliary space, e.g. $E = \mathbb{R}$), with intensity measure of the form $\nu(t, dx) = dt \otimes F(dx)$. The coefficients $b_t(\omega)$, $\sigma_t(\omega)$, $\delta(\omega, t, x)$ are subject to some regularity assumptions that we do not explain precisely here (see the referenced papers for more details). The processes X and W may be multidimensional.

We observe X at discrete times, say $0, \Delta_n, 2\Delta_n, \dots$, up to some fixed time T . We want to infer some properties of the process, or of its observed path, on the basis of these discrete observations, in the case of high frequency observations. That is, Δ_n is small, and we are in fact looking at the asymptotic $\Delta_n \rightarrow 0$.

1) Some identifiable or non-identifiable quantities. We first recall a number of quantities which can be "identified" asymptotically in this setting, that is those which can be inferred exactly from the observation of the whole path $t \mapsto X_t(\omega)$ for $t \in [0, T]$:

- 1 The drift b_t can *never* be identified, except in some very special cases.
- 2 The "law of jumps", which means the compensator of the jump measure of X (that is, the Lévy measure when X is a Lévy process) can *never* be identified either.
- 3 The volatility can be fully identified. Indeed, we know the quadratic variation

$$[X, X]_t = \int_0^t \sigma_s^2 ds + \sum_{s \leq t} (\Delta X_s)^2$$

(written here in the 1-dimensional case), since it is the limit in probability of the "realized quadratic variations" $\sum_{i=1}^{\lfloor t/\Delta_n \rfloor} (\Delta_i^n X)^2$, where $\Delta_i^n X = X_{i\Delta_n} - X_{(i-1)\Delta_n}$. Then one knows exactly $\int_0^t \sigma_s^2 ds$, and also the function $t \mapsto \sigma_t^2(\omega)$ for $t \in [0, T]$.

- 4 The jumps of X are also fully observed on $[0, T]$ in this asymptotic scheme. In particular we can decide whether X has jumps, or whether its components in the multidimensional case jump at the same times or not. We also know the quantities

$$B(r)_t = \sum_{s \leq t} |\Delta X_s|^r$$

for any $r > 0$, and in particular which ones are finite at time $t = T$.

Coming back to the discrete observation scheme, we do have "consistent estimators" for the identifiable quantities stated above. The question of inferring the volatility, or the integrated volatility (with or without jumps) has been extensively studied, so we will concentrate on inference about jumps. We deal with only two problems here, namely testing the presence of jumps, and inferring the so-called Blumenthal-Gettoor index, which is the infimum of those $r > 0$ such that $B(r)_T < \infty$. Both are the object of a joint work with Y. Aït-Sahalia.

2) Testing for jumps. The problem is to decide, on the basis of the discrete observations at stage n , in which one of the two following disjoint subsets of Ω the

observed path actually lies:

$$\begin{cases} \Omega_c = \{\omega : t \mapsto X_t(\omega) \text{ is continuous on } [0, T]\}, \\ \Omega_j = \{\omega : t \mapsto X_t(\omega) \text{ is not continuous on } [0, T]\}. \end{cases}$$

This is of course a 1-dimensional problem. The analysis is performed on the basis of the (observable) statistics

$$B(r, \Delta_n) = \sum_{i=1}^{\lfloor T/\Delta_n \rfloor} |\Delta_i^n X|^r.$$

When $r = 2$ this is the realized quadratic variation at time T . When $r > 2$, then we always have

$$(1) \quad B(r, \Delta_n) \xrightarrow{\mathbb{P}} B(r)_T < \infty.$$

Now, if X is continuous we have $B(r)_T = 0$. However in this case,

$$(2) \quad \Delta_n^{1-r/2} B(r, \Delta_n) \xrightarrow{\mathbb{P}} m_r \int_0^T |\sigma_s|^r dr,$$

where m_r is the r th absolute moment of $\mathcal{N}(0, 1)$. Note that this holds when X has no jump at all, but also in restriction to the set Ω_c when X may have jumps. Moreover in both cases above we have an associated CLT, with rate $1/\sqrt{\Delta_n}$, under suitable assumptions (essentially that the volatility process σ_t is itself an Itô semimartingale, possibly with jumps), and provided also that $r > 3$.

Now the idea is very simple. We take $r = 4$, say (any $r > 3$ would do as well). Then in view of (1) and (2), we have

$$(3) \quad T_n = \frac{B(4, 2\Delta_n)}{B(4, \Delta_n)} \xrightarrow{\mathbb{P}} \begin{cases} 1 & \text{on the set } \Omega_j \\ 2 & \text{on the set } \Omega_c. \end{cases}$$

One can even do better, due to the available CLTs: namely we construct (observable) statistics V_n^j and V_n^c which both converge in probability to a finite limit and such that the standardized statistics

$$T_n^j = \frac{1}{\sqrt{\Delta_n}} V_n^j(T_n - 1), \quad T_n^c = \frac{1}{\sqrt{\Delta_n}} V_n^c(T_n - 2)$$

converge in law, in restriction to the sets Ω_j and Ω_c respectively, to a centered variable with variance 1 for T_n^j , and to $\mathcal{N}(0, 1)$ for T_n^c . From this, it is straightforward to derive feasible tests for deciding whether the path lies in Ω_j or in Ω_c . For example, if the "null hypothesis" is that we have no jump, the critical region at asymptotic level α is

$$C_{n,c} = \{T_n < 2 - z_\alpha \sqrt{\Delta_n}/V_n^c\},$$

where z_α is the α -quantile of $\mathcal{N}(0, 1)$, that is $\mathbb{P}(U > z_\alpha) = \alpha$ where U is $\mathcal{N}(0, 1)$. Here the asymptotic level means $\alpha = \limsup_n \mathbb{P}(C_{n,c} \mid \Omega_c)$, and the test is "non-parametric" in the sense that it meets the asymptotic level for all processes X which satisfy the assumptions, regardless of the actual form of the coefficients (b, σ, δ) .

To illustrate this we did some simulations for the cases where $dX_t = \sigma X_t dW_t + X_{t-} dY_t$, where Y is a compound Poisson process, and also when $X = W + \theta Y$ with Y a Cauchy process, for various values of θ . The simulation results seem to be reasonably good. We also applied the test on the 30 DJIA stocks for all trading days of 2005, that is for about 7500 realizations of our test statistic, with the conclusion that for more than half of the days there are jumps.

3) The Blumenthal-Gettoor index. Now we turn to the inference of the Blumenthal-Gettoor index, as defined before. The following is a very preliminary report on on-going research on the subject.

