Original Paper

Active thermal conductivity of hot glass[1]

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For (semi)transparent glass, it is suggested to distinguish between an active and a passive thermal conductivity. Essentially, the latter is attached to the heat transfer through long-range photons most of which are both emitted and absorbed outside the glass volume, the former to the heat transfer through phonons and short-range photons which have an intense energy exchange with the glass.

As the temperature distribution in the glass volume can be influenced only by the heat exchange in which the glass is actively involved, it is determined by the active thermal conductivity. The sum of the active and the passive thermal conductivity, which is identical to the so-called apparent thermal conductivity, indicates the overall heat flux through the glass.

Like the apparent thermal conductivity, the active thermal conductivity can be obtained from the prototype of thermal conductivity measurements where the heat flux through a sample between two heat reservoirs of different temperatures is measured. The apparent thermal conductivity follows from the usual reduction of the measurement data, the active thermal conductivity is derived via a differentation rule. Comparing the calculation and the measurement of the temperature inside a cooling block of glass, this differentiation rule is verified.

Aktive Wärmeleitfähigkeit heißen Glases

Es wird vorgeschlagen, bei (halb)transparenten Gläsern zwischen einer aktiven und einer passiven Wärmeleitfähigkeit zu unterscheiden. Letztere beschreibt im wesentlichen den Wärmetransport durch Photonen mit langer freier Weglänge, die zum größten Teil außerhalb des Glases emittiert und absorbiert werden, erstere den Wärmetransport durch Phononen und Photonen mit kurzer freier Weglänge, die mit dem Glas intensiv Wärme austauschen.

Die aktive Wärmeleitfähigkeit beschreibt die Temperaturverteilung im Glas, da diese nur von dem Wärmeaustausch beeinflußt werden kann, an dem das Glas aktiv teilnimmt. Die Summe aus aktiver und passiver Wärmeleitfähigkeit, die mit der sogenannten anscheinenden Wärmeleitfähigkeit identisch ist, gibt den gesamten Wärmefluß durch das Glas an.

Wie die anscheinende Wärmeleitfähigkeit kann auch die aktive Wärmeleitfähigkeit aus dem prototypischen Meßaufbau für Wärmeleitfähigkeiten gewonnen werden, bei dem der Wärmefluß durch einen Probenkörper gemessen wird, der sich zwischen zwei Wärmereservoirs unterschiedlicher Temperatur befindet. Die anscheinende Wärmeleitfähigkeit folgt aus der üblichen Auswertung der Messung, die aktive Wärmeleitfähigkeit über eine Differentiationsregel. Durch den Vergleich zwischen der berechneten und der gemessenen Temperatur im Inneren eines abkühlenden Glasblocks wird diese Differentiationsregel verifiziert.

1. Introduction

Usually, the thermal conductivity is defined as the physical quantity measured by the following experiment (figure 1): Two opposite sides of a sample are held at different temperatures, at T and at $T + \Delta T$. By lateral thermal insulation, it is cared for that the resulting temperature gradient in the sample is uniform over each cross-section. The resulting heat flux is measured. With most materials, one finds a linear proportionality between the temperature gradient $\Delta T/\Delta z$, Δz being the sample thickness, and the heat flux density j. The constant of proportionality is called the thermal conductivity κ [1, p. 214],

$$j = -\kappa \cdot \frac{\Delta T}{\Delta z} . \tag{1}$$

With this linear proportionality given, heat transfer problems can be characterized by a differential equation, the heat transfer equation. It is derived from the microscopic equation corresponding to equation (1), $j = -\kappa \cdot \nabla T$, and the continuity equation $\nabla \cdot j =$ $= \varrho \cdot c_p \cdot \partial T/\partial t$ (ϱ: mass density, c_p: specific heat at constant pressure, t: time) [1, p. 215]:

$$\varrho \cdot c_p \cdot \frac{\partial T}{\partial t} = -\nabla \cdot (\kappa \cdot \nabla T) . \tag{2}$$

This equation can also be used as the starting point for thermal conductivity measurements, e.g. measurements by heat pulse methods [2]. Being the prototype of thermal conductivity measurements, the steady-state method already described, however, is better suited for principal considerations and will always be referred to in the following.

In glass, there are two mechanisms which contribute to the total heat transfer, the phononic thermal conduction and the heat transfer via thermal radiation. In (semi)transparent glasses, the particular features of the latter affect both the interpretation of the thermal conductivity measurement and the applicability of the result to heat transfer problems. To a certain extent, however,

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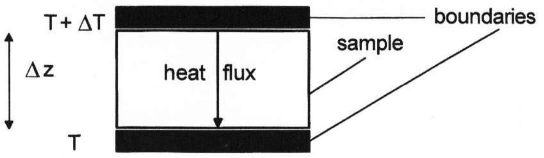

Figure 1. Principal set-up of a thermal conductivity measurement.

