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Groups and Geometries

Organised by
Martin Liebeck, London
Bernhard Mühlherr, Bruxelles
Gernot Stroth, Halle-Wittenberg

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Abstract. This was one of series of Oberwolfach workshops on *Groups and Geometries* which has taken place every 3 years for some time. There were three main inter-related themes: buildings and their relationship with algebraic groups; structure of finite groups; and applications.

Mathematics Subject Classification (2000): 20G20, 20G25, 20G40, 20D05, 20E42, 20E15, 20P05, 05E20, 05B25, 51E24, 51E12.

Introduction by the Organisers

The workshop *Groups and Geometries* was one of a series of Oberwolfach workshops on this topic which has taken place every 3 years for some time. It focused on algebraic and finite groups, their interactions with the geometry of buildings, and applications.

A particular highlight of the meeting was a celebration of the recent award of the Abel Prize jointly to John Thompson and Jacques Tits, two of the great pioneers of modern group theory and geometry, and both leading participants at many Oberwolfach meetings. The celebration took the form of two special lectures, given on the Tuesday evening. In the first, Bernd Fischer gave some personal recollections about Thompson, and spoke about his enormous influence on the development of finite group theory. The second lecture was given by Richard Weiss, who described some of the revolutionary innovations of Tits in the geometrical aspects of group theory, and also told some stories illustrating how Tits inspired people through his warmth and sense of humour.

There were 45 participants and 26 talks. These were on three main inter-related themes: buildings and their relationship with algebraic groups; structure of finite groups; and applications. Particularly pleasing was the participation of a substantial number (more than 10) of young researchers, several of whom – Alice Devillers, Silvia Onofrei, Rebecca Waldecker, Pierre-Emmanuel Caprace, Ralf Gramlich, Harald Helfgott and Nikolay Nikolov – gave talks. This demonstrates the attractiveness of the field.

The conference showed that the theories of buildings, algebraic groups, and finite simple groups and their geometries are very active areas with a great deal of interaction between them and also with other areas. People from these different areas were brought together, and their interaction was indeed very lively. The conference stands in the tradition of very successful meetings on *Groups and Geometries* at Oberwolfach.

Workshop: Groups and Geometries

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Abstracts

The Monster Group and Majorana involutions

A.A. Ivanov

Let G be the Monster group and V be a 196 884-dimensional module which is a sum of the trivial 1-dimensional module and the minimal non-trivial module (over the field of real numbers). This action preserves an algebra multiplication $\circ$ (known as the Griess algebra) and a positive definite inner product $\langle\ ,\ \rangle$. One can associate with every $2A$-involution in G a vector in V (called axial vector) so that the following proposition hold (cf. [C84]).

Proposition 1. *Let ϑ be a $2A$-involution in G and $a(\vartheta)$ be the corresponding axial vector. Then the action of ϑ on V is a Majorana involution with axial vector $a(\vartheta)$ with respect to the Griess algebra and the inner product $\langle\ ,\ \rangle$.* □

The relevant definitions are the following.

Definition 2. *Let V be a real vector space, let $\circ$ be a commutative algebra product on V, let $\langle\ ,\ \rangle$ be a positive definite symmetric inner product on V associative with $\circ$, and suppose that the* Norton!inequality

$$\langle u \circ u, v \circ v\rangle \geq \langle u \circ v, u \circ v\rangle$$

holds for all $u, v \in V$.

Let μ be an automorphism of $(V, \circ, \langle\ ,\ \rangle)$, and let $a = a(\mu)$ be a vector in V. Then μ is said to be a Majorana involution *and a is said to be an* axial vector *of μ if*

(i) *$a \circ a = a$, so that a is an idempotent;*

(ii) *V is the sum of s-eigenspaces of*

$$\mathrm{ad}_a : v \mapsto a \circ v$$

for s taken from $S = \{1, 0, \frac{1}{2^2}, \frac{1}{2^5}\}$ and the 1-eigenspace is one-dimensional spanned by a ;

(iii) *μ inverts every $\frac{1}{2^5}$-eigenvector of a and centralizes the other eigenvectors;*

(iv) *if v_s and v_t are s- and t-eigenvectors of a, where $s, t \in S$, then $v_s \circ v_t$ is a sum of eigenvectors with eigenvalues $r \in f(s,t)$, where $f : S \times S \to 2^S$ is the fusion function given by*

	1	0	$\frac{1}{2^2}$	$\frac{1}{2^5}$
1	1	0	$\frac{1}{2^2}$	$\frac{1}{2^5}$
0	0	0	$\frac{1}{2^2}$	$\frac{1}{2^5}$
$\frac{1}{2^2}$	$\frac{1}{2^2}$	$\frac{1}{2^2}$	$1, 0$	$\frac{1}{2^5}$
$\frac{1}{2^5}$	$\frac{1}{2^5}$	$\frac{1}{2^5}$	$\frac{1}{2^5}$	$1, 0, \frac{1}{2^2}$

It was known since the early stages of investigating the Monster that the $2A$-involutions form a class of 6-transpositions in the sense that the product of any two such involutions has order at most six. Furthermore, the products constitute the union of the following nine conjugacy classes:

$$1A,\ 2A,\ 2B,\ 3A,\ 3C,\ 4A,\ 4B,\ 5A,\ \text{and } 6A.$$

The orbit of the Monster acting by conjugation on the pairs of $2A$-involutions is uniquely determined by the class containing the product, so that the permutation rank of the Monster of the set of $2A$-involutions in nine. The subalgebras in (the 196 884-dimensional version) of the Monster algebra generated by the pairs of transposition axes as calculated by J.H. Conway and S.P. Norton [C84], [N96].

The following remarkable theorem proved by S. Sakuma [Sak07] gives a strong evidence that Majorana involution is a very efficient tool for studying the Monster.

Theorem 3. *Let (μ_0, μ_1) be a pair of Majorana involutions and let (a_0, a_1) be the corresponding pair of Majorana axes. Let $D \cong D_{2n}$ be the dihedral group (of order $2n$) generated by μ_0 and μ_1 and let Δ be the subalgebra generated by a_0 and a_1. Then*

(i) $n \leq 6$;
(ii) $\dim(\Delta) \leq 8$;
(iii) Δ *is isomorphic to one of the nine 2-generated subalgebras in the Griess algebra.* □

Conjecture. *The alternating group A_5 of degree five possesses exactly two Majorana representations whose dimensions are 26 and 21 and which correspond $(2A, 3A, 5A)$- and $(2A, 3C, 5A)$-subgroups in the Monster isomorphic to A_5.*

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Proper Moufang sets with abelian root groups are special

Yoav Segev

Recall that a Moufang set is a doubly transitive permutation group G on a set X, $|X| \geq 3$, such that the point stabilizer G_x contains a normal subgroup U_x (called the root group) which is regular on the remaining points and whose conjugates generate G. The notion of a Moufang set is due to Tits [T] and it is essentially equivalent to the notion of a split BN-pair of rank one and to Timmesfelds' notion of abstract rank one group [Ti].

In this talk we presented the following result.

Theorem ([S]). *Let $\mathbb{M}(U,\tau)$ be a proper Moufang set such that U is abelian. Then $\mathbb{M}(U,\tau)$ is special.*

One application of this result that we have in mind is the classification of Moufang sets with abelian root groups, establishing their close similarity to Quadratic Jordan division algebras (see, e.g. [DW] and [DS1]). The notation $\mathbb{M}(U,\tau)$ as well as the definition of "special" is explained in [DS1]. See also [DS2] for further information about Moufang sets. Recall that $\mathbb{M}(U,\tau)$ is proper if the little projective group of $\mathbb{M}(U,\tau)$ is not sharply 2-transitive.

Suppose now that $\mathbb{M}(U,\tau)$ is a Moufang set such that U is abelian. We denote

$$V_a := \{b \in U^* \mid \mu_a = \mu_b\} \cup \{0\}, \qquad \mathbf{NS} := \{a \in U^* \mid V_a \not\subseteq \{0, a, -a\}\},$$
$$I := \{a \in U^* \mid \mu_a^2 = 1\}.$$

We first observe that $\mathbb{M}(U,\tau)$ is special iff $\mathbf{NS} = \emptyset$ ([S, Corollary 2.2]). Thus if $\mathbb{M}(U,\tau)$ is not special, then $\mathbf{NS} \neq \emptyset$ and the set $\mathbf{NS}$ is the set of "non-special" elements. The proof of the Theorem then proceeds as follows. We first prove a variety of useful lemmas, some of them quite technical, that involve in a major way the μ-maps of $\mathbb{M}(U,\tau)$ ([S, §§3–6]). Again we see how important the μ-maps are in analyzing Moufang sets. Using these lemmas the proof then proceeds with the following crucial steps.

Main steps in the proof of the Theorem.

Step 1. If $a \in \mathbf{NS}$ and $\mu_a^2 = 1$, then V_a is a subgroup of U ([S, §7]).

Step 2. If μ_a is an involution, for all $a \in U^*$, then the Theorem holds ([S, §8])

Step 3. If $a \in \mathbf{NS}$, then $\mu_a^2 = 1$ ([S, §9]).

Step 4. Suppose that $\mathbb{M}(U,\tau)$ is not special. Then $\mathbf{NS} = I$ and $I \cup \{0\}$ is a sharply 2-transitive root subgroup of U ([S, §11]).

Step 5. Suppose that $\mathbb{M}(U, \tau)$ is not special. By Step 4, $I \cup \{0\}$ is a non-trivial sharply 2-transitive root subgroup of U. It quickly follows that $U = I$, so $\mathbb{M}(U, \tau)$ is sharply 2-transitive.

Remark. Some of the interesting (and pleasant) **surprises** in the proof of the Theorem are:

- That under the hypothesis that $a \in U^*$ violates "specialness" (i.e. $a \in \mathbf{NS}$) one can prove that V_a is a subgroup of U (which eventually turns out to be U, i.e. $V_a = U$).
- That under the hypothesis that $a \in \mathbf{NS}$ one can prove that μ_a is an involution. It is usually hard to prove that μ_a is involution (even when it should be true). Indeed this is one of the main difficulties in the proof of the converse of the Theorem i.e. the conjecture that asserts that a special Moufang set has abelian root groups (see [DST] for some results on this conjecture).

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A new proof of the Solvable Signalizer Functor Theorem

Paul Flavell

1. Signalizer Functors

A new proof of the following theorem of Glauberman [4] is announced:

Solvable Signalizer Functor Theorem. Let G be a finite group, A an elementary abelian r-subgroup of G with rank $m(A) \geq 3$ and θ a solvable A-signalizer functor on G. Then θ is solvably complete.

Recall that θ is a solvable A-signalizer functor on G means that for each $a \in A^{\#}$ we are given an A-invariant r'-subgroup $\theta(C(a))$ of $C(a)$ and that

$$\theta(C(a)) \cap C(b) \leq \theta(C(b))$$

for all $a, b \in A^{\#}$.

Signalizer functors were invented by Gorenstein as a tool for use in the Classification of Finite Simple Groups, see [7] for a discussion. The first solvable signalizer functor theorem was established by Gorenstein who considered the case $r = 2, m(A) \geq 5$. Goldschmidt [5], [6] improved this work, dealing with the cases r odd, $m(A) \geq 4$ and $r = 2, m(A) \geq 3$. Glauberman was the first to prove the definitive Solvable Signalizer Functor Theorem. A proof similar in outline to Glauberman's appears in the book by Kurzweil and Stellmacher [8].

Bender [3] gives a remarkably short proof in the case $r = 2$. His argument is quite different from Glauberman's. The ingredients are:

- An idea of Glauberman enabling effective use of induction.
- Bender's Maximal Subgroup Theorem.
- Glauberman's ZJ-Theorem.
- A fixed point theorem.

In attempting to generalize Bender's proof to arbitrary r, two difficulties arise: the ZJ-Theorem cannot be applied to all solvable groups of even order; and the fixed point theorem is not valid when r is a Fermat prime.

Aschbacher, in the first edition of his book *Finite Group Theory*, gives a proof of the Solvable Signalizer Functor Theorem along these lines. He uses the less powerful Glauberman Failure of Factorization Theorem as a substitute for the ZJ-Theorem and develops techniques for dealing with Fermat primes. Unfortunately the difficulties are such that the resulting proof is much more complex than Bender's. Indeed, in the second edition of *Finite Group Theory*, Aschbacher abandons the general case and presents a proof only for $r = 2$.

The proof presented here follows Bender's in outline. A recent result of the author on primitive pairs is a more suitable substitute for the ZJ-Theorem and we use Aschbacher's idea for dealing with Fermat primes. The resulting argument is similar to Aschbacher's but with several layers of complexity removed.

2. Primitive Pairs

Next we describe a new result on primitive pairs that is used in the author's proof of the Solvable Signalizer Functor Theorem. We begin with some definitions.

Definition. Let M be a group and p a prime. Then $\boldsymbol{M}$ **has characteristic** $\boldsymbol{p}$ if $C_M(O_p(M)) \leq O_p(M)$.

Definition. Let G be a group. A **weak primitive pair for** $\boldsymbol{G}$ is a pair (M_1, M_2) of distinct nontrivial subgroups that satisfy:

- whenever $\{i, j\} = \{1, 2\}$ and $1 \neq K$ char M_i with $K \leq M_1 \cap M_2$ then $N_{M_j}(K) = M_1 \cap M_2$.

