MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 2/1999

Combinatorial Optimization

10.–16.1.1999

Leitung: Rainer E. Burkard, Graz
Martin Grötschel, Berlin

Combinatorial Optimization, 10.–16.1.1999

This year's meeting on Combinatorial Optimization (CO) was organized by Rainer Burkard (Graz) and Martin Grötschel (Berlin). There were 50 participants from thirteen countries, and a considerable number of the participants visited Oberwolfach for the first time.

The structure of the meeting was different from that of earlier meetings on Combinatorial Optimization. Every day was dedicated to a major topic in the field: computational issues and large scale programs on Monday, probabilistic methods and approximation on Tuesday, modeling and solving industrial problems on Wednesday, algebraic methods in discrete optimization on Thursday, and geometry, graphs, and combinatorial optimization on Friday. The selection of these five themes aimed at covering a broad area of Combinatorial Optimization ranging from computational aspects and issues related to its applications in practice (laying on the border with OR) on one side of the spectrum, and over more classical aspects like graph theory, geometry and algebra to theoretical issues on the border to theoretical computer science (like approximation algorithms and randomization) on the other side of the spectrum. On each of the topics there were two to three one-hour invited review talks given by renowned specialists in the field. Contributions of 25 minutes were made by mostly younger researchers, presenting own results.

The new structure of the meeting was well-received by the participants. The program driven by topics was considered as a productive way to focus on major aspects of Combinatorial Optimization and to promote the exchange between young researchers and experienced scientists. The enthusiasm of the participants, the good quality of the presentations, and the friendly atmosphere, so typical of meetings in Oberwolfach, stimulated fruitful discussions and gave raise to joint research among the participants. The organizers and all participants of this meeting would like to thank the Mathematical Research Institute for the wonderful hospitality and the great service.

Abstracts of Talks

Monday, 1/11/1999

Computational Issues and Large Scale Programs

Laurence Wolsey, CORE Louvain-la-Nueve

Mixed Integer Programming and Branch-and-Cut

We survey the main features of a branch-and-cut code for mixed integer programming, including recent research on branch-and-bound strategies of Linderoth and Savelsbergh, and A. Martin, and emphasizing the questions concerning cutting plane strategies both at the top node and in the tree, such as whether the different separation routines should be run separately or simultaneously, and whether cutting should be terminated early due to tailing-off effects. We then present the special features of one such system "bc-opt", emphasizing the importance of developing interface routines linking the problem instance and the canonical structures, such as a pure integer row, on which the separation routines work.
Finally a new separation routine based on aggregation and mixed integer rounding is presented, and "bc-prod", a specialized branch-and-cut system for general lot-sizing problems, based on a combined modeling and optimization language. The latter is joint work with H. Marchand and G. Belvaux respectively.

Rüdiger Schultz, Gerhard-Mercator Universität Duisburg

Decomposition in Stochastic Integer Programming

Stochastic programs come up as deterministic equivalents to random optimization problems. With discrete probability distributions, stochastic programs typically obey a block-angular decomposition structure.
We introduce linear two-stage stochastic programs with integer requirements. With continuous probability distributions these models involve multiple integrals over implicitly given integrands that are numerically intractable. The stability analysis of stochastic integer programs, however, justifies approximation of the underlying probability measure by a weakly converging sequence of discrete distributions. In this way, the stochastic integer program turns into a large-scale block-angular mixed-integer LP. In contrast to the pure LP case now essential convexity properties of value functions are missing such that alternative approaches to decomposition are necessary. We propose a dual decomposition algorithm relying on Lagrangian relaxation of

non-anticipativity constraints and with a branch-and-bound scheme on top to regain non-anticipativity for the results of the dual optimization. Numerical experiments with an industrial problem from power production indicate some potential of the algorithm for real-life applications. Finally, enhancements of the algorithm by using test sets and extensions to the multi-stage situation are discussed.

Adam N. Letchford, Lancaster University

On Comb Separation for the TSP

We present a new class of valid inequalities, called domino-parity (DP) inequalities, for symmetric Traveling Salesman polytopes. The comb inequalities are shown to be a special case. We then give a polynomial ($O(|V|^3)$ time) exact separation algorithm for DP inequalities which works only when the point to be separated has planar support. We then present a new shrinking condition which in some circumstances enables a non-planar point to be transformed into a planar one. This shrinking condition generalizes an earlier one due to Padberg and Rinaldi.

Andreas S. Schulz, MIT; joint work with Friedrich Eisenbrand

On the Chvátal Rank of Polytopes in the 0/1-Cube

Gomory's and Chvátal's cutting-plane procedure proves recursively the validity of linear inequalities for the integer hull of a given polyhedron. The number of rounds needed to obtain all valid inequalities is known as the Chvátal rank of the polyhedron. It is well-known that the Chvátal rank can be arbitrarily large, even if the polyhedron is bounded, if it is of dimension 2, and if its integer hull is a 0/1-polytope.