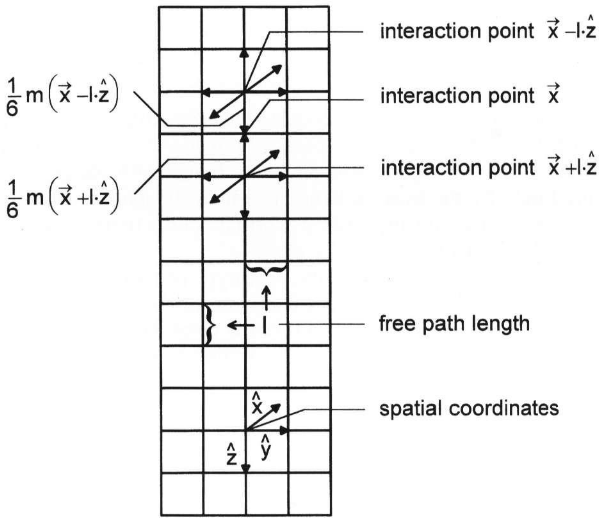

Figure 2. Microscopic mechanism of thermal conduction. It is assumed that the energy coming from each interaction point is equally distributed among the six directions indicated.

these difficulties may be overcome if one distinguishes between an active and a passive thermal conductivity, as suggested in this paper.

In the following, the characterization of both the phononic contribution to the thermal conductivity and the radiative heat transfer in opaque glasses will be recapitulated first. Considering the particular features of the radiative heat transfer in (semi)transparent glasses, the distinction between the active and the passive thermal conductivity will be introduced.

2. Phononic contribution to the thermal conductivity

For the phononic contribution, which is the major one at temperatures below about 400 °C, the microscopic theory of Debye gives [3]:

$$\kappa_{\text{phononic}} = \frac{1}{3} \cdot c_V \cdot \varrho \cdot v_s \cdot l\,, \tag{3}$$

where c_V is the specific heat at constant volume, l and v_s are the (mean) free path length and the (mean) velocity of the phonons, respectively.

To arive at this equation, one has to consider that the thermal energy of a solid is carried by vibrational excitations or phonons, which are capable of a wave-like motion through the glass volume. Due to the regular scattering and other interaction processes the phonons are subjected to, this motion is a drift process. (The interaction processes bring about a re-arrangement of the phonon distribution. Any deviation from the equal distribution of the thermal energy among all types of phonons is thus corrected. So these processes are responsible for the local thermal equilibrium.)

The distance a phonon may cover before it is subjected to the next interaction is called the free path length l. In the following, it is assumed that l is a constant number and that the interaction processes take place at fixed points in space.

To calculate the resulting heat flux density in the direction of the temperature gradient, one has to consider the net phonon flux density passing one interaction point $\vec{x}$ (figure 2). It is the difference of the flux densities coming from the two neighbouring interaction points $\vec{x}-l\cdot\hat{z}$ and $\vec{x}+l\cdot\hat{z}$. $\hat{z}$ is the unit vector with the direction of the temperature gradient. As all directions are equal at an interaction point, it can be assumed that both from $\vec{x}-l\cdot\hat{z}$ and from $\vec{x}+l\cdot\hat{z}$, one sixth of the phonons moves towards $\hat{z}$. With $m(\vec{x}-l\cdot\vec{z})$ and $m(\vec{x}+l\cdot\hat{z})$ being the corresponding phonon densities, the resulting net flux density is $(m(z+l)/6 - m(z-l)/6)\cdot v_s \approx 1/3\cdot \mathrm{d}m/\mathrm{d}z\cdot l\cdot v_s$.

As m is proportional to the local energy density e, $1/3\cdot \mathrm{d}m/\mathrm{d}z\cdot l\cdot v_s$ is equivalent to an energy flux density $j = 1/3\cdot \mathrm{d}e/\mathrm{d}z\cdot l\cdot v_s = 1/3\cdot \mathrm{d}e/\mathrm{d}T\cdot \mathrm{d}T/\mathrm{d}z\cdot l\cdot v_s$. With $\varrho\cdot c_V = \mathrm{d}e/\mathrm{d}T$ (e denotes the energy per volume, whereas c_V denotes the specific heat per mass; so ϱ has to be introduced in this equation), the resulting expression for j is:

$$j = -\frac{1}{3}\cdot c_V\cdot \varrho\cdot l\cdot v_s\cdot \frac{\mathrm{d}T}{\mathrm{d}z}\,. \tag{4}$$

This corresponds to equation (3).

3. Radiative heat transfer in opaque glass

In an opaque glass, the radiative heat transfer is similar to the phononic thermal conduction and may analogously be characterized by a radiative thermal conductivity. The thermal conductivity measured with the prototypical experiment (figure 1) – the effective thermal conductivity – is the sum of the phononic and the radiative contribution:

$$\kappa_{\text{effective}} = \kappa_{\text{phononic}} + \kappa_{\text{radiation}}\,. \tag{5}$$

With the same argument as for the phononic thermal conductivity, the following expression can be obtained for the radiative thermal conductivity [4]:

$$\kappa_{\text{radiation}} = \frac{16}{3}\cdot n^2\cdot \frac{\sigma}{k_R}\cdot T^3\,, \tag{6}$$

where n is the refractive index, σ is the Stefan-Boltzmann constant, and k_R is the Rosseland mean for the absorption coefficient (see equation (8)).

To derive this expression, one starts considering the energy flux density which is given by a similar expression as in section 2., $j \equiv -1/3 \cdot \mathrm{d}e/\mathrm{d}z \cdot l \cdot \upsilon_l/n$. This time, e is the density of the energy stored in the radiation field – as photons –, l is the inverse of the absorption coefficient k, and υ_l/n is the velocity of light (vacuum velocity of light divided by the refractive index). Usually, the absorption coefficient depends on the wavelength λ so that the energy flux density has to be calculated separately for each: $j_\lambda = -1/3 \cdot \mathrm{d}e/\mathrm{d}z \cdot l(\lambda) \cdot \upsilon_l/n = -1/3 \cdot \mathrm{d}e/\mathrm{d}z \cdot 1/k(\lambda) \cdot \upsilon_l/n$ (the wavelength dependence of n is suppressed). The integration over all wavelengths gives the total radiative energy flux density $j \equiv \int_0^\infty j_\lambda \, \mathrm{d}\lambda$.

