Mathematisches Forschungsinstitut Oberwolfach

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Singularities and Concentration Phenomena in Nonlinear Elliptic and Parabolic PDE's

January 27th – February 2nd, 2002

Organizers: H. Berestycki (Paris), B. Kawohl (Köln), Yanyan Li (Rutgers).

The meeting was dedicated to nonlinear elliptic and parabolic partial differential equations and systems with a special emphasis on:

- creation and dynamics of singularities and their asymptotics
- vortices and concentration phenomena
- Ginzburg-Landau, Allen-Cahn, Bose-Einstein models
- spike-type solutions of nonlinear elliptic equations
- classification of solutions to nonlinear elliptic equations in unbounded domains, in particular in all of space, $\mathbb{R}^n$ or in a half space, $\mathbb{R}^n_+$
- progress on a conjecture by de Giorgi about phase boundaries
- travelling waves in inhomogeneous media

The participants came from several somewhat different schools of thought in nonlinear PDE's involving variational methods, Morse theory, maximum principles, singular perturbation theory and free boundary problems. This gave rise to a very intensive scientific exchange between participants. Several speakers adjusted to the openness and dynamics of the meeting and presented ongoing research, sometimes on topics which they had not originally planned. This created a delightful and stimulating scientific atmosphere. Most participants had wide mathematical interests.

Unfortunately, two participants (Chinese citizens from the US and China) who had accepted the invitation to Oberwolfach could not come in the end as they were denied an entrance visa.

The traditional hospitality of the staff and the pleasant weather helped to make the meeting a very memorable experience.

Abstracts

Vortex energy for rotating Bose-Einstein condensates

Amandine Aftalion

For a Bose-Einstein condensate placed in a rotating trap, we set a mathematical framework of study for the Gross-Pitaevskii energy. Experimentally, when the value of the rotation is small enough, there are no singularities, but when it reaches a critical value, a line of singularities, or vortex line appears in the condensate and the particularity of this line is to be bent. We define a small parameter and give a simplified expression of the energy which only depends on the number and shape of the vortex lines. Then we check that when there is one vortex line, our simplified expression leads to solutions with a bent vortex for a range of rotational velocities and trap parameters which are consistent with the experiments.

Energy concentration for minimizers of Ginzburg-Landau functionals

Giovanni Alberti

Let u_ϵ be the minimizer, subject to suitable boundary constraints, of the simplified Ginzburg-Landau energy

$$F_\epsilon(u) := \int_\Omega |\nabla u|^2 + \frac{1}{\epsilon^2} W(u) \ ,$$

where $\epsilon > 0$, Ω is a regular domain of $I\!R^n$, $u : \Omega \to I\!R^2$, and $W(u)$ is a positive potential which vanishes on the unit circle. If we denote by λ_ϵ the corresponding energy densities (renormalized by the usual factor $|\log \epsilon|$), then the Jacobians $Ju_\epsilon := du_\epsilon^1 \wedge du_\epsilon^2$ converge (in a suitable sense) to an area-minimizing surface M of codimension 2, while the densities λ_ϵ converge (in the sense of measures) to the volume measure on M. Similar results holds also in higher codimension (provided that F_ϵ is suitably modified). In this lecture I will present some elementary computations, aimed to provide a reaonable, although non-rigorous, picture of the problem; then I will introduce some of the basic ideas which lies behind the first part of the proof, and precisely the compactness of the Jacobians Ju_ϵ, and a lower bound for the energies $F_\epsilon(u_\epsilon)$ (a posteriori, the optimal one).

Asymptotic behaviour for a class of nonlocal problems

Michel Chipot

We propose new techniques to study the asymptotic behaviour of problems of the type:

$$(1) \qquad \begin{cases} u_t - a(l(u(t)))\Delta u + u &= f(x) \quad \text{in} \quad \Omega \times I\!R^+ \\ u(\cdot, t) &= 0 \quad \text{on} \quad \partial\Omega \times I\!R^+ \\ u(\cdot, 0) &= u_0 \quad \text{in} \quad \Omega, \end{cases}$$

where

$$l(u(., t)) = \int_\Omega g(x) \ u(x, t) dx$$

and f, $g \in L^2(\Omega)$, a is some continuous function.

In particular we show that finding the associated stationary solutions to (1) reduces to find the solutions of an equation in **R**. Using some Lyapunov functions or some direct methods we are then able to establish various convergence results.

References

[1] M. Chipot: On the asymptotic behaviour of a class of nonlocal problems (to appear).
[2] M. Chipot & L. Molinet: Asymptotic behaviour of some nonlocal diffusion problems (to appear in Applicable Analysis).

On a geometric Monge-Ampère equation

Kai-Seng Chou

In this talk we gave a geometric interpretation to the following Monge-Ampère equation

$$(*)\qquad det(\nabla^2 H + HI) = \frac{f(x)}{H^{n+2}},$$

where f is a given positive function on S^n. First, consider any hypersurface in centro-affine (c.a.) geometry, the Klein geometry whose isometries are elements of $SL(n+1)$. We explained that the quantity KS^{-n-1}, where K is the Gauss curvature and S support function of the hypersurface, is invariant under $SL(n+1)$, and hence may be called the c.a. curvature of the hypersurface. Now, the c.a. Minkowski problem is: Give a positive function $f(x)$ in S^{n+1} and find necessary and sufficient conditions so that it is the c.a. curvature (as a function of the c.a. normal direction) for a convex hypersurface. It turns out this problem is equivalent to (*). Indeed, the polar body of the hypersurface whose support function solves (*) is a solution for this Minkowski problem. Concerning the solvability of (*) we stated : (a) An obstruction:

$$\int_{S^n} \nabla_\xi f(x) H^{-n-1} dS(x) = 0,$$

where ξ is any projective vector field on S^n; and (b) some sufficient conditions when $n = 1$ (see [1]).