The problem is a priori difficult, because we want to make inference about a characteristics of "very small" jumps (recall that the index describes the concentration of jumps near 0). On the other hand, we rely only on increments of the process X over small but non-vanishing intervals, and on any given interval the contribution of small jumps is "negligible" with respect to the increment of the continuous martingale part of X .

So in fact we throw away the increments that are small, meaning more or less or the order of magnitude of the increments of continuous martingale part. Then we just evaluate the proportion of "relatively small" increments among all increments which we keep. More specifically we choose a number ϖ in $(0, 1/2)$, and two numbers $0 < \alpha < \alpha'$, and set

$$U_n(\alpha) = \sum_{i=1}^{\lfloor T/\Delta_n \rfloor} 1_{|\Delta_i^n X| > \alpha \Delta_n^{\varpi}},$$

and the same for α' . The statistics of interest is then

$$S_n = \frac{\log(U_n(\alpha)/U_n(\alpha'))}{\log(\alpha'/\alpha)}.$$

Then we can show that under appropriate assumptions, to be described below, S_n is a consistent estimator for the Blumenthal-Gettoor index β of the process X , and further we have a CLT and we can derive (observable) statistics V_n which converge in probability to a finite limit, and such that $\frac{V_n}{\Delta_n^{\varpi\beta/2}}(S_n - \beta)$ converges to $\mathcal{N}(0, 1)$: so in principle we have the tools for making a sound estimation of β .

Now, unfortunately, the assumptions are rather strong. They essentially amounts to say that the compensator of the jump measure of X is of the form $dt \times F_{\omega,t}(dx)$ (this structure is already implied by the fact that X is an Itô semi-martingale), with the following structure for F : for any (ω, t) we have

$$F_{\omega,t}(dx) = \frac{1}{|x|^{1+\beta}} (a_t^-(\omega) 1_{[-a_t(\omega), 0)}(x) + a_t^+(\omega) 1_{(0, a_t(\omega)]}(x)) + F'_{t,\omega}(dx)$$

for some nonnegative processes a^+, a^-, a such that $\varepsilon < a_t(\omega) \leq 1$ and $\int_0^T (a_t^+(\omega) + a_t^-(\omega)) dt > 0$ a.s., and where $F'_{t,\omega}$ is such that $x^{\beta'} F'_{t,\omega}([-x, x]^c) \leq K$ for some constant K and all $x \in (0, 1]$, with further $\beta' < \frac{\beta}{2+\beta}$. In other words, the "Lévy measure" of X is the sum of a truncated stable Lévy measure with index β on a

non-negligible set, plus another measure whose Blumenthal-Gettoor index is smaller than the above β' .

REFERENCES

- [1] Y. Aït Sahalia, J. Jacod (2006). Testing for jumps in a discretely observed process. *Preprint*.
- [2] Y. Aït Sahalia, J. Jacod (2007). Estimating the Blumenthal-Gettoor index of the jumps of a discretely observed semimartingale. *Working Paper*.

Insurance risk problems driven by Lévy processes

ANDREAS E. KYPRIANOU

The classical Cramér-Lundberg process is nothing more than a bounded variation spectrally negative Lévy process with finite jump activity which drifts to infinity. It makes sense to model insurance risk processes with a general spectrally negative Lévy process which drifts to infinity instead. This has several advantages.

- The latter class contains the former class.
- In the case of infinite activity one interprets the small jumps as the many small claims which are offset by a compensating premium income. For example, if π is the jump measure, then claims of size $0 < x < 1$ are offset by premiums which are collected in such a way that the income in an small time interval dt is calculated as $x\pi(-dx)dt$.
- Large jumps (of magnitude greater than unity) which occur at intervals, which are spaced in such a way that they are independent and exponentially distributed, correspond to extremely big claims due to, for example, large scale disasters. This part of the Lévy process may be considered to be offset by a linear drift representing a constant stream of premiums as in the classical case.
- The inclusion of a Brownian component may be understood as a stochastic perturbation.

We discuss the age-old actuarial problem introduced by [3] of computing the optimal dividend payments on the insurance risk process until ruin. That is to say, to evaluate an optimal stream of dividend payments $L^* = \{L_t^* : t \geq 0\}$ for which

$$(1) \quad v(x) = \sup_{L \in \Pi} \mathbb{E}_x \left(\int_0^{\sigma^L} e^{-qt} dL_t \right) = \mathbb{E}_x \left(\int_0^{\sigma^{L^*}} e^{-qt} dL_t^* \right)$$

where Π is a suitable class of dividend strategies, $L^* \in \Pi$ and for each $L \in \Pi$, $\sigma^L = \inf\{t > 0 : X_t - L_t < 0\}$ is the ruin time of the aggregate process.

Within the threshold strategies, there are two types of strategies which are known to lead to optimal solutions depending on the underlying structure of the Lévy process and the class Π of permissible dividend strategies. The first type of strategy corresponds to *reflecting* X at a barrier of level $b > 0$. In that case the process $X - L$ under \mathbb{P}_x is equal in law to $b + X_t - (b - x) \vee \overline{X}_t$ under \mathbb{P}_0 where $\overline{X}_t = \sup_{s \leq t} X_s$ and we may take $L^* = (b - x) \vee \overline{X}_t - b$ when expressing the right

hand side of (1) under the measure \mathbb{P}_0 . The second type of strategy, introduced in a variant of the original problem by [4] and [1], corresponds to *refracting* X at a barrier of level $b > 0$. In short this means subtracting a linear drift off from X with rate $0 < c < \mathbb{E}_0(X_1)$ whenever the aggregate process increases above the level b . Hence the aggregate process is described by the stochastic differential equation

$$U_t = X_t - c \int_0^t \mathbf{1}_{\{U_s > b\}} ds.$$

Very little is known about the latter process for the case of Lévy processes and we present some initial fluctuation identities which are relevant to the ruin problem obtained together with Ronnie Loeffen.

Finally we conclude by mentioning some interesting ideas of [2] who produce new examples of two-sided jumping Lévy processes for which the Wiener-Hopf factorization has an explicit appearance using self similar positive Markov processes. Further, we discuss the potential of their ideas in insurance and financial mathematics.

REFERENCES

- [1] S. Asmussen, M. Taksar (1997). Controlled diffusion models for optimal dividend pay-out. *Insurance Math. Econom.* **20**, 1–15.
- [2] M. E. Caballero, L. Chaumont (2006). Conditioned stable Levy processes and the Lamperti representation. *J. Appl. Probab.* **43**, 967–983.
- [3] B. De Finetti (1957). Sur un'impostazione alternativa della teoria collettiva del rischio. *Transactions of the XVth International Congress of Actuaries* **2**, 433–443.
- [4] M. Jeanblanc, A. N. Shiryaev (1995). Optimization of the flow of dividends. *Uspekhi Mathem. Nauk.* **50**, 25–46.