If p is a prime then the primitive pair has **characteristic p** if in addition:

- for each i, M_i has characteristic p and $O_p(M_i) \le M_1 \cap M_2$.

We remark that if M_1 and M_2 are distinct maximal subgroups of the simple group G then (M_1, M_2) is a weak primitive pair.

In order to analyze primitive pairs we use some ideas of Meierfrankenfeld and Stellmacher [9], [10].

Definition ([10])**.** Let G be a group, p a prime, V a faithful GF$(p)G$-module and $A \le G$ an elementary abelian p-group. Then:

- A is **quadratic** on V if $[V, A, A] = 0$.
- A is **cubic** on V if $[V, A, A, A] = 0$.
- A is **nearly quadratic** on V if A is cubic on V and
$$[V, A] \le [v, A] + C_V(A)$$
for all $v \in V - \big([V, A] + C_V(A)\big)$.
- A is a **$2F$-offender** for G on V if $A \ne 1$ and
$$|V/C_V(A)| \le |A/A \cap O_p(G)|^2.$$

We remark that if A is quadratic then it is nearly quadratic. Moreover, if A is a $2F$-offender then $A \not\le O_p(G)$.

The following result and its proof are a presentation of work by Meierfrankenfeld and Stellmacher. The sources are [9], [10] and [8, 10.1.11, p.272].

Theorem A. Suppose that (M_1, M_2) is a weak primitive pair of characteristic p for the group G. Assume that M_1 and M_2 are p-solvable. Then there exists $i \in \{1, 2\}$, an elementary abelian p-subgroup V char M_i and $A \le O_p(M_1)O_p(M_2)$ such that, with $M_i^* = M_i/C_{M_i}(V)$, A^* is a nearly quadratic $2F$-offender for M_i^* on V.

This result is related to the ZJ, K^∞ and Failure of Factorization Theorems of Glauberman. Next we bring in a group of automorphisms.

Definition. Let R and G be groups. Then ***R* acts coprimely on *G*** if we are given a homomorphism $\theta : G \longrightarrow \mathrm{Aut}(G)$; the orders of R and G are coprime; and at least one of R or G is solvable.

Recall that for a prime p, $O_p(G)$ is the intersection of all the Sylow p-subgroups of G. By analogy:

Definition. Suppose R acts coprimely on the group G and that p is a prime. Then
$$O_p(G; R)$$
is the intersection of all the R-invariant Sylow p-subgroups of G.

We remark that there do exist R-invariant Sylow p-subgroups of G and that $C_G(R)$ acts transitively on them by conjugation. Moreover, $O_p(G; R)$ is the unique maximal $RC_G(R)$-invariant p-subgroup of G.

Whilst $O_p(G;R)$ may not be normal in G, it does in some respects behave like $O_p(G)$, as the following result demonstrates. It is convenient to embed R and G in their semidirect product RG.

Theorem B. Suppose that R acts coprimely on the group G, that p is a prime and that V is a faithful $\mathrm{GF}(p)RG$-module. Assume that G is p-solvable. Then $O_p(G;R)$ does not contain any nearly quadratic $2F$-offenders for G on V.

Corollary C. Let G be a group. The following configuration is impossible:

- (M_1, M_2) is a weak primitive pair of characteristic p for G,
- M_1 and M_2 are p-solvable and
- For each i there is a group R_i that acts coprimely on M_i and $O_p(M_1)O_p(M_2) \leq O_p(M_i; R_i)$.

Corollary D. Suppose R acts coprimely on the group G. Then there does not exist a weak primitive pair (M_1, M_2) of R-invariant subgroups with the properties:

- (M_1, M_2) has characteristic p for some prime p,
- M_1 and M_2 are p-solvable and
- $C_G(R) \leq M_1 \cap M_2$.

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Soluble Radicals

Rebecca Waldecker

Let G be a finite group and let $\mathrm{sol}(G)$ denote the soluble radical of G, i.e. the largest normal soluble subgroup of G. Paul Flavell conjectured in 2001 that $\mathrm{sol}(G)$ coincides with the set of all elements $x \in G$ such that for any $y \in G$ the subgroup $\langle x, y\rangle$ is soluble. This conjecture has been proved by Guralnick et al. in 2006, using the Classification of Finite Simple Groups [5]. As a first step towards a proof for this result which does not rely on the Classification, we attempt to show the following:

Theorem A. Let G be a finite group, let p be a prime and $P \in \mathrm{Syl}_p(G)$. Then $P \subseteq \mathrm{sol}(G)$ if and only if $\langle P, g\rangle$ is soluble for all $g \in G$.

In the following let G be a minimal counterexample to Theorem **A**, let p be a prime and let $P \in \mathrm{Syl}_p(G)$ be such that $\langle P, g\rangle$ is soluble for all $g \in G$, but P is not contained in the soluble radical of G. One of the main results so far is

Theorem B. Suppose that $C_G(P)$ is soluble. Let $\mathcal{L}$ denote the set of maximal P-invariant subgroups M of G such that
- $C_G(P) \leq M$,
- $[O(F(M)), P] \neq 1$ and
- if possible, there exists a prime $q \in \pi(F(M))$ such that $C_{O_q(M)}(P) = 1$.
If there exists a member $L \in \mathcal{L}$ such that $C_{F(L)}(P)$ is not cyclic, then $\mathcal{L} = \{L\}$.

In [1] it is proved that a group G is p-soluble if and only if for any Sylow p-subgroup P of G,$\langle P, g\rangle$ is p-soluble for all $g \in G$. This result, together with the minimality of G, already implies some restrictions for the structure of G. Let $K := O_{p'}(G)$. Then it turns out that P is cyclic of order p , that $G = PK$ and that K is characteristically simple. Moreover $K = [K, P]$. Whenever $M \in И_G(P)$ (i.e. M is a P-invariant subgroup of G) is such that $MP < G$, then $[M, P]$ is soluble. So our attention is lead to the maximal P-invariant subgroups of G and we set
$\mathcal{M} := \{M \leq G \,|\mathrm{M}$ is maximally P-invariant and $MP \neq G\}$.

One of the main ideas is to investigate the structure of the members of $\mathcal{M}$ and how they relate to each other. We first observe that, if $M \in \mathcal{M}$, then $M = P(M \cap K)$. So we have the cyclic p-group P acting on the p'-group $M \cap K$, and coprime action results apply. This yields our first starting point:

Lemma 1. Let $M \in \mathcal{M}$ be such that $P \not\leq Z(M)$. Then there exists a prime q such that $[O_q(M), P] \neq 1$.

As P is not central in G, we know that $C := C_G(P)$ is contained in a member of $\mathcal{M}$. If moreover C is solube, then whenever $C \leq M \in \mathcal{M}$, it follows that C is properly contained in M and the above lemma is applicable.

In the following, we assume that C is soluble and we focus on the subset $\mathcal{L}$ of $\mathcal{M}$ defined in Theorem **B**, i.e. $\mathcal{L}$ is the set of subgroups $M \in \mathcal{M}$ such that the following hold:

$C_G(P) \leq M$, $[O(F(M)), P] \neq 1$ and if possible, there exists a prime $q \in \pi(F(M))$ such that $C_{O_q(M)}(P) = 1$.

As mentioned above, C being soluble implies that the members of $\mathcal{L}$ contain C properly. So the second hypothesis for $\mathcal{L}$ is basically a statement about the prime 2, avoiding technical difficulties. The last hypothesis also is of a purely technical nature.

When collecting information about the elements in $\mathcal{L}$, then, unsurprisingly, the Bender Method turns out to be very useful. We refer the reader to [4] (p.110 et seq.) where a detailed exposition of it can be found. Very little work has to be done to make sure that the results can be applied in our context (where G is not simple!). The Bender Method can be brought into the picture because of the following result, due to Paul Flavell (Theorem 4.2 in [3]).

Pushing Down Lemma. Let $M \in \mathcal{M}$. If q is odd and if Q is a C-invariant q-subgroup of G contained in M, then $[Q, P] \leq O_q(M)$.

The stated version is a special case of Flavell's result, phrased for our situation and avoiding technical problems related to the prime 2 (and Fermat Primes).

To make sure that two members L_1, L_2 of $\mathcal{L}$ cannot have characteristic q for the same prime q, we apply results from [2]. In fact, this is the only place so far where the solubility of C plays a major role. Then we can successfully apply the Bender Method in order to prove uniqueness results. We start by showing that, for any $M \in \mathcal{L}$, the normaliser of certain C-invariant subgroups of $F(M)$ is contained in a unique member of $\mathcal{M}$.

The penultimate step is

Lemma 2. Let $M \in \mathcal{L}$, suppose that $|\pi(F(M))| \geq 2$ and that $q \in \pi$ is such that $C_{O_q(M)}(P)$ possesses an elementary abelian subgroup A of order q^2. Then $B := C_{F(M)}(A)$ is contained in a unique member of $\mathcal{M}$. In particular, $C_G(a)$ is contained in a unique member of $\mathcal{M}$ (namely M) for all $a \in A^{\#}$.

Theorem **B** follows from this by applying the Bender Method. So suppose that $L \in \mathcal{L}$ is such that $C_{F(L)}(P)$ is not cyclic. If $|\pi(F(L))| \geq 2$, then we can apply the previous lemma and obtain the result with tools related to coprime action. If $|\pi(F(L))| = 1$, then the analysis is more difficult and more complicated arguments arise. The main idea is to find a replacement for the previous lemma for this configuration. Theorem **B** can be read in a different way:

If $\mathcal{L}$ possesses more than one element, then for all $L \in \mathcal{L}$ the subgroup $C_{F(L)}(P)$ is cyclic. The next objective is to exclude this case. Then $\mathcal{L}$ has at most one member, and if $\mathcal{L}$ is empty, this gives strong information about the members of $\mathcal{M}$ containing C.

Acknowledgments

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Trees and euclidean buildings: coarse equivalences and Galois actions

Linus Kramer
(joint work with Richard M. Weiss)

A map $f : X \longrightarrow Y$ between metric spaces is called *controlled* if for every $r > 0$ there is an $s > 0$ such that $d(u, v) \leq r$ implies that $d(f(u), f(v)) \leq s$. If in addition preimages of bounded sets are bounded, the map is called *coarse*. Two coarse maps $g, f : X \rightrightarrows Y$ are called *equivalent* if the set of distances $d(f(u), g(u))$ is bounded. This leads to the coarse category whose objects are metric spaces with equivalence classes of coarse maps as morphisms. A *coarse equivalence* is an isomorphism in this category. If X, Y are geodesic spaces, then a coarse equivalence is the same as a quasi-isometry. If Γ is a group with finite generating sets $A, B \subseteq \Gamma$, then the identity map is a coarse equivalence between the two word metrics on Γ.

We are interested in questions of the following type: Given a coarse equivalence $f : X \longrightarrow Y$, does this imply that there is an isometry $\bar{f} : X \longrightarrow Y$ (rigidity), and if $\bar{f}$ exists, is it equivalent to f (strong rigidity)?. In these statements, one has to allow that the metric on Y is rescaled. Rigidity fails for trees, as the "infinite letters" X and H show, so additional assumptions are needed.

Theorem 1 Let T_1, T_2 be metrically complete leafless $\mathbb{R}$-trees and let $f : T_1 \longrightarrow T_2$ be a coarse equivalence. Assume that a group Γ acts on both trees, and that the induced map on the ends $\partial f : \partial T_1 \longrightarrow \partial T_2$ is Γ-equivariant. If the Γ-action on ∂T_1 is 2-transitive, then there is (possibly after rescaling) an equivariant isometry $\bar{f} : T_1 \longrightarrow T_2$ with $\partial \bar{f} = \partial f$. If T_1 has at least two branch points, then f and $\bar{f}$ are equivalent.

Based on this result, we prove the following.

Theorem 2 Let X_1, X_2 be metrically complete euclidean buildings and assume that the spherical buildings at infinity, ∂X_1 and ∂X_2, are thick. Assume that $f : X_1 \times \mathbb{R}^{n_1} \longrightarrow X_2 \times \mathbb{R}^{n_2}$ is a coarse equivalence. Then we have the following.

(i) $n_1 = n_2$ and $\partial X_1 \cong \partial X_2$.

(ii) If X_1 has no tree factors, then there is an isometry $\bar{f} : X_1 \longrightarrow X_2$.

(iii) If in addition to (ii), no factor of X_1 is an infinite cone over a spherical building, then $\bar{f}$ and f are equivalent (possibly after rescaling the metrics on the de Rham factors).

Theorem 2 generalizes Mostow-Prasad rigidity [3], Kleiner-Leeb [1] and Leeb [2]. In contrast to [1] and [2] we do not assume that the spherical building ∂X_1 is Moufang or compact. The proof relies on Tits' rigidity result [4] and uses Theorem 1 applied to the wall trees of X_1.

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Lattices in non-positively curved spaces

Pierre-Emmanuel Caprace
(joint work with Nicolas Monod)

Hilbert's fifth problem consists in finding a purely algebraic/topological characterisation of Lie groups within the category of locally compact groups. A solution has been obtained by Gleason-Montgomery-Zippin (see e.g. [Kap71]): a connected locally compact group is a Lie group if and only if it has no small subgroup, namely if it possesses a neighbourhood of the identity containing no nontrivial subgroup. Besides Lie groups, another important class of locally compact groups is provided by algebraic groups over locally compact fields, and the corresponding version of Hilbert fifth problem makes sense for these. An algebraic characterisation of p-adic analytic groups is known since the fundamental work of M. Lazard (see e.g. [DdSMS99]). On the other hand, for algebraic groups over local fields of positive characteristic the problem is still open.

