Mathematisches Forschungsinstitut Oberwolfach

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The conference was organized by Y. N. Moschovakis (Los Angeles), H. Schwichtenberg (München), and A. S. Troelstra (Amsterdam). It focused on complexity aspects of mathematical logic, in particular proof theory, lambda-calculus, complexity theory and the connection between these areas.

The formal program consisted of 24 talks and two lecture series of 3 talks each, on "Complexity-theoretic strength of higher-order linear functional programs" and "Abstraction levels and complexity", given by Martin Hofmann and Daniel Leivant. In the evenings 7 informal talks were given.

Abstracts

Complexity-Theoretic Strength of Higher-Order Linear Functional Programs
(lecture series)
Martin Hofmann

This series of talks presents complexity-theory results related to a certain functional programming language LFPL with lists and trees which through the use of linear typing as well as a novel device called "resource type" has the property that all definable functions are hereditarily non size increasing thus enabling in particular an automatic translation of the first-order fragment into the C-programming language such that the usual functional programs for e.g. sorting algorithms are mapped to equivalent C-programs which operate on their input by pointer surgery and thus modify it in-place.

We show the following results

first order LFPL	$\leftrightarrow$ ETIME
first order LFPL with tail recursion	$\leftrightarrow$ LINSPACE
LFPL	$\leftrightarrow$ EXPTIME
LFPL with structural recursion	$\leftrightarrow$ PTIME

where $X \leftrightarrow Y$ means that the characteristic functions definable in X are precisely those in Y.

The proofs use a mixture of techniques from programming language theory and computational complexity theory such as Cook's simulation of stacks, dynamic programming, finite models, logical relations, Scott domains, nondeterminism, realizability.

This complements and extends related results by Neil Jones which were for read only programs that could only read but not modify their input.

Abstraction Levels and Complexity *(lecture series)*
Daniel Leivant

Abstraction levels, covered by set existence principles, have been used in foundational studies to calibrate the strength of theorems in mathematical analysis. In this presentation we show that calibrating abstraction in second order logic leads to a natural correspondence with computational complexity classes such as polynomial time and polynomial space. This provides simple and natural characterizations of levels of feasible mathematics.

On Continuous Normalization of the lambda-Calculus
Klaus Aehlig
(joint work with Felix Joachimski)

In an extension of the untyped coinductive lambda-calculus by void constructors ("repetition rules") a primitive recursive normalization function is defined. It is continuous with respect to the natural topology on non-well-founded terms with the identity as modulus of continuity. The number of repetition rules is locally related to the number of beta-reductions necessary to reach and the number of applications within the normal form, as represented by the Böhm tree.

Bounded Arithmetic and Height Restricted Resolution

Arnold Beckmann

Height restricted resolution is a natural restriction of resolution where the height of the proof is bounded. Viewing dynamic ordinal analysis in the light of propositional proof complexity shows that polylogarithmic-height restricted resolution is strongly connected to bounded arithmetic theory $S^1_2(\alpha)$. We separate polylogarithmic-height resolution from quasi-polynomial size tree-like resolution. Inspired by this we study infinitely many sub-linear-height restricted resolution systems.

Provably Recursive Functions in the Context of GPA

Lev Beklemishev

Graded provability algebras (GPA) provide an abstract algebraic framework for proof theoretic analysis. Let $\mathcal{T}$ be a sufficiently weak fragment of Peano arithmetic. The GPA of $\mathcal{T}$ is the Lindenbaum boolean algebra of $\mathcal{T}$ equipped with the additional operators $[n]$ for each $n \in \mathbb{N}$. The operator $[n]$ sends a sentence φ to the sentence $[n]\varphi$ expressing "($\mathcal{T}$ + all true Π^0_n-sentences) proves φ". The operators $[n]$ satisfy the following identities:

$$\begin{array}{ll} [n](\varphi \to \psi) \to ([n]\varphi \to [n]\psi) = \top & [n]\varphi \to [n+1]\varphi = \top \\ [n]([n]\varphi \to \varphi) \to [n]\varphi = \top & \neg[n]\varphi \to [n+1]\neg[n]\varphi = \top \\ [n]\top = \top & \end{array}$$

These identities correspond to a polymodal logic studied by G. Japaridze in 1988. It is known to be decidable and completely axiomatizes the identities of the GPA of $\mathcal{T}$.