References

[1] Ai, K.S. Chou, Wei: Cal. Var. PDE's 13(2001), 311-337.

Solutions of finite Morse index for equations on R^n and applications

E. Norman Dancer

We consider the equation

$$-\Delta u = f(u) \text{ on } R^n . \qquad (1)$$

Here f is $C^1, f(0) \neq 0$ and $F(u) \neq F(v)$ if u and v are non-negative zeros of f where $F' = f$ (though these conditions could be weakened) and $n = 2$ or 3.

A solution u of (1) is said to be weakly stable if $E(\phi) = \int_{R^n} (\nabla\phi)^2 - f'(u)\phi^2 \geq 0$ on $C_0^\infty(R^n)$.

Theorem 1. *Assume that u is a non-negative bounded weakly stable solution of (1) such that $\int_{B_R} |\nabla u|^2 \leq CR^2$ for large R. Then u is constant.*

This is proved by using the recent techniques used to solve the De Giorgi conjecture in dimensions 2 and 3. Note that the integral condition always holds if $n = 2$. We do not know if the integral condition can be removed when $n = 3$. It can be removed if f is non-negative on $[0, \|u\|_\infty]$. This problem is of considerable interest.

A solution u of (1) is said to have finite Morse index if there is a closed subspace T of $C_0^\infty(R^n)$ of finite codimension such that $E(\phi) \geq 0$ if $\phi \in T$.

Theorem 2. *Assume that the conditions of Theorem 1 hold except that we require u has finite Morse index rather than weakly stable. Then there is a constant C such that $f(C) = 0, f'(C) \leq 0$ and $u(x) - C \to 0$ uniformly as $|x| \to \infty$.*

In many cases, one can use sub and supersolutions to prove u converges to C from above or below and then frequently one can use standard techniques to find what u's can occur.

These results can be used to study the stable positive solutions and the positive solutions of saddle point type (or of bounded Morse index) of the equation

$$-\epsilon^2 \Delta u = f(u) \ \text{ in } \ \Omega; , \ u > 0 \qquad \text{in } \ \Omega \ , \ u = 0 \qquad \text{on } \ \partial\Omega$$

for small positive ϵ (with best results for $\Omega \subseteq R^2$). In particular if $n = 2$, we can frequently prove that such solutions consist of a finite number of sharp peaks and we can then study the location of the peaks by using known techniques.

L^1 connections between equilibria of a semilinear parabolic equation

Marek Fila

Consider the problem

$$\text{(E)} \qquad \begin{cases} u_t = \Delta u + \lambda e^u, & x \in B_1(0), \ t > 0, \\ u = 0, & x \in \partial B_1(0), \ t > 0, \\ u(x, 0) = u_0(|x|), & x \in B_1(0), \end{cases}$$

where $B_1(0) = \{x \in \mathbb{R}^N : |x| < 1\}$, u_0 is a continuous function on $[0, 1]$ vanishing at $r = 1$, λ is a positive parameter and

$$3 \leq N \leq 9.$$

We discuss L^1-connections between equilibria of this problem. By an L^1-connection from an equilibrium $\phi^-(x)$ to an equilibrium $\phi^+(x)$ we mean a function $u(\cdot, t)$ which is a classical solution on the interval $(-\infty, T)$ for some $T \in \mathbb{R}$ and blows up at $t = T$, but continues to exist as a weak solution on $[T, \infty)$ and satisfies

$$u(\cdot, t) \to \phi^\pm \quad \text{as} \quad t \to \pm\infty$$

in a suitable sense.

Minimizing the entropy on a non-closed manifold

Wilfried Gangbo

(joint work with E. Carlen)

We study several constrained variational problem in the 2–Wasserstein metric for which the set of probability densities satisfying the constraint is not closed. For example, given a probability density F_0 on $\mathbf{R}^d$ and a time–step $h > 0$, we seek to minimize $I(F) = hS(F) + W_2^2(F_0, F)$ over all of the probability densities F that have the same mean and variance as F_0, where $S(F)$ is the entropy of F. We prove existence of minimizers. We also analyze the induced geometry of the set of densities satisfying the constraint on the variance and means, and we determine all of the geodesics on it. From this, we determine a criterion for convexity of functionals in the induced geometry. It turns out, for example, that the entropy is uniformly strictly convex on the constrained manifold, though not uniformly convex without the constraint. The problems solved here arose in a study of a variational approach to constructing and studying solutions of the non–linear kinetic Fokker–Planck equation.

Recent Progress on a Conjecture of De Giorgi

Changfeng Gui

De Giorgi formulated in 1978 the following

Conjecture: Suppose that u is an entire solution of the equation (0.1) with condition (0.2). Then for at least $n \leq 8$ the level sets of u must be hyperplanes, i.e. there exists $g \in C^2(R)$ such that $u(x) = g(a \cdot x)$, for some fixed $a \in I\!R^n$ with $|a| = 1$.