Due to the intense exchange of energy through absorption and emission in opaque glasses, there is strong coupling of the atomic vibrations – the phonons – and the radiation field – the photons – and thus local thermal equilibrium of both. Therefore, e_λ is given by 4π times Planck's expression [5, p. 22] for the equilibrium spectral intensity over the velocity of light, i.e. $4\pi \cdot B(\lambda, T)/(\upsilon_l/n)$.

(The spectral intensity I indicates the amount of radiation energy moving in a certain direction, per unit wavelength interval, per unit solid angle, per unit area normal to this direction, per time. The energy density e_λ is obtained from this through division by the velocity of light and integration over the whole solid angle. Because of the local thermal equilibrium, I is identical to Planck's function $B, I(\ldots, x, y, z) = B(\lambda, T(z))$, and, consequently, $e_\lambda(z) = 4\pi \cdot B(\lambda, T(z))/(\upsilon_l/n)$ is valid. Because of the coupling of temperature and radiation, a temperature gradient gives rise to a gradient of I, which results in a preferred direction of the energy transfer through alternating photon emission and absorption.)

With the relation $e_\lambda(z) = 4\pi \cdot B(\lambda, T(z))/(\upsilon_l/n)$ and $\mathrm{d}B/\mathrm{d}z = \partial B/\partial T \cdot \mathrm{d}T/\mathrm{d}z$, the total radiative energy flux density j is:

$$j = -\frac{4\pi}{3} \cdot \frac{\upsilon_l}{n} \cdot \left(\int_0^\infty \frac{\partial B(\lambda, T)}{\partial T} \cdot \frac{1}{\upsilon_l/n} \cdot \frac{1}{k(\lambda)} \, \mathrm{d}\lambda \right) \cdot \frac{\partial T}{\mathrm{d}z} . \qquad (7)$$

With the Rosseland mean

$$k_\mathrm{R} = \int_0^\infty \frac{\partial B}{\partial T} \, \mathrm{d}\lambda \Big/ \int_0^\infty \frac{\partial B}{\partial T} \cdot \frac{1}{k(\lambda)} \, \mathrm{d}\lambda \qquad (8)$$

and the identity

$$\pi \cdot \int_0^\infty B(\lambda, T) \, \mathrm{d}\lambda = n^2 \cdot \sigma \cdot T^4 , \qquad (9)$$

the expression for j can be rewritten as:

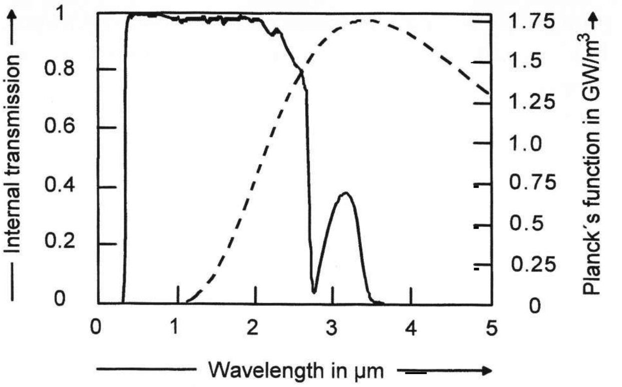

Figure 3. Internal spectral transmission of DURAN® glass, sample thickness 3 mm, at an intermediate temperature (500 °C) of the described cooling experiment (solid line). To indicate the spectral range of interest, Planck's function at the initial temperature (570 °C) of this experiment is also given (dashed line).

$$j \equiv -\frac{16}{3} \cdot n^2 \cdot \frac{\sigma}{k_\mathrm{R}} \cdot T^3 \cdot \frac{\mathrm{d}T}{\mathrm{d}z} . \qquad (10)$$

This corresponds to equation (6) for the radiative thermal conductivity.

As it has been said at the beginning of this section, the effective thermal conductivity is the phononic plus the radiative term:

$$\kappa_\mathrm{effective} \equiv \frac{1}{3} \cdot c_\mathrm{V} \cdot \varrho \cdot \upsilon_\mathrm{s} \cdot l + \frac{16}{3} \cdot n^2 \cdot \frac{\sigma}{k_\mathrm{R}} \cdot T^3 . \qquad (11)$$

Due to its last factor (T^3), the radiative term is strongly temperature-dependent. As it has been already said, it is less important than the phononic term at temperatures below about 400 °C, but equal or more important than that at higher temperatures.

4. Radiative heat transfer in transparent glass

In the following it will be shown that considering glass which is transparent for a part of the electromagnetic spectrum (figure 3), i.e. glass where the free path length of those photons is bigger than the dimension of the glass volume, it is helpful to distinguish between an active and a passive thermal conductivity.