We prove that the Chvátal rank of polyhedra featured in common relaxations of many combinatorial optimization problems is rather small; in fact, the rank of any polytope contained in the d-dimensional 0/1-cube is at most $3d^2 \log d$. This improves upon a recent result of Bockmayr et al. who obtained an upper bound of $O(d^3 \log d)$.

Moreover, we refine this result by showing that the rank of any polytope in the 0/1-cube that is defined by inequalities with small coefficients is $O(d)$. The latter observation explains why for most cutting planes derived in polyhedral studies of several popular combinatorial optimization problems only linear growth has been observed; the coefficients of the corresponding inequalities are usually small.

Finally, we provide a family of polytopes contained in the 0/1-cube the Chvátal rank of which is at least $(1+\epsilon)d$ for some $\epsilon > 0$.

Paolo Toth, DEIS, Università di Bologna; joint work with Alberto Caprara and Matteo Fischetti

Algorithms for the Set Covering Problem

The Set Covering Problem (SCP) is a main model for several important applications, including crew scheduling in railway and mass-transit companies. In this talk, we focus our attention on the most recent and effective algorithms for SCP. Different relaxations of the problem are considered, and the corresponding lower bounds analyzed. The main characteristics of both exact and heuristic approaches for SCP are outlined. Experimental comparisons of the considered algorithms on the test-bed instances of the literature, and on real-world instances, are presented.

Matteo Fischetti, Università di Padova; joint work with Juan José Salazar

Complementary Cell Suppression for Statistical Disclosure Control in Tabular Data with Linear Constraints

In this talk we provide new theoretical models and practical solution techniques for protecting confidentiality in statistical tables containing sensitive information that cannot be disseminated. This is an issue of primary importance in practice. In particular, we study the problem of protecting sensitive information in a statistical table whose entries are subject to any system of linear constraints. This very general setting covers, among others, k-dimensional tables with marginals as well as hierarchical and linked tables. In particular, we address the NP-hard problem known in the literature as the (complementary) Cell Suppression Problem. We propose a new integer Linear Programming (LP) model, and give an interesting interpretation of the (possibly fractional) optimal solution of its pure LP relaxation, in terms of range protection as opposed to cell suppression. We also describe additional inequalities used to strengthen the integer model. We introduce an effective branch-and-cut algorithm for the exact solution of the problem, which can also be used as a heuristic procedure to find near-optimal solutions. The algorithm has been embedded within tau-ARGUS, a software package for statistical disclosure control. Preliminary computational results are very promising.

Volker Kaibel, Universität zu Köln

Polyhedral Methods for (Sparse) QAPs

In the recent years, some work on polytopes that are associated with the Quadratic Assignment Problem (QAP) has resulted in structural knowledge on the problem that was successfully used within cutting plane procedures to obtain very good lower bounds and (sometimes) even optimal solutions.

Unfortunately, "successful" here refers rather to the quality of the computed bounds than to the CPU times needed for the computations. In this talk, we report on an adaptation of the polyhedral approach that exploits certain kinds of sparsity in the objective functions. We give both structural results on some accordingly projected polytopes as well as a report on computational experiments showing that this way a new possibility arises to compute good lower bounds or even optimal solutions within reasonable running times.

Manfred Padberg, New York University

Almost Perfect Matrices and Graphs

We introduce the notions of ω-projection and κ-projection that map almost integral polytopes associated with almost perfect graphs G having n nodes from $\mathbb{R}^n$ into $\mathbb{R}^{n-\omega}$, where ω is the maximum clique size in G. We show that Claude Berge's strong perfect graph conjecture is correct if and only if the projection (of either kind) of such polytopes is again almost integral in $\mathbb{R}^{n-\omega}$. Several important properties of ω-projections and κ-projections are established. We prove that the strong perfect graph conjecture is wrong if an ω-projection and a related κ-projection of an almost integral polytope with ω between 2 and $(n-1)/2$ produce different polytopes in $\mathbb{R}^{n-\omega}$.

Daniel Bienstock, Columbia University

Computational Experience with an Implementation of the ϵ-approximation Algorithm for Linear Programming

We describe experimental results with an implementation of the Plotkin-Shmoys-Tardos and Grigoriades-Khachyian exponential potential method for approximately solving linear programs to a specified accuracy. Our experiments focus on large LP-relaxations of basic network design models. On instances with approximately one million non-zeros, our approach yields a feasible solution within (roughly) 0.5% of the optimum 30 to 50 times faster than leading commercial codes can obtain a comparable solution, and several orders of magnitude faster than those codes can "solve" the problem.