The equation arises in the study of phase transition and relates to the stationay Allen-Cahn and Cahn-Hilliard equations. The conjecture is also closely related to the Bernstein problem in geometry on the complete minimal graph surfaces in the entire space. Recently the conjecture is completely proven for dimensions 2 and 3 and for some important cases in higher dimensions. However, the conjecture is still open for dimensions larger than 3. In this talk, I will explain the connection of the conjecture with phase transition and minimal surfaces, discuss in details the recent progresses, and in particular some new results on the conjecture for dimensions 4 and 5 under certain anti-symmetry conditions. I will also outline a strategy for the final resolution of the conjecture in high dimensions, in particular for dimensions 4 and 5.

Propagation of fronts in periodic and more general domains, and estimates for their speeds

François Hamel

(joint work with H. Berestycki, N. Nadirashvili)

We are concerned with some propagation phenomena for reaction-diffusion equations in periodic domains. In a periodic structure, there may exist pulsating travelling fronts connecting two given rest states. Such fronts move in a given direction with an unknown effective speed. The notion of pulsating travelling fronts generalizes that of travelling fronts for planar or shear flows. Some existence, uniqueness and monotonicity results are mentionned for two classes of nonlinear reaction terms. For one of them, there exists a semi-infinite set, bounded from below, of possible speeds of propagation. Such propagation phenomena can take place in infinite cylinders with periodic advection, periodic diffusion or periodic reaction, in cylinders with oscillating boundaries, and in more general periodic domains with a periodic array of holes and periodic coefficients. Lastly, various variational formulas and bounds for the minimal speed of propagation are given, and some possible extensions of the notion of speed of propagation in non-periodic domains are mentioned.

Reaction-Diffusion Enhancement by Convection

Steffen Heinze

In the first part we consider a convection diffusion equation with an incompressible, periodic, cellular flow field. In the rapid oscillation limit (homogenization) we provide explicit upper and lower estimates for the effective diffusivity. The rapid transport leads to enhancement of the effective diffusivity. For all values of the diffusivity the estimates are qualitatively correct. Especially for small values of the diffusivity or equivalently large Peclet numbers the estimates have the correct scaling behavior. We demonstrate that all allowed scaling laws can occur. The upper estimates also answer a problem posed by Kozlov, i.e. if it is possible to have a nonzero limit for the effective diffusivity as the original diffusivity tends to zero. This is called residual or turbulent diffusion. Our bounds exclude this possibility for Hölder continuous flow fields. The proof relies on the use of appropriate test functions which give automatically the correct size of the boundary layer and the scaling of the effective diffusivity. Since the bounds involve explicit constants we have an estimate for the range of validity of the scaling behavior for large Peclet numbers.

In the second part a diffusion convection equation with a non negative reaction term is treated (KPP type or combustion type). The time asymptotic behavior is governed by travelling wave solutions. For a shear flow convection explicit bounds for the speed of such fronts are derived. The estimates show the correct scaling for different asymptotic regimes: small diffusivity, large Peclet numbers, and rapidly oscillating flows. In particular the front speed grows linear with the Peclet number, proving a conjecture posed by Audoly, Berestycki, Pomeau. For cellular flows the enhancement was conjectured to be of order $Peclet^{1/4}$. From the first part of the talk this scaling can be confirmed in the homogenization limit.

Singular limits of an inhomogeneous reaction-diffusion equation

Danielle Hilhorst

(joint work with H. Matano, R. Schätzle)

It is well-known that some classes of nonlinear diffusion equations give rise to sharp internal layers (or interfaces) when the diffusion coefficient is small enough or the reaction coefficient is very large; the motion of such interfaces is often driven by their curvature. We consider here the inhomogeneous reaction-diffusion equation

$$u_t = \mathrm{div}(k(x)\nabla u) + \frac{h(x)}{\epsilon^2}u(1-u^2) \quad \text{in} \quad \Omega \times (0,T),$$

under homogeneous Neumann boundary conditions, where Ω is a smooth bounded domain in $\mathbb{R}^N$, ϵ is a small parameter and the coefficients h and k are smooth and strictly positive.

Our results are the following:

(1) **generation of interface:** we show, under some mild conditions on the initial data, that solutions develop an internal layer near the zeros of the initial data within a very short time; furthermore, the width of the internal layer is of order ϵ; this estimate is optimal and has not been known in higher space dimensions even for the homogeneous case;

(2) **propagation of interface:** once the layer is formed, it is expected to propagate roughly by the same motion law as the limiting interface equation; we show that this is indeed the case and that the Hausdorff distance between the limiting interface and the real internal layer remains of order ϵ as t ranges in a finite interval.

On a limiting motion and self-intersections for a geometric evolution equation

KAZUO ITO
(joint work with J. Escher, Y. Giga)

We study the following nonlocal geometric evolution equation called the *intermediate surface diffusion flow*:

$$V(t) = \Delta_{\Gamma(t)} \left(\frac{1}{D} - \frac{1}{M} \Delta_{\Gamma(t)} \right)^{-1} H(t) \qquad \text{on } \Gamma(t) \text{ for } t > 0 \qquad (*)$$

with an initial condition $\Gamma(0) = \Gamma_0$. Here $\Gamma(t)$ is an unknown, with respect to time $t > 0$ evolving closed compact oriented hypersurface in $\mathbb{R}^n$. We write $\Delta_{\Gamma(t)}$ for the Laplace-Beltrami operator on $\Gamma(t)$ with respect to the Euclidean metric. The mean curvature of $\Gamma(t)$ is denoted by $H(t)$ and $V(t)$ stands for the normal velocity of the family $\{\Gamma(t)\,;\, t > 0\}$. Moreover, $D > 0$ is a large parameter such that $D \to \infty$, and $M > 0$ is a fixed constant.