Some properties of generalised Ornstein-Uhlenbeck processes

ALEXANDER LINDNER

In the talk some results based on joint work with Jean Bertoin and Ross Maller [1], with Ross Maller [2] and with Ken-iti Sato [3] are presented. A *generalised Ornstein-Uhlenbeck process* $(V_t)_{t \geq 0}$ is defined by

$$V_t = e^{-\xi t} \left(V_0 + \int_0^t e^{\xi s} d\eta_s \right), \quad t \geq 0,$$

where $(\xi, \eta) = (\xi_t, \eta_t)_{t \geq 0}$ is a bivariate Lévy process, independent of the starting random variable V_0 . Such processes arise in a variety of situations such as risk theory, option pricing or financial time series. Lindner and Maller [2] have given a complete characterisation for which (ξ, η) a stationary version of the corresponding generalised Ornstein-Uhlenbeck process exists. In that case, the stationary marginal distribution is of the form $\int_0^\infty e^{-\xi t} dL_t$ for some other bivariate Lévy process (ξ, L) , constructed in terms of (ξ, η) . Bertoin, Lindner and Maller [1] have shown that V_∞ can never have atoms, unless it degenerates to a constant. Lindner and Sato [3] consider infinite divisibility and continuity properties of integrals of the form $V_{c,a,b} := \int_0^\infty c^{-N_t} dY_t$, where $c > 1$ and N and Y are independent Poisson

processes with parameters a and b , respectively. Then $V_{c,a,b}$ will be either absolutely continuous or continuous singular. Which of these cases occurs is shown to depend heavily on the value of c . More precisely, it is shown that $V_{c,a,b}$ is continuous singular if c is a Pisot-Vijayaraghavan number, which includes in particular all integers greater than 1. Furthermore, for every $c > 1$, $V_{c,a,b}$ is continuous singular if b/a is sufficiently small, and for every fixed a and b , $V_{c,a,b}$ is continuous singular if c is sufficiently large. On the other hand, it is shown that for Lebesgue almost every $c > 1$, $V_{c,a,b}$ will be absolutely continuous with a bounded and continuous density if b/a is sufficiently large. For independent N and Y , the law of $V_{c,a,b}$ is infinitely divisible, and if $(Z_t)_{t \geq 0}$ is a Lévy process with distribution $V_{c,a,b}$ at time 1, then it is shown that for almost every $c > 1$ there is some $T(a, b, c) \in (0, \infty)$ such that Z_t is continuous singular for every $t < T(a, b, c)$ and absolutely continuous for every $t > T(a, b, c)$. Finally, the independence assumption of N and Y is relaxed, and a complete characterisation of all bivariate Lévy processes (N, Y) whose margins N and Y are Poisson processes is given, such that $\int_0^\infty c^{-N_t-} dY_t$ is infinitely divisible. The analogue question is studied for the symmetrisation of $\int_0^\infty c^{-N_t-} dY_t$.

REFERENCES

- [1] J. Bertoin, A. Lindner, R. Maller (2006). On continuity properties of the law of integrals of Lévy processes. *Séminaire de Probabilités*. Accepted for publication.
- [2] A. Lindner, R. Maller (2005). Lévy integrals and the stationarity of generalised Ornstein-Uhlenbeck processes. *Stoch. Proc. Appl.* **115**, 1701–1722.
- [3] A. Lindner, K. Sato (2007). Continuity properties and infinite divisibility of stationary distributions of some generalised Ornstein-Uhlenbeck processes. Submitted.

To which class known distributions of real valued infinitely divisible random variables belong?

MAKOTO MAEJIMA

Recently, subdivision of the class $I(\mathbb{R}^d)$ of infinitely divisible distributions has been developed. Especially, many subclasses of $I(\mathbb{R}^d)$ can be characterised in terms of the radial component $\nu_\xi(dr)$, $r > 0$, of the polar decomposition of the Lévy measure. Distributions in the classes discussed in this talk have the densities $l_\xi(r)$, $r > 0$, such that $\nu_\xi(dr) = l_\xi(r)dr$. Depending on the properties of $l_\xi(r)$, six classes are proposed. (1) Jurek class $U(\mathbb{R}^d)$, where $l_\xi(r)$ is nonincreasing. (2) Goldie-Steutel-Bondesson class $B(\mathbb{R}^d)$, where $l_\xi(r)$ is completely monotone. (3) The class of selfdecomposable distributions $L(\mathbb{R}^d)$, where $l_\xi(r) = r^{-1}k_\xi(r)$, where $k_\xi(r)$ is nonincreasing. (4) Thorin class $T(\mathbb{R}^d)$, where $l_\xi(r) = r^{-1}k_\xi(r)$, where $k_\xi(r)$ is completely monotone.

(5) The class of type G distribution $G(\mathbb{R}^d)$, where $l_\xi(r) = g_\xi(r^2)$ with a completely monotone function g_ξ .

(6) The class $M(\mathbb{R}^d)$, where $l_\xi(r) = r^{-1}g_\xi(r^2)$ with a completely monotone function g_ξ .

Also, a mapping from $I(\mathbb{R}^d)$ (or $I_{\log}(\mathbb{R}^d)$, the class of infinitely divisible functions with log-moments) to each class can be defined, and iterating this mapping, a sequence of decreasing subclasses of each class is constructed.

Then a lot of known infinitely divisible distributions of real valued random variables are examined for determining the class to which they belong. Among many other examples, there are gamma distribution, logarithm of gamma random variables, tempered stable distribution, the distribution of limit of generalized Ornstein-Uhlenbeck process, product of standard normal random variables, the distribution of random excursion of some Bessel processes.

There remain a lot of infinitely divisible distributions to be specified.

REFERENCES

- [1] L. Bondesson (1992). *Generalized Gamma Convolutions and Related Classes of Distributions and Densities*, Lecture Notes in Statistics, No. 76, Springer.
- [2] K. Sato (1999). *Lévy Processes and Infinitely Divisible Distributions*, Cambridge University Pres.
- [3] F. W. Steutel, K. Van Harn (2004). *Infinitely Divisibility of Probability Distributions on the Real Line*, Dekker.

Stochastic calculus for convoluted Lévy processes

TINA MARQUARDT

We aim on an integration theory which allows for integrals with stochastic integrands with respect to convoluted Lévy processes in terms of the S-transform (see [1] for the case of fractional Brownian motion).

A stochastic process $M = \{M(t)\}_{t \in \mathbb{R}}$ is called a convoluted Lévy process if it can be represented by

$$M(t) = \int_{\mathbb{R}} f(t, s) L(ds), \quad t \in \mathbb{R},$$

where $f : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ is a measurable function and L is a Lévy process with second finite moments and no Brownian component.

