Original Paper

Cyclic gob weight and loading variations from stirred glass delivery systems

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With stirred glass delivery systems cyclic variations in gob weights and gob throws (the angles at which free falling gobs land in the molds) are caused by wobbly stirrers which are displaced (offset) from the delivery system's center line and rotating at speeds differing from the gobbing rate. This loss of statistical process control disappears if the stirrer's rotational speed is synchronized with the gobbing rate. However, even in a perfect setup, if the stirrer is offset from its central location, a new steady weight and a constant hooking of the gobs away from the center and toward the smaller gap side result.

Three equations are derived. One equation predicts the projected horizontal angle for the orientation of a gob as it lands in a mold. Another predicts the number of gobs before equal or almost equal weights repeat. A third equation relates weight variation to wobble and to the radial location of the rotating stirrer or needle. This equation shows the symmetry that may be expected in weight variation curves. It is used to generate a normalized theoretical weight curve by setting the averaged percent differences between the maximum and minimum weights divided by the average weight to 1 %. This curve may also be used to disclose the number of gobs per cycle.

Both loading and weight variations resulted when a non-centered, wobbling stirrer was modelled. The correlation between an experimental and a calculated weight variation curve, due to the stirrer's rotation and normalized to 1 %, was excellent. This investigation also showed that modelling could be extended to disclose changes in the weight (flow) of molten glass exiting from an orifice associated with a stirrer's rotation and alignment.

Zyklische Schwankungen des Tropfengewichts und des Durchsatzes bei Speisern mit Rührern

Bei Speisern mit Rührern werden zyklische Schwankungen des Tropfengewichts und des Auswurfs (d.h. des jeweiligen Winkels, in dem freifallende Tropfen in den Formen landen) durch azentrisch arbeitende Rührer verursacht, die aus dem Zentrum des Speisersystems herauslaufen und deren Rotationsgeschwindigkeit von der Schnittzahl abweicht. Diese Beeinträchtigung der statistischen Verfahrenssteuerung ist vermeidbar, wenn die Rotationsgeschwindigkeit des Rührers mit der Schnittzahl synchronisiert wird. Dennoch stellt sich sogar bei einer perfekten Einstellung ein neues stationäres Tropfengewicht und eine ständige Ablenkung der Tropfen von der Mitte in Richtung der Seite des kleineren Abstands ein, wenn der Rührer aus der zentralen Position herausläuft.

Drei Gleichungen wurden abgeleitet. Eine davon beschreibt den vorgesehenen horizontalen Winkel für die Lage eines Tropfens bei der Landung in der Form. Die zweite erlaubt die Berechnung der Tropfenanzahl, bevor gleiche oder fast gleiche Tropfengewichte sich wiederholen. Die dritte Gleichung stellt eine Beziehung her zwischen der Gewichtsschwankung, dem azentrischen Lauf und der radialen Positionierung des rotierenden Rührers oder Plungers. Letztere zeigt auch die Symmetrie, die beim Verlauf der Gewichtsschwankungen zu erwarten ist. Sie wird benutzt, um eine normierte theoretische Gewichtskurve aufzustellen, indem die durchschnittlichen prozentualen Abweichungen zwischen den Maximal- und Minimalgewichten, jeweils geteilt durch das Durchschnittsgewicht, auf 1 % festgelegt werden. Diese Kurve kann außerdem dazu verwendet werden, um die Anzahl der Tropfen pro Zyklus festzustellen.

Sowohl Durchsatz- als auch Gewichtsschwankungen ergaben sich aus der Modellierung eines nichtzentrierten Rührers. Die Korrelation zwischen einer experimentellen und einer berechneten Gewichtsschwankungskurve bezogen auf die Rotation des Rührers und der Gewichtsnormierung von 1 % war sehr gut. Diese Untersuchung zeigte auch, daß die auf einen rotierenden, ausgerichteten Rührer bezogene Modellierung ausgeweitet werden kann, um Veränderungen der Verarbeitbarkeit der Glasschmelze festzustellen.

1. Introduction

The quality and dimensions of all manufactured products are always subject to certain amounts of variation as a result of chance and can be followed by means of control charts. A control chart consists of a chronological plot of the quantity being recorded along with its upper and lower limits. When the process making the product is in statistical control, the plotted points fluctuate randomly about their average value and remain inside the control limits. The process is not under control when points lie outside the control limits, show trends and or cycles [1]. These abnormal data reveal the presence of unwanted, extraneously assignable causes influencing the product. Identification and removal or control of the troublesome causes result in a process which is in statistical control [2].

In all cases concerning gob orientation or fall (the horizontal angle at which gobs land in the molds) the phrase "throw of the gob" may be used. Gob orientation and weight can vary from a stirred glass gob delivery system without any adjustments being made and can be

Received March 17, revised manuscript August 1, 1997.

[1)] Now retired.

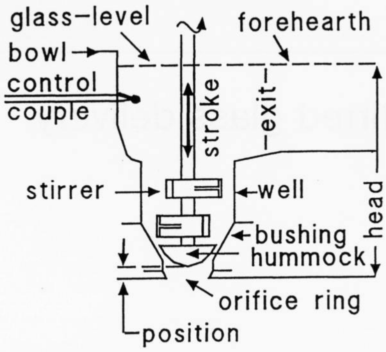


Figure 1. Sketch of a gobbing stirrer and its delivery system.

followed by means of control charts. Abnormal gobbing behavior or lack of control may result from five major causes, namely: a) changes in viscosity due to alterations in the composition of the glass, b) variations in the control temperature, c) fluctuations in the head of glass, d) mechanical problems and e) problems with stirrers or rotating needles including stroke, position, rotation, centering and wobble. Evidence supporting parts of the last four claims was presented in earlier publications [3 and 4]. The reported evidence consisted of gob weights and related variables, and each set of such data is called a weight run. The weight runs were from different production lines making different products. Consequently, there was little choice in requesting changes for a more complete study.

As reported earlier [4] changing the rotational speed (rpm) of a stirrer (i.e. the degree of synchronization) to more closely match the gobbing rate does not change the amplitude of a weight variation curve but only its period. For these cases stopping the rotation eliminated the cyclic variations in weight and loading or throw of the gobs. When rotation was restarted, new weights and gob orientations were found. In setups where synchronization is perfect no weight or loading variations due to the stirrer's rotation occur because these gobs are all cut at the same angular position of the stirrer. The work reported herein explains these problems and relates gob weight and loading variations to the angular position of the stirrer at the time of the shear cut. This angular position determines the vertical and radial gaps (clearances) between the stirrer's hummock and the adjacent bushing (figure 1). These gaps influence the weight and fall or "throw of the gobs". The influence increases as the minimum gap size decreases. While this discussion has been presented in terms of a gobbing stirrer, it is also true for a rotating needle and is not restricted to gobbing. In this report the words stirrer and rotating needle may be used interchangeably. In addition, for this work and for simplicity, additional data were obtained from a full-size model using a reciprocating, rotating needle. The remainder of this paper deals almost exclusively only with nonsynchronized stirrers. In addition, in order to restrict the number of variables, the conditions listed below were used.

