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Noncommutative Geometry and Quantum Field Theory

Organised by
Sergio Doplicher (Rom)
Mario Paschke (Leipzig)
Rainer Verch (Leipzig)
Eberhard Zeidler (Leipzig)

Oktober 23rd – Oktober 29th, 2005

Abstract. The workshop gathered experts from both mathemematics and physics working on the interrelation of Noncommutative Geometry and Quantum Field Theory, which has become one of the central topics in mathematical physics over the last decade. The talks mainly focused on the possible noncommuativity of spacetime, applications of the local index formula in quantum field theory and the significant recent progress concerning the construction of interacting quantum field theories over noncommutative spaces.

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Introduction by the Organisers

The workshop *Noncommutative Geometry and Quantum Field Theory*, organised by Sergio Doplicher (Rome), Mario Paschke, Rainer Verch and Eberhard Zeidler (Leipzig) was held Oct. 23rd– Oct 29th, 2005.
There are several motivations to study the interplay of noncommutative geometry and quantum theory. First of all, it has frequently been argued from very different perspectives that physical spacetime might be noncommutative itself, i.e. it might best be described by a noncommutative C^*-algebra $\mathcal{A}$. The first day of the workshop was therefor mainly devoted to pedagogical reviews of these arguments, and in particular the appearance of a noncommutativity of spacetime in the most prominent approaches to a quantum theory of gravity. As a possible new research direction the problem was raised to consider the algebra $\mathcal{A}$ as part of the dynamics, that is to say to generalize (the sought for quantum version of) Einstein's equations for the metric of spacetime such that they also determine the (noncommuative) topology of spacetime dynamically via the coupling to matter

fields. It has also been pointed out that such a program would stimulate new developments in mathematics.

The first and second day also saw reviews on the recent progress in pertubative constructions of quantum field theory over noncommutative spaces. It has, however, also been speculated by several speakers, that it might be possible to rigorously and nonpertubatively construct certain nontrivial interacting quantum theories over noncommuative spaces, even though this seems impossible for the corresponding theory on a commutative space.
Closely related important developments were reported, concerning Hopf algebras, progress in the Algebraic approach to Conformal Field Theory and its possible relations to a noncommutative index theory, and in the approach to Confinement and Renormalization Group in AQFT. Recent developments in String Theory, related to the subject of our meeting, were also reported.
Although quantum field theory can be formulated entirely in the language of operator algebras, geometrical entities play an essential, but not readily understood, role for the consistency and interpretation therof. Noncommutative Geometry, which studies the interrelation of operator algebras and geometry might therefor prove an indespensible tool on the way to a deeper understanding of quantum physics. A large number of talks in the workshop adressed possible applications of Noncommuative Geometry, and in particular the local index formula of Connes and Moscovici, to quantum field theory.
These applications of Noncommuative geometry require generalizations of Connes' notion of spectral triples, however. The fourth day of the meeting was therefor mainly devoted to the recent progress concerning the various generalisations required to study problems in quantum field theory, as well as to noncommutative formulations of the theory of Gravitation. The fact that the need of qantum physics will also stimulate new developments in mathematics (here the theory of operator algebras and Noncommutative Geometry), has been one of the main motivations to organise this meeting of mathematicians and physicists, which has brought about many concrete proposals for new research directions, but also accumulated many open questions. The workshop thus reflected very well the excellent prospects for progress in mathematics as well as in physics offered by this field of research.
On wednesday, after the excursion, N.P.Landsmann gave an evening talk on the classical debate between A.Einstein and N.Bohr, leading to very stimulating discussions on the foundations of quantum theory afterwards.
Discussions of this kind are rendered possible only by the unique atmosphere at the MFO. It is a pleasure to thank the director G.-M. Greuel, the admistration staff and the kitchen staff of the MFO for all their efforts to make this workshop so beneficial. We would also like to thank the state of Baden-Würtemberg, the main source of funding the MFO, and the European Union, whose support for the MFO enabled us to invite seven Ph.D. students in addition to the originally intended 26 senior researchers.

Workshop: Noncommutative Geometry and Quantum Field Theory

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Abstracts

Quantum Field Theory on Quantum Spacetime

Sergio Doplicher

At large scales spacetime is a pseudo Riemaniann manifold locally modelled on Minkowski space. But the concurrence of the principles of Quantum Mechanics and of Classical General Relativity renders this picture untenable in the small.
Those theories are often reported as hardly reconcilable, but they do meet at least in a single principle, the *Principle of Gravitational Stability against localization of events* formulated in [1, 2, 3]:
the gravitational field generated by the concentration of energy required by the Heisenberg Uncertainty Principle to localize an event in spacetime should not be so strong to hide the event itself to any distant observer - distant compared to the Planck scale.
Already at a semiclassical level, this Principle leads to *Spacetime Uncertainty Relations*, that we proposed and shown to be implemented by Commutation Relations between coordinates, thus turning Spacetime into Quantum Spacetime. The word "Quantum" is very appropriate here, to stress that noncommutativity does not enter just as a formal generalization, but is strongly suggested by a compelling physical reason.
Such an analysis leads to the following conclusions:

(i) There is no a priori lower limit on the precision in the measurement of any *single* coordinate (the apparently opposite conclusions often reported in the literature are drawn under the *implicit* assumption that *all* the space ccordinates of the event are simultaneously sharply measured);

(ii) The Space Time Uncertainty Relations emerging from the *Principle of Gravitational Stability against localization of events*, in their weak form and disregarding the contributions to the source in Einstein Equations due to the average energy momentum density in generic quantum states, can be implemented by covariant commutation relations between the coordinates, which define a fully Poincare' covariant *Basic Model* of Quantum Spacetime.

(iii) In the Basic Model of Quantum Spacetime, the Euclidean distance between two events and the elementary area have both a lower bound of the unit order in Planck units; this is quite compatible, as shown by the model, with Poincare' covariance, and not to be confused with the unlimited accuracy which is in principle allowed in the measurement of a *single* coordinate.

(iv) The Basic Model replaces the algebra of continuous functions vanishing at infinity on Minkowsky Space by a noncommutative C* Algebra $\mathcal{E}$, the enveloping C* Algebra of the Weyl form of the commutation relations between the coordinates, which turns out to be the C* Algebra of continuous functions vanishing at infinity from Σ to the C* Algebra of compact operators. Here Σ is the union of two connected components, each omeomorphic to $SL(2,\mathbb{C})/\mathbb{C}_* \simeq TS^2$. This manifold survives the large scale limit; thus *QST predicts extradimensions*, which

indeed manifest themself in a *compact* manifold $S^2 \times \{\pm 1\}$ if QST is probed with *optimally localized states*.
Quantum Field Theory on the Basic Model of Quantum Spacetime was first developed in [1]; while fully Poincare' Covariant *Free* Field Theory (as Wightman Fields on QST, or as Poincare' Covariant nets of von Neumann Algebras labelled by projections in the Borel completion of $\mathcal{E}$, which specify "noncommutative regions" in QST) can be explicitly costructed, and its violation of causality computed, all attempts to construct *interacting* QFT on QST seem to lead sooner or later to violations of Lorentz invariance, besides the inevitable violations of causality.
In the first approach to QFT on QST in [1], we gave a natural prescription, not leading to interacting fields, but to a perturbative expansion of the S - Matrix, *without manifest violations of unitarity*; but interaction required an integration over Σ, thus breaking Lorentz invariance (there is no finite invariant measure or mean Σ).

The approach based on Yang - Feldman Equation defines perturbatively covariant interacting fields, but Lorentz invariance will break a) at the level of renormalization b) at the level of asymptotic states [4, 6].
Al level a) a fully covariant procedure replaces Wick Products by *Quasiplanar Wick Products* where one subtracts only terms which are *local* and divergent on QST [6]; the above problems would riemerge with a further finite renormalization which ensures to recover the usual renormalized perturbative expansion in the large scale limit.
A more radical modification of the Wick product is suggested by the very quantum nature of Spacetime, the *Quantum Wick Product*; its use to define interactions *regularizes completely QFT in the Ultraviolet*; but the Lorenz covariance is broken here by the Quantum Wick Product itself (while no integration on Σ is needed), and the Adiabatic Limit poses serious problems [5].
Thus Lorenz breaking appears in all the above attempts through the presence of a non trivial centre of the (multipliers) Algebra of QST, whose spectrum is Σ: no finite invariant integration is possible and renormalization introduces a bad dependence on the points of Σ. More in the Reports by D. Bahns, K. Fredenhagen and G. Piacitelli in this workshop.
A New Scenario. The Principle of Gravitational Stability ought to be fully used in the very derivation of ST Uncertainty Relations, which would then depend also on the energy-momentum density of generic background quantum states; this leads to commutation relations between Spacetime coordinates depending in principle on the metric tensor, and hence on the interacting fields themselves, thus appearing as part of the equations of motions along with Einstein and matter field Equations; this new scenario proposed in [7] appears extremely difficult to formulate, but promises most interesting developments. Notably it would be related to the nonvanishing of the Cosmological Constant [7] and would explain Thermodynamical Equilibrium of the early Universe without Inflation; while QST might teach us something about dark matter if, as expected, it implies a *minimal size* for black holes, where Hawking evaporation would stop.

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Noncommutative spectral invariants and black hole entropy

Roberto Longo

(joint work with Y. Kawahigashi)

I have explained how one can use Operator Algebraic methods to define an intrinsic entropy associated with a local conformal net $\mathcal{A}$. One considers the coefficients in the expansion of the logarithm of the Trace of the "heat kernel" semigroup. In analogy with Weyl theorem on the asymptotic density distribution of the Laplacian eigenvalues, passing to a quantum system with infinitely many degrees of freedom, we regard these coefficients as noncommutative geometric invariants. Under a natural modularity assumption, the leading term of the entropy (noncommutative area) is proportional to the central charge c, the first order correction (noncommutative Euler characteristic) is proportional to $\log \mu_{\mathcal{A}}$, where $\mu_{\mathcal{A}}$ is the global index of $\mathcal{A}$, and the second spectral invariant is again proportional to c.
A further general method exists to define a mean entropy by considering conformal symmetries that preserve a discretization of S^1 and we get the same value proportional to c.
One can then make the corresponding analysis with the proper Hamiltonian associated to an interval. We find here, in complete generality, a proper mean entropy proportional to $\log \mu_{\mathcal{A}}$ with a first order correction defined by means of the relative entropy associated with canonical states.
By considering a class of black holes with an associated conformal quantum field theory on the horizon, and relying on arguments in the literature, I have indicated a possible way to link the noncommutative area with the Bekenstein-Hawking classical area description of entropy.

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Nonsmooth Symmetries and the Existence of Diffeomorphism Covariance in Chiral QFT

Mihály Weiner

(joint work with S. Carpi)

The presented talk concerned chiral components of 1+1 dimensional conformal Quantum Field Theory in the framework of Haag-Kastler nets. In this setting a chiral component is described by a net of von Neumann algebras $I \mapsto \mathcal{A}(I)$ associating to each open proper interval of S^1 a von Neumann algebra of a fixed Hilbert space, together with a unitary representation U of the group of Möbius transformations. The pair $(\mathcal{A}, U)$ is required to satisfy the natural conditions of *isotony, locality, covariance, positivity of energy, existence, uniqueness and cyclicity of vacuum*; see e.g. [3, 4] and [2] for more on axioms, basic consequences and for an introduction to some models like the ones coming from the so-called *loop-group construction*.

Diffeomorphism covariance is a feature specific to 1+1 dimensional conformal QFT and to one-dimensional chiral components. A higher dimensional field theory — for simple geometric reasons — can never exhibit such symmetries. To the contrary, most (but not all: see [5, 6] for first examples, and the construction that will be here explained) low dimensional conformal QFT models on the circle are covariant under a suitable action of $\mathrm{Diff}^+(S^1)$. As it is known, (see e.g. [7]), at the level of Wightmann-fields, the existence of diffeomorphism symmetry is essentially equivalent with the existence of a stress-energy tensor, i.e. a local field whose integral is the total energy.

One may take the existence of a stress-energy tensor as a physically motivated requirement, and thus exclude from the study all models admitting no diffeomorphism symmetry. Nevertheless, there are some reasons — more than just pure mathematical interest — to find an algebraic characterization of the existence of $\mathrm{Diff}^+(S^1)$ symmetry.

As an example, consider the following. There are a number of known methods (using half-sided modular inclusions, or, in case of Wightmann-fields, by restrictions) by which, starting from a higher dimensional QFT model, one can obtain a chiral net. Then, in a particular example, we may find that the chiral net obtained is diffeomorphism covariant. However, this is not something that we immediately see: the higher dimensional model was surely not diffeomorphism covariant, so it is not a property which is "inherited" from the original model. (Note also that the restriction of the stress-energy tensor of a higher dimensional model, by its scaling dimension, cannot be the stress-energy tensor of the lower dimensional one.)