A special case is the class of fractional Lévy processes which are defined by replacing the ordinary Brownian motion in the moving average representation of fractional Brownian motion by a general Lévy process (see [2] and [3]). However, for a large class of Lévy measures with infinite mass the corresponding fractional Lévy process cannot be a semimartingale and hence ordinary Itô calculus cannot be applied.

Our idea is to consider the Itô integral from a white noise point of view and then obtain a Skorohod integral. The first step is to define the Wick exponential

$\exp^\diamond(I_1(\eta))$ of a Wiener integral $I_1(\eta) = \int_{\mathbb{R}} \int_{\mathbb{R}_0} \eta(x, s) \tilde{N}(dx, ds)$ with respect to the compensated jump measure \tilde{N} .

Then the S -transform of a stochastic process X is defined as the integral transform given by

$$S(X)(\eta) = E^{Q_\eta}(X),$$

where $dQ_\eta = \exp^\diamond(I_1(\eta))dP$.

We define the Skorohod integral $\int_a^b X(s) M^\diamond(ds)$ to be the unique stochastic process whose S -transform is given by

$$\int_a^b S(X(s))(\eta) \frac{d}{ds} S(M(s))(\eta) ds.$$

Finally, we derive an Itô formula for convoluted Lévy processes, however this Itô formula consists of Wick-type integrals whose convergence still has to be proven. This is joint work with Christian Bender which is still in progress.

REFERENCES

- [1] C. Bender (2003). An S -transform approach to integration with respect to a fractional Brownian motion, *Bernoulli* **9**, 955–983.
- [2] T. Marquardt (2006). Fractional Lévy processes with an application to long memory moving average processes, *Bernoulli* **12**, 1009–1126.
- [3] T. Marquardt (2006). Fractional Lévy processes, CARMA processes and related topics, *PhD dissertation*, Munich University of Technology.

Modelling spot prices of electricity by non-Gaussian Ornstein-Uhlenbeck processes

THILO MEYER-BRANDIS

In recent years, privatization of electricity markets and free float of electricity prices have made the energy sector a field of increasing interest for application of stochastic models and concepts (see f. e.x. [1], [2], [3], [5], [4], [9], [10]). The aim of the talk was to present a model for electricity spot price dynamics. The standard approach in the literature is to model logarithmic electricity spot prices through a mean-reverting process (see e.g., Lucia and Schwartz [5] and Geman and Roncoroni [4]), such that in the classical Gaussian setting the spot price dynamics becomes lognormal. For such models it is notoriously difficult to derive manageable analytical expressions for the corresponding forward and futures contracts. We suggest to model the spot price dynamics directly by a sum of Ornstein-Uhlenbeck processes which, however, are non-Gaussian and possibly non-stationary. This results in a model that, while capturing the essential characteristics of spot prices to a reasonable degree, is simple enough to yield closed form expressions for electricity forward and futures contracts and other derivatives.

More precisely, let $(\Omega, P, \mathcal{F}, \{\mathcal{F}_t\}_{t \in [0, T]})$ be a complete filtered probability space, with $T < \infty$ a fixed time horizon. If $S(t)$ denotes the spot price of electricity at

time t then we set

$$(1) \quad S(t) = \mu(t) + X(t).$$

Here $\mu(t)$ is a deterministic, periodic function and the stochastic process $X(t)$ is described by the following dynamics:

$$X(t) = \sum_{i=1}^n Y_i(t)$$

where

$$(2) \quad dY_i(t) = -\lambda_i Y_i(t) dt + \sigma_i(t) dL_i(t), \quad Y_i(0) = y_i, \quad i = 1, \dots, n,$$

and λ_i are positive constants and $\sigma_i(t)$ are positive bounded functions. The processes $L_i(t), i = 1, \dots, n$ are assumed to be independent *increasing* càdlàg pure jump processes whose corresponding jump measures denoted by $N_i(dt, dz)$ have *deterministic* predictable compensators $\nu_i(dt, dz)$ of the form $\nu_i(dt, dz) = \rho_i(t) dt \nu_i(dz)$, where $\rho_i(t)$ is a deterministic function.

The main question addressed during the presentation was how to estimate the Lévy measures $\nu_i(dt, dz)$ given the observed spot price. Three potential procedures were elaborated and put under discussion. Non-linear filtering for Cox processes ([8], [7], [6]), a maximum likelihood function based on Girsanov's transform, and pseudo maximum likelihood estimation in the case of stationary OU processes. Further, a penalized least square method to filter out spikes was presented ([11]).

REFERENCES

- [1] F. E. Benth, L. Ekeland, R. Hauge, B. F. Nielsen (2003). On arbitrage-free pricing of forward contracts in energy market. *Appl. Math. Finance* **10**, 325–336.
- [2] F. E. Benth, S. Koekebakker (2005). Stochastic modeling of forward and futures contracts in electricity markets, *Preprint*.
- [3] L. Clewlow, C. Strickland (2000). *Energy Derivatives: Pricing and Risk Management*, *Lacima Publications*.
- [4] H. Geman, A. Roncoroni (2006). Understanding the fine structure of electricity prices, *Journal of Business* **79**, to appear.
- [5] J. Lucia, E. S. Schwartz (2002). Electricity Prices and Power Derivatives: Evidence from the Nordic Power Exchange. *Rev. Derivatives Research* **5**, 5–50.
- [6] V. Mandrekar, T. Meyer-Brandis, F. Proske (2007). A Bayes Formula for non-linear filtering with Gaussian and Cox noise, *Preprint*.
- [7] T. Meyer-Brandis (2006). Stochastic Feynman-Kac Equations associated to Lévy-Itô Diffusions, *Preprint*.
- [8] T. Meyer-Brandis, F. Proske (2004). Explicit Solution of a Non-Linear Filtering Problem for Lévy Processes with Application to Finance, *Appl. Math. Optim.* **50**, 119–134.
- [9] D. Pilipovic (1988). *Energy Risk*, McGraw Hill.
- [10] E. S. Schwartz (1997). The stochastic behaviour of commodity prices: Implications for valuation and hedging. *J. Finance*, **LII**(3), 923–973.
- [11] Winkler, Liebscher (2000). Smoothers for discontinuous signals, *Nonparametric Statistics* **14**.