2. Assumptions

2.1 Adjustment of shears

Shears are adjusted so cutting gobs does not influence how the gobs fall. Thus without chutes, gobs centered in the delivery system will fall and load vertically in the molds. Any deviations from a vertical, free fall are not caused by the shears but by the action of the rotating needle or stirrer.

2.2 Temperature

Temperature is assumed to remain constant. In symmetrical gobs any temperature gradients are radial so even if the core glass is the hottest, the temperature distribution in each outer glass layer is symmetrical and isothermal.

2.3 Geometry

All corresponding parts are similar and all round sections are assumed to have perfectly circular cross-sections centered on a straight center line. For the delivery system the stirring well, bushing and the orifice ring are perfectly shaped and aligned on a common axis. For the stirrer this means that any plates and/or the gobbing hummock are perfectly circular, the individual blades at any level have the same dimensions, shape and relative orientation and all parts are centered on the stirrer's shaft. For a needle the shaft is circular and its bottom (hummock) is symmetrical. In practice, if the center line of the rotating member is not straight or is cocked relative to the axis of rotation, runout occurs. Runout can be seen as a circular wobble or precession in the motion of the needle's bottom.

2.4 Physical variables

For the delivery system, fluid level, orifice ring diameter, controls for the stirrer's rotational speed, bottom position and stroke were kept constant but the stirrer's centering and wobble were changed.

3. General setup

The setup for this work is depicted in figure 2. Gobs are always cut at the same position just as the needle rises from the bottom of its stroke. Suppose the hummock's center line of wobble (C_W) is displaced a distance E (the eccentricity) towards the side where the word shears is located. Imagine a horizontal plane to be drawn through the hummock between the inside surfaces of the bushing at the elevation where the radial gap (or clearance) between the hummock and the bushing has a minimum. At this elevation the bushing has a radius R_B and that of the hummock is R_H. Project this plane to the elevation of the orifice and then on this new reference plane locate the center line of the bushing (C_B), the eccentricity (E) and the stirrer's wobble circle with center C_W and radius R_W to show the runout. Initially at t = 0 the angular position of the stirrer is at $\theta = 0°$ relative to the x-axis showing that the system has been adjusted so that C_B, E, C_W, R_H and the initial minimum gap (not shown) are on the same line and also in-line with the x-axis. The notation "shears" indicates that the center line of the shears is in the direction of the x-axis but, of course, must be at a lower elevation, and is the constant reference position at which all the gobs are severed. When the stirrer has rotated through an angle θ (as shown in figure 2), its axis is at C_H on the wobble circle and the minimum gap, identified as $G(\theta)_{Min}$, makes an angle ϕ with the x-axis as explained in section 5.2.

If rotation is stopped and then restarted, usually C_B, E and C_W are no longer in-line with the shears and a new set of initial variations result. This is equivalent to a rotation of the shears in figure 2 with revised values for θ and ϕ.

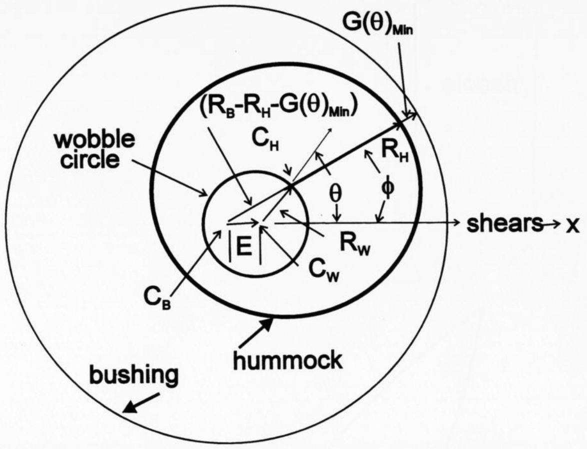


Figure 2. Pertinent variables plus the outline of the bushing and hummock at the minimum gap elevation projected downward to the orifice's level. The shears, of course, are mounted below the orifice with their center line parallel to the x-axis.

4. Special cases

4.1 Case 1

The centered, perfect parts are depicted in figure 3 with $E = R_W = 0$. At the bottom of any stroke for a perfect, rotating needle centered in a perfect delivery system, the radial clearance between the hummock and the bushing is uniform and remains constant regardless of the angular position of the hummock. For this case the minimum gap is G_0 and is equal to the clearance. At $t = 0$ let Z_0 be the reference point. For the next gob Z_0 rotates to Z_θ and the corresponding gap is G_θ which is still equal to the minimum gap. So

$$G_0 = R_B - R_H \,. \tag{1}$$

Equation (1) means that the clearance is constant so the resistance to flow of the glass does not vary with the angular position of the hummock. Consequently neither the weight nor the vertical loading of gobs (as explained later) will vary and the setup will behave like a perfectly centered, standard (nonrotating) needle with a perfect sleeve, except, of course, that visual glass striae or inhomogeneities called cords are not attenuated [5].

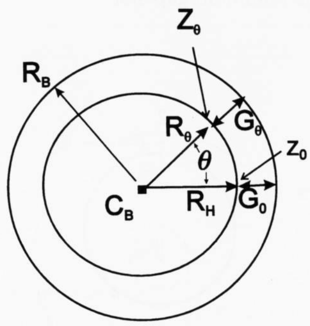


Figure 3. Gap (clearance) for case 1 with $E = R_W = 0$.

4.2 Case 2

The offset, perfect parts are shown in (figure 4 with $R_W = 0$ but $E \neq 0$. If in the previous setup the stirrer is offset from its central location, there is now an actual constant size, minimum gap between the hummock and the bushing regardless of the rotation.

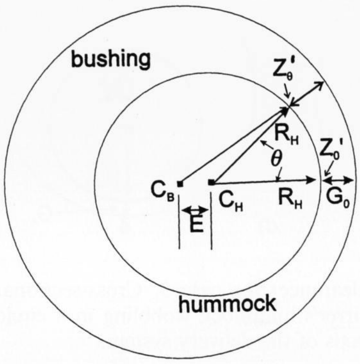


Figure 4. Gap (clearance) for case 2. Similar to figure 3 with $R_W = 0$, except center is displaced a distance E.