We show that Peano Arithmetic and its fragments $I\Sigma_n$ of Σ_n-Induction can be embedded in the GPA of $\mathcal{T}$ as filters generated by formulas of the form $\langle n\rangle\top$. The ordering relation $\varphi <_0 \psi \Leftrightarrow \psi \to \langle 0\rangle\varphi = \top$, where $\langle n\rangle = \neg[n]\neg$, is well-founded on $\mathcal{M}_0 \setminus \{\bot\}$, where $\mathcal{M}_0$ is the prime sub-algebra of the GPA. The order type of $<_0$ is ε_0. On the basis of $<_0$ we define iterated operators $\langle n\rangle^\alpha$, where α is an element of $\mathcal{T}_0 \setminus \{\bot\}$ and obtain a characterization of Π_{n+1}-consequences of elements of $\mathcal{M}_0$. In particular $\langle n+1\rangle\top$ is Π_2-conservative over $\langle 1\rangle^\alpha\top$, where α corresponds to the ordinal $\omega_n = \omega^{\omega^{\cdot^{\cdot^{\cdot^{\omega}}}}}\Big\}$ (n times). This characterizes provably total recursive functions of $I\Sigma_n$ as the ω_n-th class of the fast growing hierarchy.

Adding Choice and Uniformity to Weak Applicative Systems

Andrea Cantini

We are concerned with the classification of provably total functions within weak applicative systems, which comprise combinatory logic, extended with

1. the type W of binary strings
2. various forms of induction on W (for "positive" and gereneralized NP-conditions)

We show that the *recursive content* of all systems we consider is invariant under addition of an axiom of choice for operations and a suitable uniformity principle, restricted to positive conditions.

As to the technical tools, we apply a method, which combines a syntactical version of forcing, inspired by previous work of Coquand and Hofmann, with the use of a self-referential truth predicate and the application of realizability and cut-elimination.

Iteration of Σ-Operations in Admissible Set Theory without Foundation

Gerhard Jäger

We present a system of admissible set theory which is characterized by:

- its set existence axioms formalize a recursively inaccessible universe,
- the only induction principle available is complete induction on the natural numbers for set,
- Σ-operations can be iterated along ordinals.

This theory has the proof-theoretic ordinal $\varphi\omega 00$ and thus reflects a further aspect of meta-predicative Mahloness.

Strong Normalization Results

René David

In this talk I present an elementary proof of the strong normalization of the cut elimination procedure for the full classical natural deduction. By full I mean in the presence of all the usual connectors (conjunction, disjunction, arrow and negation) with their intuitionistic meaning. This result was first proved by P. de Groote in his TLCA'01 paper using, in particular, a CPS style translation to the simply typed lambda calculus. The proof given here is direct and purely arithmetical. I will also briefly mention the use of the same kind of method to prove the strong normalization of other systems.

Groundwork for Weak Analysis

Fernando Ferreira

Weak Analysis studies sup-exponential systems of second-order arithmetic ("analysis"). We introduce a system for feasible analysis which is able to formalize the real number system and the notion of a continuous real function of a real variable. The system is able to prove the intermediate value theorem, wherefore it follows that the real numbers form a real closed ordered field. As an application, we show that Tarski's theory of the real closed ordered fields is interpretable in Raphael Robinson's theory of arithmetic Q. We also investigate the import of Weak König's lemma in our feasible setting. Finally, we comment on stronger theories of Weak Analysis.

Remarks on the Length and Depth of Constructive Proofs

Rosalie Iemhoff

In this talk I will discuss some results and open problems concerning the length and depth of constructive proofs. The final goal and hope in this area is to find exponential lower bounds on the length of proofs in propositional logic (in terms of the formula that is proved). Here the situation for intuitionistic logic is more or less similar to the one for classical logic in that for natural systems like Gentzen calculi the best known lower bounds are only quadratic. I will briefly discuss in how far results on classical logic translate to a constructive setting. In particular, I will discuss an open problem related to the Pigeonhole Principle, which in the context of classical logic has certain properties for which it is not clear whether they hold in intuitionistic logic as well.