On proving free infinite divisibility

VÍCTOR PÉREZ-ABREU

A $d \times d$ matrix valued Brownian motion $B^d(t) = (\beta_{jk}(t))$ is an Hermitian matrix where $(\beta_{jj}(t))_{j=1}^d, (\operatorname{Re} \beta_{jk}(t))_{j < k}, (\operatorname{Im} \beta_{jk}(t))_{j < k}$ is a set of d^2 independent one dimensional Brownian motions with parameter $\frac{1}{2d}(1 + \delta_{jk})t$. Let $\{\sigma(t); t \geq 0\}$ be a one dimensional subordinator such that $\sigma(1)$ has Gamma distribution $G(\alpha, 2)$ and independent of B^d . Consider the subordinated $d \times d$ matrix $X^d(t) = (\beta_{jk}(\sigma(t)))$ with eigenvalues $\lambda_1^d, \dots, \lambda_d^d$ and mean empirical spectral measure $\mu_d = E(d^{-1} \sum_{i=1}^d \delta_{\lambda_i^d})$. It was shown in [5] that μ_d converges weakly, when d goes to infinity, to a probability measure μ which is the law of the random variable $\sqrt{\sigma_\alpha(1)}S_0$, with S_0 having the semicircle distribution in $(-1, 1)$ and independent of $\sigma(1)$. Moreover, μ has an infinitely divisible distribution if and only if $\alpha = 2$, in which case μ is the standard normal distribution.

It is well known that the semicircle law is infinitely divisible (i.d.) not in the classical but in the free sense, where it plays the role Gaussian law does in classical probability. Hence, there arises the natural question whether the above mixtures of semicircles laws are i.d. in the free sense and if in particular the Gaussian distribution itself is.

We review methods to prove free i.d. of laws based on analytic properties of the free cumulant transform [3], the Bercovici-Pata bijection between free and classical i.d. laws and the Upsilon transformation [2], and the random matrix model relation [4]. We illustrate some of these methods with the case of the power semicircle laws [1]. Finally, we discuss why the above matrix Brownian motion set up and mixtures of semicircle laws are steps towards the study of classical subordination of a free Brownian motion.

REFERENCES

- [1] O. Arizmendi-Echegaray, V. Pérez-Abreu (2007). On power semicircle laws, *Working Paper*.
- [2] O. E. Barndorff-Nielsen, S. Thorbjørnsen (2005). Classical and free infinite divisibility and Lévy processes. In: *Quantum Independent Increments Processes II*. M. Schürmann and U. Franz (eds). LNM 1866, Springer, The Netherlands.
- [3] H. Bercovici, D. Voiculescu (1993). Free convolution of measures with unbounded supports. *Indiana J. Math.* **42**, 733–773.
- [4] F. Benaych-Georges (2005). Classical and free i.d. distributions and random matrices. *Ann. Probab.* **33**, 1134–1170.
- [5] V. Pérez-Abreu (2006). A random matrix model for the Gaussian distribution. *Period. Math. Hung.* **52**, 47–65.

Non-parametric estimation of the volatility path in the presence of noise

MARK PODOLSKIJ

We consider a noisy diffusion model of the type

$$Y = X + U ,$$

where

$$X_t = X_0 + \int_0^t a_u du + \int_0^t \sigma_u dW_u, \quad t \in [0, 1]$$

and U is an i.i.d. process (independent of X), observed at time points i/n , $i = 0, \dots, n$. In this model X represents the true price process which is contaminated by microstructure noise denoted by U .

The object of our interest is the estimation of the volatility path (σ_u) and related functionals. This kind of problems has been intensively studied in a pure diffusion framework (see, for instance, [2], [3] or [1]). Quite often the estimators of the volatility path (σ_u) (in a diffusion model) are of the form

$$(1) \quad \gamma_n \sum_{i=1}^n f_n(X_{\frac{i-1}{n}}) g(\sqrt{n} \Delta_i^n X),$$

where $\Delta_i^n X = X_{\frac{i}{n}} - X_{\frac{i-1}{n}}$ and γ_n is an appropriate normalising sequence (usually f_n is a kernel function and $g(x) = x^2$).

In the noisy diffusion models we propose to use some particular quantities $\hat{X}_{\frac{i-1}{n}}$ and $\Delta_i^n \hat{X}$, based on the observations $(Y_{\frac{i}{n}})$, which correspond to $X_{\frac{i-1}{n}}$ and $\Delta_i^n X$, respectively, and which make, in some sense, the influence of the noise process U small (the detailed procedure is discussed in [4]). In the next step we define the statistic

$$\hat{\gamma}_n \sum_{i=1}^n f_n(\hat{X}_{\frac{i-1}{n}}) g(n^{1/4} \Delta_i^n \hat{X}),$$

which is obviously an analogue of the quantity given in (1). Finally, we show the consistency for this type of estimators and derive the convergence rates for different type of estimation problems.

REFERENCES

- [1] B. van Es, P. Spreij, H. van Zanten (2003). Nonparametric volatility density estimation. *Bernoulli* **9**, 451–465.
- [2] D. Florens-Zmirou (1993). On estimating the diffusion coefficient from discrete observations. *J. Appl. Probab.* **30**, 790–804.
- [3] M. Hoffmann (2001). On estimating the diffusion coefficient: parametric versus nonparametric. *Ann. I.H. Poincaré* **37**, 339–372.
- [4] M. Podolskij, M. Vetter (2006). Estimation of volatility functionals in the simultaneous presence of microstructure noise and jumps. *Working Paper*.

Issues in mathematical finance

PHILIP PROTTER

The first fundamental theorem of mathematical finance, in its current incarnation due to F. Delbaen and W. Schachermayer ([2] and [3]) states, loosely speaking, that on a filtered complete probability space $(\Omega, \mathcal{F}, \mathbb{F}, P)$ a nonnegative price process S admits no arbitrage in an economy with an interest rate identically zero, if and only if there exists an equivalent probability measure Q such that S is an \mathbb{F} martingale, or local martingale, under Q . The collection of all such probabilities Q is denoted \mathbb{Q} . The collection \mathbb{Q} is typically referred to as *the collection of risk neutral measures*. A natural question to ask is: is there a difference, in the economic meaning, if a risk neutral measure Q turns S into a martingale, or into a local martingale?

It has recently become known, thanks to the work of Loewenstein and Willard ([10]), and Cox and Hobson ([1]) among others, that in the case where S is a local martingale then the economy admits a *bubble*, where “bubble” is taken in the common language economic meaning. An alternative mathematical model of bubbles is known as *charges* and can be found in the work of Gilles and LeRoy, for example ([5] and [6]). We show in [9] that the charges approach and the local martingale approach are equivalent.

In work with Kazuhiro Shimbo and Robert Jarrow ([8]) we have shown that if one adds Robert Merton’s 1973 assumption of *No Dominance* [11], properly mathematically interpreted, then there cannot be any bubbles in a complete market. (A complete market is mathematically equivalent to the collection \mathbb{Q} being a singleton.) In informal terms, “no dominance” means that if a contingent claim can be hedged in two different ways, any rational person will prefer the cheaper method, and it is assumed all people are rational.