In the talk I presented new results pointing towards an (algebraic) characterization

of the existence of $\mathrm{Diff}^+(S^1)$ symmetry in chiral conformal QFT. In particular, one can show [9, Chapter 4] that under some algebraic condition (which, for example, in case of complete rationality, is automatically satisfied), the Möbius symmetry of the net extends to the larger geometrical group consisting of once differentiable, piecewise Möbius transformations (PCWM). The main ingredient of the proof is the modular theory of von Neumann algebras.
This larger group is "almost as big" as $\mathrm{Diff}^+(S^1)$. For example, one can show that for every $\gamma \in \mathrm{Diff}^+(S^1)$ there exists a sequence of once differentiable PCWM transformations $(\gamma)_{(n\in\mathbb{N})}$ converging uniformly to γ. (In fact, this is even true in a stronger, local sense.) But what is more important, is that this sequence can be chosen so that for *any* chiral PCWM covariant net, given that it is also $\mathrm{Diff}^+(S^1)$ covariant, we have strong convergence of the corresponding (projective) unitary operators [9, Theorem 6.1.3].
So, even if *a priori* we do not assume diffeomorphism covariance, starting from PCWM symmetry, by taking limits one should obtain $\mathrm{Diff}^+(S^1)$ symmetry. Unfortunately, this has not been achieved because of technical problems about convergence.
One may also consider the inverse problem: starting from $\mathrm{Diff}^+(S^1)$ covariance, can we obtain PCWM symmetry? To do so — as such a symmetry is based on nonsmooth (only once differentiable) transformations, one needs to evaluate the stress-energy tensor T on nonsmooth functions. In this respect, the following result is achieved in the joint work [1] with S. Carpi (see also [9, Chapter 5]): if the continuous real function on the circle f satisfies

$$\|f\|_{\frac{3}{2}} \equiv \sum\nolimits_{n\in\mathbb{Z}} |\hat{f}_n|(1+|n|^{\frac{3}{2}}) < \infty$$

where $\hat{f}_n \equiv \frac{1}{2\pi}\int_0^{2\pi} f(e^{i\theta})e^{-in\theta}d\theta$ $(n \in \mathbb{N})$, then the sum $\sum_{n\in\mathbb{Z}} \hat{f}_n L_n$ is essentially self-adjoint and so $T(f)$ can be defined. Moreover, if $f_n \to f$ in the sense of the above defined $\|\cdot\|_{\frac{3}{2}}$ norm, then

$$T(f_n) \to T(f)$$

in the strong resolvent sense.
As mentioned, at the moment there is some unresolved technical difficulties in constructing $\mathrm{Diff}^+(S^1)$ symmetry out of PCWM symmetry. Nevertheless, these nonsmooth symmetries turned out to be useful in various applications.
In particular such symmetries are directly constructed from the local algebras and the vacuum vector and so they are unique. This, by what was previously explained, can be used to show [1] that if the net is diffeomorphism covariant, its diffeomorphism symmetry is unique. (See also [9, Section 6.1] where the statement is reproved so that the regularity condition used in [1] can be dropped.) Also, using similar considerations it is proved [9, Section 6.2] that a (Möbius covariant) subnet of a diffeomorphism covariant net is automatically diffeomorphism covariant.
By these results, it is further shown that the tensor product of an infinite number of (nontrivial) nets is never diffeomorphism covariant ([1] and see also [9, Section 6.3] for the general case). Thus one can easily give new examples of nets admitting

no diffeomorphism symmetry (even strongly additive examples — note that the examples that were known before and appear in [5, 6], are *not* strongly additive). Finally, let us mention, that the recent argument of [8] showing that every locally normal representation of a diffeomorphism covariant net is automatically of positive energy, is also based on the use of nonsmooth symmetries.

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Geometric Modular Action in QFT and QFTs from Geometric Modular Action

Gandalf Lechner

As the title indicates, this talk is divided into two parts. In the first part, we give a short review of the basic concepts of Tomita-Takesaki modular theory and recall some of the well-known results on geometric modular action. In particular, we emphasize the significance of the Bisognano Wichmann theorem [1] and the theorem of Borchers [2]. These theorems state under quite general assumptions on the underlying quantum field theory that the modular group of $(\mathcal{A}, \Omega)$, where $\mathcal{A}$ is the algebra generated by observables localized in a wedge region (*i.e.* a Poincaré transform of the right wedge $W_R := \{x \in \mathbb{R}^d : x_1 > |x_0|\}$) and the vacuum vector Ω, coincides with the group of boosts leaving the wedge invariant. Furthermore, the corresponding modular conjugation is identified as the TCP operator of the theory (up to a rotation).

In the second part of the talk, we consider the inverse problem: Instead of discussing quantum field theories in which certain modular groups act geometrically as boost transformations, quantum field theories in the sense of Haag-Kastler are *constructed* from representations of the Poincaré group by using the aforementioned theorems as an input. Such an approach was followed in [3], where

Brunetti, Guido and Longo succeeded in constructing free field theories from a given representation of the proper Poincaré group in a purely quantum physical, algebraic setting. Because the emerging theories are interaction-free, their construction can essentially be carried out in the one particle space $\mathcal{H}_1$ by considering nets of localized subspaces in $\mathcal{H}_1$. Only in the final step of the construction they pass to the corresponding net of observable algebras with the help of the Weyl functor.

When trying to transfer this program to the interacting case, one meets two complications. Firstly, the representation of the Poincaré group is not a sufficient input since the interaction has to be fixed as well. Secondly, it is not possible to work on the one particle space; one has to carry out the construction on the level of observable algebras from the outset.

As the "free" and "interacting" TCP operators of a quantum field theory are related by its S-matrix S, the scattering operator suggests itself as an additional input to describe the interaction. The intended construction is thus the solution of the inverse scattering problem in quantum field theory. Schroer and Wiesbrock discovered that there is a special family of S-matrices in which the two mentioned problems can be overcome [8], namely the family $\mathcal{S}$ of factorizing S-matrices on two-dimensional Minkowski space. In contrast to the case of a general S-matrix, the kinematical constraints are in this situation strong enough to fix the most general form of S, turning it into a useful input for the inverse scattering problem.

The problem of constructing a net of observable algebras associated to a factorizing S-matrix $S \in \mathcal{S}$ can be solved as follows: In a first step, a Hilbert space $\mathcal{H}$ (depending on the chosen S) is constructed, on which the Poincaré group and Zamolodchikov's algebra (a kind of S-deformed CCR algebra) are represented. Schroer realized that these objects allow for the definition of certain wedge-localized, polarization-free quantum fields, which generate a net $W \mapsto \mathcal{A}(W)$ of wedge algebras acting on $\mathcal{H}$, which satisfies the usual axioms of isotony, Haag duality and covariance, and has the vacuum Ω as a cyclic and seperating vector for each wedge algebra [8, 5].

Furthermore, this net has the modular data as expected from the Bisognano Wichmann theorem, and in particular the modular conjugation of $(\mathcal{A}(W_R), \Omega)$ is SJ_0, where J_0 denotes the TCP operator of the free theory. We mention that it is possible to verify that the observables of this net reproduce the two-particle scattering behaviour given by the input S-matrix S.

It is an important question within this approach to the inverse scattering problem whether in the so constructed wedge local theories there also exist observables localized in bounded regions of Minkowski space, such as double cones $\mathcal{O} = W_1' \cap W_2$, where $W_1 \subset W_2$ is an inclusion of wedge regions. The maximal algebra of observables localized in $\mathcal{O}$ is $\mathcal{A}(\mathcal{O}) := \mathcal{A}(W_1)' \cap \mathcal{A}(W_2)$. Proving that this algebra is non-trivial, $\mathcal{A}(\mathcal{O}) \neq \mathbb{C} \cdot 1$, amounts to proving the existence of a local quantum field theory with a prescribed factorizing S-matrix. We report here on recent advances to this question (for some older partial results, see [6]).

We recall that the existence problem for local observables has an affirmative answer if the net of wedge algebras has the split property, and that the split property is in turn a consequence of the modular nuclearity condition [4]. The modular nuclearity condition states that the map $\mathcal{A}(W_1) \ni A \mapsto \Delta_{W_2}^{1/4} A\Omega \in \mathcal{H}$ is nuclear (Δ_{W_1} denotes the modular operator of $(\mathcal{A}(W_1), \Omega)$).
As a new result, we announce that there is a subfamily $\mathcal{S}_0 \subset \mathcal{S}$ of factorizing S-matrices, characterized in terms of the distribution of resonances of the two particle S-matrix, such that the modular nuclearity condition holds for any inclusion of wedge regions [7]. Consequently, in the corresponding model theories there exist observables localized in arbitrarily small double cones, and the inverse scattering problem has a solution for S-matrices $S \in \mathcal{S}_0$.

We conclude the talk with a more speculative outlook about possible generalizations of the described construction program to higher dimensional Minkowski space.

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Aspects of noncommutative geometry in string theory

Stefan Fredenhagen

Noncommutative geometry naturally appears in open string theory. An interaction vertex of open strings cannot be deformed such that two legs are exchanged. In going to the particle limit of string theory, it is possible to maintain this feature by a suitable rescaling of background fields. This talk should illustrate the basic mechanism in two examples.
I discussed the simplest example where a noncommutative field theory emerges in string theory: an open string moves in a flat space-time with a constant two-form B-field (see e.g. [1]). The B-field causes the space-time coordinates to satisfy non-trivial commutation relations. In the particle limit where the string tension is taken to be very large, one has to rescale the metric and the B-field to obtain a finite open string metric and a finite matrix $\theta^{\mu\nu}$ that determines the commutation relations, $[x^\mu, x^\nu] = i\theta^{\mu\nu}$.
This non-commutativity directly affects the correlation functions of vertex operators and thereby the effective field theory on the brane. The fields are functions on

the world-volume of the brane where the product is deformed to the Moyal-Weyl product determined by θ.
A second example was discussed where a non-trivial three-form field is present: open strings on the group manifold of SU(2). I focussed on maximally symmetric D-branes on SU(2) which are localised along conjugacy classes, in SU(2) these are two-spheres (and points). It turns out that, in a particle limit, the fields of the effective field theory live on a fuzzy sphere [2]. The algebra of functions is finite-dimensional which is interpreted as a truncation of the spherical harmonics at a maximal spin determined by the size of the brane. A gauge theory which is a combination of a Yang-Mills and a Chern-Simons part determines the dynamics of the brane.
These two examples should only give some impression of how noncommutative geometry appears in string theory. Further developements and the various applications e.g. to describe D-brane dynamics unfortunately could not be covered in this talk.

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Spectral Triples of Holonomy Loops

Jesper M. Grimstrup

(joint work with J. Aastrup)

A noncommutative geometry in the sense of Connes [1] is determined by a spectral triple (B, D, H) which consist of an C^*-algebra B represented on a hilbert space H on which a self-adjoint unbounded operator D, the Dirac operator, acts. The triple is normally required to satisfy a set of seven axioms proposed by Connes [2]. Ordinary Riemannian spin-geometries form a subset in this framework and are described by commutative C^*-algebras.
A special class of noncommutative geometries are the *almost commutative geometries* described by spectral triples characterized by an algebra of the form

$$B \quad = \quad C^\infty(M) \otimes B_F \ , \tag{1}$$

where B_F is a finite dimensional matrix algebra of size n. The almost commutative geometry given by

$$B_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C}) \ , \tag{2}$$

where $\mathbb{H}$ denotes quarternions, is the algebra which forms the basis of the formulation of the standard model in terms of noncommutative geometry [2].
The point we wish to stress here is that by using the language of noncommutative geometry the standard model coupled to gravity can be formulated as a

single, purely gravitational theory. This fact suggests that, within the framework of noncommutative geometry, it cannot be quantized in any straightforward manner since such a quantization would, accordingly, involve quantum gravity. On the other hand, this argument implies that the search for a suitable quantization scheme might pass through quantum gravity. This is the problem which we wish to address: How principles of noncommutative geometry might be unified with ideas on quantum gravity.

Intersecting noncommutative geometry and quantum gravity

Consider first Quantum field theory. It involves, via Feynman path integrals, integration theory over spaces of field configurations. The central object is the partition function, the generating functional for Greens functions

$$Z[J] = \int [d\Phi] \exp(-\tfrac{i}{\hbar} S[\Phi, J]) \,, \tag{3}$$

where Φ denotes the field content of the model characterized by the classical action $S[\Phi]$ coupled to external fields J. We now propose the following: Since Connes formulation of the standard model lacks a clear quantization procedure and since quantum field theory deals with integration theory over spaces of field configurations, it seems natural to try to apply the machinery of noncommutative geometry to functional spaces as a possible solution to the problem of unifying principles of noncommutative geometry with those of quantum field theory. Further, since Connes formulation of the standard model is essentially a gravitational theory we suggest to investigate a configuration space related to gravity.