Essentially, the active thermal conductivity shall determine the heat flux, which is actively transferred via phonons or photons with a short free path length. (These photons are subsequently emitted and absorbed in the glass volume so that the latter is actively involved in the transfer of them.) The passive thermal conductivity shall indicate the flux of the photons with a long free path length which therefore may pass the glass without interaction, being both generated and deleted outside the glass volume. The sum of the active and the passive contribution corresponds to what has been called apparent thermal conductivity [6]. (To match the nomenclature of section 3., the term "effective thermal conductivity" could always be used when the sum of the phononic part and a radiative contribution is addressed. This would lead, however, to long expressions like "apparent effective thermal conductivity".)

It is the latter quantity which comes out of the prototypical thermal conductivity measurement (figure 1) if the data are evaluated as usual. As it cannot be affected by those photons which are neither emitted nor absorbed in the glass, the temperature distribution in a piece of glass, however, is determined by the active thermal conductivity only. This is especially important for cooling processes.

Provided that measurements at samples with different thicknesses Δz, $\Delta z \pm \mathrm{d}z$ have been made, this quantity may be determined via the prototypical experimental set-up also. As it will be shown, the active thermal conductivity is then obtained from the different heat fluxes j, $j \pm \mathrm{d}j$ by:

$$\kappa_{\text{active}} = -\frac{1}{\Delta T} \cdot \frac{\mathrm{d}j}{\mathrm{d}\left(\dfrac{1}{\Delta z}\right)} . \tag{12}$$

As it has been mentioned, the apparent thermal conductivity follows from an "as usual"-data reduction:

$$\kappa_{\text{apparent}} = -\frac{j}{\Delta T/\Delta z} . \tag{13}$$

The passive thermal conductivity is given by the difference of the apparent and the active thermal conductivity.

Equation (12) can be derived considering how the various components of the heat flux in the prototypical thermal conductivity measurement depend on the sample thickness Δz. For this derivation, it will first be assumed that with respect to the sample thickness Δz, the absorption coefficient $k(\lambda)$ is either close to zero or almost infinite, i.e. either $k(\lambda) \cdot \Delta z \ll 1$ or $k(\lambda) \cdot \Delta z \gg 1$ is valid. The wavelengths for which the first equation is true will be indexed with t (λ_t, as transparent), the others with o (λ_o, as opaque).

Implicitly, this assumption has already been made, when the distinction between photons with a short free path length and photons with a long free path length was made. As it has been said, the temperature distribution inside a piece of glass will then be exactly determined by the heat transfer equation (2) with $\kappa = \kappa_{\text{active}}$, i.e. inserting the active thermal conductivity.

If photons with a medium free path length are involved also, the exact calculation will require a lot more of mathematical effort. In contrast to the photons with a long free path length, these photons have a non-negligible effect on the temperature distribution. In contrast to the photons with a short free path length, their local intensity is not determined by the local temperature because the coupling between them and the phonon field (to which the local temperature refers) is too small. So their intensity has to be calculated separately, which includes the solution of an integro-differential equation [7]. If this is to be avoided, a suitably defined thermal conductivity is required, which when inserted in the heat transfer equation will at least lead to a good approximation of the exact temperature distribution. Later it will be shown that the active thermal conductivity is a reasonable quantity in this case also, not only on the present assumption that either $k(\lambda) \cdot \Delta z \ll 1$ or $k(\lambda) \cdot \Delta z \gg 1$ is valid.

For the wavelengths for which $k(\lambda) \cdot \Delta z \gg 1$ is valid, the consideration leading to equation (7) for the flux density is again valid:

$$j_{\lambda_\mathrm{o}} = -\frac{4\pi}{3} \cdot \frac{\partial B(\lambda_\mathrm{o}, T)}{\partial T} \cdot \frac{1}{k(\lambda_\mathrm{o})} \cdot \frac{\mathrm{d}T}{\mathrm{d}z} . \tag{14}$$

In contrast to equation (14), the existence and the intensity of a photon flux with wavelengths for which the glass is transparent cannot be derived from the glass properties and the temperature distribution but depend on the boundary conditions. If, for instance, the glass sample is sandwiched between two black bodies, the corresponding flux density is given by [5, p. 21]

$$\begin{aligned} j_{\lambda_\mathrm{t}} &= -(\pi \cdot B(\lambda_\mathrm{t}, T + \Delta T) - \pi \cdot B(\lambda_\mathrm{t}, T)) \approx \\ &\approx -\pi \cdot \frac{\partial B(\lambda_\mathrm{t}, T)}{\partial T} \cdot \Delta z \cdot \frac{\mathrm{d}T}{\mathrm{d}z} , \end{aligned} \tag{15}$$

i.e., the difference of the surface radiations of the two black bodies. (To calculate the surface radiation, the spectral intensity has to be projected on the normal to the surface (cosine law) and integrated over the half sphere. Reflexion effects at the surface are suppressed.) In the second (approximate) equation, $\Delta T/\Delta z$ has been replaced by $\mathrm{d}T/\mathrm{d}z$ on the right side which is exactly true for a linear temperature profile $T(z)$ only.

Remarkably, the expression for j_{λ_t} almost equals the expression for j_{λ_o} if the sample thickness Δz is inserted as free path length.

So the active thermal conductivity is given by:

$$\begin{aligned} \kappa_{\text{active}} &\equiv \frac{1}{3} \cdot c_\mathrm{V} \cdot \varrho \cdot v_\mathrm{s} \cdot l + \\ &+ \frac{4\pi}{3} \cdot \int_{\lambda_\mathrm{o}} \frac{\partial B(\lambda, T)}{\partial T} \cdot \frac{1}{k(\lambda)} \mathrm{d}\lambda ; \end{aligned} \tag{16}$$

$\int_{\lambda_\mathrm{o}}$ means the integration over all wavelengths for which the glass is opaque.