However, the gobs no longer fall vertically, so that a "throw of the gob" has now been introduced. The gobs will now curl towards the minimum gap side. This can be explained by noting that on the downstroke the glass moving through the narrow (minimum) gap experiences a lot more resistance to flow than the glass moving past the other side of the needle and so the flow near the smallest gap must be smaller than elsewhere. In addition, experience has shown that the temperature in the region of the smallest clearance is easily 1 to 2 K cooler than elsewhere. The net result of the lower flow and temperature is that the gobs hook toward the low flow side (figure 5). As the size of this minimum gap (G_0') remains constant, the resistance to flow again does not vary.

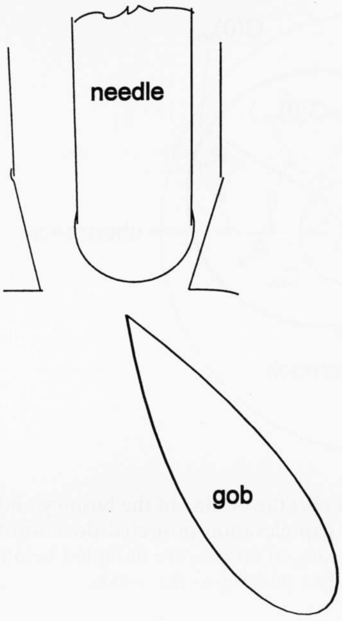
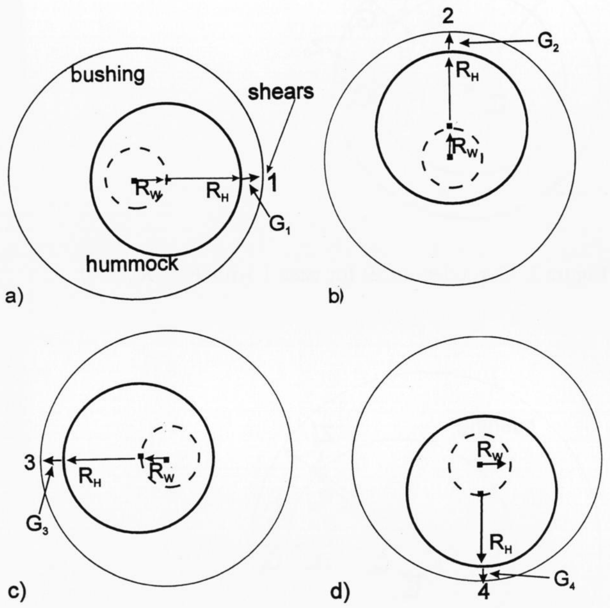


Figure 5. Gobs curl towards the minimum gap side.



Figures 6a to d. Gap (clearance) for case 3. Cross-sectional views for a needle's or stirrer's hummock wobbling in a circle which is centered on the axis of the delivery system.

Thus, although the weight has changed from its previous value for the centered case, the new weight is also constant.

Reiterating, even though the reference point Z'_0 has rotated through an angle θ to Z'_θ the minimum gap has a constant value equal to

$$G'_0 = R_B - (E + R_H) \,. \tag{2}$$

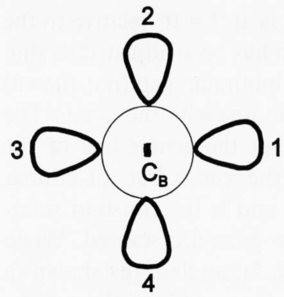


Figure 7. Relative locations of the gobs for the four positions of the hummock shown in figures 6a to d (displaced from center for clarity).

Thus, the minimum gap remains at Z'_0 and is constant for all angles as is clear from figure 4. As G'_0 is constant, the direction of "throw of the gobs" is also constant and to the right from C_B. For this setup the clearance at all points is fixed, so the flow does not vary and gob weights do not vary.

4.3 Case 3

The wobble centered on the axis of the delivery system is shown in figures 6a to d with $E = 0$ but $R_W > 0$. A stirrer wobbling in a circle whose center lies on the axis of the delivery setup, results in no weight variation, but does yield cyclic loading changes. Figures 6a to d show four cross-sectional views for the reference plane. In each figure the central square represents the axis common to the delivery system (center line of the bushing) and the other squares indicate the center line of rotation of the needle. The intermediate (heavy) circle represents the hummock and its axis is always on the circumference of the inner circle whose radius is R_W. This is the wobble circle. In figure 6a, the maximum coupling (smallest gap), is labeled G_1. Figures 6b to d depict the same setup when the needle's center line has precessed through 90, 180 and 270°, respectively. By inspection of figures 6a to d, it is clear that the size of the minimum gaps (G_1, G_2, G_3 and G_4) is the same for these four positions and (with $n = 1,2,3,4$) is

$$G_n = R_B - R_H - R_W \,, \tag{3}$$

which is independent of the angle. Thus, it follows that the fluid flowing through each configuration experiences the same forces and there is no weight variation associated with this rotation.

Viewed from above, gobs for figure 6a, on free fall, land in position 1 in figure 7, in which the gob is shown displaced radially from C_B for clarity. Similar results are found for the other positions. In actuality the gobs are not dispersed as shown but pile-up on one another with their tops overlapping at C_B. This variable "throw of the gob" is a serious defect unless chutes are used to load gobs into the molds. (From section 2.1 it is evident that the "throw of the gob" just discussed is not caused by the shears because they were adjusted not to influence the throw.)

4.4 General case

The wobble center offset from the axis of the delivery system is depicted in figures 8a to d with $E \neq 0$ and $R_W \neq 0$. A stirrer wobbling in a circle whose axis does not coincide with the axis of the delivery setup results in both cyclic weight and loading variations. Figures 8a to d are similar to figures 6a to d but include the hummock's eccentricity. For this case figure 8a has the smallest minimum gap (at Z_1) but the heaviest gob. In addition, the throw of the gob has also increased from that discussed in section 4.3 as the gap is smaller.

At $\theta = 0°$, $\phi = 0°$, according to figure 8a (angles not marked)

$$G_1' = R_B - (E + R_W + R_H) . \tag{4}$$

At $\theta = 90°$, according to figure 8b (angle not marked)

$$G_2' = R_B - R_H - (E^2 + R_W^2)^{1/2} , \tag{5}$$

and also from figure 8b

$$\phi = \tan^{-1}(R_W/E) . \tag{6}$$

For this case the reference point has rotated 90° from Z_1 to Z_2 but the minimum gap has only moved to G_2.