The lack of bubbles in complete markets leads us naturally to focus on incomplete markets, and this talk is largely based on the work of Jarrow, Shimbo and this author which is presented in [9]. This is where the models allow bubbles to appear. Another problem with the models of Loewenstein-Willard and virtually all others, is that the *birth of a bubble* is impossible. This means that any bubble the market may have at time t had to be present since the inception of the market, at time 0. It seems clear that any reasonable model should allow for “bubble birth.” We do just that in our model, which allows one to change from one risk neutral measure Q in \mathbb{Q} to a different one Q^* at a given (possibly random) time τ . It is then shown (using previous work of Delbaen and Schachermayer [4]) that it is indeed possible for some price processes S to be such that S is a uniformly integrable martingale under Q , but only a local martingale under Q^* , so that this can indeed happen. This allows for bubble birth. While our analysis of bubbles superficially applies to the study of an individual risky asset such as a stock, it can also be applied to sectors of stocks, or to sectors of the economy (such as, for example, housing prices), through an artificial device analogous to a stock index. Finally, using a recent result of Jacod and this author ([7]), or alternatively the paper of Schweizer and Wissel ([12]), one can imagine building a statistical test

to see if a market is currently experiencing a bubble. This is left open. Indeed the entire area of the real time detection of bubbles, through statistical means, is an open problem in and of itself. Other open problems are related to the theory: such as an analysis of which price process models can *a priori* allow for bubbles, and which cannot?

REFERENCES

- [1] A. M. G. Cox, D. G. Hobson (2005). Local martingales, bubbles and option prices. *Finance and Stochastics* **9** (2005), 477–492.
- [2] F. Delbaen, W. Schachermayer (1994). A general version of the fundamental theorem of asset pricing. *Math. Ann.* **300**, 463–520.
- [3] F. Delbaen, W. Schachermayer (1998). The fundamental theorem of asset pricing for unbounded stochastic processes. *Math. Ann.* **312**, 215–250.
- [4] F. Delbaen, W. Schachermayer (1998). A simple counterexample to several problems in the theory of asset pricing. *Mathematical Finance* **8**, 1–11.
- [5] C. Gilles (1988). Charges as equilibrium prices and asset bubbles. *Journal of Mathematical Economics* **18**, 155–167.
- [6] C. Gilles, S. F. LeRoy (1992). Bubbles and charges. *International Economic Review* **33**, 323–339.
- [7] J. Jacod, P. Protter (2006). Risk neutral compatibility with option prices. *Preprint*.
- [8] R. Jarrow, P. Protter, K. Shimbo (2007). Asset price bubbles in a complete market. *Advances in Mathematical Finance*. In Honor of Dilip B. Madan, 105–130.
- [9] R. Jarrow, P. Protter, K. Shimbo (2007). Asset price bubbles in incomplete markets. *Preprint*.
- [10] M. Loewenstein, G. A. Willard (2000). Local martingales, arbitrage and viability: free snacks and cheap thrills. *Economic Theory* **16**, 135–161.
- [11] R. C. Merton (1973). Theory of rational option pricing. *Bell Journal of Economics* **4**, 141–183.
- [12] M. Schweizer, J. Wissel (2006). Term structures of implied volatilities: Absence of arbitrage and existence results. *Math. Finance*, To appear.

Upsilon transform, Rudin’s equalmeasurability theorem and classes of stationary processes

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Given a σ -finite Borel measure γ on $(0, \infty)$ and a σ -finite Borel measures ρ on \mathbb{R}^d with $\rho(\{0\}) = 0$ we define a measure ρ_γ on \mathbb{R}^d by

$$\rho_\gamma(A) = \int_0^\infty \rho(Ax^{-1}) \gamma(dx) \quad A \in \mathcal{B}(\mathbb{R}^d).$$

The map $\rho \mapsto \rho_\gamma$ is called the Υ transform and we write $\rho_\gamma = \Upsilon_\gamma(\rho)$.

This general notion of Υ transform has grown out of studies of various transformations that here have the role of special cases, just to mention [1],[3], and [2]. The present talk is based on the current work with Barndorff-Nielsen and Steen Thorbjørnsen [4]. Our main interest is in the study of Υ_γ on its Lévy domain $\text{dom}_L \Upsilon_\gamma^{(d)}$ consisting all measures ρ for which ρ_γ is a Lévy measure. $\text{dom}_L \Upsilon_\gamma^{(d)}$ is a dense subset of all Lévy measures on \mathbb{R}^d and coincides with the later class if

and only if γ is a finite measure with finite second moment, see [4] and [7]. We investigate and determine various properties of Υ_γ .

An unsolved problem is to find general criteria for γ under which Υ_γ is one-to-one on its Lévy domain. The injectivity of Υ_γ can be determined in all cases of interest but there is no unified method. We can show that if γ is finite and its Mellin transform $M_\gamma(z) = \int_0^\infty t^{z-1} \gamma(dt)$ has the property that the set $\{p \in \mathbb{R} : M_\gamma(ip + 1) = 0\}$ has nonempty interior then Υ_γ is not one-to-one. But we do not know whether the converse holds. In this talk we also relate the problem of injectivity of Υ_γ to unbounded solutions of the Deny-Choquet equation.

The Υ_γ transform is not injective when $\gamma(dx) = x^{-1-\alpha} I_{(0,\infty)}(x)dx$. However, in this case Rudin's Theorem [6] yields crucial information about the relation between ρ_1 and ρ_2 from $\Upsilon_\gamma(\rho_1) = \Upsilon_\gamma(\rho_2)$. Therefore, a more general problem than the injectivity is to characterize the class $\{\rho : \Upsilon_\gamma(\rho) = \Upsilon_\gamma(\rho_0)\}$ for fixed γ and ρ_0 .

Rudin's equalmeasurability theorem and its generalizations have been crucial in the theory of integral representations of stationary stable processes [5]. It is anticipated that solutions to the problems mentioned above will make possible extensions of these results to larger classes of infinitely divisible processes.