For black bodies as boundaries, the passive thermal conductivity amounts to:

$$\kappa_{\text{passive}} = \pi \cdot \Delta z \cdot \int_{\lambda_\mathrm{t}} \frac{\partial B(\lambda, T)}{\partial T} \mathrm{d}\lambda , \tag{17}$$

$\int_{\lambda_\mathrm{t}}$ means the integration over all wavelengths for which the glass is transparent.

The sum of both the active and the passive contribution is the apparent thermal conductivity:

$$\kappa_{\text{apparent}} \equiv \frac{1}{3} \cdot c_V \cdot \varrho \cdot v_s \cdot l + \frac{4\pi}{3} \cdot \int_{\lambda_o} \frac{\partial B(\lambda, T)}{\partial T} \cdot \frac{1}{k(\lambda)} \, d\lambda + \pi \cdot \Delta z \cdot \int_{\lambda_t} \frac{\partial B(\lambda, T)}{\partial T} \, d\lambda \, . \tag{18}$$

As it has been already said, it is this apparent thermal conductivity which is measured by the prototypical experiment if the latter is evaluated according to equation (13). To obtain the active thermal conductivity, the measurement data have to be evaluated according to the differentiation rule (equation (12)). This follows from the relation of the heat flow j and the sample thickness Δz:

$$j = -\frac{1}{3} \cdot c_V \cdot \varrho \cdot v_s \cdot l \cdot \frac{\Delta T}{\Delta z} - \frac{4\pi}{3} \cdot \left[\int_{\lambda_o} \frac{\partial B(\lambda, T)}{\partial T} \cdot \frac{1}{k(\lambda)} \, d\lambda \right] \cdot \frac{\Delta T}{\Delta z} - \pi \cdot \left[\int_{\lambda_t} \frac{\partial B(\lambda, T)}{\partial T} \, d\lambda \right] \cdot \Delta T \, , \tag{19}$$

from which follows

$$-\frac{1}{\Delta T} \cdot \frac{dj}{d\left(\frac{1}{\Delta z}\right)} = \frac{1}{3} \cdot c_V \cdot \varrho \cdot v_s \cdot l + \frac{4\pi}{3} \cdot \int_{\lambda_o} \frac{\partial B(\lambda, T)}{\partial T} \cdot \frac{1}{k(\lambda)} \, d\lambda \equiv \kappa_{\text{active}} \, . \tag{20}$$

So under the condition that either $k(\lambda) \cdot \Delta z \ll 1$ or $k(\lambda) \cdot \Delta z \gg 1$ is valid, the following two theses are true: First, the temperature distribution in a piece of glass depends on the phonons and the photons with a short free path length only, the effect of which is exactly characterized by the active thermal conductivity. Second, the active thermal conductivity can be determined from the experiment of figure 1 via the differentiation rule (equation (12)).

Strictly speaking, neither statement may be made if radiation is involved the absorption coefficient of which has the order of magnitude of the sample thickness, i.e. $k(\lambda) \cdot \Delta z \approx 1$ is valid. As it has been said, the transfer of this radiation has to be calculated separately for an exact solution of the heat transfer problem. The latter will have particular features in most cases. In case of a glass which fulfills the condition that either $k(\lambda) \cdot \Delta z \ll 1$ or $k(\lambda) \cdot \Delta z \gg 1$ is valid, the temperature profile of a cooling glass plate, for instance, has a parabolic shape. If a significant amount of radiation with $k(\lambda) \cdot \Delta z \approx 1$ has to be taken into account, it is closer to a higher-order power law profile z^{2a}, $a > 1$ [8], (figure 4).

As it has been said also, the computational effort necessary for an exact solution of the heat transfer problem is immense. To avoid it, a suitably defined thermal conductivity is required which when inserted in the heat transfer equation will at least lead to a good approximation of the exact temperature distribution.

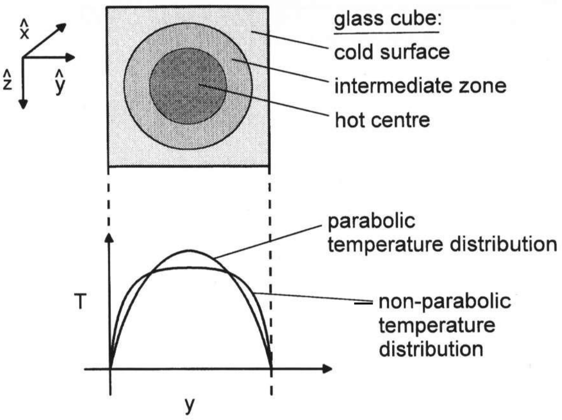

Figure 4. Temperature distribution in a cooling glass cube (the shading intensity inside is proportional to the temperature). If the heat transfer inside is exactly characterized by the active thermal conductivity, the temperature distribution is parabolic. If not, the temperature distribution has "temperature jumps" at the surface and a flat shape in the centre, both typical features of radiative heat transfer it photons are involved the free path length of which has the same dimension as the cube geometry.