At $\theta = 180°$, $\phi = 180°$, according to figure 8c (angles not marked)

$$G_3' = R_B - R_H - R_W + E . \tag{7}$$

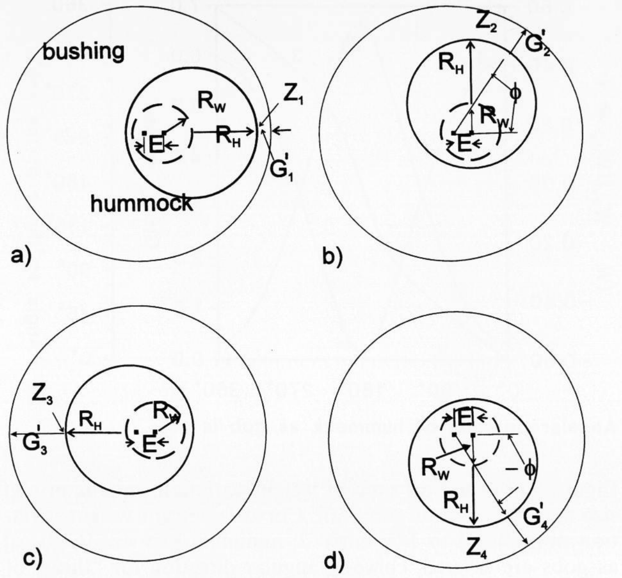
At $\theta = 270°$, according to figure 8d (angle not marked)

$$G_4' = R_B - R_H - (E^2 + R_W^2)^{1/2} , \tag{8}$$

and from the same figure

$$\phi = \tan^{-1}(-R_W/E) . \tag{9}$$

The four different values for the gaps illustrate the cyclic nature of the loading and of the weight variation for this case and substantiate the first sentence in this section.



Figures 8a to d. Similar to figures 6a to d, except the wobble circle is not centered in the well

5. Analysis

As mentioned earlier, a perfect setup does not have cyclic weight and loading variation and is not considered in the following material.

5.1 Discussion

From the discussions in section 4.1 to 4.4 it is clear that weight variation and loading variation curves are repeatable and occur because of runout and eccentricity. From experience and conjecture it is known that centering can change the violence of the "throw of the gob". In this discussion it is assumed that the needle is sufficiently close to the center of the system and its wobble is small enough, so that all possible changes such as preferential chilling at the orifice and changes in dynamic pumping (with angle) by the stirrer can be neglected. For all the different setups, at the needle's bottom position, the annular space available for the flow of glass is constant. This assumes that at the time the gob is cut, only the radial distance between the hummock and the bushing is important and that any vertical displacements associated with the changing tilt of the wobble circle can be neglected. Thus, with the assumptions listed above and in sections 2.1 to 2.4, runout and eccentricity, in this discussion, are the only variables that can cause the cyclic changes. Thus, the minimum gap at which a gob is cut may be assumed to control the variations due to rotation.

5.2 Angle ϕ

In figure 2 the arrow from C_B to the outer circle is the radius (R_B). This arrow makes an angle ϕ with the x-axis and it is perpendicular to the tangent (not shown) at that point. Likewise, the collinear arrow from C_H to the intermediate circle is the hummock's radius (R_H) and is perpendicular to the tangent (not shown) at that point. $G(\theta)_{Min}$ is the perpendicular distance between the two parallel tangents and is the minimum gap between the hummock and the bushing. Application of the trigonometric cosine law to the triangle in figure 2 and using the interior angle ϕ results in

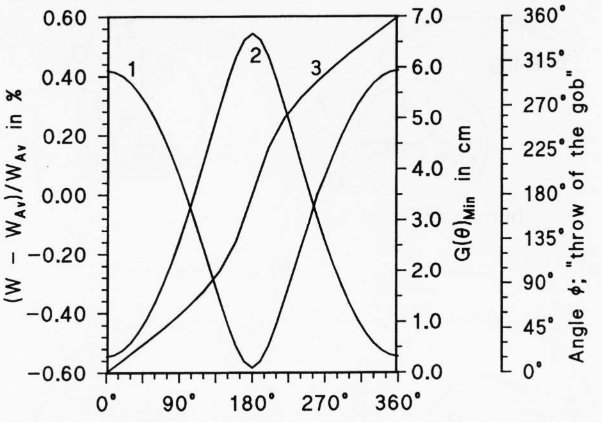


Figure 9. Theoretical percent weight variation, minimum gap size and direction of gob's fall. Curve 1: percent weight variation normalized to 1%; curve 2: minimum gap size $[G(\theta)_{\text{Min}}]$ as gobs are severed; curve 3: angular direction for "throw of the gob".

$$R_W^2 = [R_B - R_H - G(\theta)_{\text{Min}}]^2 + E^2 - 2E[R_B - R_H - G(\theta)_{\text{Min}}]\cos\phi\,, \qquad (10)$$

so

$$\cos\phi = \frac{-R_W^2 + [R_B - R_H - G(\theta_{\text{Min}}]^2 + E^2}{2E[R_B - R_H - G(\theta)_{\text{Min}}]}\,. \qquad (11a)$$

Originally, British units were used for the calculations. For a numerical example R_B, R_H, R_W and E were arbitrarily set equal to 6″, 2 − 5/8″, 2″ and 1 − 1/4″, respectively. To retain easy recognition, these numbers were not rounded-off when converted to meters and became 0.1524, 0.066675, 0.05080 and 0.03175 m, respectively. Substituting these numbers into equation (11a) yields

$$\cos\phi = \frac{-0.0015726 + [0.085725 - G(\theta)_{\text{Min}}]^2}{0.0635[0.085725 - G(\theta)_{\text{Min}}]}\,. \qquad (11b)$$

The law of sines can also be used with the triangle and yields

$$(\sin\phi)/R_W = [\sin(\pi - \theta)]/[R_B - R_H - G(\theta)_{\text{Min}}]\,. \qquad (12a)$$

Either equation (11a) or (11b) can be used to find ϕ provided the correct gap is known. For the same values of R_B, R_H, R_W and E just used, equation (12a) becomes

$$\sin\phi = 0.0508[\sin(\pi - \theta)]/[0.1524 - 0.066675 - G(\theta)_{\text{Min}}]$$

or

$$\sin\phi = 0.0508[\sin\theta]/[0.085725 - G(\theta)_{\text{Min}}]\,. \qquad (12b)$$

These equations result in the same relations for the angle ϕ found in section 4.4.

The direction for the "throw of the gob" is interesting but the minimum gap as a function of the angle is of fundamental importance.