Restricting to the simple algebraic groups, we propose to approach the problem by taking advantage of two facts which enrich the data:

- Simple algebraic groups over locally compact fields act cocompactly on non-positively curved metric spaces in the sense of Alexandrov, also called CAT(0) spaces (see [BH99] for the definition and basic theory). Indeed, semisimple groups over Archimedean local fields are nothing but semisimple Lie groups, which act on symmetric spaces in a canonical way; over non-Archimedean fields, the corresponding objects are the Euclidean buildings constructed by Bruhat and Tits.
- Semisimple groups over locally compact fields tend to contain lattices, namely discrete subgroups of finite invariant covolume.

We henceforth consider triples (G, Γ, X) consisting of a locally compact group G, a lattice Γ in G and a locally compact geodesically complete CAT(0) space on which G acts continuously, properly, effectively and cocompactly. The latter condition is equivalent to the requirement that G is a closed subgroup of $\mathrm{Is}(X)$ such that the quotient space $G\backslash X$ is compact, where $\mathrm{Is}(X)$ is endowed with its canonical structure of locally compact topological group given by the topology of uniform convergence on compacta.

Among the groups G appearing in such triples are all the non compact simple Lie groups with trivial centre, but also semisimple groups including products of the form $G = \mathrm{PSL}_n(\mathbf{R}) \times \mathrm{PSL}_n(\mathbf{Q}_p)$. Indeed, the diagonal embedding of $\Gamma =$

$\mathrm{PSL}_n(\mathbf{Z}[\frac{1}{p}])$ in G makes it a lattice, and G acts cocompactly on the CAT(0) space $X = M \times B$, where M is the symmetric space of $\mathrm{PSL}_n(\mathbf{R})$ and B the Bruhat-Tits building associated with $\mathrm{PSL}_n(\mathbf{Q}_p)$. A product of metric spaces is here endowed with the ℓ^2-metric, namely the metric defined by Pythagoras formula; with this metric any finite product of CAT(0) spaces is itself CAT(0). In fact, one should emphasize that triples as above include many examples with G non-algebraic (and Γ non-arithmetic), and even G non-linear. Indeed, there are examples of triples (G, Γ, X) where $G = \Gamma$ is a Gromov hyperbolic group, or with G non-discrete and Γ a finitely generated (or even finitely presented) simple group.

Theorem A. *Let (G, Γ, X) be a triple as above such that G/Γ is compact, Γ is irreducible and G is reducible. If Γ admits a faithful finite-dimensional linear representation (in characteristic $\neq 2, 3$), then X is a product of symmetric spaces and Bruhat-Tits buildings. In particular the socle of G is a direct product of simple algebraic groups over local fields and automorphism groups of (bi)regular trees.*

By definition, a (topological) group is called *irreducible* if no finite index closed subgroup splits nontrivially as a direct product. The *socle* of a group is the subgroup generated by all nontrivial minimal closed normal subgroups; notice that there is a priori no reason why this subgroup should be nontrivial. In fact, some information on the socle of G may be obtained with weaker assumptions than in the previous statement:

Theorem B. *Let (G, Γ, X) be a triple as above such that G/Γ is compact, Γ is irreducible and G is reducible. If Γ is residually finite, then the socle of G is a finite direct product of (topologically) characteristically simple groups, each of which is non discrete. Moreover, any nontrivial closed normal subgroup of G contains a minimal one.*

It follows in particular that G has no nontrivial discrete normal subgroup.

Among the ingredients involved in the proof of Theorem A is a detailed analysis of the full isometry group of the space X, which is completely independent of the existence of lattices. Combining this analysis with the solution to Hilbert fifth problem for Lie groups, we deduce that the connected component of the identity G° is a direct product of noncompact simple Lie groups. Applying this to the group G appearing in Theorem B, we deduce that the identity component G° is contained in the socle of G. In particular, if G° is nontrivial, then Γ possesses a linear representation over $\mathbf{R}$, which in view of Theorem A, sheds some light on the following:

Theorem C. *Let (G, Γ, X) be a triple as above such that G/Γ is compact, Γ is irreducible and G is reducible. If Γ is residually finite and G is not totally disconnected, then X is a product of symmetric spaces and Bruhat-Tits buildings. In particular the socle of G is a direct product of simple algebraic groups over local fields and automorphism groups of (bi)regular trees.*

We finish by mentioning that characterisations of the same vein have been obtained in [Mon05] in a purely algebraic context, namely without assuming that the group G act on a geometric space, but making instead stronger initial restrictions on the algebraic structure of G, e.g. that G is semisimple. Besides the interest that the present considerations might have from a purely geometrical viewpoint, we emphasize that in the results presented here the semi-simplicity of G appears as a conclusion rather than as a premise.

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On the lattice of subgroups of the special linear groups

Michael Aschbacher

First some motivation. The following question has been around for almost 30 years:

Question. Is every nonempty finite lattice isomorphic to a lattice $\mathcal{O}_G(H)$ of overgroups of H in G, for some finite group G and subgroup H of G?

One motivation for the Question comes from the following theorem:

Theorem. (Palfy-Pudlak) The following are equivalent:
(1) The Question has a positive answer.
(2) Every finite lattice is isomorphic to a congruence lattice of a finite algebra.

The answer to the Question is presumably no. Indeed let Λ be a finite lattice, 0 and ∞ the least and greatest elements of Λ, and $\Lambda' = \Lambda - \{0, \infty\}$, regarded as a graph under the compatibility relation. Define Λ to be *disconnected* if the graph Λ' is disconnected. Write $\Delta(m)$ for the lattice of subsets of an m-set, and define Λ to be a $D\Delta$-*lattice* if there exists integers $r > 1$ and $m_i > 2$, $1 \le i \le r$, such that Λ' has r connected components Λ'_i, and $\Lambda'_i \cong \Delta(m_i)'$.

Conjecture. If Λ is a $D\Delta$-lattice then there exists no finite group G and subgroup H of G such that $\Lambda \cong \mathcal{O}_G(H)$.

Aschbacher and John Shareshian have a program to prove this conjecture. They have a "reduction" to the case G almost simple, and have established the conjecture when G is an alternating or symmetric group. The next test case they consider is the case $F^*(G) \cong L_n(q)$ for some n and prime power q. In order to treat this case, one needs various fundamental results about overgroups of certain subgroups of such groups G. Here is one example due to a Caltech undergraduate, Po-Ling Loh:

Theorem. (Loh) Let G be an isotropic classical group over an arbitrary field, let G° be the subgroup generated by the unipotent elements of G, and let R be the unipotent radical of a proper parabolic of G. Then for each $X \in \mathcal{O}_G(R)$, either $G^\circ \leq X$, or X is contained in some proper parabolic subgroup of G.

We are interested in groups G such that for some n-dimensional vector space V over a finite field $F = F_q$, $Z(GL(V))SL(V) \leq G \leq \Gamma L(V)$, and $Z(GL(V)) \leq H \leq G$ such that $\mathcal{O}_G(H)$ is a $D\Delta$-lattice. To show no such group exists, we must control the overgroups of suitable normal subgroups D of maximal overgroups M of H in G.

Loh's theorem can be used to treat the case where H is reducible on V. In that case we can choose M to be a maximal parabolic, and take D to be the unipotent radical of M.

The next case of interest is when H is irreducible but imprimitive on V. Here we can take M to be the stabilizer of a direct sum decomposition $\mathcal{D} = \{V_1, \ldots, V_r\}$ of V; that is $V = V_1 \oplus \cdots \oplus V_r$. For example if $r = n$, so that $\dim(V_i) = 1$ for each i, we can take D to be the kernel of the action of $M \cap GL(V)$ on $\mathcal{D}$; that is D is a Cartan subgroup of G. In this case the following extension of a theorem of Seitz is useful. For $Y \leq G$ let $\mathcal{D}(Y)$ be the set of direct sum decompositions of V preserved by Y. There is an obvious partial order on such decompositions which appears in the next result.

Theorem. Assume X is an irreducible subgroup of G, $\mathcal{D} \in \mathcal{D}(X)$ with $|\mathcal{D}| = n$ and let D be the corresponding Cartan subgroup of G. Assume $Y \in \mathcal{O}_G(DX)$ with $SL(V) \not\leq Y$. Assume $q > 5$ and $n > 2$ if $q \leq 11$. Then there exists $\mathcal{E} \in \mathcal{D}(Y)$ such that $\mathcal{E} \leq \mathcal{D}$.

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Complete Reducibility and Separability

Ben Martin
(joint work with Michael Bate, Gerhard Röhrle and Rudolf Tange)

1. Introduction

Let G be a reductive algebraic group over an algebraically closed field k of characteristic $p > 0$. A subgroup H of G is said to be *G-completely reducible* if whenever H is contained in a parabolic subgroup P of G, there exists a Levi subgroup L of P such that $H \subseteq L$. The concept of G-complete reducibility was introduced by Serre [9, p19], motivated by the theory of buildings. It generalises the usual notion of complete reducibility, which is the special case $G = \mathrm{GL}_n(k)$.

A subgroup H of G is said to be *separable* if $\mathrm{Lie}(C_G(H))$ coincides with $\mathfrak{c}_{\mathrm{Lie}(G)}(H)$, where $C_G(H)$ denotes the centraliser of H in G and $\mathfrak{c}_{\mathrm{Lie}(G)}(H)$ denotes the centraliser of H in the Lie algebra $\mathrm{Lie}(G)$. Note that $\mathrm{Lie}(C_G(H))$ is always contained in $\mathfrak{c}_{\mathrm{Lie}(G)}(H)$, but the inclusion may be proper: for example, take $H = G = \mathrm{SL}_p(k)$.

Separability of H appears as a hypothesis in several results involving G-complete reducibility (see Section 3 below). In this talk we describe some of our recent work involving the interaction between separability and G-complete reducibility.

A subgroup H of G is either G-completely reducible or not. In either case one can say useful things. Here are some examples.

Proposition 1. [1, Cor. 3.17] *If H is G-completely reducible then the centraliser $C_G(H)$ is also G-completely reducible.*

Proposition 2. *If H is not G-completely reducible then there exists a parabolic subgroup $P = P(H)$ of G such that $P \supseteq HC_G(H)$ and H is not contained in any Levi subgroup of P.*

We call $P(H)$ from Proposition 2 the *optimal destabilising parabolic subgroup for H*. Optimal destabilising parabolics play a part in the proof of Proposition 1 and other results concerning G-complete reducibility [6], [5, Prop. 2.2], [1, Thm. 3.10, Thm. 5.8].

2. The geometric approach

Let $N \in \mathbb{N}$. The group G acts on G^N by simultaneous conjugation:

$$g \cdot (g_1, \ldots, g_N) := (gg_1g^{-1}, \ldots, gg_Ng^{-1}).$$

Let $\mathbf{h} = (h_1, \ldots, h_N) \in G^N$ and let H be the algebraic subgroup of G generated by the h_i. Note that there is no loss of generality in assuming that any subgroup H under consideration is of this form [1, Rem. 2.9, Lem. 2.10].

Theorem 3. [8, Thm. 16.4, Prop. 16.9], [1, Thm. 3.1] *H is G-completely reducible if and only if the orbit $G \cdot \mathbf{h}$ is a closed subset of G^N.*

Theorem 3 allows us to apply methods from geometric invariant theory to the study of G-complete reducibility. In particular, there is a more general notion of optimal destabilising parabolic subgroup $P(v)$ whenever we have an affine G-variety V and $v \in V$ such that the orbit $G \cdot v$ is not closed in V [4, Thm. 3.4]; the parabolic $P(H)$ in Proposition 2 is precisely $P(v)$, where $V = G^N$, $v = \mathbf{h}$ and H is the algebraic subgroup of G generated by the h_i.

3. Separability and Non-Separability

We now give three results involving G-complete reducibility which have separability of a certain subgroup as a hypothesis. Let M be a reductive subgroup of G and let H be a subgroup of M. First we recall a definition [7]: if $\mathrm{Lie}(M)$ has an M-stable complement in $\mathrm{Lie}(G)$ then we say that (G, M) is a *reductive pair*.

Proposition 4. *Suppose (G, M) is a reductive pair and H is separable in G. If H is G-completely reducible then H is M-completely reducible.*

This follows from Theorem 3 together with part (b) of the following result.

Proposition 5. *Let $\mathbf{h} = (h_1, \ldots, h_N) \in M^N$ such that H is the algebraic subgroup of G generated by the h_i. Suppose (G, M) is a reductive pair and H is separable in G. Then*

(a) *$G \cdot \mathbf{h} \cap M^N$ is a finite union of M-orbits;*
(b) *each of these M-orbits is closed in $G \cdot \mathbf{h} \cap M^N$.*

The proof is based on a nice geometric argument due to Richardson [7, Thm. 4.1]. Guralnick has shown that if $N = 1$ then part (a) holds without the hypotheses that (G, M) is a reductive pair and H is separable in G [3, Thm. 1.2].