Dieter Jungnickel, Universität Augsburg; joint work with C. Fremuth-Paege

General Matching Problems and Balanced Network Flows

It is well-known that bipartite matching problems can be viewed as suitable network flow problems which raised the question whether something similar holds for the cardinality matching problem in general graphs. This question was finally answered explicitly by Kocay and Stone (1993, 1995) who introduced a transformation to a class of special network flow problems on bipartite graphs, namely "balanced flows" on "balanced networks". In fact,

their approach allows to study general matching problems with given upper and lower bounds (b-matchings), but the algorithm they obtained for the general problem was not strongly polynomial.
We have obtained such a strongly polynomial algorithm by augmenting along minimal valid paths which results in phase ordering (similar to the Dinic algorithm in classical network flow theory); in particular, we obtain a strongly polynomial network flow algorithm which computes b-matchings in arbitrary multigraphs. To obtain these results turned out to be surprisingly difficult and required a generalisation of Vazirani's theory for the cardinality matching problem in general graphs to balanced networks. As a bonus, we also obtain a unifying theoretical framework which allows one to analyse any (cardinality) matching algorithm which makes use of augmenting paths. We note that two known such algorithms (Pape-Conradt and Kameda-Munro) turned out to be incorrect, though they presumably still provide powerful heuristics. Finally, one can also prove some interesting theoretical results, e.g. a generalization of the Gallai-Edmonds decomposition to maximum balanced flows.

Tuesday, 1/12/1999

Probabilistic Methods and Approximation

David Shmoys, Cornell University

Approximation Algorithms via Linear Programming: Rounding Algorithms

One of the oldest and most well-studied strategies for computing good solutions to hard optimization problems is to (1) formulate the problem as an integer program; (2) solve its linear programming relaxation; and (3) round the optimal fractional solution to an integer one that is nearly as good. This approach has recently been employed to derive approximation algorithms, polynomial-time algorithms with an associated performance guarantee, for a variety of combinatorial optimization problems. We shall survey some of these results for scheduling, facility location, and network design problems.

Gerhard Woeginger, TU Graz

A Survey on Polynomial Time Approximation Schemes

All the known polynomial time approximation schemes (PTAS) for difficult optimization problems can be derived from exact (but exponential time) algorithms via one of the following three approaches: (1) Simplifying the structure of the input (e.g. the Hochbaum & Shmoys PTAS for makespan on identical machines). (2) Simplifying the structure of the output (e.g. the Hochbaum & Maass PTAS for covering Euclidean point sets by unit squares). (3) Simplifying the execution of the exact algorithm by cleaning up auxiliary data (e.g. the Sahni PTAS for total weighted completion time on two identical machines). The talk explains and discusses these three approaches, and it also describes techniques for disproving the existence of a PTAS for specific optimization problems (under P not equal to NP).

Lisa Fleischer, Columbia University; joint work with Kevin Wayne

Simple Approximation Schemes for Generalized Flow

We present fast and simple fully polynomial time approximation schemes (FPTAS) for generalized versions of the maximum flow, multicommodity flow, minimum cost flow, and minimum cost multicommodity flow problems, in graphs with no flow generating cycles. These dominate the previous best known complexity bounds for all of these problems, some by as much as a factor of n^2. We believe our improvements make it practical to solve generalized multicommodity flow problems via combinatorial methods.
For the generalized maximum flow problem we obtain a $O(m^2)$ time FPTAS. Even for this well-studied version, our algorithm is faster than the previous best strongly-polynomial bound by a factor of n. In addition, for this problem we can also handle flow generating cycles, and can reduce the dependency on the error parameter from ϵ^{-2} to $\log(1/\epsilon)$.

Fabian A. Chudak, IBM T.J. Watson Research Center; joint work with David Shmoys

Improved Approximation Algorithms for Facility Location Problems

We consider the following capacitated facility location problem. There is a set of demand locations that require service from facilities; there is a set of locations at which facilities can be built, where the capacity installed at each location i is an integer multiple of u, with a cost incurred equal to the corresponding multiple of f_i. If the demand at location j is serviced by a facility at location i, then a cost of c_{ij} is incurred. The objective is to determine the capacity to build at each facility location and an assignment of demand to facilities, so as to minimize the total cost. We assume that the service costs c_{ij} are symmetric and satisfy the triangle inequality. We obtain a 3-approximation algorithm, which is a significant improvement on the previously known approximation guarantees for this problem.
Our algorithm is based on a variant of randomized rounding, which we call *randomized rounding with a backup*, since a backup solution is incorporated

to correct for the fact that ordinary randomized rounding rarely generates a feasible solution to the associated set covering problem.