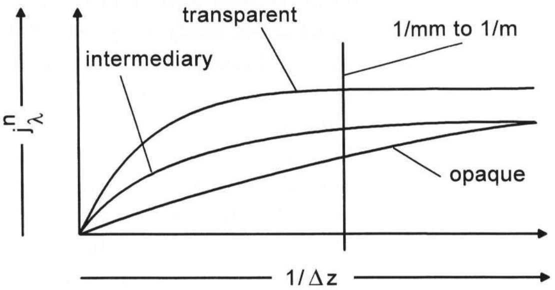

Figure 5. Normalized flux density versus inverse sample thickness for the different contributions to the heat transfer.

Remarkably, the differentiation rule (12) gives a reasonable quantity for that purpose so that the term "active thermal conductivity" may be attached to the result of equation (12) in general, not only on the assumption that either $k(\lambda) \cdot \Delta z \ll 1$ or $k(\lambda) \cdot \Delta z \gg 1$ is valid. This is not clear a priori and, therefore, a remarkable fact. It can be made plausible considering the way how the various parts of the heat flux depend on the inverse sample thickness $1/\Delta z$ (figure 5).

As it has been explained, both the phononic heat flux, j_{phononic}, and the radiative heat flux at wavelengths for which the glass is opaque, j_{λ_o}, are proportional to $\Delta T/\Delta z$. With the temperature difference ΔT having a constant value, this is equivalent to a linear relation between these two contributions and $1/\Delta z$. In contrast to this, the radiative heat flux at wavelengths for which the glass is transparent, j_{λ_t}, is independent of Δz or $1/\Delta z$ (see [9] for comparison). The latter, however, is true only as long as the condition $k(\lambda) \cdot \Delta z \ll 1$ is valid. For samples with the thickness Δz being bigger than $1/k(\lambda)$ — it is assumed that $k(\lambda)$ is not exactly equal to zero, which will never be the case in real glass — almost the whole corresponding radiation will be completely absorbed at least once on its way through the sample. Even if the

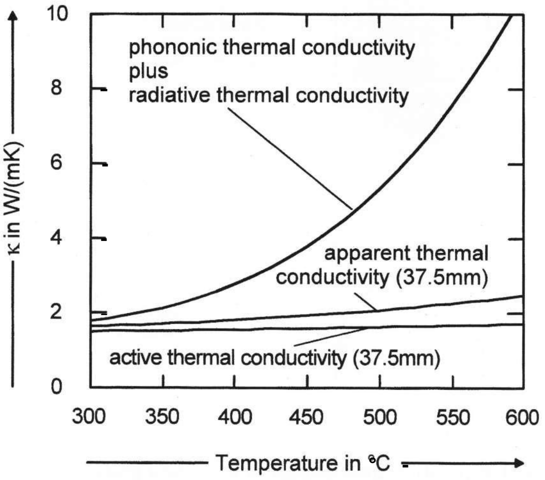

Figure 6. Thermal conductivity of DURAN® glass versus temperature for sample thicknesses 37.5 mm and infinity.

whole energy involved were re-emitted at the original wavelengths (in reality, the absorbed energy is distributed among the vibrational modes and, according to Planck's law, the whole radiation field), only one half would move on in the original direction. Because of the re-emission being isotropic, the other would be sent back to where it came from. Therefore, j_{λ_t} is not independent of the sample thickness for very large values of the latter. (The classification as opaque or transparent refers to technically relevant values of Δz.) In the limit of infinite sample thickness, there is always a similar linear relationship to $1/\Delta z$ as for j_{λ_o}. In the diagram, this results in a saturation curve with the bending point at $\Delta z(\lambda) = 1/k(\lambda)$.

In the intermediate case, the bending point of the saturation curve lies in the region of the technically relevant values of Δz, i.e. $k(\lambda) \cdot \Delta z \approx 1$ is valid for them. The corresponding flux density is indexed with "i" for intermediate: j_{λ_i}.

According to the differentiation rule (12), the slope of each of these curves is proportional to the corresponding contribution to the active thermal conductivity. The constant of proportionality is the temperature difference ΔT which according to the presupposition has a constant value for all experiments. It can be eliminated by introducing the normalized flux density j^n, which is the quotient of the heat flux density j and the temperature difference ΔT. So the active thermal conductivity is given by:

$$\kappa_{\text{active}} = -\frac{\mathrm{d}j^n}{\mathrm{d}\left(\dfrac{1}{\Delta z}\right)}\,. \tag{21}$$

At low values of $1/\Delta z$ and high values of Δz, respectively, the differentiation rule yields the sum of all zero point slopes as active thermal conductivity. (The sum of all zero point slopes is equal to the radiative thermal conductivity so that the latter is characteristic for all glasses if the thickness is infinite.) This corresponds to the fact that above a certain thickness, the glass is opaque for all wavelengths. In the opposite limit, the differentiation rule sifts the phononic contribution from all the radiative ones. This in return corresponds to the fact that below a certain thickness, the glass is transparent for all wavelengths.

At the intermediate values of Δz – the technically important ones –, the differentiation rule counts the full zero point slopes of both j^n_{phononic} and $j^n_{\lambda_o}$. As $j^n_{\lambda_t}$ has a constant amount, it is not taken into account. For j_{λ_i}, the differentiation rule yields an intermediate value. Obviously, the latter is more reasonable than any alternative. Calculating the contribution to the thermal conductivity from the difference quotient $j^n_{\lambda_i}/(1/\Delta z)$ instead of the differential quotient $\mathrm{d}j^n_{\lambda_i}/\mathrm{d}(1/\Delta z)$ means proceeding according to the definition of the apparent thermal conductivity and will lead to an overestimation of the effect of $j^n_{\lambda_i}$. Calculating the contribution of $j^n_{\lambda_i}$ from the equation $j_\lambda = -4\pi/3 \cdot \mathrm{d}B(\lambda, T)/\mathrm{d}z \cdot 1/k(\lambda)$ (which comes to calculating with the radiative thermal conductivity as defined by equation (6)) will even lead to a higher value if $1/k(\lambda) > \Delta z$ is valid. On the other hand, it is not justified to neglect this radiation completely.