Furthermore, I will talk about the depth of cut free proofs in intuitionistic logic (this is joint work with Sam Buss). We consider the standard Gentzen calculus for intuitionistic propositional logic IPC and show that every intuitionistically valid sequent $\mathcal{S}$ has a proof which depth is at most quadratic in the size of $\mathcal{S}$. We show that this bound is tight by presenting sequents of size n that have no cut free proof of depth $< n^2$. I will discuss the relation between this result and other results on this topic from the literature.

An Optimal Lower Bound for Resolution with Width two Conjunctions

Jan Johannsen

(joint work with N. S. Narayanaswamy)

A lower bound is proved for refutation of certain clause sets in a generalization of Resolution that allows cuts on conjunctions of width 2. The hard clauses are the Tseitin graph formulas for a class of logarithmic degree expander graphs. The bound is optimal in the sense that it is truly exponential in the number of variables.

Proof Mining in Analysis. Unwinding Implicit Computational Content.

Ulrich Kohlenbach

"Proof Mining" denotes the activity of extracting new numerical and computational information from prima-facie ineffective proofs by logical transformation (so-called "Proof Interpretation").

We first present general meta-theorems which guarantee the extractability of uniform effective bounds for substantial classes of proofs, in particular in analysis.

We then report on the results of two case studies

1. New uniform bounds (even implying new qualitative results) on theorems due to Istukawa, Kirk, Borwein-Reich-Shafir and Groetsch on the asymptotic regularity of Mann iteration of nonexpansive mappings.
2. (with Paulo Oliva) The first effective rate of strong unicity for best L_1-approximation by polynomials in P_n of functions $f \in \mathcal{C}[0,1]$ with optimal ε-dependency.

Feasible Arithmetic and Program Extraction

Jean-Yves Marion

This work in progress paper presents a methodology for reasoning about the computational complexity of functional programs, which are extracted from proofs.

We suggest a first order arithmetic AT^0 which is a syntactic restriction of Peano Arithmetic. We establish that the set of functions which is provably total in AT^0, is exactly the set of polynomial time functions.

Compared to other feasible arithmetics, AT^0 is surprisingly simpler. The main feature of AT^0 concerns the treatment of the quantification. The range of quantifiers is restricted to the set of *actual terms* which is the set of constructor terms with variables.

Although this paper addresses some theoretical aspects of program extraction, it is relevant for practical issue, in the long term, because certifying the computational resource consumed by a program is a challenging issue.

Contraction-Aware lambda-Calculus

Ralph Matthes

A simply-typed lambda-calculus is presented whose normal forms exactly represent the cut-free derivations of the contraction-free sequent calculus, invented independently by Vorobev, Hudelmaier, Lincoln/Shankar and Dyckhoff. Its crucial property is termination of proof search without need for loop-detection.

The proposed lambda-calculus ΛJT follows the paradigm of generalized eliminations put forward by von Plato and consequently also has permutative conversions. It will be shown that strong normalization of the beta-reductions and permutative conversions nevertheless can be established elegantly – even yielding an embedding of the system with beta-reductions only into Girard's polymorphic lambda-calculus. The full system, however, also has specific rules for the elimination of contractions. A normalization algorithm can be given by extending the method of proving admissibility of contraction in the recent JSL paper by Dyckhoff and Negri. The termination of this algorithm is still an open question.

Finally, the collapse of the Church numerals and an approach to uniform interpolation in this calculus are mentioned.

The Complexity of Computing the `gcd`

Yiannis Moschovakis

This work is inspired from work of Colson (1991) which exhibited some serious limitations of primitive recursive algorithms, and my main aim is to focus on and explain some recent contributions of Lou van den Dries. Part of the interest here lies in the methods of van den Dries, which (for the first time, I believe) employ non-standard models of arithmetic to establish results on the complexity of number-theoretic algorithms.

