Original Paper

Phasors for fluid conductors and vertical electric glass melters

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Phasors are useful in electrical engineering but they are even more useful when used to display and/or explain phenomena associated with fluid conductors. Phasor diagrams include not only the applied voltages but all the voltages found in the fluid. This has been illustrated by constructing and discussing phasor diagrams for a variety of circuits used in vertical glass melters. It has been shown that these diagrams can disclose wiring errors and/or undesirable voltage distributions. In many cases they can lead to designing improved circuits and/or alternate circuits which may have equivalent voltage distributions. Phasor diagrams have also been used as an aid in designing new circuitry with symmetrical electrical distributions. Simultaneous consideration of current and voltage phasors yields correct phase angles and equations to calculate power.

Zeigerdiagramme für flüssige elektrische Leiter sowie elektrische Glasschmelzöfen mit vertikalem Schmelzverlauf

Zeigerdarstellungen sind nützliche Hilfsmittel in der Elektrotechnik, aber noch hilfreicher, wenn sie zum Aufzeigen und/oder zur Erklärung von Phänomenen in flüssigen elektrischen Leitern eingesetzt werden. Zeigerdiagramme berücksichtigen nicht nur die angelegten, sondern auch alle in der Flüssigkeit vorhandenen Spannungen. Dies wird an einer Reihe von Schaltkreisen für Glasschmelzöfen mit vertikalem Schmelzverlauf durch das Erstellen und Diskutieren von Zeigerdiagrammen illustriert. Es wird gezeigt, daß diese Diagramme Verdrahtungsfehler und/oder unerwünschte Spannungsverläufe aufdecken können. In zahlreichen Fällen führen sie zur Entwicklung verbesserter und/oder periodischer Schaltkreise mit äquivalentem Spannungsverlauf. Zeigerdiagramme werden auch als Hilfsmittel beim Aufbau neuer Schaltkreise mit symmetrischem Spannungsverlauf genutzt. Die gleichzeitige Berücksichtigung von Strom- und Spannungsvektoren resultiert in korrekten Phasenwinkeln und Gleichungen zur Bestimmung der elektrischen Leistung.

1. Introduction

Phasors (section 10.) are useful in electrical engineering. They are even more useful for fluid electrical conductors and may be used not only to display voltages and currents but also to predict or estimate other voltages and phase distributions occurring throughout the fluid. This may improve understanding of what is occurring in the melter and to characterize its electrical circuits. Consequently, one may choose an alternate circuit which may give preferred results. It is emphasized that the phasor approach is strictly applicable only if the applied voltage(s) is sinusoidal. However, sinusoidal voltages may be assumed for design purposes and for trouble-shooting. The phasors resulting with this approach often lead to useful results.

The voltage from a secondary of a power supply is called a "phase voltage" and the current associated or caused by it is known as the "phase current". The voltage from a power supply measured at the terminals of a circuit is the "line voltage" and the current associated with it is called the "line current". When the line voltage results from a single secondary, the line and phase voltages are the same as are the line and phase currents. When the voltages from more than one secondary are applied to a circuit's terminals, the line voltage is the phasor sum of the phase voltages and the same is true for the line and phase currents.

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[1)] Now retired.

When each phase voltage or line voltage is applied to diametrically located electrodes, the melter is said to be cross-fired. When the circuits are connected to adjacent electrodes, the firing is said to be peripheral. When two secondaries furnish currents to an electrode, their phase currents combine to form a line current.

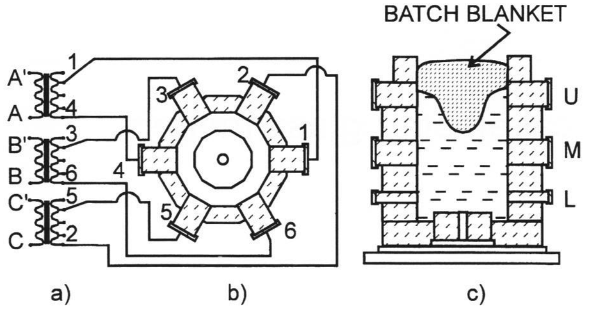
Occasionally for simplicity and to be definite a negative sign (−) may be used to indicate the tail end of a phasor and the relative polarity of the associated voltage. A positive (+) sign may be put at its head for the same reason. The inclusion of relative polarities and electrode numbers often makes it easier to construct and explain phasor diagrams.

In vertical electric melters batch or raw material is fed over a large part of the top surface, melted and refined in the upper and lower zones and then removed via an outlet near or on the bottom [1]. For vertical furnaces the outlet or delivery system is always heated electrically. Phasors for electrical delivery systems will be covered in a future report.

2. Cross-fired, vertical electric melters with in-line electrodes

2.1 Phasors for physically symmetrical, cross-fired, vertical melters

Figure 1a represents an adjustable 3-phase (3-ϕ) power supply connected to a tier of cross-fired electrodes in a Symmetrically built and operated Vertical Electric Melter (S-VEM). Figure 1b is a plan view of the melter



Figures 1a to c. Cross-fired, vertical, electric melter (VEM) with in-line electrodes, a) three-phase power supply; b) horizontal cross-section of a cross-fired S-VEM; c) vertical cross-section of a S-VEM showing electrodes and central outlet.

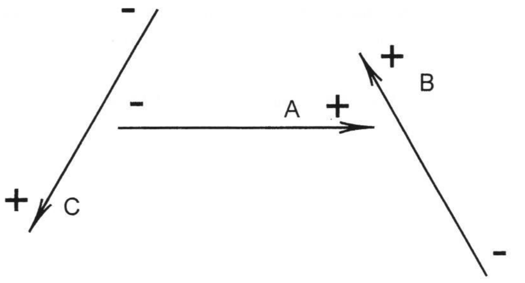


Figure 2. Directions for the power supply phasors used throughout this paper.