Eranda Çela, TU Graz; joint work with Rainer Burkard

An Asymptotic Analysis of Optimization Problems by Means of Statistical Mechanics

The analogy between combinatorial optimization and statistical mechanics has proven to be a fruitful object of study. Simulated annealing, a meta-heuristic for combinatorial optimization problems, is based on this analogy. In this talk we use the statistical mechanics formalism based on the above mentioned analogy to analyze the asymptotic behavior of a special class of combinatorial optimization problems characterized by a combinatorial conditions which is well known in the literature. Our result is analogous to results of other authors derived by purely probabilistic means: Under natural probabilistic conditions on the coefficients of the problem, the ratio between the optimal value and the size of a feasible solution approaches almost surely the expected value of the coefficients in the objective function, as the size of the problem tends to infinity. Our proof shows clearly why the above mentioned combinatorial condition which characterizes the class of the investigated problems is essential.

Martin Skutella, CORE Louvain-la-Neuve & TU Berlin; Joint work with Gerhard Woeginger

A PTAS for minimizing the total weighted completion time on identical parallel machines

We consider the problem of scheduling a set of n jobs on m identical parallel machines so as to minimize the total weighted completion time. This problem is NP-hard in the strong sense. The best approximation result known so far was a $\frac{1}{2}(1+\sqrt{2})$–approximation algorithm that has been derived by Kawaguchi and Kyan back in 1986. Our contribution is a polynomial-time approximation scheme for this problem, which settles a problem that was open for a long time. Moreover, our result constitutes the first known approximation scheme for a strongly NP-hard scheduling problem with minsum objective.

Dimitris J. Bertsimas, MIT; joint work with David Gamarnik

Asymptotically Optimal Algorithms for Job Shop Scheduling and Packet Routing

We propose asymptotically optimal algorithms for the job shop scheduling and packet routing problems. We propose a relaxation for the job shop scheduling problem called the fluid control problem, in which we replace discrete jobs with the flow of a continuous fluid. The optimal solution of the fluid control problem can be computed in closed form and provides a lower bound $C_{\max}$ to the job shop scheduling problem. We use the optimal solution of the fluid control problem to construct a feasible schedule with objective value at most $C_{\max}+O(\sqrt{C_{\max}})$. If the initially present jobs increase proportionally, then our algorithm produces a schedule with value at most $C_{\max}+O(1)$. For the packet routing problem with fixed paths the previous algorithm applies directly. For the general packet routing problem we propose a linear programming relaxation that provides a lower bound $C_{\max}$, and an algorithm that uses the optimal solution of the relaxation with objective value at most $C_{\max}+O(\sqrt{C_{\max}})$. This implies that as the total number of jobs (packets) tends to infinity, the proposed algorithms are asymptotically optimal. Unlike asymptotically optimal algorithms that rely on probabilistic assumptions, our proposed algorithms make no probabilistic assumptions, and they are asymptotically optimal for all instances with a large number of jobs (packets). In computational experiments our algorithms produce schedules which are within 1% of optimality even for problems of moderated size. In a recent application in scheduling vessels in Panama canal, the algorithm created optimal schedules.

Thomas Liebling, EPFL; joint work with Nicolas Moeri and Philip Bucker

Motif Recognition in Biomolecular Sequences

This talk describes some results of a long research collaboration with ISREC, the Swiss Institute for Applied Cancer Research. The problems at issue are how to characterize a "motif" in a family of biomolecular sequences (DNA or protein) with some biological features in common and how to efficiently discriminate between sequences that have and those that don't have the motif.

The motif characterization leads to a hard, combinatorial optimization problem, while the recognition problem is easy. A new model for motif description closely related to the Hidden Markov Model using generalized profiles and an efficient heuristic to optimize the associated parameters, given corresponding sample positive and negative "learning" sequences will be presented. First computational results demonstrate the competitiveness of this new approach.

Andreas Eisenblätter, ZIB; joint work with Ralf Borndörfer, Martin Grötschel, Alexander Martin

Assigning Frequencies in GSM-Networks

We consider the problem of assigning radio frequencies to the base station transmitters of a GSM-network. (GSM is a system for public mobile radio

telephony that is used in many countries nowadays, in particular, all over Western Europe.) In comparison to the number of transmitters only few different frequencies are available. These few frequencies have to be assigned to the transmitters (one to each) subject to many technical and regulatory constraints. Transmitters may interfere with one another if the same or adjacent frequencies are used by the transmitters. Our objective is to find a frequency assignment that, on one hand, satisfies all constraints and, on the other hand, incurs as little interference as possible.
We present a mathematical model for the *frequency assignment problem*, sketch some of the heuristics we have developed to efficiently compute assignments, and report on computational results for real-world instances. (The problem is NP-hard.)
Finally, we give an ILP formulation of the frequency assignment problem that is an extension of a known formulation of the *min k-partition problem* for edge-weighted, non-complete graphs.