Brief summary: For Φ any set of partial functions on $\mathbb{N}$, call a partial function $f : \mathbb{N}^n \to \mathbb{N}$ Φ-linear if $f(\vec{x}) = w \Leftrightarrow \exists u_1 \exists u_2 \ldots \exists u_k [u_1 = f_1(\vec{x}), \ldots, u_k = f_k(\vec{x}), w = f_{k+1}(\vec{x})]$ where the f_i are terms from $\Phi \cup \mathit{linear}$, where *linear* is the set of all partial functions of the form $f(\vec{x}) = a_0 + a_1 x_1 + \ldots + a_n x_n$, $(a_0, \ldots, a_n \in \mathbb{Q})$; these are defined when $a_0 + a_1 x_1 + \ldots + a_n x_n \in \mathbb{N}$.

Basic Lemma. If α is a primitive recursive algorithm from Φ which computes $f : \mathbb{N}^n \to \mathbb{N}$ and f is not "piecewise linear", then there is a Φ-linear partial function $\Psi(\vec{x})$ such that for infinitely many $\vec{x}$, $\Psi(\vec{x}) \downarrow$ and $\Psi(\vec{x}) \leq C_\alpha(\vec{x})$, where $C_\alpha(\vec{x})$ is the complexity of the algorithm.

Corollary with $\Phi = \emptyset$: for any primitive recursive α which computes the greatest common divisor, there is a rational $e > 0$ such that for infinitely many x, y $C_\alpha(x, y) \geq e(x + y)$.

Theorem (van den Dries): The same result with $\Phi = \{\mathtt{mod}, \mathtt{quot}, \times\}$ where `quot` and `mod` are the quotient and remainder of integer division, respectively, and $\times$ is multiplication.

On the Computational Complexity of Imperative Programming Languages

Karl-Heinz Niggl
(joint work with Lars Kirstiansen)

The present work builds on work on ramified analysis of recursion and aims at making these concepts applicable for imperative programming languages.

The starting point is a simple programming language on stacks (X, Y, Z, ...) over an arbitrary but fixed alphabet Σ. Stack programs are built from the usual basic operations `push`(a, X), `pop`(X), `nil`(X) by means of sequencing $P_1; P_2$, conditional statements `if top`$(X) \equiv a[P]$ and loops statements `foreach` $X[P]$, provided P contains no operation `push`(a, X), `pop`(X) or `nil`(X).

The operational semantics is standard, except possibly the call-by-value semantics for loop statements `foreach` $X[P]$, allowing one to inspect every symbol on the control stack X while preserving the contents of X.

The question addressed is whether one can extract information out of the syntax of a stack program so as to separate programs which run in polynomial time from programs with exponential time, and so forth.

In this talk syntactical criteria are given that separate nesting of loops which do not cause a blowup in running time from those which might do. This gives rise to a measure μ that assigns in a purely syntactic fashion to each stack program P a natural number $\mu(P)$ such that the following holds: The functions computable by a stack program of μ-measure 0 are precisely the polynomial-time computable functions.

Using the same measure for loop programs as introduced and studied by Meyer and Ritchie, one obtains in the same way the following characterization: The functions computable by a loop program of μ-measure n are exactly the functions in $\mathcal{E}^{n+2}$.

On Modified Bar Recursion and its Relation to Spector's Bar Recursion

Paulo Oliva
(joint work with Ulrich Berger)

We introduce a variant of Spector's bar recursion in finite types to give a realizability interpretation of the classical axiom of dependent choice allowing for the extraction of witnesses from proofs of $\forall\exists$-formulas in classical analysis. In this talk we also settle the relation between this new form of bar recursion and Spector's original definition.

Implicit Complexity and Bounded Arithmetic Theories

Chris Pollet

Bounded arithmetic theories are weak fragments of arithmetic useful in the study of computational complexity classes. In this talk we will discuss known techniques for doing independence proofs in these theories. We will then consider the question of classifying the Σ^b_1-definable multifunctions of a particular bounded arithmetic theory, S_2. Such a classification might prove useful in proving new independence results. We will then indicate why an implicit complexity approach to this characterization might be reasonable.