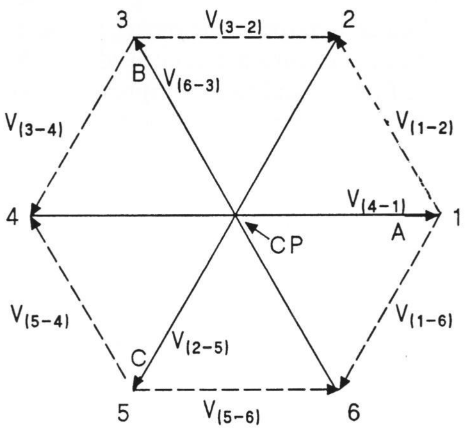


Figure 3. Applied (line) voltage phasors (solid lines) for the physically symmetrical, vertical melter (S-VEM). Dashed lines around the perimeter are the resultant (peripheral) voltage phasors between adjacent electrodes.

and shows the electrode numbers and connections for each of three tiers. It also shows the central outlet. Figure 1c is a vertical cross-section through a central plane including the outlet and the main in-line electrodes. The three sets of electrodes are called the Upper (U), Middle (M) and Lower (L) and are clearly cross-fired.

To construct its phasor diagram consider the three power supplies to be disconnected from the melter. Now adjust all three voltages (V_{AU}, V_{BU} and V_{CU}) for the upper level to V_U. The phase angles between these voltages are 120° as determined by the standard 3-ϕ connections (not shown) in the primaries of the power supplies. Arbitrarily assuming a horizontal direction for phasor A fixes the directions for phasors B and C as each makes an angle of 120° relative to the others. At this stage these phasors have the correct directions and magnitudes but are not connected together or spaced in any manner (figure 2). Now connect phase A (− to +) to electrodes no. 4 and 1. This puts electrode no. 4 at the tail-end of phasor A and electrode no. 1 at its head[2]. As the melter is symmetric its centerline must be indicated at the center point (CP) of phasor A (figure 3). Now wire phase B (− to +) to electrodes no. 6 and 3. This puts electrode no. 6 at the tail and no. 3 at head of phasor B. These connection can not change the magnitudes or directions of phasors A or B as these quantities are determined by the power supply. They can only move phasors A or B vertically and horizontally. As the melter is physically symmetrical, electrodes no. 3 and 4 influence one another in exactly the same way as electrodes no. 1 and 6. Therefore, phasors A and B must bisect each other (figure 3). Similar reasoning requires that phasor C also bisects the others.

The directed dashed lines along the perimeter of this figure show the magnitudes and directions of the voltages existing between the adjacent electrodes of the upper level and thus are phasors. While phasors 3-2, 4-CP, CP-1 and 5-6 have the same value and direction, they are not equal or the same. Each is distinct and localized and has a different meaning as each refers to different electrodes or the center point. In a similar manner note that all the other peripheral lines are also phasors.

Figure 3 shows that all the voltages between the centerline and each electrode as well as those between adjacent electrodes are equal to 1/2 V_U. In addition, the voltage differences between alternate adjacent electrodes are equal and may be calculated using the cosine law from trigonometry. Thus, for example

$$[V_{(1-3)}]^2 = [V_{(CP\text{-}3)}]^2 + [V_{(CP\text{-}1)}]^2 - 2\,V_{(CP\text{-}3)}\,V_{(CP\text{-}1)}\cos\theta\,. \quad (1)$$

For this case θ = 120°, so

$$[V_{(1-3)}]^2 = [{}^1/_2\,V_U]^2 + [{}^1/_2\,V_U]^2 - 2({}^1/_2\,V_U)({}^1/_2\,V_U)(-{}^1/_2)\,.$$

Thus

$$V_{(1-3)} = [{}^1/_2\sqrt{3}]\;V_U\,. \quad (2)$$

(Equations (1 and 2) were verified by experiment; data not shown.)

[2] In a properly designed and operated transformer the internal impedance drop may be ignored. This means that for all practical purposes the open circuit output voltage phasor for its secondary is equal to its terminal voltage phasor under load. Of course, the 3-ϕ secondary voltages could be adjusted under loads so that the terminal voltages for all three would be equal and could be used for this discussion.

From Figure 3 it is seen that CP must correlate with the centerline of the melter. Thus, the centerline is a phantom or virtual ground. If the centerline is connected to the centers of the secondaries, no current will flow between them. (This will remain true even when the other sets of electrodes are connected in a similar fashion).

Now adjust all the voltages for the middle set to V_M and connect the middle set electrodes no. 4_M and 1_M to phase A_M (− to +). Electrodes no. 4_M and 1_M have exactly the same influence on all the others. Consequently, the resultant phasor for electrodes no. 4_M to 1_M has to be collinear with phasor $A_{(4U-1U)}$ and bisect all the other phasors (figure 4). Therefore, the voltage difference between an upper and a middle in-line electrode is $1/2\,(V_U - V_M)$. A continuation of this reasoning completes the phasors for the middle set and may be used to construct the phasors for the lower set. Voltages between all the electrodes may be found in the manner already described or if the phasor diagram has been drawn to scale, voltage differences may be obtained directly from it.

The phasor diagram may also be used to help estimate the voltage at any point in the fluid. Suppose a thermocouple shall be placed at the upper tier elevation halfway between electrodes no. 1_U and 2_U. The voltage phasor at that point will have a 30° phase angle and its magnitude must necessarily be somewhat less than $1/2\, V_U$ relative to the centerline.

In a S-VEM when a line current enters the glass, it may be thought to divide into symmetrical components going to each electrode (figure 5). The side-way components cancel, so that each line current must be in-phase with its line voltage. This means that the power for any tier of electrodes is given by

$$P = 3\, I_{\text{Line}}\, V_{\text{Line}}\,. \tag{3}$$

Experiments have verified that each line current and its line voltage is in-phase for symmetrical, cross-fired melters and that equation (3) is valid; data not shown.

2.2 Geometrically symmetrical VEM with the magnitude of one line voltage changed

As an example, start with the previous melter and let the line voltage for phase A on the upper electrodes no. 4_U and 1_U be increased. As these two electrodes influence their neighbors in the same way, their old phasor can not be moved, must remain centered and can only have its length increased. Figure 6 shows the new phasor diagram from which all the voltages may be determined.