Michael Bussieck, TU Braunschweig/GAMS Development; joint work with Alexander Meeraus and Patrick Driscoll

Scheduling at United States Military Academy / West Point

We consider a particular scheduling task at the US Military Academy in West Point. The 13 academic departments of USMA offer courses for the upcoming academic year. The students or cadets create their eight-term academic program (8TAP) when they start their studies and may accommodate it later. The USMA guarantees that all cadets are able to complete their 8TAP in 4 years. Hence the timetabling of the courses must provide a schedule for each cadet that contains all requested courses without a conflict. Furthermore, the academic activities of the student must be coordinated with military and physical activities. At USMA, time periods are defined for Day-1 meeting slots (A-F) and Day-2 meeting slots (G-L), where Day-1 and Day-2 alternate over the working days.
In our particular problem the courses are already assigned to some time slots, e.g. course MA100 is offered at time slots AG, BH, CI, and DJ. Moreover each course hour (e.g. MA100, BH) has an upper bound on the enrollment of cadets which represents classroom and teacher capacities. The problem is to create a conflict free schedule for the cadets that do not result in overloaded course hours. The problem is enriched by various side constraints, e.g. the courses of a cadet's schedule must be balanced between Day-1 and Day-2.
We present a binary linear formulation of the problem. Unfortunately, the program cannot be directly solved for real world instances (4000 cadets, 30.000 8TAP entries, 900 course hours) by state-of-the-art solvers like CPLEX.

We propose a sequential heuristic that schedules small batches of cadets and tries to keep the capacity profile of the course hours. The solution of this heuristic is taken as a starting point of a set partitioning formulation in which new schedules are generated on the fly. The computer program which is completely written in GAMS will replace the legacy system for the scheduling of cadets in the academic year 1999/2.

Wednesday, 1/13/1999

Modelling and Solving Industrial Problems

Sebastian Engell, Universität Dortmund; joint work with Guido Sand and Christian Schulz

Scheduling Problems in the Processing Industry—An Engineering Perspective

The talk focuses on scheduling problem which arise in batch processing plants. Here, the necessary operations leading to one or more products are described in recipes. By the recipes, a network of material balances for raw materials, intermediates, and products is defined. Each processing step needs resources which may be used exclusively or can be shared. The transfer of batches of material also needs resources, as well as the storage of intermediates. Storage times may be limited. The processing operations are governed by continuous differential equations which however usually are abstracted into durations of the various processing steps. In many processing plants, especially when they are coupled to continuously operated sections of the plant, mixing of batches occurs. This introduces nonlinear relations for the concentrations into the models.
The problems considered can be classified into medium term planning (typically over several months up to a year) where plant capacities and demands are considered but not individual batches or processing steps; short term planning where the most important constraints are represented and operation times are considered as fixed; and on-line scheduling, where all constraints and the variations of processing times, resource availability and product properties must be considered. The first class of problem gives rise to large MILPs whereas in the other areas, nonlinear dependencies may have to be included. In general, a hierarchical approach must be used to tackle real-world problems. Important issues are to handle uncertainties in the problem formulation, including the limited knowledge about the future in a real-time setting where decisions have to be based on the available knowledge at the time

the optimization is started, the formulation of the objective function and/or constraints, and the coordination of the different layers in an hierarchical approach.
As an example, the production of expandable polystyrene is discussed in detail. A hierarchical approach with two layers was chosen. On the upper layer, a simplified process model and a coarse granularity of the time quantization is used. The uncertainty about future demands, actual product composition and processing times is modelled using a scenario tree as proposed by Römisch and Schultz for power generation problems. The optimization problem is solved as a two-stage stochastic program (cf. Carøe and Schultz). On the lower level, nonlinear models are used because of the presence of mixing in the process. Two different modeling approaches, using a fixed grid and an event driven representation where the event times are optimization variables where compared. It turned out that the event-driven representation is more effective here. The resulting nonlinear mixed-integer optimization problems were tackled using a tailored, depth-first search strategy. The relaxed nonlinear problems were solved using CONOPT. The performance of the algorithm was found to be very reasonable.
In the final remarks, the issue of modeling and the associated, usually large effort if this must be done from scratch, and the treatment of uncertainties were pointed out as key areas for future work.

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2. Christian Schulz, Sebastian Engell, and Ruediger Rudolf: *Scheduling of a Multi-Product Polymer Batch Plant.* Presented at FOCAPO, Snowbird, July 1998. To appear in Proceedings of FOCAPO 1998, AIChE. The preprint is available from the preprint server of the DFG-SPP Echtzeitoptimierung, `www.zib.de`.