Proposition 6. *Suppose H is separable in G. If* $\mathrm{Lie}(G)$ *is a semisimple H-module then H is G-completely reducible.*

In recent work [2], we studied what happens when one removes the hypothesis that H is separable from Propositions 4, 5 and 6. We suspect that Proposition 6 is false with the hypothesis removed, though we have no counterexample. We have found a different hypothesis which gives the same conclusion [2, Cor. 4.5]:

Proposition 7. *Suppose p is good for G and $[G, G]$ is simply connected or adjoint. If* $\mathrm{Lie}(G)$ *is a semisimple H-module then H is G-completely reducible.*

On the other hand, both parts of Proposition 5 — and hence also Proposition 4 — can fail without the separability hypothesis.

Theorem 8. [2, Sec. 7] *Let $p = 2$, let G be simple of type G_2 and let M be a subgroup of G of type $A_1\widetilde{A}_1$. Then (G, M) is a reductive pair, and there exists a finite subgroup H of M such that H is G-completely reducible but not M-completely reducible. Moreover, there exists $\mathbf{h} = (h_1, h_2) \in M^2$ such that $G \cdot \mathbf{h} \cap M^2$ is an infinite union of M-orbits.*

We finish with a criterion for separability of a subgroup [2, Thm. 1.2].

Theorem 9. *Let H be a connected reductive subgroup of G. If p is very good for G then H is separable.*

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Strange images of profinite groups

Nikolay Nikolov

Let G be a profinite group. By 'strange image' of G in the title I mean an image which is not continuous. The interest in such images arises in connection with the problem of closed verbal subgroups in profinite groups.

Let w be a word, i.e. an element of the free group F on k generators. The set of all values of w and w^{-1} in G is denoted by G_w. The verbal subgroup $w(G)$ is just $\langle G_w \rangle$, the subgroup of G generated by G_w. We say that w has width m in G if

$$w(G) = G_w \cdot G_w \cdots G_w \quad (m \text{ times})$$

The following is well known.

Lemma. *Let G be a profinite group and w be a word. Then the following are equivalent*

1. $w(G)$ is closed in G,

2. w has finite width in G

3. w has bounded width in the collection $\{G/N \mid N \lhd_o G\}$ of continuous finite images of G.

Clearly whenever $w(G)$ is not closed q in G then $\overline{w(G)}/w(G)$ is a strange image of $\overline{w(G)}$.

A major problem in this area is to characterize the words w which satisfy the equivalent conditions of Lemma 1 for any finitely generated profinite groups. This has been done for pro-p groups by A. Jaikin [1]

Theorem 1. *Let p be a prime and $w \in F$ be a word. Then $w(P)$ is closed in each finitely generated pro-p group P if and only if $w \notin F''(F')^p$.*

For general profinite groups Nikolov and Segal [3] proved

Theorem 2. *Suppose that w is either a basic commutator $[x_1, \dots, x_k]$ of else a d-locally finite word for some $d \in \mathbb{N}$. Then $w(G)$ is closed in all d-generated profinite groups G.*

In view of these results is is sensible to focus first on prosoluble groups.

Conjecture. *The words $w \in F$ such that $w(G)$ is closed in all finitely generated prosoluble groups are presicely those outside the union of $F''(F')^p$ for all primes p.*

By work of Segal it follows that if Conjecture 1 is false then there is a prosoluble group with a nontrivial perfect image. So we ask

Problem. Does there exist a prosoluble group with a nontrival perfect image?

By [4] such a group cannot be finitely generated.

We are interested in strange images from another direction. It follows from Theorem 2 (see [3]) that finitely generated profinite groups do not have strange finite images. Therefore they cannot have strange residually finite images. It is natural to try to extend these results and ask the following:

Blaubeuren problem. Is there a profinite group with an infinite finitely generated image?

Again by [4] the answer is No for prosoluble groups and in my talk I showed that the same answer holds for Cartesian products of finite nonabelian simple groups. This easily follows from the following

Proposition. Let G be a cartesian product of nonabelian finite simple groups. Then every simple image of G is either finite or uncountable.

The proof of Proposition 1 uses ultralimits and a theorem of Liebeck and Shalev [2] on diameters of conjugacy classes in finite simple groups.

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Conjugation Sequences

Bernd Fischer

Let a_0, b_0 be elements of a finite group G; let $a_{i+1} = a_i^{b_i}$, $b_{i+1} = b_i^{a_i}$ and $L(a_0, b_0) = \{(a_i, b_i) | i \geq 0\}$; let $L_1(a_0, b_0) = |\{a_i | i \geq 0\}|$, $L_2(a_0, b_0) = |\{b_i | i \geq 0\}|$.

Let $r = |L(a_0, b_0)|$ and $s = |L(a_r, b_r)|$; then define Type $(a_0, b_0) = [r, s]$. Types are ordered partially by $[r, s] \leq [p, q]$ iff $r \leq p$ and s divides q. If N is a normal subgroup of G then Type $(a_0N/N, b_0N/N \leq$ Type (a_0, b_0).

Let a, b be acycles of length t in the symmetric group Σ_{2l-1} generating A_{2l-1} or Σ_{2l-1}. The following table lists a few examples including the terminating group $\langle a_r, b_r \rangle$.

t	$[\ ,\ ]$	$\langle a_r, b_r\rangle$
2	[2,1]	C_2
3	[3,2]	A_4
4	[4,1]	C_4
5	[7,2]	A_5
7	[18,8]	$PGL(3,2)$
11	[16,2]	M_12
23	$[3.7.37.137, 2]$	A_24

Let $a^2 = 1 = b^3$ generate $PSL(2,p)$ or $PGL(2,p)$ such that Type $(a,b) = [r,r]$; in many cases $L_1(a,b) = L_2(a,b) = r$, but there are exceptions:

P	$[\ ,\ ]$	L_1	L_2	(L_1, L_2)
37	[190,190]	4	5	38
67	[462,462]	6	7	66
163	[378,378]	6	7	54

Lemma. *G is nilpotent iff $a, b \in G$, type $(a,b) = [r,s] \Rightarrow r = s$ and r divides $|\langle a,b\rangle/Z(\langle a,b\rangle)|$.*

Problem. Let $\mathcal{P}_n$ be the class of p-groups G such that $[a,b] \leq [p^m, p^n]$. Then $\mathcal{P}_n$ is a formation and $\mathcal{P}_n \supseteq \mathcal{C}_{n+1}$, the p-groups of class at most $n+1$.

Is there $m \in \mathbb{N}$ with $\mathcal{C}_{n+m} \supseteq \mathcal{P}_n\mathcal{C}_{n+1}$

Opposition in triality

Hans Cuypers
(joint work with Arjeh Cohen and Ralf Gramlich)

Opposition plays a crucial role in various parts of the theory of buildings. For instance the opposition relation on the set of chambers of a thick two-spherical twin building uniquely determines the Weyl distances and the Weyl codistance of that twin building, cf. [5]. For a twin building $\mathcal{T}$ the opposite chamber system Opp($\mathcal{T}$) is of particular interest for geometric group theory. Its simple connectedness generalizes the famous Curtis-Tits Theorem to groups generated by an $\mathbb{F}$-locally split two-spherical root group datum for a sufficiently large field $\mathbb{F}$, cf. [4, 8]. Moreover, the sphericity of the sub-chamber system opposite a fixed chamber implies finiteness properties of S-arithmetic subgroups of algebraic groups over local fields, cf. [1, 2, 3]. Furthermore, the simple connectedness of the system of chambers of Opp($\mathcal{T}$) fixed by a flip of $\mathcal{T}$ (i.e., an involution interchanging the two halves of the twin building isometrically) generalizes Phan's group-theoretic recognition tools [9, 10] as explained in [6, 7].

Motivated by these results related to the opposition relation we study the subsystem of those chambers of a D_4 building that are as far as possible from their image under the standard triality. In particular, we prove the following result.

Theorem 1. *Let $\mathbb{F}$ be a field containing at least three elements and let $\mathcal{C}(\mathbb{F})$ be the subsystem of chambers of the $\mathrm{D}_4(\mathbb{F})$ building which are as far as possible from their image under the standard triality τ. Then the incidence system $\mathcal{G}(\mathbb{F})$ associated to $\mathcal{C}(\mathbb{F})$ is a thick simply connected residually connected geometry admitting $\mathrm{G}_2(\mathbb{F})$ as a flag-transitive group of automorphisms. Moreover, the triality τ acts as a correlation on $\mathcal{G}(\mathbb{F})$.*

Theorem 1, Tits' Lemma, cf. [11], and the simple connectedness of the rank three residues of $\mathcal{G}(\mathbb{F})$ imply the following.

Theorem 2. *If the field $\mathbb{F}$ contains at least three elements, then the group $\mathrm{G}_2(\mathbb{F})$ equals the universal enveloping group of the amalgam consisting of the rank one and rank two parabolics of the action of $\mathrm{G}_2(\mathbb{F})$ on $\mathcal{G}(\mathbb{F})$.*

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Growth in groups and graphs

Harald Andrés Helfgott

"Growth" can mean one of many things.

(1) *Growth in graphs.* Let Γ be a graph. How many vertices can be reached from a given vertex in a given amount of time?
(2) *Growth in infinite groups.* Let A be a set of generators of an infinite group G. Let $B(t)$ be the number of elements that can be expressed as products of at most t elements of A. How does $B(t)$ grow as $t \to \infty$?

(3) *Random walks in groups.* Let A be a set of generators of a finite group G. Start with $x = 1$, and, at each step, multiply x by a random element of A. After how many steps is x close to being equidistributed in G?
(4) *More on growth in graphs: the spectral gap.* Let Γ be a graph. Consider its adjacency matrix. What lower bounds can one give for the difference between its two largest eigenvalues?
(5) *Growth in arithmetic combinatorics.* Let G be an abelian group. Let $A \subset G$. How large is $A + A$ compared to A, and why? In general, let G be a group. Let $A \subset G$. How large[1] is $A \cdot A \cdot A$ compared to A, and why?

Question (5) has been extensively studied in the abelian setting. Some time ago, I started studying it for non-abelian groups, and proved [He] that every set of generators A of $G = \mathrm{SL}_2(\mathbb{F}_p)$ grows: $|A \cdot A \cdot A| > |A|^{1+\epsilon}$, $\epsilon > 0$, provided that $|A| < |G|^{1-\delta}$, $\delta > 0$. (Here $|S|$ is the number of elements of a set S.) This answered question (1) (on growth in graphs) immediately in the case of the Cayley graph of $\mathrm{SL}_2(\mathbb{F}_p)$; the bounds obtained were strong enough to constitute the first proved case of a standard conjecture (Babai's). Questions (3) and (4) (on random walks and spectral gaps) are closely related to each other, and somewhat more indirectly to (1) and (5); the result in [He] gave non-trivial bounds for (3) and (4). These bounds were greatly improved by Bourgain and Gamburd ([BG]), who showed how to use a technique of Sarnak's [SX] to derive from the results in [He] bounds for (3) and (4) that are qualitatively optimal (sufficient to amount to an *expander graph property* for all sets of generators A of G such that (G, A) has the *large girth* property).

1. Main result

It remained to be seen whether the result in [He] on growth in $\mathrm{SL}_2(\mathbb{F}_p)$ could be generalised to other groups. Much of the work in [He] was specific to $\mathrm{SL}_2(\mathbb{F}_p)$. In [BG2], the result was generalised (in a suitably strong form) to $\mathrm{SU}_2(\mathbb{C})$; there is also a recent generalisation by O. Dinai [Di] to $\mathrm{SL}_2(\mathbb{F}_q)$, as well as results [Bo] on $\mathrm{SL}_2(\mathbb{Z}/d\mathbb{Z})$. From the point of view of the Lie algebra, all of these groups are very closely related to $\mathrm{SL}_2(\mathbb{F}_p)$. Thus, the matter of the extent to which the methods in [He] were truly flexible were remained open.

Very recently, I finished a proof of growth for $\mathrm{SL}_3(\mathbb{Z}/p\mathbb{Z})$. Part of the proof is ultimately derived from that in [He], and is likely to be valid for all semisimple groups of Lie type; part of the proof is essentially new.

Theorem 1. *Let $G = \mathrm{SL}_3$. Let $K = \mathbb{Z}/p\mathbb{Z}$, p a prime. Let $A \subset G(K)$ be a set of generators of $G(K)$.*

Suppose $|A| < |G(K)|^{1-\delta}$, $\delta > 0$. Then

$$|A \cdot A \cdot A| \gg |A|^{1+\epsilon}, \tag{1}$$

where $\epsilon > 0$ and the implied constant depend only on δ.

[1] In the non-abelian case, there are technical reasons why it makes more sense to consider $A \cdot A \cdot A$ rather than $A \cdot A$. The product $A \cdot A$ could be small "by accident".

We could, as in [He], write *let A be a subset of $G(K)$ not contained in a proper subgroup of $G(K)$* instead of *let A be a set of generators of $G(K)$*; the two statements are equivalent.

2. Consequences on diameters

By a result of Gowers, Nikolov and Pyber[2] [NP, Cor. 1 and Prop. 2],

$$A \cdot A \cdot A = \mathrm{SL}_n(K) \tag{2}$$

for $A \subset G$, $|A| > 2|G|^{1-\frac{1}{3(n+1)}}$, where $G = \mathrm{SL}_n(K)$ and $K = \mathbb{Z}/p\mathbb{Z}$.