It is clear that while this melter retains its physical (structural) symmetry, it no longer has complete electrical symmetry. Thus, the peripheral temperature distributions must vary.

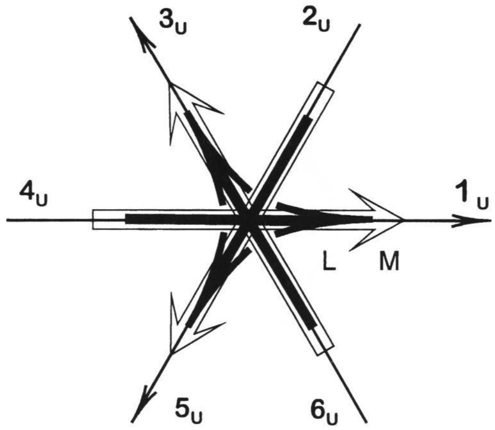


Figure 4. Upper, middle and lower circuit phasors for the cross-fired S-VEM with decreasing voltages top to bottom.

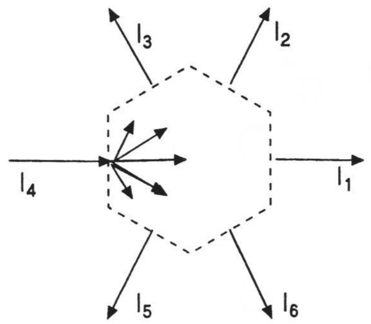


Figure 5. Line current phasors and components. I_4 is for a cross-fired S-VEM (no scale).

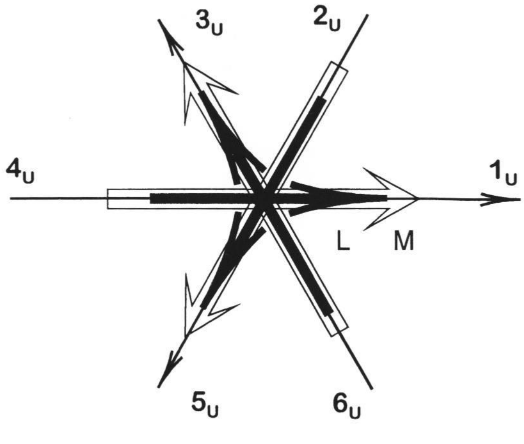


Figure 6. Phasor diagram for a symmetrically built, cross-fired VEM with an increased voltage (V_{4U-1U}) in phase A of the top tier, thus making the electrical distribution asymmetric. Note that phasor $A_{(4U-1U)}$ is longer than the others.

2.3 Phase reversal at one pair of electrodes

Occasionally by accident or misunderstanding the phasing for a pair of electrodes may be reversed. This reversal may occur because two wires have been interchanged at

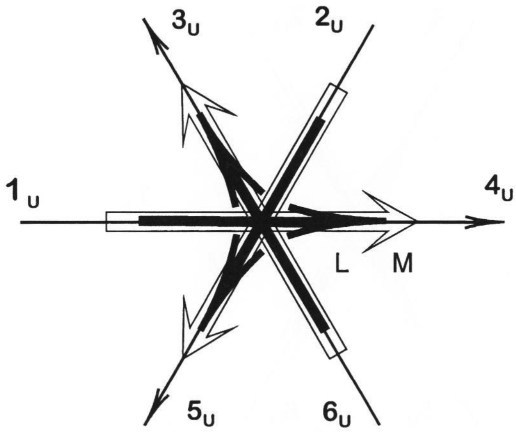


Figure 7. Phasors for a 3-tier, cross-fired VEM with leads to electrodes no. 1_U and 4_U reversed. Note interchange of electrode numbers 1_U and 4_U on phasor A.

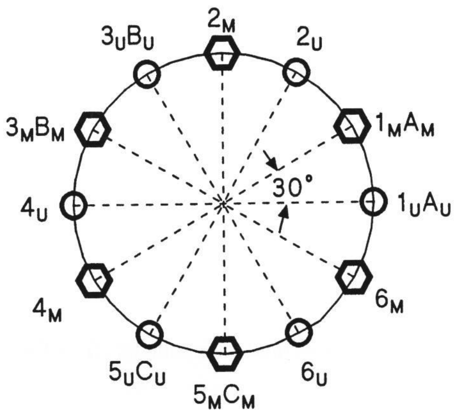


Figure 8. Electrode positions and voltage connections for a VEM with an upper (U) and a middle (M) tier of electrodes off-set by 30°. Letters A, B and C indicate relative connections and will correspond to the heads of the respective applied voltage phasors.

the electrodes, at a secondary or in the primary of the 3-ϕ power supply.

Using phase A as an example, interchange the leads to electrodes no. 4_U and 1_U. This move interchanges the electrode numbers on phasor A_U. It puts electrode no. 4_U at the head of phasor A_U and electrode no. 1_U at its tail. This move can not change the direction of any phasor including phasor A as the directions are set by the power supply. These two upper electrodes now still influence their neighbors similarly even though their influence has changed. Thus, phasor A_U must retain its position in the phasor diagram but the identifying electrode numbers on it have been interchanged (figure 7). This seemingly minor change in wiring makes a major change in the voltage distribution making it quite asymmetric. For example, the voltage between the upper level electrodes no. 1 to 2 using the cos law (equation 1) has changed from 1/2 V_U to

$$V_{(1U-2U)} = [{}^1/_2\sqrt{3}]\, V_U \tag{4}$$

while that between electrodes no. 1_U and 3_U is (by inspection)

$$V_{(1U-3U)} = {}^1/_2\, V_U\,. \tag{5}$$

In addition the voltage between upper electrode no. 1 and middle electrode no. 1 which is just below it has changed. It now is

$$V_{(1U-1M)} = {}^1/_2[V_U + V_M] \tag{6}$$

instead of its former value ${}^1/_2[V_U - V_M]$.

Clearly including electrode numbers in the phasor diagram discloses this type of wiring error quite easily.