3. Guido Sand: *Planungsmodell zur Echtzeitoptimierung einer verfahrenstechnischen Mehrproduktanlage bei Unsicherheit.* Preprint 98-28, available from the preprint server of the DFG-SPP Echtzeitoptimierung, Zuse-Institut Berlin, `www.zib.de`.

4. Claus C. Carøe and Rüdiger Schultz: *A Two-Stage Stochastic Program for Unit Commitment Under Uncertainty in a Hydro-Thermal Power System.* Preprints of the DFG-SPP Echtzeitoptimierung, `www.zip.de`.

Michael Jünger, Universität zu Köln

Some new Developments in Branch-and-Cut Algorithms for Combinatorial Optimization Problems

The purpose of this talk is to survey some recent developments in branch-and-cut algorithms for combinatorial optimization problems. Topics include a paradigm for finding cuts, along with an application to the travelling salesman problem, by Applegate, Bixby, Chvátal, and Cook, progress in sparse maximum cut computation by Jünger, Reinelt, and Rinaldi, the "small facets" paradigm by Christof and Reinelt, and a discussion and critical evaluation of these contributions. Afterwards I exemplify successful and unsuccessful branch-and-cut approaches using two examples: 2-layer straightline crossing minimization in automatic graph drawing and chromosome mapping in computational biology. Finally, I report on the object-oriented software system ABACUS - A Branch And CUt System that was developed in Köln and Heidelberg and point out its strong features in comparison to other software systems in integer programming / combinatorial optimization.

Rolf H. Möhring, TU Berlin; joint work with Andreas Fest, Andreas Schulz, Frederik Stork, and Marc Uetz

Allocating Scarce Resources in Chemical Engineering

Chemical production processes typically involve complex scheduling decisions to allocate the available production capacity and personnel to the incoming orders in such a way that different temporal constraints (delivery times, time windows between different production steps) and resource constraints (vessels, personnel) are met, and such that certain objectives (makespan, deviation from delivery dates) are minimized.
The lecture reports on a particular project together with BASF AG, Ludwigshafen, where the aim is to allocate scarce personnel to a production process such that the total production time is minimized. In mathematical terms, this leads to a large scale scheduling problem with time windows, scarce time-varying resources and makespan as objective. We present our experience with a branch and bound algorithm based on a graph theoretic model, and a Lagrangian relaxation based on an ILP formulation with time-indexed variables. We show in particular that the relaxed problem can be formulated as a min-cut problem and thus be solved efficiently. Our computational experience shows that the available real-life instances can be solved to optimality, but that there are significant duality gaps for specifically constructed hard instances.

George Nemhauser, Georgia Institute of Technology; joint work with Kelly Easton and Michael Trick

Integer Programming in Tournament Scheduling

We discuss the problem of scheduling a double round robin tournament subject to side constraints. The specific problem arises in scheduling U.S. College Basketball Conferences but similar problems arise in scheduling various sports leagues. The side constraints, which are in part dictated by having certain important games on television on certain dates, makes the problem very difficult. The goal is to find a "good" feasible solution that satisfies all of the hard constraints and comes as close as possible to satisfying several soft constraints. We present a 3 stage solution methodology that combines enumeration and integer programming. We also discuss a different solution approach that uses constraint programming. The methodology has been used to provide schedules for the Atlantic Coast Conference. The Conference has played our schedules for two years and recognizes them to be a substantial improvement over previous schedules.

Karen Aardal, Universiteit Utrecht; joint work with Bob Bixby, Cor Hurkens, Arjen Lenstra and Job Smeltink

Using Basis Reduction to Solve Systems of Linear Diophantine Equations with Bounds on the Variables

We develop an algorithm for solving a system of Diophantine equations with lower and upper bounds on the variables. The algorithm is based on lattice basis reduction. It first finds a short vector satisfying the system of Diophantine equations, and a set of vectors belonging to the null-space of the constraint matrix. Due to basis reduction, all these vectors are relatively short. The next step is to branch on linear combinations of the null-space vectors, which either yields a vector that satisfies the bound constraints or provides a proof that no such vector exists. Our algorithm is tested with good results on real-life data, and on instances from the literature.

Geir Dahl, University of Oslo; joint work with G. Storvik and A. Fadnes

Large-scale Integer Problems in Image Analysis

An important problem in image analysis is to segment an image into regions with different class-labels. This is relevant in applications in medicine and cartography. In a proper statistical framework this problem may be viewed as a discrete optimization problem. We present two integer linear programming formulations of the problem and study some properties of these models and associated polytopes. Different algorithms for solving these problems are suggested and compared on some realistic data. In particular, a Lagrangian algorithm is shown to have a very promising performance. The algorithm is based on the technique of cost splitting and uses the fact that certain relaxed problems may be solved as shortest path problems.