An obvious lack of this concept is its being derived from a one-dimensional consideration. However, this is of practical importance only in those cases when the active thermal conductivity strongly depends on the exact value of the dimension Δz. Above that, the heat transfer aspect is close to that of a glass plate in most practical cases.

5. Verification of the concept of the active thermal conductivity by a cooling experiment

Figure 6 shows the active and the apparent thermal conductivity of DURAN® glass (Schott Glaswerke, Mainz (Germany)) for sample thicknesses 37.5 mm and infinity (where both are equal and identical to the phononic thermal conductivity plus the radiative thermal conductivity as defined by equation (6)).

The necessary measurements have been carried out according to the principle given in figure 1. For a technical realization, the test specimen was sandwiched between two identical reference samples the thermal conductivity of which was known (comparative heat flux technique). This stack was placed between a heat source and a heat sink. After reaching thermal equilibrium, there was a constant heat flux through the sandwich (there were two reference samples to make sure that this was really the case) and a corresponding temperature distribution which was measured with thermocouples. From the temperature gradient along the reference samples and the thermal conductivity of them, the heat flux was calculated.

For the cooling experiment, a glass plate (DURAN® glass, 75 mm thickness) was installed in an oven with a sapphire window, equilibrated at 570 °C and subsequently cooled down at 20 K/h. Figure 7 shows the experimental set-up and figure 8 the difference between the maximum and the minimum temperature in the plate, ΔT, as well as the surface temperature as a function of the time after the start of the cooling. The three theoretical curves for ΔT have been calculated on the basis of the different thermal conductivities from figure 6, i.e. the active thermal conductivity, the apparent thermal conductivity, and the sum of the phononic thermal conductivity and the radiative thermal conductivity as defined in equation (6).

The experimental curve for ΔT was determined with a spectral radiometer (grating instrument) measuring the radiation emitted from the middle of the glass plate perpendicular to the surface. This radiation was recorded at both a wavelength for which the glass is opaque (3.6 µm) and a wavelength for which the glass is semitransparent (2.5 µm). At a typical temperature for the cooling experiment (500 °C), the corresponding absorption coefficients of DURAN® are $k(3.6\,\mu\text{m}) = 16\,\text{cm}^{-1}$ and $k(2.5\,\mu\text{m}) = 0.56\,\text{cm}^{-1}$, respectively.

Consequently, the intensity measured at 3.6 µm ($I_{3.6\,\mu\text{m}}$) exclusively derived from the glass surface. So the following equation holds:

$$I_{3.6\mu\text{m}} \equiv f_{3.6\mu\text{m}} \cdot B(T_{\text{surface}}, 3.6\,\mu\text{m}) \tag{22}$$

$f_{3.6\mu\text{m}}$ is a calibration factor allowing for the reflexion losses of the radiation at the emergence from the glass and at the sapphire windows, for the sensitivities of both the monochromator and the detector of the spectral radiometer, for the spot size measured, and for the solid angle involved. It was derived from a measurement in the equilibrated state before the cooling which must reproduce the temperature of 570 °C measured by the thermocouples of the oven control. From the subsequent measurements at 3.6 µm, the formation of the surface temperature during the cooling is derived.

If one assumes that the temperature distribution in the glass along the axis of observation is given by a parabola with a crown height ΔT, and if one takes into account, first, that the intensity of the locally emitted radiation is given by the product of the absorption coefficient and Planck's function, and, second, that the intensity emitted towards the spectral radiometer is attenuated by $\text{e}^{-k(2.5\mu\text{m})\cdot z}$ with z being the distance between the point of emission and the glass surface, the intensity measured at 2.5 µm must be given by:

$$I_{2.5\mu\text{m}} = f_{2.5\mu\text{m}} \cdot k(2.5\,\mu\text{m}) \cdot \int_0^D B(T(z), 2.5\,\mu\text{m}) \cdot \text{e}^{-k(2.5\mu\text{m})\cdot z}\,\text{d}z\,, \tag{23}$$

e.g.

$$I_{2.5\mu\text{m}} \approx f_{2.5\mu\text{m}} \cdot \left[B(T(0),\ldots) + k(2.5\,\mu\text{m}) \cdot \frac{\partial B}{\partial T} \cdot \int_0^D \Delta T \cdot \left(1 - \frac{(D/2-z)^2}{(D/2)^2}\right) \cdot \text{e}^{-k(2.5\mu\text{m})\cdot z}\,\text{d}z\right]. \tag{24}$$

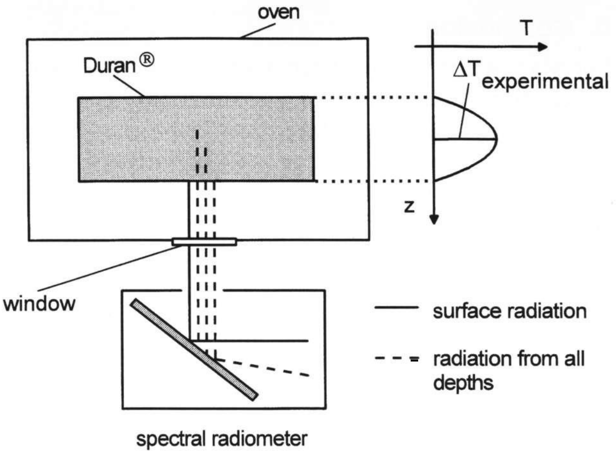

Figure 7. Set-up of the cooling experiment.