3. Angular and spatial displacement of tiers

3.1 Cross-fired VEM with tiers of six electrodes off-set by 30°

For simplicity consider a cross-fired VEM with an upper and a middle tier and ignore electrodes at other elevations. Each level has a set of six electrodes but the sets are rotated 30° relative to each other. Figure 8 depicts the layout and also includes the phasing and relative polarities. For example, the identification for upper electrode no. 1 includes 1_U and A_U. This notation emphasizes that the head of phasor A is associated with the electrode no. 1 of the upper tier. Clearly if the melter had only one tier, figure 3 could represent its phasor diagram.

Now consider the middle set which has been rotated by 30°. Nevertheless, each of its electrodes influences all the other electrodes in the same way. This means that figure 3 can represent its phasor diagram. At this stage one might be tempted to assume, erroneously, that the phasors for the middle set would have the same orientation as the electrodes. This is impossible. As emphasized previously, directions of phasors are fixed by the power supply so the two phasor diagrams must have the same orientation. Consider electrodes no. 1_M and 4_M. For a symmetrically built and operated unit, they must influence their neighbors in the same way. This requires that the two phasor diagrams have a common center and orientation (figure 9).

A casual look at this figure suggests that this VEM with its off-set electrodes is electrically symmetrical. Examination of the voltages contradicts this. For example, voltage difference between electrodes no. 1_M and 1_U (see figure 9) is

$$V_{(1U-1M)} = {}^1/_2[V_U - V_M] \tag{7}$$

and that between electrodes no. 2_U and 1_M may be found from equation (1) with $\theta = 60°$. So

$$[V_{(2U-1M)}]^2 = [{}^1/_2\, V_U]^2 + [{}^1/_2\, V_M]^2 - \\ - 2\,[{}^1/_2\, V_U]\,[{}^1/_2\, V_M]\cos 60°\,. \tag{8}$$

As a concrete example let $V_U = 200$ V and $V_M = 100$ V. With these values equation (7) yields $V_{(1U\text{-}1M)} = 50$ V and equation (8) yields $V_{(2U-1M)} = 86.6$ V. These numbers are typical, repeat around the melter, and may be confirmed from figure 9 drawn to scale.

Many of the statements made for the earlier VEM's are also true for this configuration. In addition, note that changing the elevation or separation of the tiers does not change the phasor diagrams. As the separation is changed the distance between any two electrodes in different tiers is changed. The voltage between two electrodes divided by the distance between them is the voltage gradient between them. As the electrodes get closer, the voltage gradient increases and the local heating becomes more intense. The voltage gradient for this VEM with its off-set electrodes has its maximum when all 12 electrodes are at the same elevation but heating between the electrodes is not the same.

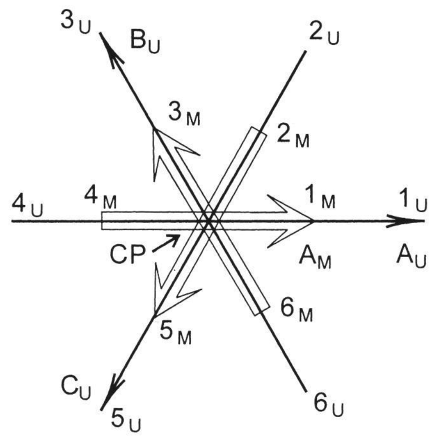


Figure 9. Phasor diagram for a cross-fired melter with level 1 and 2 electrodes off-set by 30°. Drawn with $V_U/V_M = 2$.

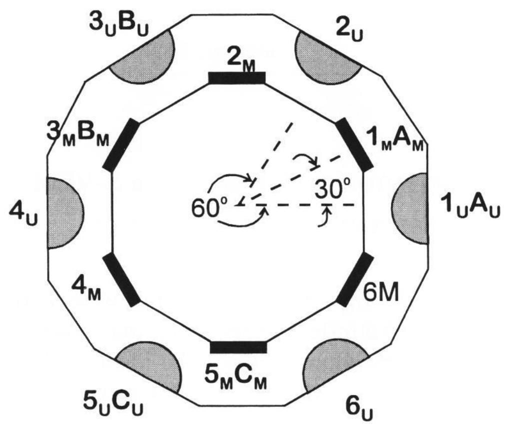


Figure 10. Sketch, top view of a two-diameter Vertical Electric Melter showing inside contour and differently shaped electrodes.

3.2 S-VEM's with two tiers of electrodes located at different diameters

To generalize cross-fired VEM's consider the melter just discussed but let the six electrodes of the middle set be located at a smaller diameter than those of the upper set. Thus, there is a jog in the walls of this melter between the two sets of electrodes and a resultant shelf at that elevation. In addition, to be even more general, let the electrodes of the middle set be flat while those of the upper set are hemispherical (figure 10). A repetition of the argument just concluded would again result in a voltage phasor diagram similar to that of figure 9. This must be true as the applied voltage phasors depend only on the line voltages at the electrodes and not on their shapes, immersions or locations as long as the melter is built and operated to retain physical symmetry.

A 3-dimensional salt water model was used to obtain representative data. Figure 10 is a partial plan view of the model showing its inside wall contours and the shape of the electrodes and their locations. For this case the model was adjusted arbitrarily to operate at a current ratio whose average was 2.13. Table 1 includes the actual data and average values. For an average value of 27.23 V applied to the upper set of electrodes, the peripheral voltage difference between the upper adjacent electrodes should have been $V_U/2 = 27.23/2 = 13.62$ V. This calculated value compares well with the V value of 13.88 V. In a similar way the average peripheral voltage difference between the electrodes of the middle set should have been $V_M/2 = 15.97/2 = 7.99$ V. Again this number compares well with the observed value of 7.88 V. From equation (7) an average voltage between an upper and a middle electrode should have been $V_{(U\text{-}M)} = (27.23-15.97)/2 = 5.63$ V. This agrees well with the average experimental value of 5.65 V. The other average peripheral value between the two sets was from equation (8) equal to 11.85 V as shown by

$$[V_{(2U-1M)}]^2 = [{}^1\!/_2\, V_U]^2 + [{}^1\!/_2\, V_M]^2 - \\ - 2[{}^1\!/_2\, V_U]\,[{}^1\!/_2\, V_M] \cos 60° = 11.90 \text{ V}.$$

This agrees well with the average measured value of 11.80 V.