The purpose of this talk is to show that the problem $(*)$ admits a unique local solution $\Gamma^D := \{\Gamma^D(t)\,;\, t \in [0,T]\}$ of class $C^{1,2+\alpha}$ on a *common* existence interval $[0,T]$ (i.e. $[0,T]$ is independent of $D \geq 1$), and that this solution Γ^D converges for $D \to \infty$ in $C^{1,2+\alpha}$ to the unique solution of the *averaged mean curvature flow*. Moreover, we show that the flow $(*)$ can drive an embedded hypersurface into self-intersections for every $D > 0$.

On a Nonlinear Diffusion Equation with Singular Coefficient

SHOSHANA KAMIN

We study some special self-similar solutions of the equation

$$\text{(1)} \qquad \frac{1}{|x|^\alpha} u_t = (u^m)_{xx}, \quad x \in I\!R, \quad t > 0$$

and use them for the description of the behaviour of the solutions for some other problems.

I. (Joint work with V.A. Galaktionov, R. Kersner and J.-L. Vazquez) Consider the following problem

$$\text{(2)} \qquad \rho(x) u_t = (u^m)_{xx}, \qquad u(x,0) = u_0(x)\ .$$

Suppose that $u_0 \in L^\infty(I\!R)$, $\lim_{|x|\to\infty} u_0(x) = 0$ and $\rho(x)$ behaves like $\frac{1}{|x|^\alpha}$ for large $|x|$. Let $\alpha \in (1,2)$. It is proved that as $t \to \infty$ the solution of the Cauchy problem (2) approaches the self-similar solution $U(x,t)$ of the singular "limit" equation (1),

$$u(x,t) \to U(x,t) = f\left(\frac{x}{t^{1/2-\alpha}} \right)\ .$$

II. (Joint work with P. Rosenau) Dipole self-similar solution of equation (1) is used for the study of the convergence to a travelling wave for the problem

$$\text{(3)} \qquad u_t = (u^2)_{xx} + u - u^2, \qquad u(x,0) = u_0(x)\ .$$

The polar cone of the set of convex functions and applications to the regularity of solutions for variational problems subject to a convexity constraint

T. Lachand-Robert
(joint work with G. Carlier)

We first study the minimizers, in the class of convex functions, of an elliptic functional with nonhomogeneous Dirichlet boundary conditions. We prove C^1 regularity of the minimizers under the assumption that the upper envelope of admissible functions is C^1 (see CPAM 2001).

Another way to study this sort of problem is to consider the polar cone of the set $K = \{\nabla u; u \text{ convex}\}$, for the L^2 scalar product. This polar cone is $K^- = \mathbb{R}_+ \overline{\text{co}}(S - \text{id})$, where S is the set of measure-preserving maps. We prove that any L^2 vector field can be written as $\nabla u + p$, with u convex, $p \in K^-$, and $< \nabla u, p >= 0$. This implies new expression of the Euler-Lagrange equation associated with minimization problems in the set of convex functions, as well as new regularity results for these.

Travelling waves in quasiperiodic media and their associated flow on a torus

Hiroshi Matano

Travelling waves in heterogeneous media are gaining more and more attention in various fields of science such as ecology, physiology and combustion theory. They have also become an important subject of mathematical studies in the past decade. However, most of those theoretical studies have been focused on spatially periodic cases, and little is known about the nature of traveling waves in aperiodically varying media.

In this lecture I introduced the precise notion of travelling waves in spatially quasi-periodic (or almost periodic) diffusive media and discussed basic properties of such travelling waves, mainly for bistable type diffusion equations of the form

$$u_t = u_{xx} + b(x)f(u) \quad (x \in \mathbf{R},\, t > 0),$$

or their higher dimensional versions.

More precisely, I have shown that each travelling wave defines a flow on a torus and that the behavior of this associate flow characterizes the nature of the travelling wave.

Let me also point out that in Fisher-KPP type diffusion equations, a certain eigenvalue problem on a torus may play a crucial role in the estimate of the speed of travelling waves as well as in the existence proof. This part is an ongoing joint work with H. Berestycki and F. Hamel.

Blow-up solutions for L^2 critical KdV

Franck Merle

We consider the problem

$$u_t + (u_{xx} + u^5)_x = 0 \quad x \in I\!R, \ t > 0,$$

with initial data $u_0 \in H^1(I\!R)$.

We prove existence of blow-up solutions for negative energy and describe the speed and the shape of blow-up for $E_0 < 0$, $\displaystyle\int u_0^2 \leq \int Q^2 + \delta_0$, where Q is the soliton of speed 1.

On the Singularities for the Obstacle Problem

Régis Monneau

The obstacle problem has been extensively studied in the literature. A simple example is the minimisation of the convex functional

$$\int_\Omega |\nabla u|^2 + 2u$$

over the convex set of functions

$$K = \{u \in H^1(\Omega), \quad u = g \text{ on } \partial\Omega, \quad u \geq 0 \text{ on } \Omega\}$$

where Ω is a smooth bounded open set in $\mathbf{R}^n$, and g is a smooth positive function. The free boundary is the set

$$\partial \{u = 0\}$$

Quite recently the problem was revitalized by a work of Caffarelli where it is in particular proved that the singular set of the free boundary can be contained in a smooth hypersurface of Ω. Based on a beautiful work of Weiss , we have recently proved a *new monotonicity formula for singular points*, providing generalizations to more general obstacle problems. Using this monotonicity formula we have proved in dimension $n = 2$ a conjecture of Schaeffer (1974) which claims that the free boundary is generically smooth. This conjecture stays open in dimensions $n \geq 3$.