Together with (2), the main theorem implies results on diameters. The *diameter* of a graph Γ is

$$\max_{v_1, v_2 \in V} (\text{shortest distance between } v_1 \text{ and } v_2),$$

where V is the vertex set of Γ. We are especially interested in the diameters of *Cayley graphs*. The *Cayley graph* $\Gamma(G, A)$ of a pair (G, A) (where G is a group and $A \subset G$) is defined to be the graph that has G as its set of vertices and $\{(g, ag) : g \in G, a \in A\}$ as its set of edges. It is easy to see that the diameter $\mathrm{diam}(\Gamma(G, A))$ of a Cayley graph $\Gamma(G, A)$ is the least integer k such that

$$G = \{I\} \cup A \cup (A \cdot A) \cup \cdots \cup (\underbrace{A \cdot A \cdots A}_{k \text{ times}}).$$

If A is a set of generators of G, then, by definition, every element of G can be expressed as a product of elements of $A \cap A^{-1}$; when G is finite, this implies that every element of G can be expressed as a product of elements of A, i.e., the diameter $\mathrm{diam}(\Gamma(G, A))$ of the Cayley graph $\Gamma(G, A)$ is finite. The question remains: how large can the diameter $\mathrm{diam}(\Gamma(G, A))$ be in terms of G and A?

The following statement is known as *Babai's conjecture*.

Conjecture ([BS])**.** *For every non-abelian finite simple group G and any set of generators A of G,*

$$\mathrm{diam}(\Gamma(G, A)) \ll (\log |G|)^c, \tag{3}$$

where c is some absolute constant and $|G|$ is the number of elements of G.

Until recently, there was no infinite family of groups G for which the conjecture was known for all A. In [He], I proved Babai's conjecture for $G = \mathrm{SL}_2(\mathbb{Z}/p\mathbb{Z})$ and all A. I shall now prove the conjecture for $G = \mathrm{SL}_3(\mathbb{Z}/p\mathbb{Z})$.

[2]Gowers [Gow] proved a statement from which (2) quickly follows, as was pointed out by Nikolov and Pyber; see [NP]. The results in [Gow] and [NP] are of a general nature; with the aid of standard lower bounds on the dimensions of complex representations of SL_n, the special cases SL_2 and PSL_n were worked out in [Gow] and [NP], respectively. More general statements can be found in [BNP]. A weaker version of (2) for $n = 2$ was proven in [He, Key proposition, part (b)].

Corollary 1 (to the main theorem and (2)). *Let p be a prime. Let $G = \mathrm{SL}_3(\mathbb{Z}/p\mathbb{Z})$. Let A be a set of generators of G. Then*

$$\mathrm{diam}(\Gamma(G,A)) \ll (\log|G|)^c, \tag{4}$$

where c and the implied constant are absolute.

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Root shadow spaces

Arjeh M. Cohen

(joint work with Gábor Ivanyos and Dan Roozemond)

Let D be a Dynkin diagram and let J be the set of nodes adjacent to the node used to build the extended Dynkin diagram of D. A *root shadow space* is the J-shadow space of a building of type D. These spaces are parapolar spaces with the property that, for each point p and symplecton S, the set of points in S collinear to p is not a singleton. Professor Ernest Shult has recently reduced the hypotheses needed for his characterization with Kasikova of certain root shadow spaces as parapolar spaces to this single property besides the known rank conditions and the requirement that the diameter of the collinearity graph be at least 3.

This result helps to provide a geometric proof that every simple Lie algebra of finite dimension over a field of characteristic at least 5 containing an element x such that the image of ad_x is the space spanned by x is either the 5-dimensional Witt algebra over a field of characteristic 5 or a classical Lie algebra.

See [Arjeh M. Cohen, Gábor Ivanyos, and Dan Roozemond, *Simple Lie algebras having extremal elements*, `http://arxiv.org/abs/0711.4268`]. It also leads to an alternative proof of parts of Timmesfeld's characterization of systems of abstract root groups, [F.G. Timmesfeld. *Abstract root subgroups and simple groups of Lie-type*, Monographs in Mathematics, vol. 95, Birkhäuser, 2001]; see [A.M. Cohen & G. Ivanyos, *Root filtration spaces from Lie algebras and abstract root groups*, J. Algebra **300** (2006) 433–454] and [A.M. Cohen, G. Ivanyos, *Root shadow spaces*, European J. Combinatorics, **28** (2007) 1419–1441].

Strong involutions in finite Lie type groups of odd characteristic

Cheryl E. Praeger
(joint work with Alice C. Niemeyer and Frank Lübeck)

The motivation for this work originated in a question from Charles Leedham-Green and Eamonn O'Brien concerning the analysis of a Monte Carlo algorithm to construct a certain type of involution in a finite classical group. Finding such an involution is a key step in their new Las Vegas algorithm [2] for constructing standard generators for a finite n-dimensional classical group H in odd characteristic in its natural action. The required involution was one with fixed point subspace of dimension between $n/3$ and $2n/3$, a so-called *strong involution*.

They construct a strong involution by making random selections from H to find a *strong preinvolution*, that is an even-ordered element h such that the involution in $\langle h \rangle$ is strong. The number of random selections, and hence the complexity of this procedure, depends on the proportion of strong pre-involutions in H. Leedham-Green and O'Brien showed in [2, Theorem 8.1] that this proportion is at least c/n for some constant c, so that $O(n)$ random elements needed to be tested.

In [1, Theorem 1.1] (see Theorem 1 below), we improve the lower bound to $c/\log n$, thus enabling the number of random elements tested to be reduced to $O(\log n)$. We work with a wider class of finite Lie type groups H than in [2], including projective groups. For $I \subset H$ a subset of involutions in H, let

$$\textsc{PreInv}(H; I) = \{h \in H \mid \ |h| \text{ is even, } h^{|h|/2} \in I\} \qquad (1)$$

that is, the set of elements of H which "power up" to an involution in I. We denote by $\mathrm{GSp}_{2\ell}(q)$, $\mathrm{GU}_n(q)$ and $\mathrm{GO}_n(q)$ the general symplectic, unitary, and orthogonal groups, respectively, that is, the groups preserving the relevant forms up to a scalar multiple; $\mathrm{GO}^{\pm}_{2\ell}(q)^0$ denotes the connected general orthogonal group - the index 2 subgroup of $\mathrm{GO}^{\pm}_{2\ell}(q)$ that does not interchange the two $\mathrm{SO}^{\pm}_{2\ell}(q)$-classes of maximal isotropic subspaces.

Theorem. *Let q be a power of an odd prime and ℓ an integer with $\ell \geq 2$. Let S, X, n be as in one of the lines of Table 1, so that n is the dimension of the natural representation of X. Let H satisfy $S \leq H \leq X$ and let $I \subset H$ be the set of involutions which have a fixed point subspace of dimension r with $n/3 \leq r < 2n/3$. Then*

$$\frac{|\textsc{PreInv}(H; I)|}{|H|} \geq \frac{1}{5000 \log_2 \ell}.$$

Moreover, if $Z_0 \leq Z(X)$, $\overline{I} := IZ_0/Z_0$, and $\overline{L} := LZ_0/Z_0$ for $L \leq X$, then $\overline{S} \leq \overline{H} \leq \overline{X}$, and $|\textsc{PreInv}(\overline{H}; \overline{I})|/|\overline{H}| \geq 1/(5000 \log_2 \ell)$.

S	X	n
$\mathrm{SL}_{\ell+1}(q)$	$\mathrm{GL}_{\ell+1}(q)$	$\ell+1$
$\mathrm{SU}_{\ell+1}(q)$	$\mathrm{GU}_{\ell+1}(q)$	$\ell+1$
$\mathrm{Sp}_{2\ell}(q)$	$\mathrm{GSp}_{2\ell}(q)$	2ℓ
$\mathrm{SO}_{2\ell+1}(q)$	$\mathrm{GO}_{2\ell+1}(q)$	$2\ell+1$
$\mathrm{SO}^{\pm}{}_{2\ell}(q)$	$\mathrm{GO}_{2\ell}{}^{\pm}(q)^0$	2ℓ

Table 1. Table for Theorem

Type of X	G_2	${}^2G_2$	${}^3D_4$	F_4	E_6	${}^2E_6$	E_7	E_8
c	.375	.578	.578	.333	.328	.328	.168	.353
Type of X	A_ℓ	${}^2A_\ell$	B_ℓ	C_ℓ	D_ℓ	${}^2D_\ell$		
Values for ℓ	$1,2,3,4$	$2,3,4$	$3,4$	$2,3,4$	4	4		
c	.171	.187	.134	.134	.105	.132		

Table 2. Table for Theorem

We also deal with groups of Lie type of small rank in odd characteristic (including all exceptional simple types) and I a subset of involutions with any fixed type of centralizer.

Theorem. *Let $X = X_\ell(q)$ be a finite group of Lie type of rank ℓ defined over a field of odd order q, such that X and a positive real number c are as in one of the cases of Table 2. Let I be a conjugacy class of involutions in X. Then $|\mathrm{PreInv}(X;I)|/|X| \geq c$.*

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Primitive permutation groups of bounded orbital diameter

Katrin Tent

(joint work with Martin Liebeck, Dugald Macpherson)

In this talk I will present a classification of infinite classes of finite primitive groups with a uniform bound on the diameters of all orbital graphs. It is well known that a group action is primitive if and only if all orbital graphs are connected. The starting point for our considerations was the problem that being a primitive group or a connected graph is in general not a first-order property. However, if there is a uniform bound on the diameters of orbital graphs, this can be expressed by a first order sentence.

Our aim is to classify all finite primitive groups with a given bound on the diameters of their orbital graphs. We use the classification of the finite simple group, but only in a weak sense, namely we assume that the number of sporadic groups is finite.

If $\mathcal{C}$ is an infinite family of finite primitive groups, by applying the O'Nan-Scott theorem and passing to an infinite subset, we may assume that the members of $\mathcal{C}$ are of one of the following types:

(1) affine;
(2) almost simple of unbounded L-rank;
(3) almost simple of bounded L-rank;
(4) simple diagonal actions;
(5) product actions;
(6) twisted wreath actions.

It is fairly easy to see that up to isomorphism there are only finitely many primitive groups of twisted wreath type for any given bound on the diameters.

We obtain the following theorem (Here, the L-rank denotes the untwisted Lie rank, or n if the group in question is A_n. By t-bounded classical type and standard t-action we mean some fairly canonical induced actions.)

Theorem 1. *Let $\mathcal{C}$ be an infinite class of finite primitive permutation groups of one of the types* $(1)-(6)$ *above.*

(1) *If $\mathcal{C}$ consists of affine groups, then the diameters of the orbital graphs of $\mathcal{C}$ are bounded essentially (assuming that groups contain scalars) if and only if these are all of t-bounded classical type, for some bounded t.*

(2) *If $\mathcal{C}$ consists of almost simple groups of unbounded L-ranks, then the diameters of the orbital graphs of $\mathcal{C}$ are bounded (essentially) if and only if the socles of groups in $\mathcal{C}$ of sufficiently large L-rank are alternating or classical groups in standard t-actions, where t is bounded.*

(3) *If $\mathcal{C}$ consists of almost simple groups G of bounded L-rank and the diameters of the orbital graphs of $\mathcal{C}$ are bounded , then point stabilizers G_x have unbounded orders; moreover, if G has socle $G(q)$, of Lie type over $\mathbb{F}_q$, and G_x is a subfield subgroup $G(q_0)$, then $|\mathbb{F}_q : \mathbb{F}_{q_0}|$ is bounded.*

Conversely, if $\mathcal{C}$ is a class consisting of primitive almost simple groups G of bounded L-rank such that

(i) *point stabilizers G_x $(G \in \mathcal{C})$ have unbounded orders, and*

(ii) *if $G \in \mathcal{C}$ has socle $G(q)$, of Lie type over $\mathbb{F}_q$, and G_x is a subfield subgroup $G(q_0)$, then $|\mathbb{F}_q : \mathbb{F}_{q_0}|$ is bounded.*

Then the class $\mathcal{C}$ is bounded.

(4) *If $\mathcal{C}$ consists of primitive groups G of simple diagonal type, then the diameters of the orbital graphs of $\mathcal{C}$ are bounded if and only if these have socles of the form T^k, where T is a simple group of bounded L-rank and k is bounded.*

(5) *If $\mathcal{C}$ consists of primitive groups (X, G) of product action type, where $X = Y^k$ and $G \leq H$ wr S_k for some primitive group (Y, H), then then the diameters of the orbital graphs of $\mathcal{C}$ are bounded if and only if k is bounded, and (Y, H) has bounded diameter.*

(6) *No bounded class $\mathcal{C}$ consists of primitive groups (X, G) of twisted wreath type.*

For example, the theorem tells us that if $\mathcal{C}$ consists of the groups $E_8(q)$ (q varying) acting on the coset space $E_8(q)/X(q)$ for some maximal subgroup $X(q)$ arising from a maximal connected subgroup $X(K)$ of the simple algebraic group $E_8(K)$, where $K = \bar{\mathbb{F}}_q$ (for example $X(K) = D_8(K)$ or $A_1(K)$), then the diameters of all the orbital graphs are bounded by an absolute constant.