Thus, values found using the voltage phasor diagram (figure 9) as a guide agreed very well with the observed results.

This model was also used to check other variations involved moving all the electrode connections for the middle set relative to the upper set. In the first of these variations corresponding phases were wired to electrodes of the two sets which were 90° apart as viewed from the top of the figure. For example, electrodes no. 1_U and 2_M (figure 10) were connected to the same phase. In the next variation corresponding phases were wired to electrodes 150° apart again viewed from above.

In all cases voltages predicted by the phasor diagrams were correct. Line currents were in-phase with their voltages and power for each tier could be found with equation (3).

Table 1. Voltage and current data from a two-diameter model with two sets of electrodes staggered by 30° (figure 10)

phase voltage in V		electrode		
upper circuit	middle circuit	number	upper current in A	middle current in A
A−A′ 27.2	A−A′ 16.3	1 and 4	4.1 and 4.2	1.9 and 1.9
B−B′ 27.2	B−B′ 16.0	3 and 6	4.1 and 4.1	2.0 and 1.9
C−C′ 27.3	C−C′ 15.6	5 and 2	4.0 and 4.0	1.8 and 2.0
average: 27.23	average: 15.97		average: 4.08	average: 1.92
average (V_U/V_M): 1.71			average (I_U/I_M): 2.13	

peripheral voltage in V			
upper circuit	middle circuit	upper to middle	upper to middle
$V_{(1U-2U)} = 14.0$	$V_{(1M-2M)} = 7.8$	$V_{(1U-1M)} = 11.8$	$V_{(1U-6M)} = 5.4$
$V_{(2U-3U)} = 13.8$	$V_{(2M-3M)} = 8.0$	$V_{(2U-2M)} = 11.8$	$V_{(2U-1M)} = 6.0$
$V_{(3U-4U)} = 13.9$	$V_{(3M-4M)} = 8.0$	$V_{(3U-3M)} = 11.8$	$V_{(3U-2M)} = 5.6$
$V_{(4U-5U)} = 14.0$	$V_{(4M-5M)} = 7.7$	$V_{(4U-4M)} = 11.8$	$V_{(4U-3M)} = 5.5$
$V_{(5U-6U)} = 13.7$	$V_{(5M-6M)} = 8.0$	$V_{(5U-5M)} = 11.8$	$V_{(5U-4M)} = 6.0$
$V_{(6U-1U)} = 13.9$	$V_{(6M-1M)} = 7.8$	$V_{(6U-6M)} = 11.8$	$V_{(6U-5M)} = 5.4$
average: 13.88	average: 7.88	average: 11.80	average: 5.65

4. Phasor diagram-aided design of a S-VEM

4.1 12 electrodes and a symmetric voltage distribution

In all preceding sections phasor diagrams were constructed for given electrode arrangements and connections. The reverse procedure will now be illustrated. A phasor diagram which has the desired characteristic will be chosen and used to help design circuitry to match.

Based on the discussion in section 3., it seems desirable to consider 12 electrodes fired to give a symmetric voltage distribution. The object is to design a symmetrical melter with 12 electrodes spaced 30° apart and with a symmetric voltage distribution. In one of the designs all 12 electrodes can be at one elevation [2]. Of course more electrical equipment will have to be added for the required circuity.

As a start, physical symmetry is assured by planning a "cylindrical" tank with 12 symmetrically placed electrodes. These could be at one elevation but, for now, two elevations will be used with the angular arrangement shown in figure 8 but with the second set identified as the lower one and with different phasing, i.e. electrical connections for the lower set. From the preceding material and from past experience it is known that electrical symmetry can be obtained with six electrodes if a physically symmetrical tank is cross-fired (figure 3).

Now referring to figure 8 the mission would be accomplished if the phasors for one set could be rotated by 30° relative to the other. This can be done with proper connections in the power supply but not by just interchanging leads. It is preferred to retain the phasor directions shown in figure 2 and for the upper six electrodes figure 3 is still valid.

4.2 Construction of figure 11a (as described in the following four paragraphs)

First, imagine a convenient size reference circle (not shown) representing the outer wall of the melter and locate the six upper electrode numbers on it (figure 11a).

Second, for the three applied upper voltage phasors let their lengths be equal to the diameter of this reference circle and draw them similar to those in figure 3 using the subscript U for the electrodes.

Third, imagine a second arbitary (smaller) reference circle (not shown), locate and label the angular position of its six electrodes with the subscript L as shown.

Fourth, draw arrows from position 3_L to 1_L, from 1_L to 5_L and from 5_L back to 3_L. From geometry it is known these three arrows have the same length and form an equilateral triangle. Note that the top arrow of these three is parallel to phasor A while the other two are parallel to phasors C and B, respectively.

4.3 Construction of figure 11b

Because of their directions and locations the last three arrows are labeled as phasors A_U, C_U and B_U, respectively. The question now is whether these three proposed phasors are real or a figment of one's imagination and can equipment actually furnish them. Starting with A_U it is noted that it has a direction, a magnitude and a separate spatial location. All these quantities can be obtained from a separate secondary connected to phase A of a power supply (top transformer, figure 11b). However, at this stage the spatial location A_U is not yet set. Now complete the delta in figure 11b with two more identical transformers adjusted to the same voltage magnitude and show their secondary voltage phasors, C_U and B_U. Note the clockwise rotation for these three phasors. This closed delta, with its electrode connections, fixes the location of the three subject phasors and ties them together. As similar electrical relations will be maintained for all the electrodes, this phasor triangle must be centered as shown (figure 11a). (The triangular representation with the three transformers in figure 11b makes the required connections obvious.) Figure 11b is now complete.

4.4 Construction of figure 11c

Now draw directed arrows 4_L to 6_L, 6_L to 2_L, and 2_L back to 4_L (figure 11a). Without repeating the previous argument, draw figure 11c and show the connections for the three remaining electrodes no. 4_L, 6_L and 2_L. Note the counterclockwise rotation used for this triangle and again note that all the voltage for it were kept constant, i.e. their phasors also form an equilateral triangle. These three additional secondaries also form a closed delta.