Symmetry and other qualitative properties of solutions of semilinear elliptic equations

Filomena Pacella

We study the symmetry properties of the solutions of the semilinear elliptic problem

$$\begin{aligned} -\Delta u &= f(x,u) \qquad \text{in } \Omega \\ u &= g(x) \quad \text{on } \partial\Omega, \end{aligned}$$

where Ω is a bounded symmetric domain in $I\!R^N$, $N \geq 2$, and $f : \Omega \times I\!R \to I\!R$ is a continuous function of class C^1 in the second variable, g is continuous and f and g are somehow symmetric in x.

We show that all solutions of the above problem of index one are axially symmetric when Ω is an annulus or a ball, $g \equiv 0$ and f is strictly convex in the second variable.

Moreover we are able to prove that if the solution of index one is not radially symmetric then all critical points are located on the simmetry axis.

To do this we prove that the nonnegativity of the first eigenvalue of the linearized operator in the caps determined by the symmetry of Ω, is a sufficient condition for the symmetry of the solution, when f is a convex function.

This condition is stable under "small perturbation" in the sense that it allows to prove that the symmetry of the solutions is preserved under a small symmetric perturbation of the domain.

References

[1] F. Pacella, Symmetry results for solutions of semilinear elliptic equations with convex nonlinearities, J. Funct. Anal. (to appear).

[2] M. Grossi, F. Pacella and S.L. Yadava, Symmetry results for perturbed problems and related questions, Top. Meth. Nonlin. Anal.(to appear).

Continuity of the blow-up time and a priori bounds for solutions in superlinear parabolic problems

Pavol Quittner

We consider parabolic problems of the form

$$
(1) \qquad \begin{cases} u_t - \Delta u &= f(u), & x \in \Omega,\ t > 0, \\ u &= 0, & x \in \partial\Omega,\ t > 0, \\ u(x,0) &= u_0(x), & x \in \Omega, \end{cases}
$$

where Ω is a domain in $I\!R^n$ with a smooth compact boundary $\partial\Omega$ and f is a locally Lipschitz continuous function which is superlinear at infinity. Problem (1) is well posed in a suitable Banach space X ($X = L^\infty(\Omega)$, for example). Denote by $u(t, u_0)$ the solution of (1) at time t and by $T_{\max}(u_0)$ the maximal existence time of this solution. We show that for a large class of functions f with subcritical growth, the function $T_{\max} : X \to (0, \infty]$ is continuous (this need not be true in the supercritical case). The result is based on a priori estimates of the form $\|u(t, u_0)\|_X \leq C(\|u_0\|_X, \delta)$ for any $t \in [0, T_{\max}(u_0) - \delta)$. We also discuss other applications of these estimates: blow-up rates for blowing-up solutions and existence of positive periodic solutions if $f = f(t, u)$ is periodic in t.

A sharp Sobolev inequality on Riemannian manifolds

Tonia Ricciardi

(joint work with YanYan Li)

For $n \geq 3$ and $2^* = 2n/(n-2)$, let

$$
(1) \qquad K^{-1} = \inf \left\{ \frac{\|\nabla u\|_{L^2(I\!R^n)}}{\|u\|_{L^{2^*}(I\!R^n)}} \ :\ u \in L^{2^*}(I\!R^n) \setminus \{0\},\ |\nabla u| \in L^2(I\!R^n) \right\}.
$$

We prove the following sharp Sobolev inequality:

Theorem 1 Let (M, g) be a smooth compact Riemannian manifold without boundary of dimension $n \geq 6$. There exists a constant $A > 0$, depending on (M, g) only, such that for all $u \in H^1(M)$ there holds:

$$
(2) \qquad \|u\|^2_{L^{2^*}(M,g)} \leq K^2 \int_M \left\{ |\nabla_g u|^2 + c(n) R_g u^2 \right\} dv_g + A\|u\|^2_{L^{\bar{r}}(M,g)},
$$

where 2^* and K are defined above, $c(n) = (n-2)/[4(n-1)]$, $\bar{r} = 2n/(n+2) = 2^{*\prime}$, R_g is the scalar curvature of g.

Remark

Theorem 1 is *sharp*, in the sense that one can neither replace K by any smaller number, nor replace R_g by any $R_g + f$ with $f \in C^0$ negative somewhere. Moreover, if (M, g) is not locally conformally flat, one cannot replace $\bar{r}$ by any smaller number.

For locally conformally flat manifolds we have:

Theorem 2

Let (M, g) be a smooth compact locally conformally flat Riemannian manifold without boundary of dimension $n \geq 3$. There exists a constant $A > 0$, depending on (M, g) only, such that for all $u \in H^1(M)$ there holds:

$$\|u\|^2_{L^{2^*}(M,g)} \leq K^2 \int_M \left\{ |\nabla_g u|^2 + c(n) R_g u^2 \right\} dv_g + A \|u\|^2_{L^1(M,g)}. \tag{3}$$

Nonlinear Schrödinger equations with Hardy potential and critical nonlinearities

Didier Smets

We study a time independent nonlinear Schrödinger equation with an attractive inverse square potential and a non autonomous nonlinearity whose power is the critical Sobolev exponent, the domain being the whole $\mathbb{R}^N$. A particular attention is paid to the blow-up possibilities, i.e. the critical points at infinity of the corresponding variational problem. Due to the strong singularity in the potential, these are of two kinds. A complete existence result is obtained in dimension 4, after a detailed analysis of the gradient flow lines in the spirit of the work of A. Bahri.