In fact, a configuration space suitable for our purposes has already been investigated in the literature. Loop Quantum Gravity (LQG) [3] is an attempt to quantize general relativity using methods of canonical quantization. The configuration space relevant for LQG is a space $\mathbb{A}$ of $SU(2)$ connections which are interpreted as certain spin-connections living on a 3-dimensional hyper-surface. This surface emerges from a foliation of 4-dimensional space-time which is needed for the quantization procedure.

Central to LQG is an algebra of Wilson loops $W(L)$ which form an abelian algebra of observables on the space of connections

$$\begin{aligned} W(L) : \mathbb{A} &\rightarrow \mathbb{C} \,, \\ \nabla &\rightarrow Tr\,\mathrm{Hol}(L, \nabla) \,, \end{aligned} \tag{4}$$

where $\mathrm{Hol}(L, \nabla)$ is the holonomy of the connection ∇ along the loop L and Tr is the trace with respect to the representation of the group. One of the advantages of this formulation is that it permits a natural implementation of diffeomorphism invariance in a way that leads to a countable structure, including a separable hilbert: Roughly, the set of Wilson loops form certain labeled, oriented graphs of increasing complexity and, up to diffeomorphisms, only the structure of graphs is relevant. This structure is countable [4][1].

We believe that there exist a natural intersection between LQG and noncommutative geometry: Instead of using Wilson loops we suggest to study the noncommutative algebra of holonomy loops themselves. By avoiding the trace the gauge symmetry of local lorentz transformations is preserved. Further, since the objective is to apply the machinery of noncommutative geometry to the functional space, rather than a canonical quantization procedure, we propose to consider space-time as a whole and avoid a foliation. Thus, we consider an algebra of space-time holonomy loops

$$\begin{aligned} L : \mathbb{A} &\rightarrow G \, , \\ \nabla &\rightarrow \mathrm{Hol}(L, \nabla) \, , \end{aligned} \tag{5}$$

where G is the symmetry group. Since compactness of the gauge group is at present needed for the analysis, we are at first forced to consider Euclidean gravity with, for example,

$$G = SO(4) \; .$$

Finally, rather than postulating constraints on the Hilbert space, such as the Hamilton constraint in LQG, we suggest to apply the spectral action principle [5]: To seek physical information in the spectrum of the Dirac operator.
This intersection of LQG and NCG contains all the ingredients we are looking for: Integration theory over a functional space related to gravity involving a natural noncommutative algebra.
The holonomy loops are matrix valued and can be heuristically argued to entail an almost commutative algebra in a classical limit characterized by a single space-time geometry [6], that is, a single point in $\mathbb{A}$. This is encouraging and provides us with the hope/vision/fantasy that low energy physics characterized by an almost commutative algebra may arise as the classical limit of a pure quantum gravity.

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[1] In fact, it turns out that so-called extended diffeomorphisms are required to obtain countable structures. Extended diffeomorphisms permit finitely many non-smooth points [4].

Noncommutative Geometry and Gravity

Paolo Aschieri

The study of the structure of spacetime at Planck scale, where quantum gravity effects are non-negligible, is one of the main open challenges in fundamental physics. Since the dynamical variable in Einstein general relativity is spacetime itself (with its metric structure), and since in quantum mechanics and in quantum field theory the classical dynamical variables become noncommutative, one is strongly lead to conclude that noncommutative spacetime is a feature of Planck scale physics.
A first question to be asked in this context is whether one can consistently deform Riemannian geometry into a noncommutative Riemannian geometry. In [1] we address this question by considering deformations of the algebra of functions on a manifold obtained via a quite wide class of $\star$-products. In this framework we successfully construct a noncommutative version of differential and of Riemannian geometry, and we obtain the noncommutative version of Einstein equations.
Even without physical motivations, the mathematical structure of deformed spaces is a challenging and fruitful research arena. It is very surprising how well $\star$-noncommutative structures can be incorporated in the framework of differential geometry.

The $\star$-products we consider are associated with a deformation by a twist $\mathcal{F}$ of the Lie algebra of infinitesimal diffeomorphisms on a smooth manifold M. Since $\mathcal{F}$ is an arbitrary twist, we can consider it as the dynamical variable that determines the possible noncommutative structures of spacetime. Examples of the noncommutative spacetime structures we obtain include the Moyal-Weil (or θ-constant) noncommutative space and the quantum (hyper)plane $xy = qyx$.
The twists $\mathcal{F}$ is an element $\mathcal{F} \in U\Xi \otimes U\Xi$, where $U\Xi$ is the universal enveloping algebra of the Lie algebra of vectorfields (infinitesimal diffeomorphisms). Since vectorfields act on functions, forms and tensorfields, using the twist $\mathcal{F}$ we canonically deform these spaces into the $\star$-noncommutative spaces of functions, forms and tensorfields. The Lie algebra of vectorfields is similarly deformed to a $\star$-Lie algebra (in the spirit of [2] and [3]). Furthermore we show that this deformed Lie algebra has a deformed action on the noncommutative spaces of functions, forms and tensorfields. We have thus constructed a tensor calculus that is covariant under infinitesimal noncommutative diffeomorphisms. (In the special case of θ-constant noncommutativity, if we choose the preferred coordinate system $[x^\mu, x^\nu] = i\theta^{\mu\nu}$ we recover the results of [4]).
The $\star$-covariant derivative is then defined in a global coordinate independent way. Locally the $\star$-covariant derivative is completely determined by its coefficients $\Gamma^\sigma_{\mu\nu}$. Using the deformed Leibniz rule for vectorfields we extend the $\star$-covariant derivative to all type of tensorfields.
Having the covariant derivative it is easy to guess the expression for the noncommutative curvature and torsion. Then one has to show that these operators are well defined noncommutative tensors. This is done by showing that they are (left)

$A_\star$-linear maps on vectorfields, where $A_\star$ is the space of noncommutative functions. Also the noncommutative Ricci tensor is singled out by requiring it to be a (left) $A_\star$-linear map.
Finally the metric is an arbitrary $\star$-symmetric element in the $\star$-tensorproduct of 1-forms $\Omega_\star \otimes_\star \Omega_\star$. Using the $\star$-pairing between vectorfields and 1-forms the metric is equivalently described as an $A_\star$-linear map on vectorfields, $(u, v) \mapsto g(u, v)$. The scalar curvature can then be defined and Einstein equations on $\star$-noncommutative space are obtained. The requirement of $A_\star$-linearity uniquely fixes the possible ambiguities arising in the noncommutative formulation of Einstein gravity theory. We have a deformed gravity theory because we can impose reality conditions on the spaces of noncommutative functions, vectorfields and tensorfields, so that for example the metric tensor is real.

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Renormalization of Euclidean Quantum Field Theories

Harald Grosse

(joint work with Raimar Wulkenhaar)

The idea that the problems of quantum field theory may be cured by deforming space-time goes back to Snyder. After Alain Connes developed noncommutative geometry a new development set in. I personally learnt Fuzzy spaces through interactions with John Madore and we did formulate simple models on such spaces already in 1992. Interesting regularizations resulted, we were able to approximate quantum field theory by finite degrees of freedom preserving symmetries[1]. In such models like the Fuzzy sphere, Fuzzy CP^N, Fuzzy tori etc. an embedded sequence of matrix algebras is found, such that the action converges in a particular sense towards the action of a continuum model. This way one introduces a cutoff. We did extend these ideas to handle nontrivial topological contributions and certain supersymmetric models without violating supersymmetry.
Removing it again leads back the well-known divergences.
But additional problems may result: The Feynman rules for a field theory on the canonical deformed Euclidean four dimensional space has been worked out by Filk in 1995. Feynman diagrams split into planar and nonplanar ones. The planar diagrams show the same UV-divergences like they occur in the undeformed theory, but the nonplanar diagrams turn out to be convergent for non-exceptional

momenta. For exceptional momenta new divergences in the infrared region occur. This phenomenon is called the IR/UV mixing. No general scheme has been found to handle these divergences.
In our work we analyzed this problem for a scalar field theory. In a first step we transformed the model to a matrix base: The interaction term becomes a matrix product and no oscillations occur anymore. Next we used renormalization ideas of Wilson and Polchinski and deduced a power counting law for ribbon graphs drawn on a Riemann surface of a particular genus with marked points. The iterative solution of the Polchinski equation can now be handled. We realized that the decay properties of the free propagator are essential for renormalzability. Since the resulting free Greens function does not have the necessary decay property, we did put the system into a box through the coupling to an oscillator potential. The resulting model has now four relevant/marginal operators in the action and we have been able to show that the model becomes renormalizable. Our proof follows ideas of Wilson and Polchinski, we proofed first a power counting theorem for general Ribbongraphs, this allowed us to identify all graphs drawn on a Riemannsurface of nonzero genus and those with higher boundary components as irrelevant[2]. Only two and four point functions remained. But there are still infinitely many. Next we followed the old ideas of Bogoliubov and used a discrete difference procedure to identify relevant/marginal operators: There remained only four, just those, with which we started with. This is the only model which shows this nice feature [3]. Calculation of the beta function shows special features of the models at the self-dual point, particular interactions become integrable [4].
At the workshop various other model calculations were presented, all of them show the IR/UV mixing problem. Through interactions with Edwin Langmann we realized that the addition of magnetic fields may work too.
But many problems remain and have been addressed during this workshop: The transformation to Lorentzian signature is unclear, how to handle other models especially the formulation of a renormalizable noncommutative gauge model is not settled. How much we may learn concerning the quantization of gravity is of course unclear too.
At this workshop all contributions were of great interests for me, the program was chosen coherently and the atmosphere was great. I enjoyed all contributions.

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Renormalisation of the Noncommutative Gross-Neveu Model

FABIEN VIGNES-TOURNERET
(joint work with Razvan Gurau, Vincent Rivasseau and Raimar Wulkenhaar)

Historically noncommutative spacetimes have been introduced in order to cure the ultraviolet divergences of quantum field theories. This attempt remained more or less out of main stream theoretical physics until the discovery that noncommutative quantum field theories (NCQFT) arise in certain compactifications of M-theory [1] or as limiting cases of string theory [2, 3]. Then these theories became fashionable and the hope of a completely finite QFT was revived. Unfortunately NCQFT are not only divergent but also exhibit a new kind of divergences. This phenomenon called UV/IR mixing makes the theories not renormalisable [4, 5, 6]. Such an entanglement between the ultraviolet and infrared sectors appears in non-planar graphs. For *non exceptionnal* incoming momenta, these graphs are UV-finite. But inserted into a bigger loop they diverge for small momenta (IR). By inserting enough such dangerous graphs into loops, one can make any amplitude divergent. This is typically the sign of a non renormalisable theory. Nevertheless Harald Grosse and Raimar Wulkenhaar proved recently that there is a way to define a *renormalisable* noncommutative field theory. They began by proving a power-counting theorem for non-local matrix models [7]. Then they were able to prove the perturbative renormalisability of the ϕ^4-theory on the two and four dimensionnal Moyal spaces [8, 9] by rewriting the theory in the matrix base [10]. The key-point was to notice that the usual noncommutative extension (1) of the ϕ^4-theory is not covariant under the Langmann-Szabo duality [11].

$$S = \int dx\, \frac{1}{2}\phi\left(-\Delta + m^2\right)\phi + \frac{\lambda}{4!}\phi\star\phi\star\phi\star\phi \tag{1}$$

That is the action functional has not the same expression in x- and in p-space. The vertex $\phi^{\star 4}$ is self-dual but the kinetic term is not. The Laplacian p^2 transforms into x^2 which doesn't belong to the action. It seems that the Langmann-Szabo duality is related to the renormalisability of the theory. In fact H. Grosse and R. Wulkenhaar added that harmonic potential term (2) which rendered the theory renormalisable. The breaking of the translation invariance seems to be the price to pay for a renormalisable theory.

$$S = \int dx\, \frac{1}{2}\phi\left(-\Delta + +\Omega^2\tilde{x}^2 + m^2\right)\phi + \frac{\lambda}{4!}\phi\star\phi\star\phi\star\phi \tag{2}$$

Actually the original proof of the perturbative renormalisability of this duality-covariant noncommutative ϕ^4-theory used numerical studies of the propagator. V. Rivasseau, R. Wulkenhaar and myself proved rigorous bounds on its propagator [12]. Moreover we performed a multi-scale analysis of this theory and gave a shorter proof of its power-counting. We used the notion of dual graphs which allowed to solve in a very simple way the constraints on the indices of the propagator. These constraints come from the rotational invariance of the theory. The final aim of that work was to put the renormalisability proof of H. Grosse and R. Wulkenhaar

in a suitable setting for future constructive purposes [13, 14].
One can now ask how general is the procedure of Grosse and Wulkenhaar? Can we rely on the Langmann-Szabo duality to build renormalisable noncommutative field theories? In a first step before a complete answer, we study the Gross-Neveu model on the Moyal plane (3). Note that we wrote here only one of the four possible quartic interactions with fixed sign of the coupling constant λ.

$$S=\int dx\,\bar{\psi}\frac{1}{2}\left(\gamma_\mu p^\mu+\Omega\gamma_\mu\tilde{x}^\mu+m\right)\psi+\frac{\lambda}{4!}\bar{\psi}\star\psi\star\bar{\psi}\star\psi \tag{3}$$

As usual the star-product interaction makes the vertex oscillating. The different amplitudes are no longer absolutely convergent which is harder to handle. Then we decided to use again the matrix base. The main advantage of this base is the very simple form of the vertex which becomes Tr ϕ^4. Its drawback is the complexity of the propagator. On $\mathbf{R}^2$ the power-counting of the Gross-Neveu model is half the ϕ^4_4's one and the behaviour of its propagator is approximately the same as in ϕ^4_4. We found that the propagator of the noncommutative Gross-Neveu model does not behave as the ϕ^4_4's one. The bounds we proved in [12] are no longer valid. For example, there is a region in the indices where the propagator "decreases" in only two directions in place of three [15]. That means we can only sum over two indices of the propagator. This fact will modify the proof of the power-counting but doesn't modify the power-counting itself [16]. Moreover one and two-loop computations let us think that the model is renormalisable. The proof to all orders is in progress [16].