Normally a single primary will be used for each pair of (separate but identical) secondaries resulting in identical voltages for each double-delta.

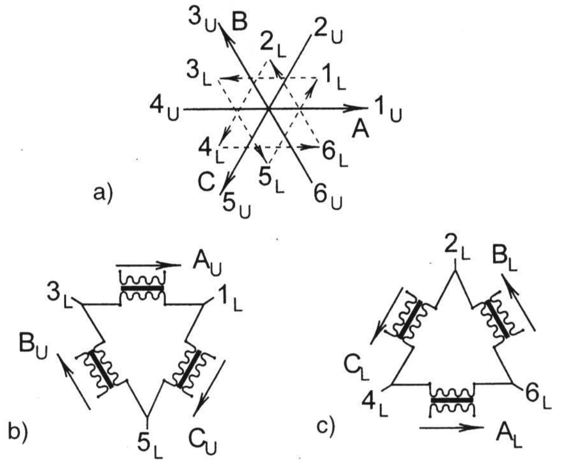
Figure 11a is now complete. It has one set with six cross-fired electrodes with a given voltage. It has a second set of six electrodes connected to a double-delta which may be controlled to the same or a second voltage.

As noted earlier in this report, the voltage phasor diagram will not change even if the melter's size is changed, its diameter is different at each tier or if the separation between the tiers is changed. Thus, the objective of a 12 electrode-tier melter with symmetrical electrical distribution has been met. From this work it is clear that additional tiers may be added.

From figure 11 and its discussion, it is clear that the voltages for the cross-fired set and that for the double-delta set may be changed independently. For any given voltage ratio between the sets the voltage difference between any two diametrically placed electrodes of a set is constant. Thus, in effect there can be a 12-electrode cross-fired tier which may have two different voltages. It is stated without proof that each line current is in-phase with the voltage difference between the corresponding diametrically opposed electrodes. As an example the line (not phase) currents measured at electrodes no. 2_L and 5_L are equal and in-phase with the voltage between these two electrodes even though they are connected to different deltas.

4.5 Three/six-electrode units

Note that a double-delta secondary can be used by itself to power a six-electrode melter and the phasor diagram for it is the star in figure 11a. Further note that the three electrodes of a delta may be used independently or may be at a different elevations from those of the other delta. For these cases the applied voltages are peripheral and not cross-fired. It can be shown (as will be done with equations (10 to 14)) that power for a double-delta, with six electrodes at a single level is given by



Figures 11a to c. Diagrams used as aids in designing VEM's, a) phasor diagram constructed while developing an electrically symmetrical melter with 12 electrodes. Solid lines are the applied cross-fired voltage phasors for the upper set, dashed lines are the applied delta-fired voltage phasors for the lower set; b) delta-connected secondaries for electrodes no. 1, 3 and 5 of the lower set; c) delta-connected secondaries for electrodes no. 2, 4 and 6 of the lower set.

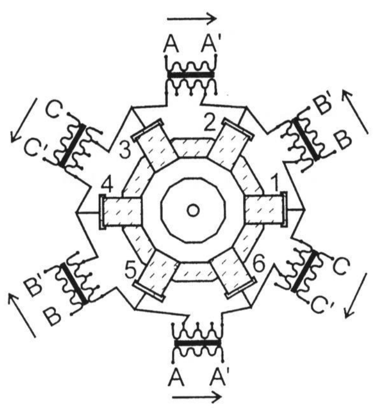


Figure 12. Plan view of a S-VEM and a 3-ϕ circuit for peripheral heating.

$$P = 2[\sqrt{3}]\, I_{\text{Line}}\, V_{\text{Phase}} \,. \tag{9}$$

5. Alternate circuits from phasor diagrams

This was illustrated in section 4. by setting the phasors of the upper set to zero and examining the remaining phasor diagram. A second example will be developed using figure 3 as the base. In this figure the phasor diagram was drawn with the applied voltage obtained from a known circuit. In this section, the reverse procedure will be followed. It will be recalled that the cross-fired applied voltage phasor diagram of figure 3 consists of three mutually bisecting straight lines with 60° between them. This phasor diagram resulted from the wiring shown in figures 1a to c. For specified 3-phase applied line voltages, specific 3-phase line currents resulted. Of

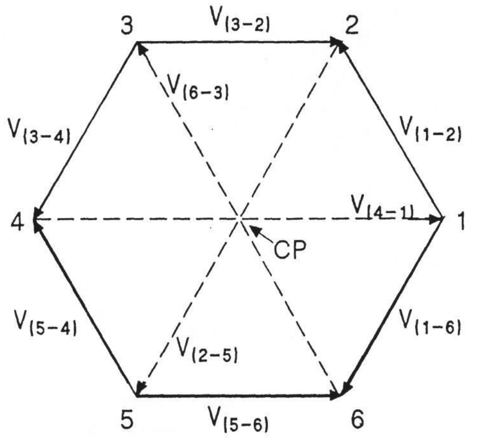


Figure 13. Applied peripheral voltage phasors (solid lines), resultant cross-fired voltage phasors (dashed lines).

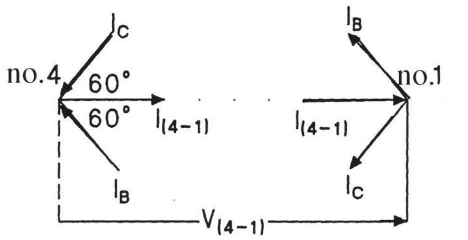


Figure 14. Current phasors for electrodes no. 4 and 1 of figure 13 with applied peripheral firing.

course, the corresponding currents and voltages were in-phase. For this symmetrically constructed and operated melter, six peripheral voltage phasors also resulted as shown by the dashed lines in figure 3.