5.3 Angle θ

The equation for the minimum gap as a function of θ can be found from the same triangle but the exterior angle θ is now used. θ is the angle between R_W and the x-axis and is the angle that the stirrer has turned. The trigonometric law now yields

$$[R_B - R_H - G(\theta)_{\text{Min}}]^2 = E^2 + R_W^2 + (2ER_W)\cos\theta\,. \qquad (13)$$

So

$$G(\theta)_{\text{Min}} = R_B - R_H - [E^2 + R_W^2 + (2ER_W)\cos\theta]^{1/2}\,. \qquad (14)$$

Of course, this equation shows that the minimum gap depends on the angular position of the hummock and the other pertinent variables when the gob is cut.

For a numerical example R_B, R_H, R_W and E were again arbitrarily set equal to 0.1524, 0.066675, 0.05080 and 0.03175 m, respectively, and then substituted into equation (14). So

$$G(\theta)_{\text{Min}} = 0.085725 - 0.0599058[1 + 0.898877\cos\theta]^{1/2}\,. \qquad (15)$$

When θ increases by multiples of 2π, the value of $\cos\theta$ remains unchanged, and the value of $G(\theta)_{\text{Min}}$ repeats showing that this function has a period of 2π. When θ is equal to any $j \cdot \pi \pm \delta$ (δ = any arbitrary angle and j = 0,1,2, etc.), the value of $G(\theta)_{\text{Min}}$ remains unchanged showing symmetry about $\theta = j \cdot \pi$ (curve 2, figure 9).

5.4 Graph of the angle for the "Throw of the gob"

This angle was found by using equations (12b and 14) which were solved simultaneously with a simple computer program (not shown). The angle ϕ is plotted as curve 3 in figure 9. It is a periodic function of the angle θ at which the gob is cut for the general case in which E and $R_W > 0$. It is noted that $\phi = 0°$ when $\theta = 0°$ and lags θ for $0° < \theta < 180°$. When $\theta = 128.68°$, $\phi = 90°$. Also $\phi = 180°$ for $\theta = 180°$ and leads θ for $180° < \theta < 360°$. At $\theta = 231.32°$, $\phi = 270°$, etc.

5.5 Weight versus minimum gap or angle

A simple assumption relating weight with the angular position of the needle is that gob weight, $W(\theta)$, is a linear function of the minimum gap at which the gob is cut, i.e.

$$W(\theta) = m\,G(\theta)_{\text{Min}} + b \qquad (16)$$

where m and b are appropriate constants that can be evaluated. As it is desirable to consider weight variations, this equation may be rewritten as

$$W(\theta) = W_{\text{AV}} + m[G(\theta)_{\text{Min}} - G_{\text{AV}}] \qquad (17)$$

where W_{AV} is the average weight and G_{AV} is equal to $G(\theta)_{\text{Min}}$ averaged over a complete cycle (see section 11.). In equation (16), b has been replaced by

$$b = W_{AV} - m\,G_{AV}\,. \qquad (18)$$

Equation (17) shows that $W(\theta) = W_{AV}$ when $G(\theta)_{Min} = G_{AV}$. (Note that the assumptions used to write equation (17) determine how b is defined by equation (18). Different assumptions will result in different forms for equation (18).) In practice, with a fixed physical structure, W_{AV} and G_{AV} can be changed by varying the temperature of the stirred glass, its chemical composition and its level (head, see figure 1) above the orifice of the delivery system. W_{AV} and G_{AV} can also be varied by any changes in the stirrer's rotational speed, direction of rotation, centering, gobbing rate as well its elevation (vertical position) and/or stroke [1]. In other words, W_{AV} and G_{AV} can be changed by varying the value of any of the parameters in equation (14).

From equation (17) divided by W_{AV}

$$[W(\theta) - W_{AV}]/W_{AV} = m[G(\theta)_{Min} - G_{AV}]/W_{AV}\,. \qquad (19)$$

Define the maximum and minimum gob weights as W_{Max} and W_{Min} and the maximum and minimum gap sizes as G_{Max} and G_{Min}. Using the fact, as explained earlier, that the gob weight has its maximum value, W_{Max}, when $G(\theta)_{Min}$ has its minimum value, equation (17) gives

$$(W_{Max} - W_{AV})/W_{AV} = m[G_{Min} - G_{AV}]/W_{AV}\,. \qquad (20)$$

Likewise

$$(W_{Min} - W_{AV})/W_{AV} = m[G_{Max} - G_{AV}]/W_{AV}\,. \qquad (21)$$

Subtracting equation (21) from equation (20) yields

$$(W_{Max} - W_{Min})/W_{AV} = m[G_{Min} - G_{Max}]/W_{AV}\,. \qquad (22)$$

It is desirable to normalize the total percent weight fluctuation to one percent. With this information

$$\frac{W_{Max} - W_{Min}}{W_{AV}}\,100\,\% = \frac{m[G_{Min} - G_{Max}]}{W_{AV}}\,100\,\% = 1\,\%. \qquad (23)$$

This results in

$$m = W_{AV}/(100[G_{Min} - G_{Max}])\,. \qquad (24)$$

On this basis equation (19) becomes

$$\frac{W(\theta) - W_{AV}}{W_{AV}}\,100 = \frac{G(\theta)_{Min} - G_{AV}}{G_{Min} - G_{Max}}\,. \qquad (25)$$

Thus theoretically, the normalized percent weight fluctuation for any gob can be found by evaluating the right hand side of equation (25). The results for a complete cycle are plotted in figure 9 as curve 1. From this curve it can be seen that the peak for the minimum weights is narrower than that for the maximum weights. Figure 9 can be used to predict a normalized percent weight variation curve for any actual or predicted weight run. This requires determining the change in the angular position of the stirrer between successive gobs cut or to be cut. This procedure is illustrated for the weight run discussed in this article in section 5.8.

5.6 General relation between rpm and gpm

When rpm and gpm are known, the actual angles at which gobs are cut can be calculated. With these angles equation (15) can then be used to calculate the corresponding minimum gaps. The closer the angles at which gobs are cut, the closer the gob weights will repeat. In this section, gob and the matching turn numbers for gobs cut close to 0°, 360°, 720°, etc. will be calculated.