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Locality in Quantum Field Theory on the Noncommutative Minokwski space

Dorothea Bahns

Starting point of our investigations is the free massive quantum field ϕ on the noncommutative Minokowski space $\phi(q) = \int d\mu(k) \left(a(k) \otimes e^{-ikq} + a^*(k) \otimes e^{ikq}\right)$ with $kq = k_\mu q^\mu$, $k \in \mathbb{R}^4$ as defined in [1]. Here, $a(k)$ and $a^*(k)$ denote the ordinary annihilation and creation operators, $d\mu(k)$ is the invariant measure on the mass shell, and q^μ, $\mu = 0, 1, 2, 3$, denote the quantum coordinates as defined in [1], i.e. we have $[q^\mu, q^\nu] = iQ^{\mu\nu}$ and the spectrum of the central commutator Q is fixed in a Lorentz invariant way (though throughout this paper, a point θ is chosen from this spectrum in order not to obscure the essential point). After evaluation in a suitable state ω on the algebra generated by the quantum coordinates, we have an operator $\phi(\omega)$ on Fock space with $\phi(\omega) = \varphi(\psi_\omega)$, where φ is a field on *ordinary* Minkowski space, evaluated in a testfunction ψ_ω defined by $\hat{\psi}_\omega(k) := \omega(e^{ikq})$ (the hat denoting Fourier transform).
Unfortunately, for the same reason as in ordinary field theory, the product of two such fields is ultraviolet divergent, as the function $\hat{\psi}_{\theta,\omega}(k,p) := \omega(e^{ikq}\ e^{ipq})$ (which also depends on θ by the appearance of the so-called twisting $\exp\left(-\frac{i}{2}k\theta p\right)$ arising from the Weyl relation) decreases quickly only in $k+p$. We therefore have to come up with a good principle to define a sensible renormalization procedure.
In ordinary field theory this is the principle of locality, which on the noncommutative Minkowski space has various generalizations [2, 3]. The one I am concerned with in this talk is based on the following observation. First we define a regularized field $\phi_f(\omega) := \varphi(\psi_\omega \times f)$, where f is a suitable test function on $\mathbb{R}^4$ and where the $\times$ denotes ordinary convolution (choosing the δ-distribution for f, we recover $\phi(\omega)$). Formally, we may then write $\phi_f(q) = \int dx\ \phi(q + xI)\ f(x)$, where $x \in \mathbb{R}^4$, and where I denotes the identity in the algebra of quantum coordinates (the symbol I is dropped in what follows). This definition gives rise to well-defined products of fields $\phi_f^{\otimes n}(q) = \int dx_1 ... dx_n\ \prod \phi(q + x_i)\ f(x_1, \ldots, x_n)$. Now consider

the Wick product of three such fields,

$$:\prod_{i=1}^{3}\phi(q+x_i): = \prod_{i=1}^{3}\phi(q+x_i) - \Delta_+(x_1-x_2)\phi(q+x_3) - \Delta_+(x_2-x_3)\phi(q+x_1)$$
$$- \int d\mu(k)\ e^{-ik(x_1-x_3)}\ \phi(q+x_2+\theta k)\ ,$$

where Δ_+ denotes the ordinary 2-point function. To calculate the above, we have passed to momentum space and used the *ordinary* commutation relations of annihilation and creation operators. It is obvious that for coinciding points, i.e. evaluation of $:\prod_{i=1}^{3}\phi(q+x_i):$ in $f(x_1,x_2,x_3) = g(x_1)\delta(x_2-x_1)\delta(x_3-x_1)$, the first two subtraction terms become ill-defined as usual. However, the third term yields $\int dx\ \phi(q+x)\ \int d\mu(k)\ g(x-\theta k) =: \phi_{\gamma(g)}(q)$ which, after evaluation in a state ω, is an operator on Fock space[1]. We therefore do not have to subtract it in order to give meaning to the left hand side of the equation. The crucial point now is that this term *must not* be subtracted since it violates even a *minimalistic* locality principle: take g to be a testfunction with compact support, then we find that supp $\gamma(g) \not\subset$ supp g. It was shown in [4], that the supports of g and $\gamma(g)$ may even become disjoint.

In [2] we have given a definition of modified Wick products, the so-called quasiplanar Wick products, where only counterterms subject to this minimalistic locality principle of not increasing the support of testfunctions are admitted. Without quoting the details here, let me point out that such a product $\vdots\prod_{i=1}^{n}\phi(q+x_i)\vdots$ is recursively defined as

$$\phi(q+x_1)\vdots\prod_{i=2}^{n}\phi(q+x_i)\vdots - \sum_{k=1}^{[\frac{n}{2}]}\mathcal{C}_{2k}(\theta;x_1,\ldots,x_{2k})\vdots\prod_{i=2k+1}^{n}\phi(q+x_i)\vdots$$

where the $\mathcal{C}_{2k}(\theta;x_1,\ldots,x_{2k})$ are *numerical* distributions (certain vacuum expectation values of $2k$ fields) which, apart from the lowest order ($k=1$, where $\mathcal{C}_2 = \Delta_+$) generally depend on θ. Note that in the limit of coinciding points with $g=\delta$, all counterterms are of the form $c\ \phi(q)^k$ with an (as usual infinite) constant c.

The proof that these products remain well-defined in the limit of coinciding points is quite technical. The idea is to show that all terms which are not subtracted in a quasiplanar Wick product compared to the respective ordinary Wick product are finite in the noncommutative setting. This involves an analytic continuation to handle oscillatory integrals which – as it turns out – renders the Schwartz functions unsuitable as test functions. We are preparing a publication [5], where the correct domain of definition for the quasiplanar Wick products is given.

Last but not least, I would like to comment on the so-called infrared-ultraviolet mixing problem. This effect (an ultraviolet finite graph giving rise to serious infrared divergences when inserted into a larger graph) has so far been observed only in field theories on a noncommutative Euclidean flat space, and seems (see,

[1] This is even true when $g=\delta$, where this contraction yields $\int dp\,\hat{\phi}(p)\,\Delta_+(\theta p)\,e^{-ipq}$, since $\Delta_+(\theta p)$ is bounded (and even quickly decreasing in some p-directions) for $p^2=m^2$.

for instance [6, 7, 8]) to be absent in (unitary) approaches on the noncommutative Minkowski space, such as the Hamiltonian approach. This is not surprising in view of the fact that the relation between this Euclidean formulation and the Minkowski version discussed here remains obscure: not only is there no Osterwalder-Schrader positivity theorem, but the Wick rotation itself has not been defined in a sensible way – and the Minkowskian version one naively guesses from the Euclidean formulation violates the optical theorem, unless the time variable commutes with all space variables.
Generally, the Euclidean field theory used in the literature does not seem to be of much use for understanding the structure of divergences in the noncommutative Minkowskian regime. A simple example is easily provided: in the ordinary Euclidean formulation, the vacuum expectation value of 4 fields where the 1^{st} and 3^{rd} and the 2^{nd} and 4^{th} field are contracted (a 2-tadpole), $\int dkdp \, \frac{1}{p^2+m^2} \, \frac{1}{k^2+m^2} \, e^{ik\theta p}$, is finite for a nondegenerate antisymmetric 4×4-matrix θ (to see this, use Schwinger parameters). But this is not true on the noncommutative Minkowski space, where in the limit of coinciding points with $g = \delta$, this contraction yields the formal expression $\int d\mu(p) \, \Delta_+(\theta p)$ (see discussion above), which is ill-defined as $\Delta_+(\theta p)$, while bounded for $p^2 = m^2$, does not decrease quickly in all p-directions.

However, we do find unusual infrared effects also on the noncommutative Minkowski space: employing the quasiplanar Wick products in the framework of the Yang-Feldman equation in massive scalar interacting theories, the asymptotic behaviour is seriously modified. In fact, the asymptotic fields have a mass which depends on the momentum and the modification of the dispersion relation is larger at smaller momenta (see [2] and J. Zahn's contribution).
Moreover, the infrared behaviour of massless fields is such that they cannot be renormalized using quasiplanar Wick products. To see this, consider again the Wick product of 3 fields. Analogously to the above discussion we find 2 ordinary contractions $D_+(x_1 - x_2)\phi(q + x_3)$ and $D_+(x_2 - x_3)\phi(q + x_1)$, while the third one yields

$$\int dp \; \delta(p^2)\theta(p_0) \; D_+(\theta p) \; \hat{\varphi}(p) \; e^{ipq}$$

with the Heaviside function $\theta(p_0)$. Now, by the same argument as used above, this term does not fulfill our minimalistic locality requirement, but contrary to the massive case, it is no longer finite. In fact, $D_+(\xi) = \frac{1}{|\vec{\xi}|} \; (\delta(\xi_0 + |\vec{\xi}|) - \delta(\xi_0 + |\vec{\xi}|))$ and for $\xi = \theta p$, $p^2 = 0$, the term above contains a product of δ-distributions. The ill-definedness occuring at $p = 0$, it is an *infrared* effect.

This does of course not at all put an end to the programme! Our hope still is that in massive scalar theories, all divergences (in the Hamiltonian or the Yang-Feldman approach) may be absorbed using quasiplanar counterterms only.
However, the strange infrared effects indicate that we should investigate models of noncommutativity other than those with central commutator – which is also

quite necessary if one wishes to incorporate gravity at some point. One investigation along these lines, where the effects of noncommutativity become smaller at larger distances has been pursued in the context of star products and deformation quantization [9].

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Perturbative QFT on DFR quantum spacetime

Gherardo Piacitelli

In a seminal paper [1], Doplicher, Fredenhagen and Roberts (DFR) laid down their basic model for a fully covariant, flat quantum spacetime, where the commutation relations among the coordinates q^μ are suggested by a stability condition of spacetime under localization alone (see also the report of S. Doplicher in this same workshop). By mimicking in a covariant way the Weyl–von Neumann quantization $f \mapsto f(q)$, a covariant twisted product $\star_Q$ of symbols (defined by $(f \star_Q g)(q) = f(q)g(q)$) was there proposed for the first time as a nonlocal replacement of the usual pointwise product of functions. A few years later, a reduced (non covariant) component $\star_\sigma$ of the DFR covariant twisted product became widely used[1] in the literature concerning noncommutative geometry, under the name of "Moyal product" (often the notation $\theta^{\mu\nu}$ is preferred to the original $\sigma^{\mu\nu}$).
In that paper, an approach to the perturbation theory of quantum (neutral, Klein–Gordon) fields was also proposed. The recipe was based on three ingredients:

(1) An ordinary local quantum field $\phi(x)$ (A "second quantized" field) is replaced by its DFR quantization ("third quantization") $\phi(q) = \int dk\check{\phi}(k) \otimes e^{ik_\mu q^\mu}$ understood as a formal element of $\mathfrak{F} \otimes \mathcal{E}$, where $\mathcal{E}$ is the localization C*-algebra generated by the coordinates, and $\mathfrak{F}$ is the field algebra.

[1] This is actually the same product only if the integral form is taken, or if its formal differential expansion is restricted to real–analytic functions.