The Axiom of Choice in Constructive Set Theory

Michael Rathjen
(joint work with Sergei Tupailo)

The axiom of choice does not have an unambiguous status in constructive mathematics. On the one hand it is said to be an immediate consequence of the constructive interpretation of the quantifiers. Any proof of $\forall x \in a\, \exists y \in b\, \phi(x, y)$ must yield a function $f : a \to b$ such that $\forall x \in a\, \phi(x, f(x))$. This is certainly the case in Martin-Löf's intuitionistic theory of types. On the other hand, from the very earliest days, the axiom of choice has been criticized as an excessively non-constructive principle even for classical set theory. Moreover, it has been observed that the full axiom of choice cannot be added to systems of constructive set theory without yielding constructively unacceptable cases of excluded middle. On the other hand, it has been shown by Peter Aczel that constructive set theory has a canonical interpretation in Martin-Löf's type theory and that this interpretation also validates several choice principles, e.g., the axiom of countable choice and the axiom of dependent choices.

The main aim of the talk is to present joint work with Sergei Tupailo in which we give a characterization of a restricted class of set-theoretic statements realizable in Martin-Löf type theory. This class contains all the statements that figure in ordinary mathematics. The realizable statements turn out to be those provable in an extension of CZF via the so-called $\Pi\Sigma$-axiom of choice.

Classes, Trees, Objects and Interaction

Anton Setzer
(joint work with Peter Hancock)

We introduce the concept of a state-dependent interactive program in dependent type theory. A world, i.e., the signature of such a system, is given by a quadruple $(\mathrm{S}, \mathrm{C}, \mathrm{R}, \mathrm{n})$, where S is the set of states, $\mathrm{C} : \mathrm{S} \to \mathrm{Set}$ is the set of interactive commands, the program can execute, $\mathrm{R} : (s : \mathrm{S}, c : \mathrm{C}(s)) \to \mathrm{Set}$ is the set of answers from the real world to such commands, and $\mathrm{n} : (s : \mathrm{S}, c : \mathrm{C}(s), r : \mathrm{R}(s, c)) \to \mathrm{S}$ is the next state of the system, after that command has been executed. Interactive programs are then possibly non-well-founded trees, with nodes labeled by states and commands, and branching degree being the set of responses to such a command, such that the states are in accordance with n. They can be seen as well as elements of the final coalgebra of the functor $\mathrm{F} = \lambda X, s.\Sigma c : \mathrm{C}(s).\Pi r : \mathrm{R}(s, c).X(\mathrm{n}(s, c, r))$ on the category of presheaves over S.

Strictly positive functors are equivalent to the functors mentioned before and we introduce type theoretical rules for final coalgebras over such functors: The introduction rule states that if $A : \mathrm{S} \to \mathrm{Set}$ and $f : (s : \mathrm{S}, A(s)) \to \mathrm{F}(A, s)$, then $\mathrm{intro}(A, f) : (s : \mathrm{S}, A(s)) \to \mathrm{F}^{\infty}(s)$, the elimination rule that there exists some $\mathrm{elim} : (s : \mathrm{S}, \mathrm{F}^{\infty}(s)) \to \mathrm{F}(\mathrm{F}^{\infty}, s)$, and the equality rule that $\mathrm{elim}(s, \mathrm{intro}(A, f, s, a)) = \mathrm{F}(\mathrm{intro}(A, f))(s, f(s, a))$. A computationally more efficient, but otherwise equivalent introduction rule uses as type of f $(s : \mathrm{S}, A(s)) \to \mathrm{F}(\mathrm{F}^{\infty}(A + \mathrm{F}^{\infty}), s)$.

We sketch a model of the resulting type theory. We then indicate why the above mentioned more efficient introduction rule extends Coquand's principle of guarded recursion. We now look at examples of state-dependent coalgebras: the set of strictly increasing streams and bisimulation.