REFERENCES

- [1] O. E. Barndorff-Nielsen, S. Thorbjørnsen (2004). A connection between free and classical infinite divisibility. *Inf. Dim. Anal. Quantum Prob.* **7**, 573–590.
- [2] O. E. Barndorff-Nielsen, M. Maejima, K. Sato (2004). Some classes of multivariate infinitely divisible distributions admitting stochastic integral representation. *Bernoulli* **12**, 1–33.
- [3] O. E. Barndorff-Nielsen, S. Thorbjørnsen (2006). Regularising mappings of Lévy measures, *Stoch. Proc. Appl.* **116**, 423–446.
- [4] O. E. Barndorff-Nielsen, J. Rosiński, S. Thorbjørnsen (2006). General Υ -transformations, *In preparation*.
- [5] J. Rosiński (1995). On the structure of stationary stable processes, *Annals of Probability* **23**, 1163–1187.
- [6] W. Rudin (1976). L^p isometries and equimeasurability, *Indiana Univ. Math. J.* **25**, 215–228.
- [7] K. Sato (2005). Unpublished notes.

A multivariate extension of the Ornstein-Uhlenbeck stochastic volatility model

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Over the last years the Ornstein-Uhlenbeck (OU) type stochastic volatility model introduced in [2] has been studied in detail and successfully applied to model univariate financial return series. Recently [3] introduced OU type processes taking values in the positive semi-definite matrices. Currently we are using these processes as stochastic volatility processes in a multivariate extension of the OU type stochastic volatility model in [4].

Let $Y = (Y_1, Y_2, \dots, Y_d)^T$ be the d -dimensional vector of the logarithmic price processes Y_1, Y_2, \dots, Y_d of d stocks (or currencies etc.). Then the dynamics of

$Y = (Y_t)_{t \in \mathbb{R}^+}$ are given by the stochastic differential equation

$$dY_t = (\mu + \Sigma_{t-}\beta)dt + \Sigma_{t-}^{1/2}dW_t$$

where $\mu, \beta \in \mathbb{R}^d$ and W is a d -dimensional standard Brownian motion. The OU type stochastic volatility process Σ is defined as the unique stationary strong solution of

$$d\Sigma_t = (A\Sigma_{t-} + \Sigma_{t-}A^T)dt + dL_t$$

where A is a $d \times d$ matrix whose eigenvalues have strictly negative real part and L is a matrix subordinator (i.e. a Lévy process in the symmetric $d \times d$ matrices having positive semi-definite increments, see [1]) with finite logarithmic moment. It turns out that this model is highly tractable analytically and most of the appealing features of the univariate model are preserved.

In this talk the probabilistic structure of this model is studied in detail and explicit expressions for the second order structure (of the integrated volatility, the returns and “squared” returns) are obtained. Furthermore, a state space representation for the discrete observations as well as first estimation results are presented.

REFERENCES

- [1] O. E. Barndorff-Nielsen, V. Pérez-Abreu (2007). Matrix subordinators and related Upsilon transformations. *Theory Probab. Appl.*, To appear.
- [2] O. E. Barndorff-Nielsen, N. Shephard (2001). Non-Gaussian Ornstein-Uhlenbeck-based models and some of their uses in financial econometrics (with discussion). *J. R. Stat. Soc. B Statist. Methodol.* **63**, 167–241.
- [3] O. E. Barndorff-Nielsen, R. Stelzer (2007). Positive-Definite Matrix Processes of Finite Variation. *Probab. Math. Statist.* **27**, To appear.
- [4] C. Pigorsch, R. Stelzer (2007). A Multivariate Extension of the Ornstein-Uhlenbeck Stochastic Volatility Model. *In preparation*.

Regularizing mappings of Lévy measures (Upsilon transformations)

STEEN THORBJØRNSEN

For σ -finite measures γ on $(0, \infty)$ and ρ on \mathbb{R} we consider the measure ρ_γ on \mathbb{R} defined by

$$\rho_\gamma(B) = \int_0^\infty \rho(t^{-1}B) \gamma(dt),$$

for any Borel set B . Denoting by $\mathfrak{M}_L(\mathbb{R})$ the class of Lévy measures on \mathbb{R} , the Upsilon mapping Υ_γ associated to γ is defined by

$$\Upsilon_\gamma: \rho \mapsto \rho_\gamma: \text{dom}_L \Upsilon_\gamma \rightarrow \mathfrak{M}_L(\mathbb{R}),$$

where the *Lévy domain* $\text{dom}_L \Upsilon_\gamma$ of Υ_γ is given by

$$\text{dom}_L \Upsilon_\gamma = \{\rho \in \mathfrak{M}_L(\mathbb{R}) \mid \rho_\gamma \in \mathfrak{M}_L(\mathbb{R})\}.$$

It turns out that $\text{dom}_L \Upsilon_\gamma = \{0\}$ unless γ is itself a Lévy measure, and Υ_γ has full Lévy domain, i.e. $\text{dom}_L \Upsilon_\gamma = \mathfrak{M}_L(\mathbb{R})$, if and only if γ is a finite measure with finite second moment. If γ_1, γ_2 are Lévy measures on $(0, \infty)$, then the inclusion

$\text{dom}_L \Upsilon_{\gamma_1} \subseteq \text{dom}_L \Upsilon_{\gamma_2}$ is equivalent to the existence of a positive constant C such that

$$\int_0^\infty (1 \wedge s^2 t^2) \gamma_2(dt) \leq C \int_0^\infty (1 \wedge s^2 t^2) \gamma_1(dt), \quad (s \in (0, \infty)).$$

The Lévy range $\text{ran}_L \Upsilon_\gamma$ of Υ_γ is defined naturally by

$$\text{ran}_L \Upsilon_\gamma = \{ \Upsilon_\gamma(\rho) \mid \rho \in \text{dom}_L \Upsilon_\gamma \},$$

and then the inclusion $\text{ran}_L \Upsilon_{\gamma_1} \subseteq \text{ran}_L \Upsilon_{\gamma_2}$ is equivalent to the condition that γ_1 itself belongs to $\text{ran}_L \Upsilon_{\gamma_2}$.

If γ is a finite measure with finite second moment (the case of full Lévy domain), then the mapping Υ_γ gives rise, via the Lévy-Khintchine representation, to a mapping $\Upsilon^\gamma: \mathcal{ID}(\mathbb{R}) \rightarrow \mathcal{ID}(\mathbb{R})$, where $\mathcal{ID}(\mathbb{R})$ denotes the class of all infinitely divisible probability measures on \mathbb{R} . Specifically, Υ^γ may be defined in terms of cumulant transforms by the identity

$$C_{\Upsilon^\gamma(\mu)}(x) = \int_0^\infty C_\mu(xt) \gamma(dt), \quad (x \in \mathbb{R}, \mu \in \mathcal{ID}(\mathbb{R})),$$

where C_ν denotes the cumulant transform of an infinitely divisible probability measure ν . From the formula above it is easy to establish a number of appealing properties of Υ^γ among which we mention the following:

$$\Upsilon^\gamma(\mu_1 * \mu_2) = \Upsilon^\gamma(\mu_1) * \Upsilon^\gamma(\mu_2), \quad \Upsilon^\gamma(D_c \mu) = D_c \Upsilon^\gamma(\mu), \quad \Upsilon^\gamma(\delta_c) = \delta_{M_1(\gamma)c},$$

where $D_c \mu(B) = \mu(c^{-1}B)$ and $M_1(\gamma)$ is the first moment of γ . In addition, Υ^γ is continuous and closed with respect to weak convergence. As a consequence of these properties, the range $\Upsilon^\gamma(\mathcal{ID}(\mathbb{R}))$ of Υ^γ is a subset of $\mathcal{ID}(\mathbb{R})$ which contains the Dirac measures, is closed under convolution and scaling by constants, and which is topologically closed with respect to weak convergence. We mention finally that $\Upsilon^\gamma(\mu)$ may be realized as the distribution of a stochastic integral with respect to the Lévy process associated to μ .