(2) The interaction Hamiltonian density is then replaced by $:\phi(q)^n: = :\phi^{n\star}(q):$. Since the commutation relations allow for sharp localization in time, at the cost of total delocalization in space, it makes sense to integrate the Hamiltonian density over {t=0}, and to define an effective, perturbative, non local theory on Minkowski spacetime which, in the interaction representation, amounts to take an interaction Hamiltonian of the form

$$H_I(t) = g \int dx_1 \cdots dx_n \; K_t(x_1, \ldots, x_n) : \phi(x_1) \cdots \phi(x_n) :, \tag{1}$$

for a suitable kernel K_t (note that, by a suitable choice of the kernel, we might recover the usual local theory).

(3) Then define the perturbative solution in terms of the Dyson series (alone or inserted into the Gell–Mann & Low formula).

See the report of S. Doplicher in this same workshop for the sequel of this story. Here, we concern ourselves with the use of diagrams.

The basic problem is that, *as in the local case*, the time ordering of the Hamiltonians is taken according to the evolution parameter t, which however, *contrary to the local case*, is not the same as time components ${x_j}^0$ of the argument of each field $\phi(x_j)$ (see equation (1)). This apparently prevents the possibility of absorbing the time ordering into a suitable propagator, and seems to force us to consider different diagrams for different arrangements of times. A considerable effort has been devoted by many authors to this problem; they developed a complicate diagrammatic language to cope with this (apparent) obstruction. The unfortunate fact was that (apparently) the Wick theorems for the reduction of time ordered and mixed products of fields were lost, and all one was left with was the Wick reduction of ordinary Wick products.

Quite surprisingly, Denk and Schweda [2] were able to absorb the time ordering in the definition of a non local propagator. Not only the result was surprising, but the method they used, too. Actually, this was obtained by a complicate algebraic game of ad hoc manipulations of the two-points functions arising in the Wick reduction of ordinary Wick products, while adding together all similar diagrams corresponding to different orderings of the time parameters.

The reason why those authors did not recognize the very simple reason why this was possible may be that they did not consider Wick ordered Lagrangeans, so loosing the total symmetry, together with the possibility of further simplifying the expressions. But in my opinion, there was a psychological obstruction; I myself found it truly by chance, and many other authors missed it as well; this simple remark could have been done already ten years ago. The cause should be searched for in the tendency to overlook the role of locality, which is deeply rooted in the soul of any physicist working with quantum field theories.

The very simple underlying reason why one may incorporate the time ordering in the choice of a (non local) propagator is that the Wick theorem for the reduction of time ordered products has nothing to do with locality; the original proof [3] of Wick does not rely on locality, and its method can be used in the nonlocal setting considered here as well, with minor changes [4, 5].

As a consequence, Feynman and Dyson diagrams can be reobtained with no difficulties; exactly the same diagrams that are so efficient in the local case to label the various contribution to the S-matrix can be used in the present case.
Indeed, this should raise some (more) concerns about the current interpretation of Feynman diagrams as pictorial representations of local processes; it seems instead that diagrams only represent the interplay between time dependent perturbation theory and the CCR algebra, and none of these concepts has anything to do with locality.

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Dispersion relations in Noncommutative Quantum Field Theory

Jochen Zahn

We study Quantum Field Theory on the Noncommutative Minkowski space introduced by Doplicher, Fredenhagen and Roberts [1]. In the Yang-Feldman approach to Noncommutative Quantum Field Theory (NCQFT) [2, 4], the so-called UV/IR-mixing [3] shows up as a distortion of the dispersion relation in the infrared [5]. In the case of the ϕ^4 model this effect has been shown to be quite strong. The aim of this work is to study this phenomenon in other models in order to arrive at testable predictions. We present (partial) results for the ϕ^3 and Wess-Zumino model and Quantum Electrodynamics (QED).
We extract dispersion relations from the two-point function of the, perturbatively defined, interacting field, $\langle \phi_{int}(f)\phi_{int}(h)\rangle$. For a localized mass term as interaction, $S_{int} = M\mathrm{Tr}(g\phi^2)$, this can be done rigorously. In the adiabatic limit $g \to 1$ one finds, at first order in M, the expected result [6],

$$\int \mathrm{d}^4k \ \hat{f}(-k)\hat{h}(k)\theta(k^0)M\delta'(k^2-m^2).$$

In the interacting case, one finds, at 1-loop order, a similar expression, but with M replaced by a formal expression $\Sigma(k^2,(k\sigma)^2)$. Here σ is an element of the joint spectrum of the commutator of the coordinates q^μ. Such two-point functions may be described with nonlocal, i.e. momentum dependent, mass and field strength terms $M((\sigma k)^2) = -\Sigma(m^2,(\sigma k)^2)$, $Z((\sigma k)^2) = \frac{\partial}{\partial k^2}\Sigma(m^2,(\sigma k)^2)$. It is also possible to extract the group velocity.

The ϕ^3-model is only logarithmically divergent on the ordinary Minkowski space. Thus, one may hope that the distortion of the dispersion relation in the noncommutative case is much weaker than for the ϕ^4 model. This expectation is verified by our calculation: For realistic values of the mass and coupling constant, the group velocity is only slightly altered, by a distortion of the order 10^{-6} [7].
In the Feyman-graph approach to NCQFT, supersymmetric theories have a much better UV/IR-behavior [8]. It is natural to ask if this is still true in the Yang-Feldman approach. It turns out that this is indeed the case for the Wess-Zumino model. There is no momentum dependent mass and thus also no change in the group velocity. Only the field strength normalization becomes momentum dependent. However, it is not clear how this might show up in experiments.
Since the dispersion relations for the photon can be measured very precisely, QED is the ideal testbed for Lorentz violating effects. We start a study of the dispersion relations in NCQED. Since Electrodynamics on the Noncommutative Minkowski space is an interacting theory, already pure electrodynamics is interesting. For the Yang-Feldman formalism one needs equations of motion, so we have to break gauge invariance. Since the theory is nonlinear, we have to use the BRST-formalism to do this consistently.
When quantizing the theory a subtlety arises. As interaction term, there will be commutators like $[A^\mu, \partial_\mu A^\nu]$ in the equation of motion. If the usual quantization procedure is used, such terms do not have the right commutative limit, i.e. they do not vanish in this limit. The term mentioned above will even diverge badly. This problem can be cured by a redefinition of the product of quantum fields on the Noncommutative Minkowski space. It will lead to a nonassociative structure, but it can be shown that no ambiguities arise.
Unfortunately, due to the vanishing mass, the photon 2-point function $\langle A^\mu A^\nu \rangle$ at 1-loop order is hard to compute. Preliminary calculations indicate that the nonplanar part of $\Sigma^{\mu\nu}$ contains terms proportional to $\sigma^{\mu\lambda} k_\lambda \sigma^{\mu\rho} k_\rho$ and that the effects are huge, since the underlying UV-divergence is quadratic.

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Computer-friendly presentation of the combinatorics of renormalization in QFT

José M. Gracia-Bondía

Combinatorics of renormalization in perturbative quantum field theory is anti-represented in terms of infinite matrices, that can be truncated at convenience. The prescription is derived from an algebraic form of the "Birkhoff decomposition" in the Hopf-wise description of renormalization by Connes and Kreimer; and so gives a partial answer to the question of practical usefulness of Hopf algebra for computations in quantum field theories. Our approach helps to resolve the tension between the "additive" and "multiplicative" sides of renormalization: the recursive diagrammatic subtraction of subdivergences is the outcome of a multiplicative process indeed; the latter, however, because of triangularity of the representation, recursively calls additions over. For more details than given here and pertinent references, look over the article [1].

In the Connes-Kreimer formalism, Feynman diagrams are organized in a Hopf algebra H_F of graphs. The linear space H_F is the algebra of polynomials with connected Feynman graphs as indeterminates, multiplication being simple juxtaposition of graphs. On H_F there is a coproduct $\Delta : H_F \to H_F \otimes H_F$, serving to encode the superficially divergent subgraphs, by setting $\Delta(\Gamma) := \sum_{\Gamma'} \Gamma' \otimes \Gamma/\Gamma'$, in the standard notation for graphs, subgraphs and cographs. Now, Feynman rules are understood as linear and multiplicative maps of H_F into a *Rota–Baxter* algebra V (commutative, with unit) of quantum amplitudes; and, as advertised, the disentangling of subdivergences is formulated as a factorization problem.

Let us ponder the paradigmatic example. Consider Laurent series

$$S(\epsilon) = \frac{a_{-n}}{\epsilon^n} + \frac{a_{-n+1}}{\epsilon^{n-1}} + \cdots + \frac{a_{-1}}{\epsilon} + a_0 + a_1\epsilon + \cdots .$$

With the ordinary multiplication, they form a commutative algebra V with unit. Consider further the operation $K_- \equiv K$ that picks out the pure pole part

$$K[S](\epsilon) = \frac{a_{-n}}{\epsilon^n} + \frac{a_{-n+1}}{\epsilon^{n-1}} + \cdots + \frac{a_{-1}}{\epsilon},$$

and the operation $K_+ := \mathrm{i}d - K$ keeping the finite part,

$$K_+[S](\epsilon) = a_0 + a_1\epsilon + \cdots .$$

The projector condition $K^2 = K$ ensures that the intersection between $K(V)$ and $K_+(V)$ is zero. The product of two elements of $K(V)$ remains in $K(V)$ —and likewise for $K_+(V)$. The key property

$$K[S_1]\,K[S_2] = K\big[K[S_1]S_2 + S_1K[S_2] - S_1S_2\big]$$

is easy to check. It makes K a Rota–Baxter operator; also K_+ is a Rota–Baxter operator. All this applies in particular to series corresponding to dimensionally regularized Feynman integrals in the MS-scheme (as our arguments are purely combinatorial, we need not worry about the precise form of the a_i coefficients), with K_+ the subtraction map.

Let then (V, K) be a commutative Rota–Baxter algebra, and consider the space $Hom(H, V)$ of linear maps from H to V; this is an algebra with the convolution operation, given (in Sweedler's notation) by $f \star g = \sum f(a_{(1)})\, g(a_{(2)})$. The multiplicative (that is, product-respecting) elements of $Hom(H_F, V)$, with V the algebra of Feynman amplitudes, are of particular interest. Clearly they are determined by their action on the subspace F of connected graphs. We construct an antirepresentation Ψ of $Hom(H_F, V)$ by infinite triangular matrices with entries in V by taking the composition

$$\Psi[f] : V \otimes F \xrightarrow{id_V \otimes \Delta} V \otimes H_F \otimes F \xrightarrow{\mathrm{id}_V \otimes f \otimes \mathrm{id}_F} V \otimes V \otimes F \xrightarrow{m_V \otimes \mathrm{id}_F} V \otimes F,$$

where m_V is just multiplication on V. The plot works because the external structure of the cographs Γ/Γ' is the same as that of Γ, so Δ actually sends F into $H_F \otimes F$. Thus for any $f \in Hom(H_F, V)$ a connected graph is sent by $\Psi[f]$ into a linear combination of connected graphs with coefficients in V, corresponding to the same n-point function. One easily verifies $\Psi[f \star g] = \Psi[g]\Psi[f]$. With the operator K given by $\mathrm{K}[f](a) := K[f(a)]$, the space $Hom(H_F, V)$ in turn becomes a (noncommutative) Rota–Baxter algebra; then $\Psi[\mathrm{K}[f]] = K[\Psi[f]]$, with K the matrix Rota–Baxter map.

Let finally $\tilde{\varphi} \in Hom(H_F, V)$ be the Feynman rule, which is multiplicative. Denote $\varphi := \Psi[\tilde{\varphi}]$. This will be a unipotent matrix. The practical construction of φ goes as follows. Recall that if $\Gamma_i \subseteq \Gamma_j$ is a superficially divergent subgraph of Γ_j, the *cograph* Γ_j/Γ_i is obtained by shrinking Γ_i to a vertex within Γ_j. Chosen an n-point function, the spaces of vectors on which the matrices act are spanned by the corresponding (superficially divergent, *connected*, amputated) Feynman graphs. A basis $\Gamma_1, \Gamma_2, \Gamma_3, \ldots$ for such a space can be ordered in many ways, the only conditions being that $\Gamma_1 = \emptyset$ —the empty diagram— and that each cograph of any Γ_l occurs in the basis as some Γ_m with $m < l$. It is then convenient to order by number of loops —or vertices, if we work on coordinate space. The order within a given loop-number sector is immaterial. Once the external structure and the basis are fixed, we fill up the entries of a matrix by the rule: for $i \neq j$,

$$\varphi_{ij} = \sum_{\Gamma'} \text{(unrenormalized) amplitude of } \Gamma' \text{ if } \Gamma_i \simeq \Gamma_j/\Gamma',$$

otherwise $\varphi_{ij} = 0$. Upper triangularity is clear, since $\varphi_{ij} = 0$ if $i > j$. We set $\varphi_{ii} = 1$ for all i. Note that Γ' need not belong to the basis list (it might be disconnected, for one thing). Let $\tilde{\varphi}(\Gamma')$ be the unrenormalized amplitude of Γ'. We just said that the coefficient of Γ_i in $\varphi(\Gamma_j)$ is $\sum_{\Gamma'} \tilde{\varphi}(\Gamma')$ for $\Gamma_i \simeq \Gamma_j/\Gamma'$.