The main point of the argument in proving that such classes do indeed have bounded orbital diameter uses fairly deep results about the model theory of pseudofinite fields and difference fields and the fact that ultraproducts of primitive groups are primitive if and only if all orbital diameters are bounded.

Here is an application of the theorem: recall that a *distance-transitive* graph is one for which the automorphism group is transitive on pairs of vertices at any given distance apart. Thus a finite distance-transitive graph is an orbital graph for the automorphism group (acting on the vertex set) in which the diameter is equal to one less than the permutation rank.

Corollary 2. *There is a function $f : \mathbb{N} \to \mathbb{N}$ such that the following holds. Let G be a finite almost simple group with socle $G(q)$ of Lie type over $\mathbb{F}_q$, and of L-rank r. Suppose G acts primitively on a set X, with G_x a non-parabolic subgroup, and suppose there exists a (non-complete) distance-transitive graph on X with automorphism group containing G. Then $q < f(r)$.*

The Phan-type theorem for finite Chevalley groups

Ralf Gramlich

The main purpose of my talk was to state the success of the project called *Phan theory.* It has been initiated in [1]. A survey on the methods can be found in [6].

Phan-type theorems can be considered as an analogue of the famous Curtis-Tits Theorem. The latter states (cf. e.g. [5, Theorem 2.9.1]) that a Chevalley group K of rank at least three equals the universal enveloping group of the amalgam of fundamental rank one and two subgroups of K; this system of fundamental rank one and two subgroups is called a *Curtis-Tits system* of K.

In case K is a non-twisted Chevalley group over a field of square order, let G be the subgroup of K fixed by the product of the Chevalley involution (with respect to the above choice of a fundamental system) and the field involution. Then the intersections of the fundamental rank one and rank two subgroups of G with K equal twisted Chevalley groups of rank one and two; the system of these subgroups of G is called a *Phan system* of G. A *weak Phan system* of an abstract group is a generating collection of subgroups of that abstract group which, as an amalgam, is isomorphic to a Phan system of a twisted Chevalley group.

The Phan-type Theorem over finite fields. *Let $q \geq 3$, let Δ be a spherical Dynkin diagram of rank at least three, and let G be a group with a weak Phan system of type Δ over $\mathbb{F}_{q^2}$. Then G is isomorphic to a quotient of*

- $\mathrm{SU}_{n+1}(q^2)$, *if* $\Delta = A_n$ *and* $q \geq 4$
  (Bennett, Shpectorov [3], Phan [11])*;*
- $\mathrm{Spin}_{2n+1}(q)$, *if* $\Delta = B_n$ *and* $q \geq 4$
  (Bennett, G., Hoffman, Shpectorov [2], G., Horn, Nickel [9])*;*
- $\mathrm{Sp}_{2n}(q)$, *if* $\Delta = C_n$
  (G., Hoffman, Shpectorov [10], G., Horn, Nickel [8])*;*
- $\mathrm{Spin}^{\pm}_{2n}$, *if* $\Delta = D_n$ *and* $q \geq 4$, *of plus type if* n *even, of minus type if* n *odd*
  (G., Hoffman, Nickel, Shpectorov [7], Phan [12])*;*
- *the universal Steinberg-Chevalley group of type* ${}^2E_6(q^2)$, *if* $\Delta = E_6$ *and* $q \geq 4$
  (Devillers, G., Mühlherr [4], G., Hoffman, Mühlherr, Shpectorov 2005, Phan [12])*;*
- *the universal Steinberg-Chevalley group of type* $E_7(q)$, *if* $\Delta = E_7$ *and* $q \geq 4$
  (Devillers, G., Mühlherr [4], G., Hoffman, Mühlherr, Shpectorov 2005, Phan [12])*;*
- *the universal Steinberg-Chevalley group of type* $E_8(q)$, *if* $\Delta = E_8$ *and* $q \geq 4$
  (Devillers, G., Mühlherr [4], G., Hoffman, Mühlherr, Shpectorov 2005, Phan [12])*;*
- *the universal Steinberg-Chevalley group of type* $F_4(q)$, *if* $\Delta = F_4$ *and* $q \geq 13$
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Classification of Buildings – a local approach

Mark Ronan

In the first part of this talk I summarised the classification of buildings under three broad headings: spherical, affine, other. Spherical buildings, whose apartments are tilings of a sphere, arise naturally from groups of Lie type. Affine buildings, whose apartments are tilings of Euclidean space, arise naturally from groups of Lie type over fields–such as p-adic fields–having a discrete valuation. The affine building yields the usual spherical building 'at infinity'. Other types of building–for example hyperbolic buildings–arise from Kac-Moody groups.

In his 1974 book on buildings Jacques Tits classified all irreducible spherical buildings of rank at least 3. In the process he showed that the local structure determines the global structure; for instance, the local structure for a building of type E_8 is determined by a commutative field, and there is exactly one such building for each field. These buildings can be created independently of the groups by a simple combinatorial construction given by Ronan and Tits in their 1987 paper, *Building Buildings*. This then allows Tits's methods to be used to create the groups, independently of the Lie theory.

In a 1986 paper entitled *Immeubles de type affine*, Tits also classified irreducible affine buildings of rank at least 4. Their local structure does not determine their global structure, but the classification uses the spherical building at infinity, which has rank one less than the affine building; for example an affine building of type E_9 has a spherical building of type E_8 at infinity.

Other types of buildings, such as E_{10}, arise from Kac-Moody groups, and E_{10} buildings are not known from any other source. Possibly their local structure determines their global structure, and to investigate this question I discussed twin buildings, where we already know from a theorem of Mühlherr and Ronan in 1995 that the local structure does determine the global structure. A Kac-Moody group yields a twin building, so it is plausible that buildings of type E_{10} are twinnable. With this in mind I gave conditions under which a twinning from a single chamber (a rank 0 twinning) can be extended to a full twinning. This involved the following

two theorems, in which the term 'non-fragile' means that if the rank 2 residues are Moufang then they cannot be of type $Sp_4(2)$, $G_2(2)$, $G_2(3)$ or ${}^2F_4(2)$. Condition H is a technical condition that is both necessary and sufficient for a rank 0 twinning to extend to a rank 1 twinning.
Theorem 1: If Δ is a 2-spherical, non-fragile building satisfying condition H, then every rank 0 twinning extends uniquely to a rank 1 twinning.

Theorem 2: Let Δ be a 3-spherical, non-fragile building satisfying condition H. Then every rank 0 twinning extends uniquely to a twinning of Δ with another building Δ'. In particular Δ is twinnable.

Condition H is not only sufficient, but also necessary, for a rank 1 extension. Of course this work begs the question as to whether one can create a rank 0 twinning, and this remains entirely open.

Fixed point sets and Lefschetz modules for sporadic simple groups

Silvia Onofrei
(joint work with John Maginnis)

The present work investigates various properties of the reduced Lefschetz modules. The underlying simplicial complexes arise in a natural way from the group structure and are relevant to the mod p cohomology and to the modular representation theory of the group. We are specially interested in those complexes which can be related to p-local geometries for the sporadic simple groups.

Terminology. Let G be a finite group and p a prime dividing its order. An element of order p in G is called *p-central* if it lies in the center of a Sylow p-subgroup of G. A subgroup Q of G is a *p-radical* subgroup if $Q = O_p(N_G(Q))$. The subgroup Q is *p-centric* if its center $Z(Q)$ is a Sylow p-subgroup of $C_G(Q)$. A group G is said to have *characteristic* p if $C_G(O_p(G)) \leq O_p(G)$. A group is said to have *local characteristic* p if $C_H(O_p(H)) \leq O_p(H)$ for all p-local subgroups H of G. A group has *parabolic characteristic* p if all p-local subgroups which contain a Sylow p-subgroup of G have characteristic p.

A *collection* $\mathcal{C}$ is a family of subgroups of G which is closed under conjugation by G and it is partially ordered by inclusion. The *subgroup complex* $\Delta = \Delta(\mathcal{C})$ associated to $\mathcal{C}$ is the simplicial complex whose simplices are proper inclusion chains in $\mathcal{C}$. For a simplex σ in Δ let G_σ denote its isotropy group. Also let Δ^Q denote the elements in Δ fixed by Q, a subcomplex which affords the action of $N_G(Q)$. In what follows $\mathcal{B}_p(G)$ will denote the Bouc collection of nontrivial p-radical subgroups.

For $\mathcal{C}_p(G)$ a collection of p-subgroups of G denote by $\widehat{\mathcal{C}}_p(G)$ the collection of subgroups in $\mathcal{C}_p(G)$ which contain p-central elements in their centers. We call $\widehat{\mathcal{C}}_p(G)$ the *distinguished* $\mathcal{C}_p(G)$ *collection.* We shall refer to the subgroups in $\widehat{\mathcal{C}}_p(G)$ as *distinguished subgroups.*

In [3], we obtained various results about equivariant homotopy equivalences involving categories of distinguished p-subgroups as well as the categories defined by Dwyer (the orbit category and the category whose objects are the monomorphisms to G), relevant to the study of homology decompositions for the classifying space BG of G. These homotopy equivalences have been proven under one of a list of three hypotheses about G which are valid for most of the sporadic groups.

Let k denote a field of characteristic p. The *reduced Lefschetz module* is given by the alternating sum of the chain groups; it can also be described using induced modules:

$$\widetilde{L}_G(\Delta, k) = \sum_{\sigma \in \Delta/G} (-1)^{\dim(\sigma)} \mathrm{Ind}_{G_\sigma}^G k \; - \; k$$

where Δ/G denotes the orbit complex of Δ.

Fixed point sets of p-elements. This section contains the three main results presented in this talk. They describe the structure of the fixed point sets under the action of elements of order p; the details of the proofs can be found in [4]. Let $\Delta = \Delta(\widehat{\mathcal{B}}_p(G))$ denote the complex of distinguished p-radical subgroups in G.

Proposition 1. *Let G be a finite group of parabolic characteristic p. Set $Z = \langle z \rangle$ with z a p-central element in G. Then the fixed point set Δ^Z is $N_G(Z)$-contractible.*

For an element t which is not of central type, the homotopy type of the corresponding fixed point set is determined by the group structure of its centralizer $C = C_G(t)$.

Proposition 2. *Let G be a finite group of parabolic characteristic p. Let t be a noncentral element of order p and set $T = \langle t \rangle$. Assume that $O_p(C)$ contains a p-central element. Then the fixed point set Δ^T is $N_G(T)$-contractible.*

Theorem A. *Assume G is a finite group of parabolic characteristic p. Set $T = \langle t \rangle$ with t an element of order p of noncentral type in G. Suppose that the following hypotheses hold:*

(1) *$O_p(C)$ does not contain any p-central elements;*
(2) *The quotient group $\overline{C} = C/O_p(C)$ has parabolic characteristic p.*

Then there is an $N_G(T)$-equivariant homotopy equivalence $\Delta^T \simeq \Delta(\widehat{\mathcal{B}}_p(\overline{C}))$.

The proof of the theorem is quite technical and a few pages long and requires a combination of several homotopy equivalences. It relies on:

- techniques of poset homotopies; in particular strings of equivariant poset maps which are homotopy equivalent to the identity map;
- the p-local structure of the group G, properties of groups of characteristic p and local characteristic p.

Information about fixed point sets leads to details about the vertices of indecomposable summands of the reduced Lefschetz module, as can be seen from the following result:

Proposition. [5, Robinson] The number of indecomposable summands of $\widetilde{L}_G(\Delta, k)$ with vertex Q is equal to the number of indecomposable summands of $\widetilde{L}_{N_G(Q)}(\Delta^Q, k)$ with the same vertex Q.

With Δ the complex of distinguished p-radical subgroups in G, set $\widetilde{L} = \widetilde{L}_G(\Delta, k)$, the associated reduced Lefschetz module. If G has parabolic characteristic p, then $\widetilde{L}$ is projective relative to those p-subgroups which do not contain any p-central elements; therefore the vertices of indecomposable summands of $\widetilde{L}$ are also among such subgroups.

An example: the Fischer group $\mathbf{Fi_{22}}$ and $\mathbf{p = 2}$. Consider the sporadic simple group Fi_{22}, which has parabolic characteristic 2 and has three conjugacy classes of involutions, denoted $2A, 2B$ and $2C$ in the Atlas[1]. The class $2B$ is 2-central. Their centralizers are $C_{Fi_{22}}(2A) = 2.U_6(2)$, $C_{Fi_{22}}(2B) = (2 \times 2_+^{1+8} : U_4(2)) : 2$ and $C_{Fi_{22}}(2C) = 2^{5+8} : (S_3 \times 3^2 : 4)$.

We consider the simplicial complex Δ whose vertex stabilizers are four maximal 2-local subgroups of Fi_{22}:

$H_1 = (2 \times 2_+^{1+8} : U_4(2)) : 2$ $\qquad$ $H_2 = 2^{5+8} : (S_3 \times A_6)$

$H_3 = 2^6 : Sp_6(2)$ $\qquad$ $H_4 = 2^{10} : M_{22}$

The complex Δ (the standard 2-local geometry for Fi_{22}) is G-homotopy equivalent to the complex of 2-centric and 2-radical subgroups, and since Fi_{22} has parabolic characteristic 2, this is equal to the complex of distinguished 2-radical subgroups; for details we refer the reader to Benson and Smith [2, Sections 8.16 and 9.4].