Let

$$n_g/t_g = \text{gpm} \qquad (26a)$$

or

$$t_g/n_g = \text{time per gob} \qquad (26b)$$

and let u_s equal the rotational speed of the stirrer in turns per minute. Therefore,

$$u_s = 360\,n_s/t_s\,. \qquad (27)$$

Let the angle the stirrer turns through per gob be γ, then

$$\gamma = u_s t_g/n_g = (360\,n_s/t_s)\,(t_g/n_g) \qquad (28a)$$

or

$$\gamma = 360\,(\text{rpm}/\text{gpm})\,. \qquad (28b)$$

Let g_m be the integral number of gobs cut during an integral (complete) number of turns N_{int} of the stirrer. The difference in the angular position at which the gob is cut and the angular position of the stirrer is

$$\Delta = [360\,N_{int} - \gamma g_m]\,. \qquad (29a)$$

This difference may be positive or negative. With equation (28b) and using absolute values, it may be rewritten as

$$|\Delta/360| = |[N_{int} - (\text{rpm}/\text{gpm})\,g_m]| = \text{Var} \le \varepsilon \ge 0 \qquad (29b)$$

where Var is defined to be $|\Delta/360|$ and ε is any desired, arbitrary number(s). Division of this inequality by the positive number g_m results in

$$[N_{int}/g_m - (\text{rpm}/\text{gpm})] \le \varepsilon/g_m = \varepsilon' \ge 0 \qquad (29c)$$

where ε' is also arbitrary. Examination of this inequality shows that if the arbitrary numbers ε and ε' are large enough, one or more values of the ratio of the integral number of turns over number of gobs cut (N_{int}/g_m) will be close enough to rpm/gpm to satisfy this inequality.

Table 1. Variations for corresponding integral values of gobs and turns for which weights (and throws) will or almost will repeat versus ε in the range ($1 = g_m \leq 50$) with ($1 < N_{int} \leq 65$).

g_m	N_{int}	$\varepsilon = 0.08$	$\varepsilon = 0.06$	$\varepsilon = 0.04$	$\varepsilon = 0.02$	$\varepsilon = 0$
10	9	0.02	0.02	0.02	0.02	–
19	17	0.06	–	–	–	–
20	18	0.04	0.04	0.04	–	–
29	26	0.04	0.04	–	–	–
30	27	0.06	0.06	–	–	–
39	35	0.03	0.03	0.03	–	–
40	36	0.08	–	–	–	–
49	44	0.01	0.01	0.01	0.01	–

Explanation: Variations = Var = $[N_{int} - (\text{rpm/gpm})\, g_m] \leq \varepsilon \geq 0$. Division of this inequality by the positive integer, g_m, results in $[N_{int}/g_m - (\text{rpm/gpm})] \leq \varepsilon/g_m \geq 0$. When $\varepsilon = 0$ both of the above inequalities become equations. For this case (N_{int}/g_m) is exactly equal to (rpm/gpm). Otherwise, when $\varepsilon \neq 0$ and from either inequality, it is clear that the smaller ε and/or ε/g_m is, the closer the ratio of the integral number of turns over the (integral) number of gobs cut (N_{int}/g_m) is to (rpm/gpm). This statement is further explained in the discussion following equation (29b) in the text.

Clearly, as the value ε or ε' decreases, some of these ratios will be excluded. For small enough values of ε and ε', all ratios will be excluded unless N_{int}/g_m is identically equal to rpm/gpm. For this later case the inequality (equation (29c)) will be satisfied for $\varepsilon = \varepsilon' = 0$. For large values of ε or ε' there may be many values (pairs) of N_{int} and g_m which will satisfy the inequalities (equations (29b and c)). As smaller values of ε (or ε') are chosen, the number of pairs of of N_{int} and g_m which will also satisfy the inequalities' decrease (table 1). Other examples are discussed in section 5.7. In the above inequalities (equations (29b or c)), N_{int} is the number of the turn during which gobs numbered g_m are cut. For values of N_{int} and g_{int} for which dif or Var or ε or $\varepsilon' = 0$, the weight and throw will repeat exactly. When there is a mismatch in synchronization between gob cutting and stirrer rotation, $|N_{int} - (\text{rpm/gpm})g_m|$ ε and $\varepsilon' > 0$ and the weight and throw will change for each gob. The smaller the value of ε and ε', the closer weight and throw will repeat. Some examples will now be discussed.

5.6.1 rpm are multiples of gpm

Synchronization occurs when rpm is any exact multiple of gpm. All gobs will then be cut at the identical angular position of the stirrer. Consequently, no weight or loading differences due to stirrer's rotation can occur.

5.6.2 Integral values of rpm and gpm

From physical consideration if rpm < gpm, then some gob weights and throws must be different from others. For example, if rpm = 1 and gpm = 3, then three gobs are cut for each revolution of the stirrer. During the first turn, the hummock will have turned through 120° for the first gob, 240° for the second gob and 360° for the third gob. As the gaps for the original gob and the following two are different, the weights and throws must also be different. As long as the other variables remain unchanged, weights and throws will repeat exactly every third gob as these will be cut at identical angles.

For the same example equation (29b), is rewritten as

$$\Delta/(360\, g_m) = [N_{int}/g_m - (\text{rpm/gpm})] = \text{Var}/g_m \geq \varepsilon/g_m \geq 0.$$

With rpm/gpm = 1/3, this equation can be satisfied with $\varepsilon = 0$ by equating N_{int}/g_m to 1/3 or 2/6 or 3/9, etc. From this it is again clear that weights and throws will repeat every third gob. $\Delta = 360[N_{int} - (\text{rpm/gpm})\, g_m])$ will now be used to study the same example with more detail.

For the first turn $N_{int} = 1$ and three gobs (g_1, g_2 and g_3) are associated with it. So for the first gob

$$\Delta = 360[1 - 1\,(1/3)] = 240°.$$

For the second gob

$$\Delta = 360[1 - 2\,(1/3)] = 120°.$$

For the third gob

$$\Delta = 360[1 - 3\,(1/3)] = 0°.$$

For the fourth, fifth and sixth gobs $N_{int} = 2$ and the differences are

$$\Delta = 360[2 - 4\,(1/3)] = 240°,$$
$$\Delta = 360[2 - 5\,(1/3)] = 120°,$$
$$\Delta = 360[2 - 6\,(1/3)] = 0\,.$$

With this procedure it is again clear that weight and throw will repeat for multiples of 3.

With this example it is conaluded that when rpm and gpm are integers and rpm < gpm, then cyclic weight and loading variations must occur.

5.7 Determination of gob and turn numbers for which weights and throw will or almost will repeat

For this example let rpm = 28.2 and gpm = 31.4, so that rpm/gpm = 0.8980892. With these numbers the angle at which a gob is cut can be found. Then equations (14 and 17) can be used to determine the gap and the gob weight, respectively. It is assumed that the starting gob in a weight run is cut at the zero angular position of the stirrer where the gob weight is a maximum.