A new circuit will now be designed to apply these six peripheral, equal magnitude voltages. It will need six separate secondaries with two secondaries connected to each of the three phases. As an aid to visualize the wiring, each secondary will be drawn parallel to its phase direction. Figure 12 is a plan view of a S-VEM and the proposed electrical connections required for the electrodes at one elevation. In this sketch voltage (A−A′) is applied to electrodes no. 3 and 2 (instead of to no. 4 and 1). Next the voltage (A−A′) from a second secondary is applied to electrodes no. 5 and 6. Then the (B−B′) voltages from two separate secondaries are applied to electrodes no. 1 and 2 and to no. 5 and 4, resepctively. The wiring is completed by using the other two separate secondaries with voltages, (C−C′). After these six peripheral voltages have been adjusted to the same magnitude, their phasor diagram may be represented by the hexagon of figure 13. Solid lines in this figure represent the applied peripheral voltage phasors while the dashed lines

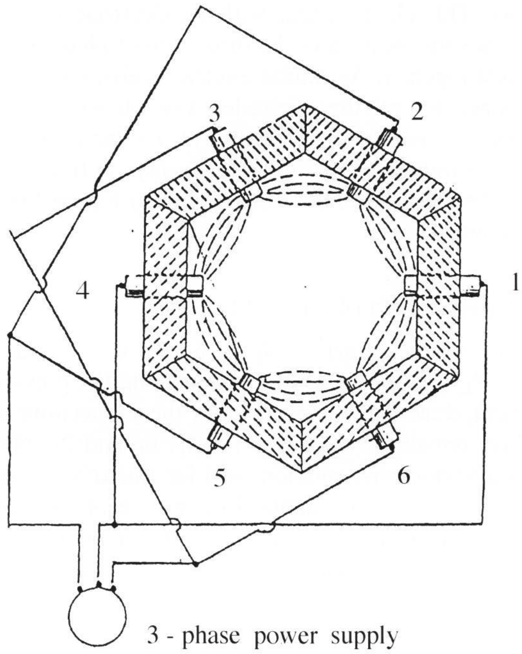
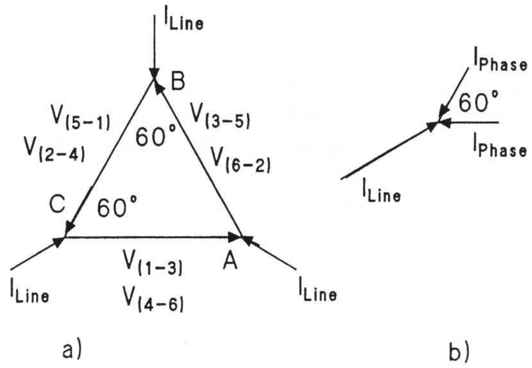


Figure 15. S-VEM with "pure" peripheral heating, i.e. with no cross-fired components.



Figures 16a and b. Current and voltage phasors for a symmetric unit wired per figure 15, a) applied voltage and line current phasors showing "pure" peripheral firing, i.e. no electric currents between diametrically placed electrodes; b) representative line and phase current phasors.

represent the resulting cross-fired phasors. This hexagon and the polygon in figure 3 are identical. Thus, a new equivalent circuit has been designed.

Some comments are in order and refer only to S-VEM's. Each secondary has a phase current which must be in-phase with its (phase) voltage as molten glass acts like an ohmic resistor. At each electrode two equal magnitude phase currents combine into a line current and the opposing components of the phase currents cancel. Thus, each line current has a current phasor which is the phasor sum of two phase currents. An example of these current phasors is shown in figure 14 which refers to electrodes no. 4 and 1. For each of these electrodes their phase currents are drawn parallel to their corresponding voltage phasors and make angles of 60° with the x axis. As the voltage magnitudes are equal and for S-VEM's, these two phase currents have equal magnitudes and their lateral components cancel. This leaves the line current, $I_{(4-1)}$, in-phase with $V_{(4-1)}$ and thus, represents the resulting cross-firing current between these two electrodes. Thus, measuring line currents and voltages at electrodes is not necessarily sufficient to distinguish between S-VEM's.

For cross-fired S-VEM's line currents must be in-phase with their respective applied voltage and these voltages may be adjusted independently. This will be true regardless of their magnitudes. On the other hand, this is not true with peripheral firing. With unequal voltages opposing lateral components of the phase currents do not cancel and thus, line currents are no longer in-phase with the corresponding voltages. (This conclusion may be visualized by examining figure 14). In addition, as the polygon of figure 13 must remain closed, changing one applied peripheral voltage causes changes in at least the other corresponding phase voltage. In general, an appreciable change in one phase voltage will cause significant alterations in the others.

6. Peripheral firing with no cross-firing components

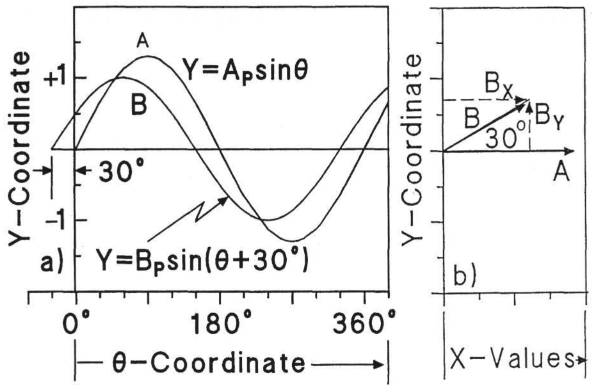
All the circuits discussed until now have electric currents between diametrically opposed electrodes and all the others. The electric current between diametrically opposed electrodes can be eliminated if they have identical voltages (phases and magnitudes). This can be realized with a single closed-delta 3-ϕ circuit whose secondaries are connected to alternate electrodes while diametrically placed electrodes are connected in parallel [3] (figure 15). From this sketch it can be seen that electrodes no. 1 and 4 are wired to one terminal of the power supply, while no. 2 and 5 are connected to another terminal and no. 3 and 6 are connected to the remaining terminal. Thus, electrodes no. 1, 3 and 5 are connected for delta-firing and electrodes no. 4, 2 and 6 are also delta-fired in exactly the same way. Each of these deltas have the same applied voltage phasor diagram (figure 16a). This figure also shows the three line currents and these make 120° angles with the adjacent voltage phasors. From this diagram it follows that the line currents are not in-phase with the applied voltages. Each line current has two phase currents (figure 16b) and each phase current is in-phase with the appropriate phase voltage. From this it follows that the magnitudes of these currents are related by