Now, the Rota–Baxter property ensures there is a factorization

$$\varphi = \varphi_+ \varphi_-^{-1}, \tag{1}$$

where the factors $\varphi_- \in 1 + K[M(V)]$, $\varphi_+ \in K_+[M(V)]$ (with an obvious notation for matrices with entries in V) are also unipotent, multiplicative and unique. This may be called a matrix Birkhoff decomposition. The formula for the matrix of renormalized amplitudes φ_+ is

$$\varphi_+ = 1 - \mathrm{K}_+\big[(\varphi^{-1} - 1)\varphi_+\big];$$

the recursive series solving this equation terminates. With the help of some symbolic programming, it can be obtained at one stroke; note that Bogoliubov's 'preparation' map is not invoked explicitly here.

As a consequence of the above, the renormalization of the Lagrangian's parameters by counterterms must take place by composition of series. This calls for rewriting the whole renormalization process in terms of Faà di Bruno algebras and Lagrange reversion. Some other open problems in the framework presented here concern the adaptation of the entire formalism to regularization-free schemes like BPHZ and Epstein–Glaser renormalization. The massive BPHZ scheme rather obviously possess a Rota–Baxter property. For Epstein-Glaser renormalization, one apparently requires an associate module structure. The massless models pose the puzzling problem of reconciling manifest Lorentz invariance of the renormalization map with the multiplicative property.

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Matrix Quantum Mechanics and Soliton Regularization of Noncommutative Field Theory

Fedele Lizzi

There are several good reasons for which physicists often approximate spaces, or rather functions on a space, with matrices. The most obvious one is the possibility to solve problems numerically, but equally important is the fact that a theory may simply loose its meaning at very short distances (effective theories). The straightforward *lattice* approximation, approximating a space with a set of topologically disconnected points, and functions with arrays of numbers multiplied componentwise, looses totally the information of the underlying space, and in particular nearly totally destroys the of the problem. And of course a lattice approximation for noncomutative spaces does not make any sense.

In my talk I discussed how to approximate field theories on the noncommutative torus, the archetypical noncommutative (compact) geometry. It is the algebra of elements $a = \sum_{n,m} a_{nm} U_1^n U_2^m$ generated by two unitary generators with the relation: $U_1 U_2 = e^{2\pi i\theta} U_2 U_1$. It is possible to study field theories on a NC-Torus with the use an integral (trace) defined as $\int a := a_{0,0}$, and two derivatives $\partial_i U_j = 2\pi i \delta_{ij} U_i$ No sum on i.

The first approximation discussed is based on work by Pimsner and Voiculescu [1]

and has been reported in [2]. I described how and to what extent the noncommutative two-torus can be approximated by a tower of finite-dimensional matrix geometries. The approximation is carried out for both irrational and rational deformation parameters by embedding the C^*-algebra of the noncommutative torus into an approximately finite algebra. The construction is a rigorous derivation of the recent discretizations of noncommutative gauge theories using finite dimensional matrix models, and it shows precisely how the continuum limits of these models must be taken. We clarify various aspects of Morita equivalence using this formalism and describe some applications to noncommutative Yang-Mills theory.
Recall that the noncommutative torus, as well as the ordinary torus, cannot be the inductive limit of a finite dimensional algebra (for example because it has non trivial K_1 group). It is however possible to *embed* the algebra in a larger approximately finite algebra.
With Landi and Szabo we have proven that while that the limit of all matrix elements of the elements of the approximating algebra converge (in the weak sense) to the matrix elements of the full algebra. This also provides a sense for which physical expectation values have a meaning regardless of the fact that θ is rational or irrational. We have also shown that all Morita equivalent noncommutative tori are subalgebras of the same approximatively finite algebra. One of the drawbacks of the approximation is however the fact that there are no version of the two derivatives, and only translation operators can be approximated.
In the second part of the talk, based again on work with Landi and Szabo [3, 4], I used an approximation of the noncommutative torus due to Elliott and Evans [5]. This is based on projectors and partial isometries of the algebra and approximates the noncommutative torus with a sequence of subalgebras isometric to the algebra of matrix valued functions on two circles. Even if the matrices are finite, the algebra is not finite dimensional, hence the limit can be inductive.
After a description of the matrix approximation, illustrated by the pictures of the Wigner tranforms of the projectors and partial isometries, I built and integral and two *approximate* derivations which approximate the derivations of the full algebra. They are approximate in the sense that they close the Leibnitz rule only in the limit, but can be expressed solely in terms of the matrix algebra. Thus we approximate field theories with a matrix quantum mechanics. This is applied to the perturbative dynamics of scalar field theory, to tachyon dynamics in string field theory, and to the Hamiltonian dynamics of noncommutative gauge theory in two dimensions. We also described the adiabatic dynamics of solitons on the noncommutative torus and compare various classes of noncommutative solitons on the torus and the plane.

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θ-deformations and UV/IR mixing

Victor Gayral

The aim of this talk is to study the manifestations of the ultra-violet/infra-red entanglement phenomenon for (Euclidean) scalar QFT on a class noncommutative manifolds, the θ- or isospectral deformations. They are curved-space generalizations of Moyal planes and noncommutative tori, introduced in [1, 2] in the periodic case and generalized for non-periodic actions in [3, 4] using a Rieffel's twisted product approach [5].

The construction of isospectral deformations goes as follow. Let (M, g) be an n-dimensional Riemannian manifold (non-compact a priori) endowed with an isometric action $\alpha : \mathbb{R}^l \to \mathrm{Isom}(M, g)$. There is basically two distinct situations to consider. When the action is effective ($\ker\alpha = \{0\}$), we assume the action to be proper. In this case, it will imply that the action is also free. For periodic action ($\ker\alpha \simeq \mathbb{Z}^l$), α factorizes through a torus and the factorized action is automatically proper but no longer free.

Given now Θ a real $l\times l$ skew-symmetric matrix, one can define the twisted product $\star_\Theta$ as a bilinear map on $C_c^\infty(M)$ with value on $C^\infty(M) \cap L^\infty(M, \mu_g)$:

$$\big(f \star_\Theta g\big)(p) := (2\pi)^{-l} \int_{\mathbb{R}^{2l}} d^l y\, d^l z\, e^{-iy.z}\, f(\alpha_{-\frac{1}{2}\Theta y}(p))\, g(\alpha_z(p)).$$

To properly define algebras from this product in the non-periodic case, one has to move to bigger function spaces since this product is non-local on the orbits of the action (non-preservation of supports). This noncommutative product inherits of all the properties of the Moyal one, in particular the ordinary integral with Riemannian volume form μ_g is a trace.

We can then define generically a classical functional action for a real scalar field $\varphi \in C_c^\infty(M)$, by

$$S[\varphi] := \frac{1}{2} \int_M \mu_g \varphi(p)\,(\triangle + m^2)\varphi(p) + \frac{\lambda}{k!}\varphi^{\star_\Theta k}(p),$$

where $\triangle$ denotes the scalar Laplacian. The one-loop regularized effective action (sums of one-loop 1PI Feynman diagrams in external field) can be written as

$$\Gamma_{1l}^\epsilon[\varphi] = -\frac{1}{2}\int_\epsilon^\infty \frac{dt}{t}\,\mathrm{Tr}\big(e^{-tH} - e^{-tH_0}\big),$$

where H is the effective potential (i.e. the operator on $L^2(M, \mu_g)$ whose distributional kernel is given by the second functional derivative of the classical action) and

$H_0 = \triangle + m^2$. In dimension four with a quartic interaction term ($n = 4 = k$), one gets $H = \triangle + m^2 + \frac{\lambda}{3!}(L_{\varphi\star_\Theta\varphi} + R_{\varphi\star_\Theta\varphi} + L_\varphi R_\varphi)$, where L_f (resp. R_f) denotes the operator of left (resp. right) twisted multiplication by f. Using then a (modified) heat kernel expansion, one obtains two types of contribution with highly different behavior. The first one invokes traces of the form $\mathrm{Tr}(L_f e^{-tH_0})$ or $\mathrm{Tr}(R_f e^{-tH_0})$. It corresponds to the planar sector of the theory and gives ordinary UV divergences. The non-planar sector is understood here in term of contributions of traces with mixed left and right twisted multiplication operators $\mathrm{Tr}(L_f R_g e^{-tH_0})$. The latter traces have a nicer behavior for $t \to 0$. This is due to the presence of off-diagonal heat kernel in the Feynman integrals. However, this sector exhibits a UV/IR mixing. In particular, the regularizing character of the product of left and right twisted multiplication operators depends highly on the geometrical data. For non-periodic deformations, when the rank of the deformation matrix is lower than the dimension of the manifold (thus for all examples expect non-degenerate Moyal planes), the non-planar sector remains divergent but with non-local divergences. This implies that the theory is not renormalizable in the ordinary sense. For periodic deformations, using a Peter-Weyl decomposition of the field (induced by the torus action), we first see that only the (total) zero mode is affected by the mixing if the deformation parameters are irrational and satisfy a Diophantine condition (meaning that $\|k\theta\|_{\mathbb{T}}^{-1}$ is a temperate sequence for $k \in \mathbb{N}\setminus\{0\}$). There is another type of mixing, due to the possible existence of fixed points for the action. Indeed, if the function $d_g^{-2}(.,\alpha_z(.))$ (where d_g is the Riemannian distance function) is not locally integrable for some non-trivial group element z, then there is a new non-local singularity. In such a case, the theory turns out also to be non renormalizable. If the two previous conditions are satisfied, the UV/IR singularities are proportional to $\int_M \mu_g\varphi \int_M \mu_g\varphi$, $\int_M \mu_g\varphi \star_\Theta \varphi \int_M \mu_g\varphi \star_\Theta \varphi$ and $\int_M \mu_g\varphi \star_\Theta \varphi \star_\Theta \varphi \int_M \mu_g\varphi$. Thus the quantum model is not stable with respect to the renormalization.

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Remarks on the Relation between Spin and Statistics

Florian Scheck
(joint work with Mario Paschke, Nikolaos Papadopoulos, Andrès Reyes)

The famous spin-statistics theorem of Fierz and Pauli [1] states that identical particles with *integer* spin obey *Bose-Einstein* statistics, particles with *half-integer spin* obey *Fermi-Dirac* statistics. This quantum statistics manifests itself in many domains of physics, from exchange interactions in atoms and nuclei, condensed matter phenomena, astrophysics, to Bose-Einstein condensation. The simple example of two particles with spin s and orbital angular momentum $\ell = 0$ or $\ell = 1$ shows that the statistics is determined by $(-)^{2s}$, a phase that comes from the symmetry relation

$$\big(sm_2, sm_1|JM\big) = (-)^{2s-J}\big(sm_1, sm_2|JM\big)$$

of SU(2) Clebsch-Gordan coefficients and, hence stems from the rotation group SO(3) over $\mathbb{R}^3$. Yet, the field theoretic proof of the theorem makes essential use of Lorentz–covariance and of micro–causality.
The essential phase being due solely to the rotation group, and stimulated by a controversial work by M. Berry and J.M. Robbins [2] we address the question under which conditions the theorem could be proven in nonrelativistic quantum mechanics. Exchanging two or more identical particles with all their attributes immediately leads to questions of topology of the configuration space [3]. For example, for n identical particles in d-dimensional space-time one defines the space

$$\widetilde{Q} := \big(\mathbb{R}^{d-1}\big)^n \setminus \Delta\ , \quad \text{with } \Delta = \big\{(x_1, \dots, x_n)|x_i = x_j \text{ for at least one pair}\big\}$$

Since the particles are indistinguishable all permutations are equivalent. Therefore, the true configuration space is

$$Q := \widetilde{Q}/S_n\ , \quad \text{with } S_n \text{ the symmetric group.}$$