Equation (29b) is now used to find the succeeding gob and turn numbers for which the weights will be equal to or close to the original weight. The smaller ε is in this equation, the closer weights (and throws) will repeat. To find the values g_m and N_{int} for $\varepsilon = 0$, consider rpm/gpm = 28.2/31.4 = 282/314 = 141/157. No smaller ratio of integers is possible. With this ratio becomes equation (29b) becomes

$$|N_{int} - (141/157)g_m| = \text{Var} = \varepsilon = 0\,.$$

Obviously the solution is $g_m = 157$ and $N_{int} = 141$. Thus, depending on the accuracy of the measured vales of rpm and gpm, the first exact repetition of weight will occur for gob no. 157 and turn no. 141 and others will occur at multiples of these numbers. Equation (29b) was also solved to find other pairs of (g_m, N_{int}) with other values of ε. Table 1, generated with a computer, shows the values of g_m, N_{int} and Var for g_m ranging from 1 to 50 and N_{int} going from 1 to 65 for various values of ε. For $\varepsilon = 0.08$, eight sets of (g_m, N_{int}) were found. The first set is (10, 9) with Var = 0.02. The second and third sets are (19, 17) with Var = 0.06 and (20, 18) with Var = 0.04. The smaller value of Var for the third set means that the angle at which this gob is cut is closer to a multiple of 360° than that for the second set. This means that the first two peak gob weights will be ten gobs apart. The fourth and fifth sets are (29, 26) with Var = 0.04 and (30, 27) with Var = 0.06. Thus set (29, 26) will be cut at a more favorable angle than set (30, 27), so that its peak gob weight will be nine gobs away from the previous one. As smaller values of ε are used, fewer sets will be found. Thus, with an examination of table 1, gob order for peak weights is 0, 10, 29, 39 and 49. This shows a separation of nine and ten gobs between adjacent peaks. In addition, the different values of Var for these sets show that their peak weights are also different and that the weight of gob no. 49 is closer to that of the starting gob than the others.

Equation (29b) was now examined for (g_m, N_{int}) varying up through (195, 215) with $\varepsilon = 0.08$, 0.04 and 0. Similar conclusions resulted including the observation that multiples of 49 gobs also yielded weights quite close to the maximum gob weight.

From this work it was clear that weight (and throw) would repeat approximately every ninth or tenth gob and that the peak weights in a weight run would only occasionally repeat exactly.

6. Experimental percent weight variation curve

From theory it is known that a properly designed and operated model can be used to study gobbing representing a stirred glass delivery system [3]. Thus, a full-size model (figure 1) was used to simulate the delivering of gobs of a typical production line making borosilicate glassware. Its gobbing stirrer was offset from the delivery system's center line and had a pronounced wobble. The rpm and gpm were determined by timing 100 turns and 100 gobs, giving an rpm of 28.2 and a gobbing rate of 31.4 gpm. Gob collection was started when the angular position of the stirrer was apparently aligned to give one gob ahead of a maximum weight. This means, of course, that for the first maximum weight gob, the angle θ was as close to zero as could be realized experimentally. Thus the weight run started with gob number (−1).

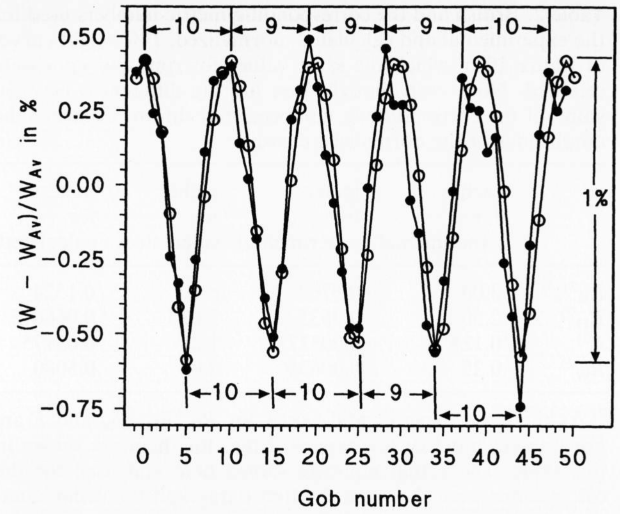


Figure 10. Percent weight variation curves normalized to 1 %. Solid circles represent experimental values, open circles are calculated points.

The cyclic gob-loading variation, predicted by theory, was clearly evident. In addition, weights varied from the average by about 6 %. The average was calculated with the following equation in which $N = 49$,

$$W_{AV} = \sum_{i=-1}^{N} W_i/(N+2) \,. \tag{31}$$

The cycles in the resultant curve repeated every nine to ten gobs. For comparison with a calculated curve, the spread was normalized to 1 %. This was done by algebraically subtracting the average of the lower peaks from the average of the upper peaks and dividing by the average weight. This percent spread was then made equal to 1 %. This 1 % spread and the normalized experimental weight curve (with solid circles) are shown in figure 10. In appearance this weight curve is similar to many others found in production runs [4]. From figure 10 it also appears that the lower peaks are sharper than the upper peaks as suggested earlier. It is also noted that the maximum weights of the gobs are different as suggested earlier, while the minimum value at gob no. 44 was the lowest. In practice, parts are not perfect, temperature distributions may not be ideal and other unpredictable perturbations may occur as stated in the introduction. Therefore, weight and loading variation curves will always show some degree of asymmetry. Of course, the better the parts are made and aligned, and the smaller the random changes in the operating variables are, the smaller the fluctuations in the cyclic variations will be.

7. Theoretical percent weight variation curve

7.1 Calculation

From the rpm and gpm values, the angle θ can be determined and by equations (12a and 14), the angle ϕ and

Table 2. British and the corresponding metric numbers used for the experimental and calculated, normalized, 1 % weight curves of figure 10 in which the same values for rpm and gpm were retained. Note, even though there are big differences between some of the corresponding numbers, they do not influence the amplitudes of the normalized curves.

	inches for experimental	meters for experimental	inches for calculated	meters for calculated
R_B[2)]	3.00	0.0762	6.00	0.1524
R_H[2)]	2.50	0.0635	2.625	0.066675
E[2)]	0.125	0.003175	1.25	0.03175
R_W[2)]	0.25	0.00635	2.00	0.5080

[2)] As the exact experimental values for R_B, R_H, R_W and E are not known, more or less typical values for them are shown in this table. The actual numbers shown here and used for the calculations were shown in the first paragraph following equation (11a). Note the large differences between the numbers relative to the experiment and to those used in the calculations. Nevertheless, the normalized experimental and calculated percent weight variation curves show excellent agreement (see figure 10).

the minimum gap can be calculated. Thus, theoretical curves of gob weights and throws can be constructed. From these curves the number of gobs at which weights and throws almost repeat can be seen directly. However, only the theoretical gob weight curve was constructed.