$$I_{\text{Line}} = 2\, I_{\text{Phase}} \cos 30° \tag{10}$$

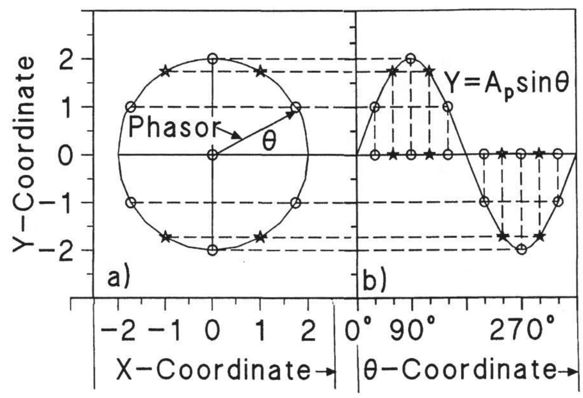
or

$$I_{\text{Line}} = [\sqrt{3}]\, I_{\text{Phase}} \,. \tag{11}$$

Power (P) for this circuit is



Figures 17a and b. Relations between two sinusoidal curves, a) curve A lags curve B by a phase angle β = 30°; b) phasor representation of the two sinusoids. Note that the phasor for curve A lies along the x axis so its y component is equal to zero: B_X and B_Y are the x and y components of phasor B.



Figures 18a and b. Construction of a sinusoidal wave using a phasor, a) angular positions of a phasor (adjusted to the peak value of the sinusoid) used to construct a sine curve; b) sine curve generated by the counter clockwise rotation of this phasor.

$$P = 6\, I_{\text{Phase}}\, V_{\text{Phase}} \,. \tag{12}$$

So using equation (11), the power may be calculated from

$$P = 2(3)^{1/2}\, I_{\text{Line}}\, V_{\text{Phase}} \,. \tag{13}$$

7. Conclusions

Symmetrically built and operated cross-fired VEM's with in-line electrodes have simple phasor diagrams which show equal voltages between adjacent electrodes and between other similarly placed electrodes. Increasing the voltage on a pair of electrodes increases the length of the corresponding phasor and the peripheral voltages now alternate between two values. If there is a phase reversal for a pair of electrodes, the phasor diagram appears unchanged but the identifying electrode numbers are switched on that pair's phasor and the melter is now electrically asymmetric. Off-setting the electrodes between two tiers by 30° does not change the voltage phasor diagram but the peripheral voltage between tiers alternates. In all cases symmetrical changes in the size of a melter, in the radial or vertical location or separation of the tiers leaves the voltage phasor diagram unchanged.

Voltage phasor diagrams can be used to help estimate the voltage at any point in the electrically conducting fluid. It was also shown that phasors can help in trouble-shooting. Lastly the method of designing an electric circuit with the help of phasor was exemplified.

8. Nomenclature

8.1 Symbols

A	phasor
A	amplitude of a sine wave
B	phasor
B	amplitude of a sinusoidal wave
C	phasor
CP	center point
grd	ground
I	current in A
P	power in W
V	voltage in V
x	horizontal coordinate
y	vertical coordinate
β	phase angle
θ	angle
ϕ	phase

8.2 Subscripts

L	lower
M	middle
P	peak value of a sinusoid
U	upper
x	component
y	component

*

The author whishes to thank Corning Inc. for permission to publish this material and to thank Dr. R. W. Palmquist, Mr. R. V. Shafer, Mr. R. A. Murnane and others for their help.

9. References

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[3] Preston, F. W.: Method and apparatus for refining fusible material. US pat. no. 3 583 861. June 8, 1971.

[4] Floyd, T. L.: Principles of electric circuits. 2nd ed. Columbus, OH: Merrill 1985. p. 345–359.

10. Appendix [4]

Alternating (ac) voltages are very carefully generated by electric power companies to have excellent sinusoidal wave forms which may be represent by the equation

$$V = V_A \sin(\theta + \beta) \tag{14}$$

where V is the instantaneous value of the sine wave, V_A is its peak or maximum value, θ is its angular position with respect to time and β, its phase angle, refers to its position relative to a coordinate system. Figure 17a shows two graphs of equation (14). Curve A starts at the origin and its β value is zero. Curve B leads curve A by the angle β which has been chosen equal to 30°. Theta (θ) is shown along the x axis. Figure 17b has two directed lines: Each has a fixed length value and a fixed angular position. The one along the x axis has $\beta = 0°$ and a length equal to the root mean square (rms) of the peak value of curve A. This is phasor A. The other line with $\beta = 30°$ lies above it and has a value equal to rms of the peak value of curve B. This is phasor B. Phasor B's (x) and (y) components are also shown. These two phasors represent the two sinusoids of figure 17a. A sinusoid plotted with $x-y$ coordinates as in figure 17a shows the instantaneous values as a function of θ (or time). A phasor may represent only the magnitude of rms and the phase angle and often only these two values are needed and wanted. A phasor may also be adjusted to represent the positive peak value of a sinusoid or its average value over a positive half cycle value.

A phasor does not throw away any ac information as it can be used to generate its sinusoid. To do this, a rms phasor is converted to its peak value by multiplying it by $\sqrt{2}$ and rotating it (figure 18a). At each angular position, its θ value and its projection on the vertical axis are used to locate the corresponding point in rectangular coordinates (figure 18b).

A phasor diagram may also be rotated as a whole to a new fixed angle without changing the magnitudes of the voltages or their relative phases (i.e. the angles between the phasors remain unchanged). Similarly a phasor or phasor diagram may be translated as a whole in the $x-y$ plane without changing the relevant electrical values. Phasor diagrams are always located in a single plane even when they refer to the voltages or electrical currents in a three-dimensional figure. The phasor sum (similar to vector sum) of the voltage phasors around any complete path or circuit is always equal to zero. In addition, the phasor sum of the current phasors meeting at any point is always equal to zero. (Note that phasor sums are not the same as scalar sums.)

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