Finally we look at the relationship of the concepts of object-oriented programming and the notions introduced above: interfaces can be seen as the command/response set of an interactive program (however the order between command and response is interchanged), objects are interactive programs (i.e. elements of the final coalgebras) and classes are functions from some set to the set of objects (given by the constructors of that class).

Safe Weak Minimization Revisited

DIETER SPREEN

Minimization operators of different strength have been studied in the framework of "predicative (safe) recursion". In this talk a modification of these operators is presented. By adding the new operator to those used by Bellantoni-Cook and Leivant to characterize the polynomial-time computable functions one obtains a characterization of the nondeterministic polynomial-time computable multifunctions. Thus, the generation of the nondeterministic polynomial-time multifunctions from the deterministic polynomial time functions parallels the generation of the computable functions from the primitive recursive functions.

Algorithms and Recursive Abstract State Machines

ROBERT F. STÄRK

Y. Gurevich has shown that if an algorithms satisfies i) the sequential time postulate, ii) the abstract state postulate and iii) the uniformly bounded exploration postulate, then its one-step transformation can be computed by an Abstract State Machine (ASM). Many recursive algorithms, however, do not exactly fit into Gurevich's framework. Therefore, we introduce a logic for sequential-time, deterministic, recursive ASMs with parallel function updates. Unlike other logics for ASMs which are based on dynamic logic, our logic is based on an atomic predicate for function updates and on a definedness predicate for the termination of the evaluation of transition rules. We do not assume that the transition rules of ASMs are in normal form. Instead, we allow structuring concepts including sequential composition and possibly recursive submachine calls. We show that several axioms that have been proposed for reasoning about ASMs are derivable in our system. The main technical result is that the logic is complete for hierarchical (non-recursive) ASMs. In fact, for hierarchical ASMs, the logic is a definitional extension of first-order predicate logic.

Relating Toposes and First Oder Set Theories

THOMAS STREICHER

(joint work with Awodey, Butz, Simpson)

From the early seventies it is known that bounded Zermelo set theory (bZ) is equiconsistent with Higher Order Arithmetic (HOA), the logic of toposes with NNO (natural numbers object). In our work we show more, namely that every topos $\mathcal{E}$ appears as the full subcategory of sets of a model of a weak class theory (bounded intuitionistic Zermelo set theory with strong collection and urelements, called biZ + (Coll)).

The key idea is to show that every topos is equivalent to one with a good system of inclusions $\mathcal{I}$ saying which types are in the subset relation. Using $\mathcal{I}$ we construct a forcing model for biZ + (Coll) conservative over the original topos.

A Proof-Theoretic Characterization of the Basic Feasible Functionals

Thomas Strahm

In this talk we will present a simple classical Feferman-style self-applicative theory PT and exhibit its relationship to the Melhorn-Cook-Urquhart basic feasible functionals BFF. This class of functionals has proven to be a robust candidate for the notion of type two feasibility. It is shown that a type two functional provably converges in PT iff it is basic feasible. Moreover, PT naturally contains well-known first and higher order systems of classical bounded arithmetic. Extensions of this work to other complexity classes will be sketched.

Non-Standard Construction of Gibbs States

Alasdair Urquhart

The main results of this talk concern a construction for equilibrium states in mean field models of generalized spin systems. For finite-range interactions, there is a well known construction due to Van Hove that defines the thermodynamic limit for such systems. In the case of weak, infinite-range interactions such as occur in mean field models, this procedure seems not to apply, so that textbook discussions of such models as the Curie-Weiss model usually omit discussion of the meaning of the thermodynamic limit in this case. In this talk, I sketch how non-standard analysis can be used to give a meaning to the thermodynamic limit. The construction allows a general definition of Gibbs state by employing the DLR equations; the Gibbs states defined in this way can be decomposed into pure states.

Proof Theory and Analytic Number Theory

Andreas Weiermann

We investigate the fine structure of several combinatorial independence results using analytic combinatorics. This approach applies to the Paris Harrington theorem, the Hydra game, the Goodstein process and Friedman style miniaturizations of well-orderedness and well-partial orderedness principles. For example, the strength of Kruskal's theorem can be measured in terms of $\frac{1}{\log_2(\alpha)}$ where $\alpha = 2.95576\ldots$ is Otter's tree constant.