The calculation was started one gob early, so the gob numbered (0) was cut with $\theta \approx 0$. For equation (29a) successive gobs were cut at successive values of γ where

$$\gamma = 360\,\text{rpm/gpm} = 360(28.2/31.4) = 323.2°/\text{gob}\,. \quad (32)$$

With this value for γ, (inequality) equation (29b) with ε ranging from 0.04 to 0.01 shows that the gob weights will almost repeat every ninth or tenth gob (as shown in section 6.) in agreement with the experimental results.

The normalized theoretical percent weight variation values for gobs numbered -1 to 49 can be found from curve 1 in figure 9 at $\gamma = (i) \cdot (323.2°)$, where $i = -1$ to 51. These values can also be found more accurately and easily by substituting the γ's into equation (25) with $G_{AV} = 0.0298$ as evaluated in section 11. The resultant points are plotted in figure 10 as open circles.

7.2 Discussion

Normalization of the weight run curves changes the magnitude of the variations in weight due to eccentricity and wobble but leaves the angular variations intact. Thus, the periodicity of these curves is only a function of the angle (γ) irrespective of eccentricity and wobble and other dimensions. This statement is confirmed by noting the widely different values used for the experimental and calculated values (table 2) of R_B, R_H, R_W and E to construct the curves shown in figure 10, while the same values were retained for rpm and gpm.

The normalized percent weight curves extend over several cycles in which the gob weights almost repeat. The calculated curve based on the experimental rates, namely 28.2 rpm and 31.4 gpm correlates very well with the experimental one. The maximum weight was off by +1 gob at the third and fourth cycles and by +2 gobs in the fifth and sixth cycles. Agreement for the minimum weights was excellent. Changing the ratio rpm/gpm = 28.2/31.4 by using 28.4 for the rpm or by 0.7 % made the fit much poorer. Using 28.0 rpm shifted the disparity in the opposite direction. With rpm = 28.1 or by a shift of 0.35 % in the ratio, the maximum weight was off by +1 gob at the tops of the fourth, fifth and sixth cycles and the bottom of the cycles no longer fit as well. It was concluded that the calculated curve based on the experimental values of 28,2 rpm and 31.4 gpm resulted in the best fit and that the disparities were due to small perturbation in the experiment and to possible but unknown asymmetries in the equipment.

8. Conclusions

It was concluded that the model could be used to study weight and loading variations due to a stirrer's wobble and eccentricity, and moreover, that these normalized variations could be calculated. It was also clear that the variations could be eliminated if all the gobs were cut at the same angular position.

9. Nomenclature

9.1 Symbols

b	appropriate constant
C_B, C_H, C_W	centers for bushing, hummock and wobble, respectively (figure 2)
E	eccentricity, offset distance (figure 2)
$G(\theta)_{Min}$	minimum gap at angle θ (figure 2)
G_{AV}	average gap size
G_{Max}	maximum gap size
G_{Min}	minimum gap size
G_n	size of minimum gap independent of angle
G_0, G_θ	gaps (figure 3)
G'_0	gap (figure 4)
G_1, G_2, G_3, G_4	gaps (figures 6a to d)
G'_1, G'_2, G'_3, G'_4	gaps (figures 8a to d)
g_m	gob number
m	appropriate constant
N_{int}	integral number of turns
n_g	number of gobs
n_s	number of turns of the stirrer
R_B, R_H, R_W	radii for bushing, hummock and wobble, respectively
t	time
t_g	time for gob
t_s	time for the stirrer in min
u_s	rotational speed of the stirrer
W	gob weight
Z_0, Z_θ	reference points (figure 3)
Z'_0, Z'_θ	reference points (figure 4)
Z_1, Z_2, Z_3, Z_4	reference points (figures 8a to d)

γ	angle at which a gob is cut for the theoretical and experimental weight runs
δ	arbitrary size angle
ε	arbitrarily chosen number
θ	angular position of stirrer as gob is cut
ϕ	angle for "throw of the gob"

9.2 Subscripts

B	bushing
g	gob
H	hummock
i, n, m	index numbers
W	wobble
0	reference point at start

9.3 Abbreviations

AV	average
gpm	gobs per minute
rpm	rotation per minute
Var	variation (equation (29b))

*

The author thanks Corning Inc. for permission to publish this material. In addition thanks are extended to Dr. R. W. Palmquist and to Mr. J. H. Bruns for their help with this manuscript.

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11. Appendix: Theoretical evaluation of G_{AV}

Theoretically, this average is defined as

$$G_{AV} = \left(\int_0^{2\pi} G(\theta)\,d\theta\right) / \int_0^{2\pi} d\theta = (1/(2\pi)) \int_0^{2\pi} G(\theta)\,d\theta \tag{33}$$

With equation (14), equation (33) yields

$$G_{AV} = R_B - R_H - (1/2\pi) \int_0^{2\pi} \sqrt{R_W^2 + E^2 + 2ER_W\cos\theta}\,d\theta\,. \tag{34}$$

For a numerical example, R_B, R_H, R_W and E were arbitrarily set equal to 0.1524, 0.066675, 0.0508 and 0.03175 m, respectively, and substituted into equation (34) yielding

$$G_{AV} = 0.085725 - 0.0095343 \int_0^{\pi} \sqrt{1 + 0.898877\cos\theta}\,d\theta\,. \tag{35}$$

Integrals of the type found in equation (35) are known [6]. After various transformations letting $\theta = 2x$, $d\theta = 2dx$ and then with the identity $\cos(2x) = 1 - 2\sin^2(x)$, the integral became

$$\int_0^{2\pi} \sqrt{1 + 0.898877\cos\theta}\,d\theta = 2.7559949 \cdot \int_0^{\pi} \sqrt{1 - 0.9467457\sin^2(x)}\,dx\,. \tag{36}$$

This last integral is an elliptic integral of the second kind and its values are tabulated in [7]. Thus, equation (35) may be solved in this manner or very easily using a computer. With equation (36) equation (35) becomes

$$G_{AV} = 0.085725 - 0.0262765 \int_0^{\pi} \sqrt{1 - 0.9467457\sin^2(x)}\,dx \tag{37}$$

and after evaluation of the integral

$$G_{AV} = 0.0298\,. \tag{38}$$

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