Some recent results on viscous Hamilton-Jacobi equations

Philippe Souplet

We report on various questions concerning Hamilton-Jacobi equations with viscosity

$$\begin{aligned} u_t - \Delta u &= a|\nabla u|^p, \quad t > 0, \quad x \in I\!R^N \qquad (p > 0,\ a \neq 0) \\ u(0,x) &= u_0(x), \quad x \in I\!R^N. \end{aligned} \tag{1}$$

1. Singular initial data (joint with M. Ben-Artzi and F. Weissler, to appear in J. Math Pures et Appl. 81 (2002)).

We address the question of local existence for singular initial data $u_0 \in L^q$ $(1 \leq q < \infty)$ or u_0 measure, for $p \geq 1$. We obtain an almost complete classification regarding local (non-)existence and (non-)uniqueness, introducing the critical exponent $q_c = N(p-1)/(2-p)$ ($q_c = \infty$ if $p \geq 2$) and distinguishing the repulsive ($a > 0$, $u_0 \geq 0$) and attractive ($a < 0$, $u_0 \geq 0$) cases.

2. Growth of mass (joint with Ph. Laurençot, to appear in J. d'Analyse Math.).

We consider finite mass solutions of (1) with $a > 0$ and $p \geq 1$. When $u_0 \in L^1$, $u_0 \geq 0$ (and $u_0 \in W^{1,\infty}$, say), it is known that (1) has a global, nonnegative solution, which remains in L^1 for all $t > 0$. Moreover, it is easy to see that the mass $I(t) = \|u(t)\|_1$ is a nondecreasing function. A natural question is then to determine whether $I_\infty = \lim_{t\to\infty} I(t)$ is finite or not. The answer is as follows: if $p \leq p_N$, then $I_\infty = \infty$ for all u_0 $(\not\equiv 0)$; if $p_N < p < 2$, then $I_\infty < \infty$ if u_0 is suitably small, but $I_\infty = \infty$ also occurs for some u_0; if $p \geq 2$, then $I_\infty < \infty$ for all u_0.

3. Finite-time extinction (joint with S. Benachour, Ph. Laurençot and D. Schmitt, to appear in Asympt. Anal.).

It is well-known that for $p \in (0,1)$, all positive solutions of

$$u_t - \Delta u + u^p = 0, \quad t > 0, \quad x \in I\!R^N \qquad (u_0 \in L^\infty), \tag{2}$$

extinct in finite time (i.e., $u(T,.) \equiv 0$ for some $T > 0$). We study the same question for positive solutions of (1) with $a < 0$ and $p \in (0,1)$. In contrast with (2), the extinction property for (1) depends crucially on the order of decay of u_0.

Variational Principles for Propagation Speeds in Inhomogeneous Media

Angela Stevens

(joint work with Steffen Heinze, George Papanicolaou)

An important problem in reactive flows is how to estimate the speed of front propagation, especially when inhomogeneities are present. A variational characterization of the front speed for reaction-diffusion-advection equations in periodically varying heterogeneous media is proved. This formulation makes it possible to calculate sharp estimates for the speed explicitly. The method can be applied to any problem obeying a maximum principle. Three examples are analyzed in detail: a shear flow problem, a problem with rapidly oscillating coefficients and a discretized diffusion problem. In all cases the effects of the inhomogeneous medium on the speed are discussed in comparison to the homogeneous problem. For the shear flow problem, enhancement of the speed results.

Green function estimates and their consequences

Guido Sweers

Probably the most famous singularity for elliptic problems is the one of the fundamental solution for the laplace operator: $c_n |x-y|^{2-n}$ if $n \geq 3$. If one is looking for solution operator for $-\Delta u = f$ in Ω with $u = 0$ on $\partial\Omega$ then one has to combine this singularity with the zero Dirichlet bounday condition. How does the singularity survive?

Writing the solution of this Dirichlet Laplacian by $u(x) = \int_\Omega G(x,y) f(y)\, dy$, with G the Green function, and knowing that the maximum principle implies that G is positive, one should be able to derive an optimal two-sided estimate by a positive function. Indeed such has been done:

Theorem 1.

Let Ω be a bounded domain in $\mathbb{R}^n$ with a smooth boundary $\partial\Omega$, then

$$\begin{array}{lcll} G(x,y) & \sim & |x-y|^{2-n} \min\left(1, \frac{d(x)d(y)}{|x-y|^2}\right) & \text{if } n \geq 3, \\ G(x,y) & \sim & \log\left(1 + \frac{d(x)d(y)}{|x-y|^2}\right) & \text{if } n = 2, \\ G(x,y) & \sim & \sqrt{d(x)\, d(y)} \min\left(1, \frac{d(x)d(y)}{|x-y|^2}\right)^{\frac{1}{2}} & \text{if } n = 1, \end{array}$$

where $a(x,y) \sim b(x,y)$ means $0 < c_\Omega \leq \frac{a(x,y)}{b(x,y)} \leq C_\Omega < \infty$ for all $x, y \in \Omega$, and $d(x)$ denotes the distance to the boundary: $d(x) = \inf\{|x - x^*|\,; x^* \in \partial\Omega\}$.

Remark

For $n \geq 3$ the estimate from above is due to Widman, [3]. The estimates from below are essentially due to Zhao, [4] and [5]. See also [2].