Thursday, 1/14/1999

Algebraic Methods in Discrete Optimization

Robert Weismantel, Otto-von-Guericke-Universität Magdeburg

Augmentation Algorithms, Test Sets, and Hilbert Bases

Augmentation algorithms iteratively move from one feasible point of an integer program to another feasible point that attains a better objective function value. We discuss the result that with a polynomial number of augmentation steps an optimal solution of the integer program can be achieved. We also discuss a mathematical framework to design such algorithms. This first issue leads naturally to a study of test sets that may be derived from Hilbert bases of pointed rational polyhedral cones. We survey the application of Hilbert bases in integer programming and discuss recent results about these objects. In particular we disprove the integer Caratheodory property by showing that in general at least [7n/6] vectors from a Hilbert basis in an n-dimensional cone are needed to express any integer point in the cone as a non-negative integer combination of the Hilbert basis elements.

Günter M. Ziegler, TU Berlin

On the Geometry of "Bad" Linear Programs

The geometry (and combinatorics) of linear programs has many different aspects—some of them surveyed in this lecture under the following headings:

1.) "Real" Linear Programs
Small linear programs can be *completely* analyzed in their combinatorial structure; we present a method to compute 2-dimensional shadows, plus a complete analysis of the "afiro" problem in the POLYMAKE framework.

2.) Deformed Products
Virtually all exponential examples ever constructed for simplex algorithms can be viewed, constructed and analyzed as "deformed products".

3.) Extremal Problems
Among the major open problems connected to the geometry of linear programs and the simplex algorithm are
— the Hirsch conjecture,
— the existence of polynomial pivot rules, and
— the (monotone) upper bound conjecture.

Rekha R. Thomas, Texas A&M University; joint work with Sekran Hosten

Group Relaxations in Integer Programming

In this talk I present certain structural results concerning extended group relaxations of the family of integer programs $IP_{A,c}(b) := \min\{cx : Ax = b, x \geq 0, \text{integer}\}$ as the right hand side vector b is varied. These results are obtained as translations of algebraic results regarding the associated primes of initial ideals of lattice ideals. We show in particular that the extended group relaxations of the family $IP_{A,c}$ come in chains and that the length of a maximal such chain is at most $\min(d, 2^{n-d} - (n-d+1))$.

Michele Conforti, Università di Padova; joint work with Ajai Kapoor and Bert Gerards

A Theorem of Truemper

An important theorem due to Truemper characterizes the graphs whose edges can be labelled so that all chordless cycles have prescribed parities. This theorem has since proved an essential tool in the study of balanced matrices, graphs with no even length chordless cycle and graphs with no odd length chordless cycle of length greater than 3. In this talk we prove this theorem in a novel and elementary way and we derive some of its consequences. In particular, we show how to obtain Tutte's characterization of regular matrices.

Gerard Cornuejols, Carnegie-Mellon University

On Ideal Binary Clutters

A binary clutter is the set of circuits of a binary matroid that intersect with odd parity a fixed subset of elements. Let A denote the 0/1-matrix whose rows are the incidence vectors of these odd circuits. The clutter is ideal if the polyhedron $Ax \geq 1, x \geq 0$ is integral. Examples of ideal binary clutters are st-paths in a graph, T-joins in a graph, or odd circuits in a weakly bipartite graph. Seymour conjectured in 1977 that a binary clutter is ideal if and only if it does not contain any of three specified minors. In joint work with Bertrand Guenin, we show that a binary clutter is ideal if it does not contain one of five specified minors, namely Seymour's three minors plus two others. Since the last two minors are both ideal, our result does not contradict Seymour's conjecture.

Peter Hammer, Rutgers University; joint work with Stéphane Földes

Pseudo-Boolean Functions

A pseudo-Boolean function is a real-valued function with binary (0/1) variables. It is well known that every pseudo-Boolean function has a polynomial expression. We discuss in this talk the possibility of representing every pseudo-Boolelan function as the sum of a real number and a "disjunctive pseudo-Boolelan expression". A "disjunctive pseudo-Boolean expression" is the maximum of positively weighted elementary conjunctions. We define implicants and prime implicants of pseudo-Boolean functions and prove that the consensus method of Blake and Quine can be extended to pseudo-Boolean functions, produces all their prime implicants, as well as the minimum of the function. In an analogous way, we define conjunctive pseudo-Boolean expressions, implicata, prime implicata, and extend the resolution method for their determination.
We define quadratic, Horn, monotone, submodular and supermodular pseudo-Boolean functions, show that the minimization of the first two classes of functions can be done in polynomial time, and provide syntactic characterizations of the normal forms of all these classes of functions.