A Fragment of System F Characterizing the Functions Provably Recursive in ID_n *(informal talk)*

Klaus Aehlig

A subsystem of the polymorphic lambda-calculus is presented that captures precisely the functions provably total in the system ID_n of n iterated inductive definitions. The restriction is that every subtype of the form $\forall\alpha\tau$ occurring in the typability derivation must not contain free type variables and that the nesting depth of type quantifiers is at most $n+1$.

The proof that every term typable in that way denotes a function provably total in ID_n proceeds in two steps: a natural proof in a fragment of second order arithmetic with the same restriction for second order variables and a cut-elimination proof for that system which can be formalized in ID_n. As ID_n can be embedded into that system of second order arithmetic a proof theoretic characterization thereof is obtained, which might be of independent interest.

The proof that every function provably total in ID_n can be defined in the said fragment of system F is an interpretation of the proof (embedded in second order arithmetic) as terms.

Turing machines with non-strict oracles are Turing incomplete *(informal talk)*

Ulrich Berger

We consider type 2 functionals $F\colon (\mathbb{N}_\perp^k \to \mathbb{N}_\perp) \times \mathbb{N}_\perp^n \to \mathbb{N}_\perp$ where $\mathbb{N}_\perp$ is the flat domain of partial natural numbers and $\mathbb{N}_\perp^k \to \mathbb{N}_\perp$ is the set of monotone functions from $\mathbb{N}_\perp^k$ to $\mathbb{N}_\perp$.

Let x, y and z range over $\mathbb{N}_\perp$ and i, m over $\mathbb{N}$. The primitive recursive functionals of type 2 are defined by projection $F(g,\vec{x}) = x_i$, use of base functions f, that is $F(g,\vec{x}) = f(F_1(g,\vec{x}),\ldots,F_m(g,\vec{x}))$, "oracle calls", that is $F(g,\vec{x}) = g(F_1(g,\vec{x}),\ldots,F_k(g,\vec{x}))$ and primitive recursion:

$$\begin{array}{lcl} F(g,\vec{x},\perp) & = & \perp \\ F(g,\vec{x},0) & = & F_0(g,\vec{x}) \\ F(g,\vec{x},m+1) & = & F_1(g,\vec{x},m,F(g,\vec{x},m)) \end{array}$$

The μ-recursive functionals are obtained by adding the scheme

$$F(g,\vec{x}) = \mu m[F_1(g,\vec{x},m) = 0] = \begin{cases} m & \text{if } F_1(g,\vec{x},m) = 0 \\ & \text{and } F_1(g,\vec{x},i) > 0 \text{ for all } i < m \\ \perp & \text{if no such } m \text{ exists} \end{cases}$$

On the other hand, the recursive functionals are obtained (sloppy speaking) by allowing in the defining schemes for the primitive recursive functionals the function symbol F to occur on the right hand side. An example of a recursive functional is

$$\begin{array}{l} H\colon (\mathbb{N}_\perp^2 \to \mathbb{N}_\perp) \to \mathbb{N}_\perp \to \mathbb{N}_\perp \\ H(g,x) = g(x, H(g,x+1)) \end{array}$$

In other words, $H(g,x) = g(x, g(x+1, g(x+2, \ldots$. It is shown that this functional is not μ-recursive by proving the following main lemma: Let F be μ-recursive and $\vec{x}$ such that $F(\perp^1,\vec{x}) = \perp$, where the oracle $\perp^1$ always returns $\perp$. Then there is an $m \in \mathbb{N}$ such that for all g with $F(g,\vec{x}) \neq \perp$ there are $m_1, m_2 < m$ such that $g(m_1,m_2) \neq \perp$.