Also for iterated Dirichlet Laplacian such estimates are known. For example for the biharmonic operator with Navier boundary conditions, $\Delta^2 u = f$ in Ω with $u = \Delta u = 0$ on $\partial\Omega$ the Green function $G^2(x,y)$ satifies the following estimates:

Theorem 2. [1]

Let Ω be a bounded domain in $\mathbb{R}^n$ with a smooth boundary $\partial\Omega$, then

$$\begin{array}{lcll} G^2(x,y) & \sim & |x-y|^{4-n} \min\left(1, \frac{d(x)d(y)}{|x-y|^2}\right) & \text{if } n \geq 5, \\ G^2(x,y) & \sim & \log\left(1 + \frac{d(x)d(y)}{|x-y|^2}\right) & \text{if } n = 4, \\ G^2(x,y) & \sim & \sqrt{d(x)\,d(y)} \min\left(1, \frac{d(x)d(y)}{|x-y|^2}\right)^{\frac{1}{2}} & \text{if } n = 3, \\ G^2(x,y) & \sim & d(x)\,d(y) \log\left(1 + \frac{1}{|x-y|^2 + d(x)d(y)}\right) & \text{if } n = 2, \\ G^2(x,y) & \sim & d(x)\,d(y) & \text{if } n = 1. \end{array}$$

Remark For higher iterates no new estimating functions appear; only the numbers n shift.

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Approaching a partial differential equation of mixed elliptic-hyperbolic type

Giorio Talenti

Suppose an isotropic, non-conducting, non-dissipative medium and a monochromatic electromagnetic field interact in absence of electric charges. Let n and ν denote the refractive index and the wave number, respectively. Here n is a scalar real-valued field, whose reciprocal is proportional to the relevant velocity of propagation through the medium, and ν is a large positive parameter, whose reciprocal is proportional to the length of waves involved. The following Helmholtz equation

$$\Delta U + \nu^2 n^2 U = 0 \tag{1}$$

is an archetype of those partial differential equations that ensue from Maxwell's system and model the subject matter mathematically. A distinctive feature of (1) is *stiffness* — the order of magnitude of ν is significantly greater than that of the other coefficients involved.

An expansion, which represents solutions to (1) asymptotically as $\nu \to +\infty$, originates from WKBJ method and reads thus

$$U \simeq \exp(i\nu S) \sum_{k=0}^{\infty} A_k \cdot (i\nu)^{-k}. \tag{2}$$

Here S and A_k are scalar fields, independent on ν. The former, named *eikonal,* is real-valued and governed by

$$|\nabla S|^2 = n^2;$$

the latter is complex-valued and governed by the so-called *transport equations.*

Inference built upon expansion (2) amounts to geometrical optics. Though successful in describing both the propagation of light and the concurrence of caustics via the mechanism of rays, geometrical optics is inherently unable to account for those phenomena, such as the development of evanescent waves, that take place beyond a caustic.

A more powerful asymptotic expansion, which is apt to represent solutions to (1) on *both sides of a caustic,* simultaneously in the region covered by geometric optical rays and in the opposite region where geometrical optics breaks down, is provided by a theory of Kravtsov and Ludwig. In case the caustic involved is smooth and convex, such an expansion reads

$$U \simeq e^{i\nu v}\Big\{\mathrm{Ai}(\nu^{2/3}u) \sum_{k=0}^{\infty} A_k \cdot (i\nu)^{-k} + i\nu^{-1/3}\mathrm{Ai}'(\nu^{2/3}u) \sum_{k=0}^{\infty} B_k \cdot (i\nu)^{-k}\Big\}. \tag{3}$$

Here u, v, A_k, B_k are scalar fields, independent on ν; u and v are real-valued, A_k and B_k are complex-valued; Ai denotes the *Airy function.*

Properties of Ai inform us that the right-hand side of (3) oscillates rapidly where u is negative, approaches smoothly a limit if u approaches 0, quenches fast where u is positive. Therefore (3) matches geometrical optics in the region where u is negative and predicts the occurrence of damped waves in the region where u is positive; a caustic take place on the level surface where $u = 0$.

Assembling (1) and (3) results in

$$\begin{array}{l} u\ |\nabla u|^2 - |\nabla v|^2 + n^2 = 0 \\ \nabla u \cdot \nabla v = 0 \end{array} \tag{4}$$

— a fully nonlinear, first-order partial differential system governing u and v.

In the present paper we sketch some lineaments of (4) in the case where the space dimension equals 2, i.e. we let x and y denote rectangular coordinates in the Euclidean plane and investigate the following system

$$\begin{array}{lcl} u\ (u_x^2 + u_y^2) - v_x^2 - v_y^2 + n^2(x, y) & = & 0 \\ u_x v_x + u_y v_y & = & 0. \end{array} \tag{5}$$

An Inverse Problem in Neurology

Alfred Wagner
(joint work with Steve Cox)

We consider a one dimensional nerve fibre $[0,a]$ of fixed length a. At an initial time we apply a known stimulus $v_x(0,a)$ where $v(x,t)$ is the potential drop across the fibre at place x and time t. After Hodgkin and Huxley the evolution of $v(x,t)$ is given by

$$v_t = v_{xx} + f(v) - w \quad \text{in} \quad Q_T := (0,a)\times(0,T) \tag{1}$$

$$w_t = \delta v - \gamma w \quad \text{in} \quad Q_T \tag{2}$$

where a and T are positive real numbers. We fix the boundary conditions

$$v_x(0,t) = g_1(t) \qquad 0 \le t \le T \tag{3}$$

$$v_x(a,t) = 0 \qquad 0 \le t \le T \tag{4}$$

and initial conditions

$$v(x,0) = g_2(x) \qquad 0 \le x \le a \tag{5}$$

$$w(x,0) = 0 \qquad 0 \le x \le a. \tag{6}$$

We are interested in the following problem:

Find the nonlinearity f as the unique solution of the overdetermined boundary value problem (1) - (6) and

$$v(0,t) = h(t). \tag{7}$$

We will prove the following theorem:

Theorem: *Suppose $g_1 \in C^{2+\alpha}([0,T])$, $g_2 \in C^{3+\alpha}([0,a])$ and $h \in C^{2+\alpha}([0,T])$ for $\alpha \in (0,\frac{1}{2})$ and h and g_1 sufficiently large in norm. Assume also that they satisfy all necessary compatibility conditions, and that $g_{2xx} \ge 0$ for $0 \le x \le a$ holds. Then for any $T > 0$ there exists a unique solution*

$$(v,w,f) \in X_\alpha \times X_\alpha \times C^{1+\alpha}(I\!R)$$

to the inverse problem (1) - (7). Here

$$X_\alpha := \{u \in C^{2+\alpha,1+\alpha}(\overline{Q}_T) : u_x, u_t \in C^{2+\alpha,1+\alpha}(\overline{Q}_T)\}$$

where $Q_T := (0,a)\times(0,T)$.

Existence and Stability Of Spiky Pattern For Reaction-Diffusion Systems

Juncheng Wei

We consider the following reaction-diffusion system in R^n, $n = 1,2$,

$$\begin{cases} A_t = \epsilon^2 \Delta A - A + \dfrac{A^2}{H}, \; x \in \Omega \\ \tau H_t = D\Delta H - H + A^2, \; x \in \Omega \\ A > 0, H > 0 \\ \dfrac{\partial A}{\partial \nu} = \dfrac{\partial H}{\partial \nu} = 0 \text{ on } \partial\Omega \end{cases} \tag{8}$$

In particular we are interested in the role of D on the existence and stability of peaked solutions. It is known that when D is $+\infty$ (shadow system case), there are many boundary and interior peaked solutions, but only one of them is stable. The main issue of this talk is to study the situation when we move D from $+\infty$ to $O(1)$. We present two interesting results: first there exist a sequence of critical values

$$D_K = \begin{cases} \dfrac{1}{K^2(\log\sqrt{3})^2} & \text{if } n = 1, \\ \\ \dfrac{|\Omega| \log \frac{1}{\epsilon}}{2\pi K} & \text{if } n = 2, \end{cases}$$

such that when $D < D_K$, $K-$peaked solutions are stable and when $D > D_K$, K-peaked solution is unstable.

Our second result concerns the existence and stability of **asymmetric** patterns: we show that there exists multiple asymmetric patterns. They are generated by exactly two types-type **A** and type **B**.

We also discussed the existence of **clusters**.

Blow-up phenomena in degenerate parabolic equations

Michael Winkler

We consider positive solution to

$$(1) \qquad \begin{aligned} u_t &= u^p u_{xx} + u^q \quad \text{in } (-L/2, L/2) \times (0,T) \\ u_{|\partial\Omega} &= 0 \\ u_{|t=0} &= u_0, \end{aligned}$$

where $p, q \geq 1$. We show that blow-up in finite time occurs if $q = p+1$ and $L > \pi$, or if $q > p+1$ and u_0 is sufficiently large. At $t = T$, the "apparent mass" $\dfrac{u(t)}{\max u}$ of such a solution concentrates on a subset S of Ω with measure: greater than or equal to π if $q = p+1$, or zero if $q > p+1$.

We also study the effect of a further source term of gradient type, leading to the equation

$$u_t = u^p u_{xx} + u^q + \gamma u^r u_x^2,$$

$p, q \geq 1$, $r \geq 0$, $\gamma > 0$.

Some inequalities related to isoperimetric inequalities with partial free boundary

Meijun Zhu

The main purpose of this paper is to prove a sharp Sobolev inequality in the exterior of a convex bounded domain. There are two ingredients in the proof: One is the observation of some new isoperimetric inequalities with partial free boundary, and the other is an integral inequality (due to Duff) for any nonnegative function under Schwarz equimeasurable rearrangement. These ingredients also allow us to establish some Moser-Trudinger type inequalities, and obtain some estimates on the principal frequency of a membrane with partial free boundary, which extend early results of Nehari and Bandle for two dimensional domains to the one for any dimensional domains (dimension ≥ 2).

On a simplified 1D model of fluid-solid interaction

Enrique Zuazua
(joint work with J.L. Vázquez)

In this lecture we present some recent joint work with J. L. Vázquez on a simple model in one space dimension for the interaction between a fluid and a solid mass. The fluid is governed by the viscous Burgers equation and the solid mass, which shares the velocity of the fluid, is accelerated by the difference of pressure at both sides of it. We describe the asymptotic behavior of solutions for integrable data using energy estimates and scaling techniques. We prove that the asymptotic profile of the fluid is a self-similar solution of the Burgers equation with an appropriate mass and we describe the parabolic trajectory of the solid mass. We also prove that, asymptotically, the difference of pressure to both sides of the mass vanishes. Finally, we consider the case of a finite number of masses. We show that they may not collide in finite time.

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