We shall use the notation from the Modular Atlas homepage, where φ_i denotes an irreducible module of Fi_{22} and $P_{Fi_{22}}(\varphi_i)$ is its corresponding projective cover.

Proposition 3. *Let Δ be the standard 2-local geometry for Fi_{22}.*

(a) *The fixed point sets Δ^{2B} and Δ^{2C} are contractible.*

(b) *The fixed point set Δ^{2A} is equivariantly homotopy equivalent to the building for the Lie group $U_6(2)$.*

(c) *There is precisely one nonprojective summand of the reduced Lefschetz module, it has vertex $\langle 2A \rangle$ and lies in a block with the same group as defect group.*

(d) *As an element of the Green ring:* $\widetilde{L}_{Fi_{22}}(\Delta) = -P_{Fi_{22}}(\varphi_{12}) - P_{Fi_{22}}(\varphi_{13}) - 6\varphi_{15} - 12 P_{Fi_{22}}(\varphi_{16}) - \varphi_{16}$.

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A Shadow of O'Nan

Inna (Korchagina) Capdeboscq
(joint work with Richard Lyons)

In this talk we continue the discussion of characterization of various bicharacteristic finite simple groups G in the sense of [KoL] and the earlier papers [KoS], [KoLS]. The strategy is part of the GLS revision project [GLS1], but expanded to the case $e(G) = 3$ to make the GLS project fit with the Aschbacher-Smith Quasithin Theorem [AS].

We use the following notation: G is a finite simple group, p is an odd prime, $m_p(X)$ is the p-rank of an arbitrary group X, $m_{2,p}(G)$ is the maximum value of $m_p(N)$ over all subgroups $N \leq G$ such that $O_2(N) \neq 1$, and $e(G)$ is the maximum value of $m_{2,p}(G)$ as p ranges over all odd primes. Moreover $m_p^I(G)$ is the maximum value of $m_p(C_G(z))$ as z ranges over all involutions of G.

We fix an odd prime p and set

$$\mathcal{H}(G) = \{H \leq G | H \text{ is a 2-local subgroup of } G \text{ and } m_p(H) = m_{2,p}(G)\}$$

The groups that we consider in this paper satisfy the following conditions:

$$m_{2,p}(G) = e(G) = 3 \text{ and } m_p^I(G) \leq 2. \qquad (H1)$$

We state our theorem, tie it in with the main theorem of [KoL] to obtain a corollary, and then discuss the technical terminology in the theorem.

Theorem 1. *Suppose that G satisfies the following conditions:*

(1) *G is a finite $\mathcal{K}$-proper simple group;*
(2) *G has restricted even type; and*
(3) *For some odd prime p, G satisfies $(H1)$ and has weak p-type.*

Then $p = 3$ and there exists $H \in \mathcal{H}$ such that $F^*(H) = O_2(H)$. Moreover, for any $H \in \mathcal{H}(G)$ and any $B \leq H$ such that $B \cong E_{3^3}$, there is a hyperplane B_0 of B such that $L_{3'}(C_G(B_0)) \cong A_6$.

The conclusion of Theorem 1 implies that G satisfies all the hypotheses of Theorem 1.2 of [KoL]. That theorem in turn yields that G has the structure asserted in the corollary, or $G \cong Sp_8(2)$ or $F_4(2)$. But these last two groups do not satisfy the assumption $m_3^I(G) \leq 2$. Indeed, in both, the centralizer of a long root involution is a parabolic subgroup P with Levi factor isomorphic to $Sp_6(2)$, so $m_3^I(G) \geq m_3(Sp_6(2)) = 3$. Thus we have the following corollary.

Corollary 2. *If G satisfies the assumptions of Theorem 1, then $G \cong A_{12}$ or G has the centralizer of involution pattern of F_5.*

The $\mathcal{K}$-proper assumption in Theorem 1 means that all proper simple sections of G are among the known simple groups, as is appropriate for the inductive classification [GLS1].

The hypothesis that G is of weak p-type [KoL] means that:

For every $x \in G$ of order p such that $m_p(C_G(x)) \geq 3$, and for every component L of $E(C_G(x)/O_{p'}(C_G(x)))$, $L \in \mathcal{C}_p$, and $O_{p'}(C_G(x))$ has odd order.

Here $\mathcal{C}_p$ is an explicit set of quasisimple $\mathcal{K}$-groups defined for any odd prime p (cf. [GLS1; p.100]).

The term "restricted even type" is defined on page 95 of [GLS1]. This definition implies that if z is an involution of G with centralizer $C = C_G(z)$, then any component L of C lies in the set $\mathcal{C}_2$ defined in [GLS1;p.100].

It is somewhat arbitrary that the definition of $\mathcal{C}_2$ excludes the covering groups $4L_3(4)$. This is because the sporadic group $O'N$, in which the centralizer of an involution has such a component, in GLS emerges from the analysis of groups of odd type in [GLS6]. Nevertheless, our assumptions in Theorem 1 inevitably lead toward the situation in which $F^*(C_G(z))$ is a covering group of $L_3(4)$ by Z_4, and this situation is prevented only by the definition of $\mathcal{C}_2$. In Bender's terminology, $O'N$ is a "shadow" group in our setup.

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Strongly p-embedded subgroups

Christopher Parker

(joint work with Gernot Stroth)

For p a prime and G a finite group, a subgroup H of G is *strongly p-embedded* in G if the following two conditions hold.

(i) $H < G$ and p divides $|H|$; and

(ii) if $g \in G \setminus H$, then p does not divide $|H \cap H^g|$.

One of the most important properties of strongly p-embedded subgroups H is that they contain $N_G(X)$ for any non-trivial p-subgroup X of H. In the final phases of the project orchestrated by Meierfrankenfeld, Stellmacher and Stroth to understand the groups G of local characteristic p [3], a simple subgroup H of G is often constructed which is strongly p-embedded in G. In this environment, the group G is what is known as a *$\mathcal{K}$-proper group.* That is G is a group in which every proper subgroup has its composition factors from among the simple groups listed in the Classification of Finite Simple Groups. Notice that every group with cyclic Sylow p-subgroups possesses a strongly p-embedded subgroup or has a non-trivial normal p-subgroup. Thus any investigation of groups with a strongly p-embedded subgroup with p odd must have some condition on the p-rank of H.

Suppose that G is a finite group and H is a strongly p-embedded subgroup of G. If $p = 2$, then Bender's strongly 2-embedded theorem [1], shows that, if G is a simple group, then $G \cong \mathrm{PSL}_2(2^n)$, $n \geq 2$, $\mathrm{PSU}_3(2^n)$, $n \geq 2$ or ${}^2\mathrm{B}_2(2^{2n+1})$, $n \geq 1$. Indeed, if the p-rank of G is at least 3, then a consequence of the Classification of Finite Simple Groups is that the simple groups with a strongly p-embedded subgroup are precisely the Lie Type groups of rank 1 defined in characteristic p (See [2, Theorem 7.6.1]).

Our main theorem is as follows.

Theorem 1. *Suppose that G is a finite $\mathcal{K}$-proper group, p is an odd prime and that H is a strongly p-embedded subgroup of G such that $H \cap K$ is of even order for any non-trivial normal subgroup K of G. Assume that $O_{p'}(H) = 1$ and that $m_p(C_H(t)) \geq 2$ for every involution t of H. Then either $F^*(G) \cong \mathrm{PSU}_3(p^n)$ for some $n \geq 2$ or, $p = 3$ and $F^*(G) \cong {}^2\mathrm{G}_2(3^{2n-1})$ for some $n \geq 2$.*

Considering G as in the theorem, we may suppose that H is not strongly 2-embedded for the groups with a strongly 2-embedded subgroup do not satisfy our hypothesis. Thus there is an involution $t \in H$, such that $C_G(t) \not\leq H$. The structure of $C_H(t)$ is easy to understand. In particular, our condition on the p-rank of $C_H(t)$ and the fact that H is strongly p-embedded in G imply that $O_{p'}(C_G(t)) \leq H$ and that $X = F^*(C_G(t)/O_{p'}(C_G(t)))$ is a simple group with a strongly p-embedded subgroup with p-rank at least 2. Using the fact that G is $\mathcal{K}$-proper and $O_{p'}(H) = 1$, we know the possibilities for X. This begins to limit the possibilities for $C_G(t)$ and thus for G. One of the first interesting consequences of this observation is the somewhat surprising fact that $C_G(s) \not\leq H$ for all involutions $s \in H$. The proof of Theorem 1 involves a detailed investigation of the possibilities for $C_G(t)$ and takes strikingly different directions depending upon whether H has components or not.

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Codistance

Hendrik Van Maldeghem
(joint work with Alice Devillers & Bernhard Mühlherr)

Let (W, S) be a Coxeter system and $\mathcal{B} = (\mathcal{C}, \delta)$ a building of type (W, S). A *codistance* on $\mathcal{B}$ is a function $f : \mathcal{C} \to W$ such that, for all $s \in S$ and P an s-panel of $\mathcal{C}$, there exists $w \in W$ with $f(x) \in \{w, ws\}$ for all $x \in P$ and P contains a unique chamber with f-value the longest word of the two, see [2].

As an example, if $\mathcal{B}$ is half of a twin building and x is a chamber in the other half, the twinning restricted to x is a codistance on $\mathcal{B}$.

A natural question is whether the class of examples just given is unique. In other words, does a codistance on a building imply a twinning?

It is easy to see that every tree without finite ends admits a codistance, but a twin tree is necessarily bi-regular. Hence the above question must be answered in the negative in general. This example shows that we must restrict ourselves to the 2-spherical case (i.e., all rank 2 residues are spherical). Here, one is tempted to conjecture that the answer is positive, certainly considering the fact that p-adic affine buildings do not admit any codistance, see [3].

In this note, we present a result that answers the above question positively for 3-spherical buildings under some mild conditions (basically saying that the building is locally large enough). Here is the theorem.

Theorem: *Let $\mathcal{B}_- = (\mathcal{C}_-, \delta_-)$ be a thick building of 3-spherical type (W, S). Assume that the following two conditions hold.*

- (lco) *If R is a rank 2 residue containing a chamber c, then the set of chambers opposite c inside R is connected.*
- (lsco) *If R is a rank 3 residue containing a chamber c, then the set of chambers opposite c inside R is simply 2-connected.*

If there exists a codistance function $f : \mathcal{C}_- \to W$, then there exists a building $\mathcal{B}_+ = (\mathcal{C}_+, \delta_+)$ and a mapping $\delta^* : (\mathcal{C}_- \times \mathcal{C}_+) \cup (\mathcal{C}_+ \times \mathcal{C}_-) \to W$ such that the following two statements hold.

a) *$(\mathcal{B}_-, \mathcal{B}_+, \delta^*)$ is a twin building.*
b) *There exists a chamber $c \in \mathcal{C}_+$ such that $\delta^*(c, x) = f(x)$ for all $x \in \mathcal{C}_-$.*

The conditions (lco) and (lsco) are not too restrictive. In practice, only condition (lsco) puts a restriction on the building $\mathcal{B}_-$. Indeed, if the diagram of $\mathcal{B}_-$ is not connected, then we can apply the theorem to each non-spherical connected component. Noting that every spherical connected component trivially admits a twinning, we can thus restrict to irreducible non-spherical 3-spherical buildings $\mathcal{B}_-$. If the diagram is simply laced, then (lco) is void, and (lsco) is equivalent with requiring that each panel contains at least 4 chambers; in the other case the only rank 2 residues are, besides projective planes, generalized quadrangles, and there is at least one rank 3 residue with a C_3 diagram. It is well known that a building of type C_3 satisfies (lsco) whenever it corresponds to an embeddable polar space and panels contain at least 16 chambers (but this is only a sufficient condition). In this case, however, Condition (lco) is automatically satisfied.

In the course of the proof of the theorem one uses filtrations to show that the set of chambers with codistance the identity is simply 2-connected (see the paper by Alice Devillers in these proceedings). The simply 2-connectivity allows one to construct adjacent codistances (of any type), and that is a rather technical part of the proof. Eventually, all codistances that one can construct this way form the second half of a twin building. Details are given in [1].

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Filtrations of buildings

Alice Devillers
(joint work with Bernhard Mühlherr)

Filtrations of buildings have been used by Abels, Abramenko [2, 1] in order to study finiteness properties of groups acting on buildings. In that work, buildings were considered as simplicial complexes. In this talk, we presented a chamber system version of their fundamental tool, known as Brown's criterion. That 'translation' of Brown's criterion to chamber systems turns out to be extremely useful for several applications described below.

We describe here our theorem [3], as well as two major applications.

We refer to [3] for the definitions of chamber system, residue and simple 2-connectedness.

Let I be a set and let $\mathcal{C} = (C, (\sim_i)_{i\in I})$ be a chamber system over I. The example to have in mind here is the chamber system point of view on buildings of type (W, S), with $I = S$.

In the following we denote the set of non-negative integers by $\mathbf{N}$.

A *filtration* of $\mathbf{C}$ is a family $\mathcal{F} = (C_n)_{n\in\mathbf{N}}$ of subsets of C such that the following holds.