A Recursion Theorem for Nonwellfounded Trees *(informal talk)*

Wilfried Buchholz

Let $\mathbb{T} := \mathbb{N}^{<\omega} \to \mathbb{N}$. We consider systems of equations $(*)$ $\mathsf{F}_i = \mathbf{t}_i(X_1,...,X_m,x_1,...,x_k)$ $(i = 0,...,p)$ where $\mathsf{F}_0,...,\mathsf{F}_p$ are function variables of type $\mathbb{T}^m \times \mathbb{N}^k \to \mathbb{T}$, and $\mathbf{t}_1,...,\mathbf{t}_p$ are terms build up (respecting types) from the variables $X_1,...,X_m$ of type $\mathbb{T}$ and $x_1,...,x_k$ of type $\mathbb{N}$ by means of the following function symbols:

– $\mathsf{F}_0,...,\mathsf{F}_p$;

– $\mathsf{hd} : \mathbb{T} \to \mathbb{N}$, $\mathsf{tl} : \mathbb{T} \times \mathbb{N} \to \mathbb{T}$, $\mathsf{cons} : \mathbb{N} \times (\mathbb{N} \to \mathbb{T}) \to \mathbb{T}$;

– finitely many symbols for primitive recursive functions $f : \mathbb{N}^n \to \mathbb{N}$.

(The formation rule for cons reads: $r : \mathbb{N}$, $t : \mathbb{T} \implies \mathsf{cons}(r, \lambda x.t) : \mathbb{T}$.)

The symbols hd, tl, cons come with a fixed interpretation, namely $\mathsf{hd}(\alpha) = \alpha(\langle\rangle)$, $\mathsf{tl}(\alpha,n) = \lambda\sigma.\alpha(\langle n\rangle * \sigma)$, $\mathsf{cons}(a,f)(\langle\rangle) = a$, $\mathsf{cons}(a,f)(\langle n\rangle * \sigma) = f(n)(\sigma)$.

We establish a syntactic condition $\mathcal{C}$ such that if $(*)$ satisfies $\mathcal{C}$, then $(*)$ has a unique solution $F_0^\omega,...,F_p^\omega : \mathbb{T}^m \times \mathbb{N}^k \to \mathbb{T}$, and moreover the functionals $\lambda\vec{\alpha},\vec{a},\sigma.F_i^\omega(\vec{\alpha},\alpha)(\sigma)$ $(i = 0,...,p)$ are primitive recursive.

An application of this result to proof theory is the following: One easily verifies that Mints' continuos cut-reducing operator $\mathcal{R}_1$ (on possibly nonwellfounded derivations of ω-arithmetic) can be defined by an equation system $(*)$ satisfying our condition $\mathcal{C}$, and therefore $\mathcal{R}_1$ is primitive recursive (hence continuous).

Proof Theory and Explicit Substitution *(informal talk)*

Roy Dickhoff

We outline a proof of the strong normalization of a system of cut-reduction rules for Herbelin's sequent calculus, the system being strong enough to simulate β-reduction of the λ-calculus; the cut rules may in fact be considered as explicit substitution constructors.

An Elementary Fragment of System F *(informal talk)*

Jan Johannsen
(joint work with Klaus Aehlig)

A fragment of second-order lambda calculus (System F) is defined that characterizes the elementary recursive functions. Type quantification is restricted to be non-interleaved and stratified, that is, the types are assigned levels, and a quantified variable can only be instantiated by a type of smaller level, with a slightly liberalized treatment of the level zero.

Contraction-Aware lambda-Calculus. Examples and Discussion. *(informal talk)*

Ralph Matthes

After a review of the definition of the calculus, it is shown that every Church numeral above 1 reduces to numeral 1 in the calculus. An example is given that local confluence does not hold. The discussion is meant to see relations with certain functor category semantics and with calculi of explicit substitution.

The Borel Hierarchy in Intuitionistic Mathematics *(informal talk)*

Wim Veldman

A subset X of Baire space $\mathcal{N}$ is called *positively Borel* if and only if it may be obtained from basic open sets by the operations of countable union and countable intersection. We show how to formulate and prove an intuitionistic Borel Hierarchy Theorem. We also mention some surprising examples of sets that are not positively Borel.

Edited by Klaus Aehlig

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