The results outlined above were obtained as parts of the joint project [2] with Ole Barndorff-Nielsen and Jan Rosinski.

REFERENCES

- [1] O. E. Barndorff-Nielsen, M. Maejima, K. Sato (2004). Some classes of multivariate infinitely divisible distributions admitting stochastic integral representation. *Bernoulli* **12**, 1–33.
- [2] O. E. Barndorff-Nielsen, J. Rosinski, S. Thorbjørnsen (2007). General Υ -transformations. (In preparation).
- [3] O. E. Barndorff-Nielsen, S. Thorbjørnsen (2004). A connection between free and classical infinite divisibility. *Inf. Dim. Anal. Quantum Prob.* **7**, 573–590.
- [4] O. E. Barndorff-Nielsen, S. Thorbjørnsen (2005). Classical and Free Infinite Divisibility and Lévy Processes. In U. Franz and M. Schürmann (Eds.): *Quantum Independent Increment Processes II. Quantum Lévy processes, classical probability and applications to physics*. Heidelberg, Springer, 33–160.
- [5] O. E. Barndorff-Nielsen, S. Thorbjørnsen (2006). Regularising mappings of Lévy measures, *Stoch. Proc. Appl.* **116**, 423–446.

Detecting common arrival of jumps in multivariate high-frequency data

VIKTOR TODOROV

We develop tests for detecting common arrival of jumps in a discretely-observed two-dimensional Itô semimartingale X . Our interest is in determining whether the jumps in the individual elements of X arrive together on a given interval $[0, T]$, where T is a **fixed** positive number. Correspondingly, the sample probability space can be split in the following three sets

$$\begin{aligned}\Omega_T^{(j)} &= \{\omega : \text{on } [0, T] \text{ the process } \Delta X_s^1 \Delta X_s^2 \text{ is not identically } 0\} \\ \Omega_T^{(d)} &= \{\omega : \text{on } [0, T] \text{ at least one of the processes } \Delta X_s^1 \text{ and } \Delta X_s^2 \\ &\quad \text{is not identically } 0, \text{ but the process } \Delta X_s^1 \Delta X_s^2 \text{ is}\} \\ \Omega_T^{(c)} &= \{\omega : \text{on } [0, T] \text{ the processes } X^1 \text{ and } X^2 \text{ are continuous}\}\end{aligned}$$

The goal of this paper is to determine whether the sample path is in the set $\Omega_T^{(d)}$ or $\Omega_T^{(j)}$, based on equidistant observations of X at times $0, \Delta_n, \dots, [T/\Delta_n]\Delta_n$.

To define the tests we need the following preliminary notation as in [3]. For any measurable function f on \mathbb{R}^2 we set

$$V(f, \Delta_n)_T = \sum_{i=1}^{[T/\Delta_n]} f(\Delta_i^n X), \quad \Delta_i^n X = X_{i\Delta_n} - X_{(i-1)\Delta_n}.$$

In the tests we use also the following two functions

$$f(x) = x_1^2 x_2^2 \quad \text{and} \quad g(x) = x_1^2 + x_2^2.$$

To test the null hypothesis of “disjoint jumps” ($\omega \in \Omega_T^{(d)}$) we propose the following test statistic

$$\Phi_n^{(d)} = \frac{V(f, \Delta_n)_T}{V(g, \Delta_n)_T}.$$

Under fairly general conditions $\Phi_n^{(d)}$ converges in probability (as $\Delta_n \rightarrow 0$) in restriction to $\Omega_T^{(d)} \cup \Omega_T^{(j)}$ to the following

$$\frac{\sum_{s \leq T} (\Delta X_s^1 \Delta X_s^2)^2}{\sum_{s \leq T} (\Delta X_s^1)^4 + \sum_{s \leq T} (\Delta X_s^2)^4},$$

which is 0 if and only if the null hypothesis of “disjoint jumps” is true.

When the null hypothesis is of “joint jumps” ($\omega \in \Omega_T^{(j)}$) we use the following test statistic

$$\Phi_n^{(j)} = \frac{V(f, k\Delta_n)_T}{V(f, \Delta_n)_T}, \quad \text{for } k = 2, 3, \dots$$

Under fairly general conditions $\Phi_n^{(j)}$ converges in probability (as $\Delta_n \rightarrow 0$) in restriction to $\Omega_T^{(j)}$ to 1. At the same time in restriction to $(\Omega_T^{(j)})^c$ it converges to quantity which is a.s. different from 1 (and “close” to k).

We derive the asymptotic behavior of $\Phi_n^{(d)}$ and $\Phi_n^{(j)}$ as $\Delta_n \rightarrow 0$ and use this result to conduct testing for the two null hypothesis of “disjoint jumps” and “joint jumps” respectively.

Monte Carlo experiments show the power and size of the tests in empirically realistic settings.

Finally, the tests developed in this paper concern the situation where we have perfect observation of X at discrete time. Instead, it is possible that X is observed with noise, e.g. market microstructure noise in high-frequency financial data. Therefore we need to analyze the behavior of the developed tests in such scenarios as well.

This is joint work with Jean Jacod.

REFERENCES

- [1] Y. Aït Sahalia, J. Jacod (2006). Testing for jumps in a discretely observed process. *Preprint*.
- [2] O. E. Barndorff-Nielsen, S. E. Graversen, J. Jacod, M. Podolskij, N. Shephard (2006). A central limit theorem for realised power and bipower variations of continuous semimartingales. In *From Stochastic Calculus to Mathematical Finance*, the Shiryaev Festschrift, Yu. Kabanov, R. Liptser, J. Stoyanov eds, pp. 33-69, Springer Verlag, Berlin.
- [3] J. Jacod(2006). Asymptotic properties of realized power variations and related functionals of semimartingales. *Preprint*.

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