One shows: If Q is the manifold corresponding to a classical configuration space of a given system then the inequivalent scalar quantizations of this system are in bijective correspondence with the characters of the fundamental group of Q. This relationship implies that in $d = 4$, i.e. in the case we study and where $\pi_1(Q) \simeq S_n$, there is only the Fermi–Dirac or Bose–Einstein alternative. This is not so in lower dimensions where the braid group is relevant and where the topology allows for anyon statistics.
Once the Fermi–Dirac or Bose–Einstein alternative is decided for spin $s = 0$, the theorem is established for all spins. In the light of these remarks one is lead to ask the following questions: Is there a geometric proof of the spin–statistics theorem? And is it true that the theorem holds under less restrictive assumptions? [4]
In the case $n = 2$, i.e. of two identical particles with spin s, $\tilde{Q} = \mathbb{R}^3 \times \mathbb{R}_+ \times S^2$, by dropping the center-of-mass motion and using polar coordinates, may be reduced to S^2 so that the true configuration space reduces to $\mathbb{R}P^2$. By pull-back from $\mathbb{R}P^2$ to S^2 the sections describing spin states may equivalently be studied over the sphere. Concentrating on the two particle case we reformulate the Berry–Robbins

approach in a model independent manner by studying the action of SU(2) on S^2, making use of the algebras $\mathcal{A}_+$ and $\mathcal{A}_-$ of symmetric and antisymmetric functions on S^2, respectively, and of projective modules [5], [6]. We show, in particular, that Berry and Robbins' *uniqueness* condition for the wave "function" is problematic and should rather be replaced by requiring the relevant sections to be *well-defined*. Using a basis where the angular momenta of the two particles are coupled to J, and yielding a phase $(-)^{2s-J+K}$ upon interchange, with K the spin-statistics sign, a trivial vector bundle η^{BR} over S^2 can be defined. The *physical* vector bundle lives on $\mathbb{R}P^2$, however. The action of $\mathbb{Z}_2$ on η^{BR}

$$\sigma \in \mathbb{Z}_2 \,:\, \tau(\sigma)\big(r, |JM(r)\rangle\big) := \Big(\sigma \cdot r, (\text{sign }\sigma)^{2s-J+\tilde{K}}|JM(\sigma r)\rangle\Big)$$

contains the term $\tilde{K}$ which must be chosen even or odd integer. One then sees that a section over $\mathbb{R}P^2$ can be physical only if $K = \tilde{K}$ is chosen. This shows that deciding the Fermi–Dirac or Bose–Einstein alternative at the level of scalar quantum mechanics fixes spin and statistics for all $s > 0$.
The spin-statistics relation rest to a large extent on the topology of the physical manifold, perhaps more than on locality and covariance. However, in order to prove the theorem in the nonrelativistic context one needs one more physically motivated input. What that could be is not clear at this time.

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Quantum coordinates of an event

Klaus Fredenhagen

What is the physical motivation for the description of spacetime by noncommutative spaces? The oldest arguments, going back to Heisenberg and Snyder, were motivated by the desire to avoid the divergences of quantum field theory, a hope which has not been fulfilled in most examples. Another motivation came from the fact that the noncommutative framework allows a geometric interpretation of the Higgs field as a component of a gauge field. This observation is at the roots of the work of Connes et al.. A different motivation came from the analysis of the expected uncertainties of localization measurements due to the simultaneous validity of the principles of quantum physics and general relativity, and the fact that these uncertainties can be derived from a suitable noncommutative structure. Further motivations came from string theory and also from magnetic systems in solid state physics, but also from the wish to broaden the framework for possible physical theories. The strongest argument for noncommutativity of spacetime might be that spacetime is an observable object which should be governed by the principles of quantum physics. It is, however, not obvious how quantum observables can be associated to spacetime itself.

In the following note, I will describe an idea which works at least in some simple examples. The idea is a generalization of the idea for the description of time observables in quantum mechanics [2]. There a positive operator A is chosen which marks the effect whose time of occurence is to be determined. If α describes the action of the time translation group by automorphisms of the algebra of bounded Hilbert space operators, assumed to be pointwise strongly continuous, we may define the total duration of the event by the integral

$$B = \int dt \alpha_t(A).$$

The Hilbert space decomposes into a direct sum of three subspaces, one on which B vanishes (hence the effect does never happen), one in which the effect takes infinitely long time (formally $B = \infty$) and a third subspace on which B is a positive selfadjoint unbounded operator with zero kernel. On the latter subspace a meaningful time observable can be constructed in terms of a positive operator valued measure P, where we asoociate to the interval I the operator

$$P(I) = B^{-\frac{1}{2}} \int_I dt \alpha_t(A) B^{-\frac{1}{2}}$$

If the time translations are implemented by a selfadjoint Hamiltonian with absolutely continuous nondegenerated spectrum $S \subset \mathbb{R}$, we may give an explicit formula for P in terms of the integral kernel $a(E, E')$ (assumed to be smooth) of A with respect to the realization of the Hilbert space as the space of L^2 functions on S,

$$(P(I)f)(E) = (2\pi)^{-1} \int dE' a(E,E)^{-\frac{1}{2}} a(E,E') a(E',E')^{-\frac{1}{2}} f(E') \int_I dt e^{it(E-E')}.$$

The first moment of the measure P takes the familiar form

$$T = \frac{1}{i}\frac{d}{dE} + g_A$$

where the derivative operator is to be chosen with Dirichlet boundary conditions and g_A is the multiplication operator with a function depending on the chosen effect A which can be computed from its integral kernel a. Note that, contrary to projection valued measures, positive operator valued measures are not completely determined by their first moments.

The idea above can immediately be generalized to describe the spacetime localization of events in Minkowski space. We only have to replace the time translation group by the translation group of Minkowski space. The crucial ingredient is the choice of a suitable positive operator A which marks an event in spacetime.

We consider a free scalar field. As our event we take the collision of 2 particles, represented by the positive sesquilinear form

$$\alpha_x(A) = a^*(x)^2 a(x)^2$$

where $a(x)$ is the annihilation operator for a particle at the point x. We restrict our considerations to the 2-particle space. There such a collission occurs only in the s-channel, i.e. when the relative angular momentum vanishes. Therefore the subspace on which the localization of the event can be defined consists of states which are completely characterized by their total momentum. We may thus identify this space with the space of L^2 functions on the spectrum of the momentum opertor on the 2 particle subspace, i.e. the set

$$H^+_{>2m} = \{p \in \mathbb{R}^4, p^2 \geq 4m^2, p_0 > 0\} \ .$$

The arising operator valued measure P has the density

$$(P(x)\Phi)(p) = (2\pi)^{-4} \int_{H^+_{>2m}} d^4k e^{i(k-p)x} \Phi(k).$$

The first moments are the maximally symmetric operators

$$\frac{1}{i}\frac{\partial}{\partial p_\mu}$$

with Dirichlet boundary conditions.

We see that in this example the quantized spacetime is obtained by a completely positive map from Minkowski space. The arising noncommutative space may be called the Töplitz quantization of Minkowski space. (In a similar way, the quantized time axis is the Töplitz quantization of the real axis, if the energy spectrum is the positive half axis.)

We now want to repeat the same operation for the free scalar field on the noncommutative Minkowski space characterized by a symplectic form σ. Formally, the operator characterizing the event is

$$A(q) = a^*(q)^2 a(q)^2$$

where $q = (q^\mu)$ denote the noncommutative coordinates with commutation relations

$$[q^\mu, q^\nu] = i\sigma^{\mu\nu}$$

and where

$$a(q) = \int d\mu(k) a(k) e^{ikq}$$

with the standard annihilation operators $a(k)$ for a particle with momentum k and the Lorentz invariant measure μ on the mass shell. As in [1] the noncommutative Minkowski space is defined as the Weyl algebra associated to the symplectic form σ.

We may define for every positive functional ω on this algebra the operator

$$B(\omega) = \int d\mu(k_1) d\mu(k_2) d\mu(k_3) d\mu(k_4) \ldots$$
$$\ldots a^*(k_1) a^*(k_2) a(k_3) a(k_4) \omega(e^{-iqk_1} e^{-iqk_2} e^{iqk_3} e^{iqk_4}) \; .$$

We restrict as before the operators to a subspace of the 2 particle space. On the orthogonal complement of this subspace all the operators $B(\omega)$ vanish. The subspace may again be identified (as a representation space of the translation group) with $L^2(H^+_{>2m})$. The integral kernel $b_\omega(k,p)$ of $B(\omega)$ on this space is

$$b_\omega(k,p) = c(k) c(p) \omega(e^{-ikq} e^{ikp})$$

with a positive function c. Let now tr denote the trace on the Weyl algebra. Every positive element T of the Weyl algebra induces a positive functional by

$$\omega_T(C) = \mathrm{tr} CT \; .$$

We obtain a completely positive map P from the Weyl algebra into the operators on our Hilbert space by

$$P(T) := B(\mathrm{tr})^{-\frac{1}{2}} B(\omega_T) B(\mathrm{tr})^{-\frac{1}{2}}$$

P is unit preserving. If T arises by Weyl quantization from a function f on Minkowski space, we obtain

$$(P(T)\Phi)(p) = \int_{H^+_{>2m}} dk e^{i\sigma(k,p)} \hat{f}(p-k) \Phi(k)$$

The selfadjoint generators q^μ of the Weyl algebra are mapped onto the symmetric operators

$$P(q^\mu) = \frac{1}{i} \frac{\partial}{\partial p_\mu} + \sigma^{\mu\nu} p_\nu$$

with Dirichlet boundary conditions. On a dense domain, these operators have the same commutation relations as the generators q^μ, but these relations cannot be exponentiated to yield the Weyl relations. Instead the quantized noncommutative Minkowski space is the image of the noncommutative Minkowski space under a completely positive map.

One may compose the map P with a positive map Q from the algebra of functions on Minkowski space into the Weyl algebra and thus obtain a localization observable associated to Minkowski space. A convenient choice of such a map arises from an optimally localized state on the noncommutative Minkowski space. Such a state is of the form

$$\omega_x(e^{ikq}) = e^{-||k||^2} e^{-ikx}$$

with a quadratic form $||k||^2$ on Minkowski space which is minimal with respect to the inequality

$$\sigma(k,p)^2 \leq 4||k||^2||p||^2 \ .$$

The positive map is then obtained by

$$Q(f) = (2\pi)^{-4} \int dx f(x) \int dk e^{-||k||^2} e^{ik(q-x)}$$

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Exact solution of quantum field theory models on noncommutative phase spaces

Edwin Langmann

Two years ago we (together we R. Szabo and K. Zarembo) studied a quantum field theory model of bosons in an external field $B_{\mu\nu}$ and with a 4-point interaction defined with the Moyal $*$ product such that $[x_\mu, x_\nu]_* = 2i\theta_{\mu\nu}$. We showed, in particular, that the model is exactly solvable at the special point where the matrices $(B_{\mu\nu})$ and $(\theta_{\mu\nu})$ are inverse to each other. The exact definition of this model requires a regularization (cut-off), and it therefore nicely illustrates interesting non-trivial features of the quantum field theory limits where the cut-off is removed.

In my talk I gave an overview of our previous work [1,2,3], and I described some interesting issue concerning the QFT limit which we did not resolve in our work at that time [1]: the limit which we studied breaks certain symmetries of the model, and there seems to exist another limit which respects all symmetries. Related work of Grosse and Wulkenhaar [4] on a closely related models suggests that the limit of main interest for this model is indeed this latter limit. The solution of the model in this latter limit is a technical challenge since it seems no longer possible to use the matrix model techniques which were the key to our solution in [1]. The talks of Grosse and F. Vignes-Tourneret at this meeting made clear to me that it would be very interesting to find the explicit solution of the model in this latter limit, and I now plan to return to this question in the near future.

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(Modular) Index Theory of Cuntz Algebras

Adam Rennie

(joint work with Alan Carey, John Phillips)

This work shows how the Fredholm index can be computed for algebras without trace.

The theory of Fredholm operators in type II von Neumann algebras was initiated by Breuer, [1, 2], and extended in [7, 4, 5]. Furthermore, [4, 5] extended the local index theorem of Connes-Moscovici, [6], to the general semifinite setting.

When one does not have a trace, the problems become similar to those one encounters in type III von Neumann algebra theory. There are many C^*-algebras without trace for which the general modular theory does not immediately apply, but for which our methods work perfectly well.

Whilst we phrase the brief description below in terms of the Cuntz algebras, the results are true in much greater generality.

To begin, we let O_n be the Cuntz algebra generated by $n \geq 2$ isometries such that $\sum_{j=1}^{n} S_j S_j^* = 1$, where 1 is the identity. The algebra O_n has a unique KMS state ϕ, with associated flow given by $S_j \to n^{-it} S_j$. We let $L^2(O_n, \phi)$ be the GNS Hilbert space, and so obtain the unbounded operator Δ such that $\Delta^{it} a \Delta^{-it}$ is the flow for the KMS state. Since this flow is an action of S^1, which is compact, we obtain a faithful positive expectation $\Phi : O_n \to F$ where F is the fixed point algebra for the KMS flow. Observe that ϕ is a trace on F, and $\phi = \phi \circ \Phi$.