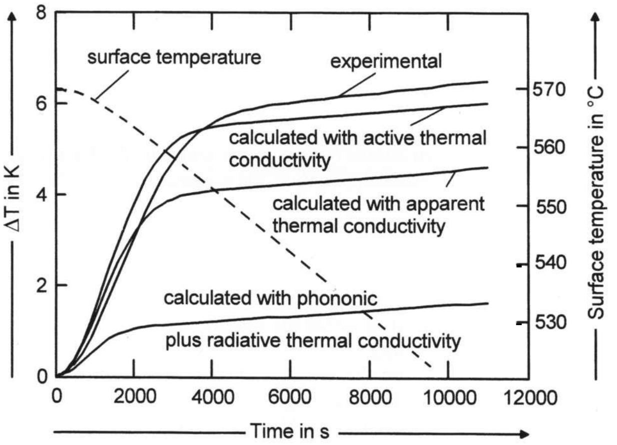

Figure 8. Experimental and calculated values for the difference between maximum and minimum temperature of a cooling glass plate (ΔT: solid lines) as well as the surface temperature measured (dashed line). Approximately 500 s after the start of the experiment, the surface temperature decreases synchronously with the oven temperature. For ΔT, the time offset after which a steady state is reached amounts to about 3000 s. This steady state is not characterized by a constant value of ΔT but a sligthly increasing one, reflecting the decreasing value of the thermal conductivity (whatever definition for the thermal conductivity is taken).

D is the thickness of the glass plate (75 mm). The calibration factor was again derived from an advance measurement at the equilibrated glass when $\Delta T = 0$ was valid. From the subsequent measurements of $I_{3.6\mu\text{m}}$ and $I_{2.5\mu\text{m}}$, the formation of ΔT with time was determined.

For the theoretical curves, the temperature profile along the axis of observation was calculated with the surface temperature measured as boundary condition. As expected, the correspondence between the experimental curve and the theoretical curve calculated with the sum of the phononic thermal conductivity and the radiative thermal conductivity is least. The correspondence with the theoretical curve calculated with the apparent thermal conductivity is not satisfactory also. The best fitting theoretical curve is derived with the active thermal conductivity.

6. Conclusion

For glass which is completely transparent for a part of the electromagnetic spectrum and completely opaque for the rest of it, the temperature distribution is exactly determined by the active thermal conductivity, which can be obtained from a differentiation rule. Remarkably, the active thermal conductivity determined via this differentiation rule leads to a good approximation of the temperature distribution inside the glass even if there are spectral bands where the coefficient of absorbance has the same order of magnitude as the inverse geometrical dimension of the glass.

Strictly speaking, the characterization of the heat transfer problem by a single parameter — however defined — is not possible in this case. On the other hand, the active thermal conductivity determined via the differentiation rule gives a good result if one only needs an approximate temperature information.

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7. Symbols

B	Planck's function
c_p	specific heat at constant pressure
c_V	specific heat at constant volume
e	energy density
e_λ	derivative of the energy density of the radius with respect to the wavelength
I	spectral intensity
j	heat flux density
j^n	normalized heat flux density
j_phononic	phononic heat flux
j_λ	derivative of the radiative heat flux with respect to the wavelength
j_{λ_i}	radiative heat flux at wavelengths for which the glass under consideration is neither opaque nor transparent
j_{λ_o}	radiative heat flux at wavelengths for which the glass under consideration is opaque
j_{λ_t}	radiative heat flux at wavelengths for which the glass under consideration is transparent
k	absorption coefficient
k_R	Rosseland mean for the absorption coefficient
l	free path length
m	density of either phonons or photons
n	refractive index
T	absolute temperature
v_l	vacuum velocity of light
v_s	mean phonon velocity
x, y, z	spatial coordinates
$\hat{x}, \hat{y}, \hat{z}$	unit vectors in the direction of the spatial coordinates
$\vec{x}$	radius vector
κ	thermal conductivity
κ_active	active thermal conductivity of a (semi)transparent glass
κ_apparent	apparent thermal conductivity of a (semi)transparent glass
$\kappa_\mathrm{effective}$	effective thermal conductivity of an opaque glass
κ_passive	passive thermal conductivity of a (semi)transparent glass
κ_phononic	phonon contribution to the thermal conductivity
$\kappa_\mathrm{radiation}$	radiation contribution to the thermal conductivity of an opaque glass
λ	(optical) wavelength
λ_i	wavelength for which the glass under consideration is neither opaque nor transparent
λ_o	wavelength for which the glass under consideration is opaque
λ_t	wavelength for which the glass under consideration is transparent
ϱ	density
σ	Stefan-Boltzmann constant
ΔT	temperature difference
Δz	difference in sample thickness
$\mathrm{d}/\mathrm{d}x$	operator of total differentiation with respect to the variable x
$\partial/\partial x$	operator of partial differentiation with respect to the variable x
∇	Nabla operator

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