- (F1) $C_n \subset C_{n+1}$ for all $n \in \mathbf{N}$,
- (F2) $\bigcup_{n\in\mathbf{N}} C_n = C$,
- (F3) for each $n > 0$ if $C_{n-1} \neq \phi$ then there exists an index $i \in I$ such that for each chamber $c \in C_n$ there exists a chamber $c' \in C_{n-1}$ which is i-adjacent to c.

A filtration $\mathcal{F} = (C_n)_{n\in\mathbf{N}}$ is called *residual* if for each $\emptyset \neq J \subset I$ and each J-residue R the family $(C_n \cap R)_{n\in\mathbf{N}}$ is a filtration of the chamber system $\mathcal{R} := (R, (\sim_j)_{j\in J})$.

For each $x \in C$ we put $|x| := \min\{\lambda \in \mathbf{N} \mid x \in C_\lambda\}$. For a residue R of C we put $|R| := \min\{|x| \mid x \in R\}$ and $\text{aff}(R) := \{x \in X \mid |x| = |R|\}$. Note that $C_0 = \text{aff}(C)$ (assuming that $C_0 \neq \emptyset$).

We say that $\mathcal{F}$ satisfies Condition (*lco*), resp. (*lsco*), if aff(R) is a connected, resp. simply 2-connected, subset of the chamber system $\mathcal{R} = (R, (\sim_j)_{j\in J})$ for all $J \subseteq I$ and all J-residue R.

Theorem A. *Suppose that the residual filtration $\mathcal{F} = (C_n)_{n\in\mathbf{N}}$ of the chamber system $\mathcal{C} = (C, (\sim_i)_{i\in I})$ satisfies (lco), (lsco) and $C_0 \neq \emptyset$. Then the following are equivalent:*

a) *$\mathcal{C}$ is simply 2-connected;*
b) *$(C_0, (\sim_i)_{i\in I})$ is simply 2-connected.*

We explained partly the proof of this result in our talk.

This theorem was crafted with an application to Phan's Theory in mind. Let $\Delta = (\mathcal{B}_+, \mathcal{B}_-, \delta_*)$ be a twin building of type (W, S), provided with a flip τ (see [3] for a definition). We choose an injection $w \mapsto |w|$ from W into $\mathbf{N}$ such that $l(x) < l(y)$ implies $|x| < |y|$ for all $x, y \in W$ and such that $|1_W| = 0$. We define $C_n := \{x \in C_+ \mid |\delta_*(x, x^\tau)| \leq n\}$. Then the family $\mathcal{F}_\tau := (C_n)_{n\in\mathbf{N}}$ is a residual filtration of the chamber system $\mathcal{C}_+$, and so our Theorem gives a local criterion to show that $C_0 = \tau^{\text{op}}$ is simply 2-connected. Ralf Gramlich explained in his talk why this result is important for Phan's Theory.

Another very recent application concerns codistances on a building. A *codistance* on a building $\mathcal{B}$ is a function $f : \mathcal{C} \to W$ such that, for all $s \in S$ and P an s-panel of $\mathcal{C}$, there exists $w \in W$ with $f(x) \in \{w, ws\}$ for all $x \in P$ and P contains a unique chamber with f-value the longest word of the two. Suppose f is a codistance on $\mathcal{B}$, then defining $C_n := \{x \in C \mid |f(x)| \leq n\}$ (where $|\ |$ is as above), we get a residual filtration $\mathcal{F}_f := (C_n)_{n\in\mathbf{N}}$. Assuming local criteria, we then get that $C_0 = f^{\text{op}}$ is simply 2-connected. This is a crucial step in showing that a building admitting a codistance is 'twinnable' [4], as explained in his talk by Hendrik Van Maldeghem.

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Applications of the Gowers trick

László Pyber
(joint work with N. Nikolov and in part with L. Babai)

Answering an 1985 question of Babai and Sós [BS] Gowers [Gow] shoved that the group $\Gamma = \mathrm{PSL}(2,p)$ has no product-free subsets of size $\geq c|\Gamma|^{\frac{8}{9}}$ for some $c > 0$. He obtained this as a consequence of the following general result.

Theorem: *Let G be a group of order n, such that the minimal degree of a nontrivial representation is k. If A, B, C are three subsets of G such that $|A||B||C| > \frac{n^3}{k}$, then there is a triple $(a,b,c) \in A \times B \times C$ such that $ab = c$.*

The starting point of [NP] is the following surprising consequence.

Corollary 1. [NP]. *Let G be a group of order n, such that the minimal degree of a nontrivial representation is k. If A, B, C are three subsets of G such that $|A||B||C| > \frac{n^3}{k}$, then we have $A \cdot B \cdot C = G$. In particular, if, say, $|B| > \frac{n}{k^{\frac{1}{3}}}$, then we have $B^3 = G$.*

Corollary 1 apart from its intrinsic interest, seems to be an extremely useful tool.

For groups of Lie type rather strong lower bounds on the minimal degree of a representation are known [LS].

Combining these bounds with Corollary 1 e.g. for $L = \mathrm{PSL}(n,q)$ we obtain the following.

Proposition 2. *Let B be a subset of size at least $2|L|/q^{\frac{n-1}{3}}$. Then we have $B^3 = L$.*

A slightly weaker result in the case of $\Gamma = \mathrm{PSL}(2,p), p$ prime was obtained earlier by Helfgott [He1]. The result proved in [He1] plays an important role in proving the main result of [He1]; namely that the diameter of any Cayley graph of Γ is bounded by $(\log p)^c$ for some constant c.

Recently Helfgott [He2] (resp. Dinai [Di]) has obtained similar polylogarithmic bounds for the diameters of Cayley graphs of $\mathrm{PSL}(3,p)$ (resp. $\mathrm{PSL}(2,p^\alpha)$) using (among many other tools) Proposition 2.

In [BNP] several extensions of Corollary 1 are obtained. These can be used to prove the following results.

Theorem A. [BNP] *Let G be a nonabelian finite simple group. For a group word w let $W = w(G)$ denote the set of values of w in G.*

Then the probability that for three random elements y_1, y_2, y_3 of W we have $y_1, y_2, y_3 = g$ is $(1 + o(1))|G|^{-1}$ for all $g \in G$.

This implies a deep result of Shalev [Sh]; if G is a large enough simple group than we have $W^3 = G$.

The proof of Theorem A rests on estimates for $|w(G)|$ obtained in [LSh] and [LP].

Theorem B. [BNP]
Let G be finite simple group in Lie(p). Then G is a product of 5 Sylow p-subgroups.

Earlier Liebeck and Pyber [LP] have proved that 25 Sylow p-subgroups suffice.

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Reporter: David Stewart

Participants

Prof. Dr. Michael Aschbacher
Dept. of Mathematics
California Institute of Technology
Pasadena, CA 91125
USA

Dr. Barbara Baumeister
Institut für Mathematik
Freie Universität Berlin
Arnimallee 2-6
14195 Berlin

Prof. Dr. Rieuwert Blok
Dept. of Mathematics
Bowling Green State University
Bowling Green, OH 43403
USA

Prof. Dr. John van Bon
Dipt. di Matematica
Universita della Calabria
Arcavacata di Rende
I-87036 Rende CS

Prof. Dr. Alexandre Borovik
Department of Mathematics
The University of Manchester
Oxford Road
GB-Manchester M13 9PL

Dr. Inna Capdeboscq
Mathematics Institute
University of Warwick
GB-Coventry CV4 7AL

Dr. Pierre-Emmanuel Caprace
CNRS - I.H.E.S.
Le Bois Marie
35, route de Chartres
F-91440 Bures-sur-Yvette

Prof. Dr. Arjeh M. Cohen
Department of Mathematics and
Computer Science
Eindhoven University of Technology
Postbus 513
NL-5600 MB Eindhoven

Dr. Hans Cuypers
Department of Mathematics and
Computer Science
Eindhoven University of Technology
Postbus 513
NL-5600 MB Eindhoven

Prof. Dr. Tom De Medts
Dept. of Pure Mathematics and
Computer Algebra
Ghent University
Krijgslaan 281
B-9000 Gent

Dr. Alice M. Devillers
Dept. de Mathematiques
Universite Libre de Bruxelles
CP 216 Campus Plaine
Bd. du Triomphe
B-1050 Bruxelles

Prof. Dr. Bernd Fischer
Fakultät für Mathematik
Universität Bielefeld
Universitätsstr. 25
33615 Bielefeld

Prof. Dr. Paul Flavell
School of Maths and Statistics
The University of Birmingham
Edgbaston
GB-Birmingham B15 2TT

Dr. Ralf Gramlich
Fachbereich Mathematik
TU Darmstadt
Schloßgartenstr. 7
64289 Darmstadt

Dr. Harald Helfgott
Department of Mathematics
University of Bristol
University Walk
GB-Bristol BS8 1TW

Dipl.Math. Max Horn
Fachbereich Mathematik
TU Darmstadt
Schloßgartenstr. 7
64289 Darmstadt

Prof. Dr. Alexander A. Ivanov
Department of Mathematics
Imperial College London
Huxley Building
180, Queen's Gate
GB-London SW7 2BZ

Prof. Dr. Linus Kramer
Mathematisches Institut
Universität Münster
Einsteinstr. 62
48149 Münster

Dr. Dimitri Leemans
Dept. de Mathematiques
Universite Libre de Bruxelles
CP 216 Campus Plaine
Bd. du Triomphe
B-1050 Bruxelles

Prof. Dr. Martin W. Liebeck
Department of Mathematics
Imperial College of Science
Technology and Medicine
180 Queen's Gate, Huxley Bldg
GB-London SW7 2BZ

Prof. Dr. Kay Magaard
School of Maths and Statistics
The University of Birmingham
Edgbaston
GB-Birmingham B15 2TT

Prof. Dr. Hendrik Van Maldeghem
Department of Pure Mathematics and
Computer Algebra
Ghent University
Galglaan 2
B-9000 Gent

Prof. Dr. Benjamin M.S. Martin
Dept. of Mathematics and Statistics
University of Canterbury
Private Bag 4800
Christchurch 1
NEW ZEALAND

Prof. Dr. Bernhard Mühlherr
Universite Libre de Bruxelles
Service Geometrie
C.P. 216
Bd. du Triomphe
B-1050 Bruxelles

Dr. Nikolay Nikolov
Department of Pure Mathematics
Imperial College London
South Kensington Campus
GB-London SW7 2AZ

Prof. Dr. Silvia Onofrei
Department of Mathematics
Kansas State University
Manhattan, KS 66506-2602
USA

Prof. Dr. Christopher W. Parker
School of Maths and Statistics
The University of Birmingham
Edgbaston
GB-Birmingham B15 2TT

Dr. Gemma Parmeggiani
Dipartimento di Matematica Pura
e Applicata
Universita di Padova
Via Trieste, 63
I-35121 Padova

Prof. Dr. Cheryl E. Praeger
School of Mathematics & Statistics
The University of Western Australia
35 Stirling Highway
Crawley 6009 WA
AUSTRALIA

Dr. Laszlo Pyber
Alfred Renyi Mathematical Institute
of the Hungarian Academy of Science
Realtanoda u. 13-15
H-1053 Budapest

Prof. Dr. Mark A. Ronan
Dept. of Mathematics, Statistics
and Computer Science, M/C 249
University of Illinois at Chicago
851 South Morgan St.
Chicago, IL 60607-7045
USA

Prof. Dr. Peter J. Rowley
Department of Mathematics
The University of Manchester
Oxford Road
GB-Manchester M13 9PL

Dr. Abdul Q. Sami
Department of Mathematics
Michigan State University
Wells Hall
East Lansing, MI 48824-1027
USA

Prof. Jan Saxl
Dept. of Pure Mathematics and
Mathematical Statistics
University of Cambridge
Wilberforce Road
GB-Cambridge CB3 0WB

Dipl.-Math. Marcel Schmidt
Institut für Mathematik
Naturwissenschaftliche Fakultät III
Universität Halle-Wittenberg
06099 Halle

Prof. Dr. Yoav Segev
Dept. of Mathematics
Ben-Gurion University of the Negev
Beer Sheva 84 105
ISRAEL

Dipl. Math. Andreas Seidel
Institut für Mathematik
Naturwissenschaftliche Fakultät III
Universität Halle-Wittenberg
06099 Halle

Prof. Dr. Sergey V. Shpectorov
School of Maths and Statistics
The University of Birmingham
Edgbaston
GB-Birmingham B15 2TT

Prof. Dr. Bernd Stellmacher
Mathematisches Seminar
Christian-Albrechts-Universität Kiel
Ludewig-Meyn-Str. 4
24118 Kiel

David Stewart
Department of Mathematics
Imperial College of Science
Technology and Medicine
180 Queen's Gate, Huxley Bldg
GB-London SW7 2BZ

Prof. Dr. Gernot Stroth
Institut für Mathematik
Naturwissenschaftliche Fakultät III
Universität Halle-Wittenberg
06099 Halle

Prof. Dr. Katrin Tent
Fakultät für Mathematik
Universität Bielefeld
Postfach 100131
33501 Bielefeld

Prof. Dr. Franz-Georg Timmesfeld
Mathematisches Institut
Universität Gießen
Arndtstr. 2
35392 Gießen

Dr. Rebecca Waldecker
School of Maths and Statistics
The University of Birmingham
Edgbaston
GB-Birmingham B15 2TT

Prof. Dr. Richard M. Weiss
Dept. of Mathematics
Tufts University
503 Boston Ave.
Medford, MA 02155
USA