Martin Loebl, University Karlovy Praha

A Theory of Pfaffian Orientations

We present several results concerning the generating functions of cuts, even subgraphs and perfect matchings of a graph.

Franz Rendl, Universität Klagenfurt; joint work with Michele Goemans

Semidefinite Programs and Association Schemes

We consider Semidefinite Programs (SDP), where the matrices defining the problem all arise from some association scheme. We show that in this case the SDP can be solved through an ordinary Linear Program. The key property used in the proof is the fact that matrices from an association scheme commute. As an application we show that a basic SDP relaxation for Max-Cut has a closed form solution, if the underlying graph is contained in some association scheme.

Ulrich Faigle, Universiteit Twente; joint work with W. Kern and J. Knipers

On the Computation of the Nucleolus of a Cooperative Game

A cooperative game is characterized by a function $c\colon 2^N \to \mathbb{R}$ s.t. $c(\emptyset) = 0$. The nucleolus is the unique vector $\eta(c) \in \mathbb{R}^N$ that yields the lexicographic maximum of the ordered vector of excesses

$$(e(S_1, x), e(S_2, x), \ldots) \quad \text{for } S_i \in 2^N \setminus \{\emptyset, N\}\,, \text{ where } e(S_i, x) \leq e(S_{i+1}, x).$$

Using Maschler's algorithm and the ellipsoid method, one can compute the nucleolus for several classes of games assuming that

$$\min\{e(S, x)\colon S \in 2^N \setminus \{\emptyset, N\}\} \quad \text{can be computed efficiently.}$$

Friday, 1/15/1999

Geometry, Graphs and Combinatorial Optimization

Petra Mutzel, Max-Planck-Institut für Informatik

Optimization Problems in Automatic Graph Drawing

This talk surveys the main optimization problems in Graph Drawing. In the first part of my talk I will give an introduction to the field of graph drawing. We will see examples of state diagrams, data models, and network graphs arising in projects with Siemens AG, Molkerei Alois Müller GmbH & Co, and Genesys GmbH.

In the second part of my talk we will focus on some optimization problems arising in the planarization method. Here, the idea is producing a crossing minimal to pological embedding of the given graph, substituting the crossings by introducing artificial vertices and then using a planar graph drawing algorithm. Topics in clude: the maximum planar subgraph problem, the planar augmentation problem, the compaction problem, optimization over all embeddings in a given planar graph, the incremental crossing number problem, the crossing number problem, and the Conway thrackle conjecture.

András Frank, Eötvös Loránd University

Applications of Relaxed Submodularity to Connectivity Problems

In this talk we overviewed several type of relaxations of submodular functions such as intersecting, crossing, skew submodular, and showed how general results concerning these functions can be applied to graph and hypergraph connectivity problems. Two recent results of the area are as follows.

1. (jointly with Zoltan Kiraly) Every 4-edge-connected graph has a strongly connected orientation in which the in-degree of all but possibly one node is odd.

2. Given a digraph with two specified nodes s and t, it is possible to add at most γ new edges so that in the augmented digraph there are k edge-disjoint paths from s to t and there are k edge-disjoint paths from t to s if and only if $k - \rho(X)$ is at most γ for every subset X of nodes containing exactly one of the nodes s and t, AND $k - \rho(S) + k - \rho(T)$ is at most γ for every pair of subsets S, T so that s is in S, t is in T and S, T are either disjoint or their complements are disjoint. Here $\rho(Z)$ denotes the number of edges entering Z.

Günter Rote, TU Graz

Optimal Triangulations

Triangulations are needed in a variety of contexts:

1. triangulations of a *polygon* or a polytope are often a first step of other algorithms;
2. triangulations of a *point set* are used for interpolating data or surfaces;
3. triangulations of a *domain* are needed for finite element algorithms.

I have given a survey of optimal triangulations (with respect to various optimization criteria) in the areas (1) and (2). I have mainly concentrated on the *minimum-weight triangulation* (MWT) problem of computing a triangulation of a planar point set with smallest total length, a problem whose complexity status is a famous open question, although the methods are applicable in other cases as well. I have briefly discussed higher-dimensional problems and the difficulties encountered there.
As for algorithms, I have first mentioned attempts to attack the problem by integer linear programming; then I have discussed lower bounds for the length of the MWT that follow from a lemma about matchings between two triangulations and their use in a branch-and-bound scheme; finally I have described the so-called LMT-skeleton, which consists of all edges in a point set that must belong to every triangulation that is *locally minimal* with respect to edge flips. Together with a preprocessing phase which quickly excludes most of the $O(n^2)$ possible edges, this has led to efficient programs that can compute the MWT for general point sets in practice (for example 40,000 points in a couple of minutes).

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