Now use the expectation to define an F-valued inner product on O_n by $(a|b) := \Phi(a^* b)$. This makes O_n a right F-inner product module, and we let X be the C^*-module completion. Let $End_F(X)$ be the algebra of adjointable endomorphisms of the right F-module X, and observe that they act on $L^2(O_n, \phi)$. Then

Proposition 1. *Let $\mathcal{N} = (End_F(X))''$ be the weak closure of the endomorphisms acting on $L^2(O_n, \phi)$. Then there exists a faithful normal semifinite trace $\tau : \mathcal{N} \to$ $\mathbf{C}$ such that*

$$\tau(\Theta_{x,y}) = \phi((y|x)) \quad \text{for all } x, y \in X,$$

where $\Theta_{x,y} z := x(y|z)$. Moreover, the operator $D = \log \Delta$ is affiliated with $\mathcal{N}$.

Writing $D = \log n \sum_{k \in \mathbf{Z}} k \Phi_k$, we have $\tau(\Phi_k) = n^k \quad$ for all $k \in \mathbf{Z}$. This is terrible, and does not even allow θ-summability hypotheses. However if we define, for $T \in \mathcal{N}$, $\tau_\Delta(T) := \tau(\Delta T)$ then we have $\tau_\Delta(\Phi_k) = 1 \quad$ for all $\quad k \in \mathbf{Z}$. Now we have reasonable summability, but have lost the trace property. Nevertheless, on the algebra $\mathcal{M} \subset \mathcal{N}$ generated by the Φ_k and the left action of F, τ_Δ is trace. Thus we can apply the Carey-Phillips spectral flow formula, [3], to compute the spectral flow between D and $D + M$ where M is any self-adjoint element of $\mathcal{M}$. In the tracial setting we would expect to pair with unitaries, but here we have an additionl requirement on the unitaries, namely that they map $L^\infty(D)$ into $\mathcal{M}$.

Definition 1. *Let u be a unitary over O_n. We say that u satisfies the* **modular condition** *with respect to Δ if both the operators*

$$u\Delta^{-1}u^*\Delta, \quad u^*\Delta^{-1}u\Delta$$

are in (a matrix algebra over) the algebra F. We denote by U_Δ the set of modular invertibles. Two modular unitaries are modular homotopic if they are connected by a continuous path of modular unitaries.

It turns out that the modular homotopy classes of modular unitaries form an abelian semigroup, which is ordinary K_1 when $\Delta = Id$. We call this semigroup modular K_1. If u is a modular unitary then uDu^* is affiliated to $\mathcal{M}$, and we can prove an index theorem. First observe that if $v \in O_n$ is a partial isometry with range and source projections in F, then

$$u_v = \begin{pmatrix} 1 - v^*v & v^* \\ v & 1 - vv^* \end{pmatrix}$$

is a modular unitary. Now O_n is generated by partial isometries $S_\mu S_\nu^*$, where $S_\mu = S_{\mu_1} \cdots S_{\mu_m}$ and $S_\nu = S_{\nu_1} \cdots S_{\nu_k}$. Then

Theorem 1. *If u is a modular unitary then the spectral flow from D to uDu^* depends only on the modular K_1 class of u. For $v = S_\mu S_\nu^*$, the spectral flow from D to u_vDu_v is given by*

$$sf(D, u_vDu_v) = res_{s=0}\tau_\Delta(u_v[D, u_v](1 + D^2)^{-1/2-s}) \tag{1}$$

$$= (m - k)\left(\frac{1}{n^k} - \frac{1}{n^m}\right) \geq 0 \tag{2}$$

The 2-linear functional

$$(a^0, a^1) \to res_{s=0}\tau_\Delta(a^0[D, a^1](1 + D^2)^{-1/2-s})$$

is a twisted (b, B) cocycle for O_n.

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The geometry of gauge fields on toric noncommutative manifolds

Walter van Suijlekom
(joint work with Giovanni Landi)

We develop Yang-Mills theory on toric noncommutative manifolds which were introduced in [3] (see also [2]) . These noncommutative spaces M_θ are defined as deformations of a Riemannian manifold M carrying an action of $\mathbb{T}^n$: this torus is deformed to a noncommutative torus $\mathbb{T}^n_\theta$ [5] with θ a matrix of deformation parameters. We start by recalling their construction and derive a simplified form of the Connes-Moscovici local index formula [4] on these noncommutative spaces. We then focus on two such noncommutative manifolds and construct a noncommutative principal Hopf fibration $S^7_{\theta'} \to S^4_\theta$ with structure group $SU(2)$, starting with the algebras $\mathcal{A}(S^4_\theta), \mathcal{A}(S^7_{\theta'})$ of polynomials on them. The algebra $\mathcal{A}(S^7_{\theta'})$ carries an action of $SU(2)$ by automorphisms and we identify the subalgebra consisting of invariants under this action with $\mathcal{A}(S^4_\theta)$. This gives a one-parameter family of Hopf fibrations, where θ' is expressed in terms of θ.
We construct the $\mathcal{A}(S^4_\theta)$-bimodules associated to all finite-dimensional representations V of $SU(2)$ as the collection of "equivariant maps from $S^7_{\theta'}$ to V" with respect to the action of $SU(2)$, and define connections on them. We prove that these modules are finite projective by explicit construction of projections. This allows for a computation of the indices of Dirac operators having coefficients in these noncommutative vector bundles.
We develop Yang-Mills theory on S^4_θ by defining a Yang-Mills action functional in terms of the curvature of a connection and derive that the connections with (anti-)selfdual curvature are minima of this action: such connections are called instantons. Starting with the basic instanton given in [3], gauge non-equivalent instantons are obtained by acting on it by twisted infinitesimal conformal transformations, encoded in the Hopf algebra $U_\theta(so(5,1))$. The Hopf subalgebra $U_\theta(so(5))$ is made of twisted infinitesimal symmetries under which the basic instanton is invariant. This leads to a five-parameter family of (infinitesimal) instantons. Finally we prove, by using an index theoretical argument as in [1], that this family is in fact the complete set of (infinitesimal) charge 1 instantons.
Finally, we sketch how to generalize Yang-Mills theory from S^4_θ to any four-dimensional toric noncommutative manifold M_θ. Let $P \to M$ be a G-principal bundle, where G is a semisimple Lie group. We assume that the action of the torus $\mathbb{T}^2$ on M can be lifted to P, in such a way that this lifted action commutes with the action of G on P. This allows for the definition of the two algebras $C^\infty(M_\theta)$ and $C^\infty(P_\theta)$ as toric noncommutative manifolds. The inclusion $C^\infty(M_\theta) \subset C^\infty(P_\theta)$

can be understood as a noncommutative principal bundle: $C^\infty(P_\theta)$ carries an action of G by automorphisms in such a way that $C^\infty(M_\theta)$ forms the subalgebra consisting of elements in $C^\infty(P_\theta)$ that are invariant under the action of G.
We define the associated vector bundles $P_\theta \times_G V$ for all finite-dimensional representations V of G as $C^\infty(M_\theta)$-bimodules of G-equivariant maps from P_θ to V; these modules are again finite projective. Finally, we define a Yang-Mills action functional and find that instantons, i.e. connections with selfdual or anti-selfdual curvature, are minima of this action.

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Noncommutative Geometry of Quantum Spaces

Andrzej Sitarz

1. Introduction

Noncommutative geometry offers a remarkable possibility to enhance the meaning of *geometry* beyond the standard classical setup of spaces and manifolds. Therefore, it appears to be an attractive tool in the quest for a better understanding of space-time structure at very small scale, in the hope to find a good unified description of gravitation and quantum field theory. However, despite the fact that one can consistently introduce geometry *without points* we are still very far from understanding the crucial notions like *noncommutative manifolds*, for example. So far, we are learning by examples, trying to figure out how the relevant constructions should look like. The most important problems, which cannot be neglected in the physical approach, are, for instance, the choice of the differential calculus (which is by no means canonical in the case of noncommutative algebras) or the definition of metric.
The approach of *spectral triples* proposed by Alain Connes [1, 2] proposes a solution to the problem by setting a definition of *noncommutative manifold* described in terms of a suitable Hilbert space representation of an algebra together with the Dirac operator, which encompasses both the differential data and the metric aspects. The spectral triple data, which is based on the classical formulation of Dirac operator in the geometry of spin manifolds, has been applied to noncommutative tori, isospectral deformations and finite matrix algebras. The task to include also

the examples of quantum geometry, noncommutative q-deformations of manifolds, has been completed only recently [4, 3, 5].

2. Towards the q-geometry: main result

Leaving aside the task of a detailed discussion of the foundations of spectral triples and referring the reader to the textbook [7], let us briefly present the result, which could be stated as the following existence theorem:

Theorem 2. *(see* [5]*)*
There exists an equivariant (algebraic) spectral triple on the algebra $\mathcal{A}(SU_q(2))$ such that all algebraic axioms are satisfied with the exception of the commutant axiom and the order-one condition, which are satisfied up to compact operators. The Dirac operator is unique up to rescaling and shift by a constant and has the same spectrum as in the classical case ($q = 1$).

Instead of going into the technical details, let me explain the phrasing of the above statement. First of all, we have obtained only a first step, which is the construction of the Dirac operator. We call this *algebraic spectral triple* as it involves basically relations within the operator algebra. The task of pursuing the analytical aspects of the spectral geometry of quantum spaces still lies ahead. The constructed spectral geometry is the *equivariant case*[10], as we were using the immense $\mathcal{U}_q(su(2)) \times \mathcal{U}_q(su(2))$ symmetry of the $\mathcal{A}(SU_q(2))$ algebra. This made the task possible but restricts us only to the "round" Dirac operator (corresponding to the spherically symmetric metric). Finally, in the course of the construction, we had to overcome a significant problem, which resulted in changing of some of the axioms - we no longer require them to be exact but only satisfied up to certain ideal of compact operators.
Thus, our data of q-spectral geometry involves the polynomial algebra $\mathcal{A}(SU_q(2))$, which is $\mathcal{U}_q(su(2)) \times \mathcal{U}_q(su(2))$ module algebra and the equivariant representation π of $\mathcal{A}(SU_q(2))$ on a Hilbert space $\mathcal{H}$. If ρ is the representation of the $\mathcal{U}_q(su(2)) \times \mathcal{U}_q(su(2))$ Hopf algebra, then the equivariance means that on a dense subspace the following holds for all $a \in \mathcal{A}(SU_q(2))$ and $h \in \mathcal{U}_q(su(2)) \times \mathcal{U}_q(su(2))$

$$\rho(h)\pi(a) = \pi(h_{(1)}) \triangleright a)\rho(h_{(2)}).$$

Finally, an equivariant unbounded Dirac operator D (that $D\rho(h) = \rho(h)D$ holds) and the equivariant reality structure J (which is the polar part of an operator satisfying $T\rho(h) = \rho(Sh)^*T$ complete the data in the odd case.
The most significant deviation from the so far accepted axioms was that the reality operator J no longer maps the algebra to the its commutant, but rather to its Paschke dual [9]:

$$\forall a, b \in \mathcal{A}(SU_q(2)): \quad [\pi(a), (J\pi(b)J)] \in \mathcal{K}_q,$$

where $\mathcal{K}_q$ is a certain ideal of compact operators (generated by operators with spectrum of exponential decay).
The order-one condition holds also only up to this ideal:

$$\forall a, b \in \mathcal{A}(SU_q(2)): \quad [[D, \pi(a)], (J\pi(b)J)] \in \mathcal{K}_q.$$

3. Open problems

As mentioned before, the results achieved so far appear to be a part of the bigger puzzle, which is still to be studied. Some of the challenges, still lying ahead are crucial if we want tu fulfil the dream of understanding the quantum geometry. Let me list some of them:

- C^∞ **algebra**. Working with the polynomial algebra is sufficient to get the spectral data, however, to complete the project it is necessary to get a good insight into the C^∞ algebra. In particular, it would be interesting to calculate its cyclic homology.
- **Paschke duality**. The apparently *ad hoc* introduced Paschke duality is a notion used in K-homology. What is its geometric role in the case of studied q-geometry? What is the role of the geometric ideal $\mathcal{K}_q$?
- **Hochschild cycle**. One of the standard requirements of the spectral geometry axioms is the existence of Hochschild cycle, whose image plays the role of the volume form. So far, this axiom has not been fulfilled here. Could it be satisfied or modified?
- **Local index**. The local index calculations have been successfully performed in the $SU_q(2)$ case [6]. The obtained cyclic cocycle, is, however, only an image of a one-cyclic cocycle. On the other hand, the *twisted cyclic cohomology* [8] gives a possibility of a nontrivial three-cyclic cocycle of the algebra. Is there a canonical way to obtain it from the the spectral data?
- **Differential calculi and the metric.** What is the differential calculus obtained from the spectral data? What freedom do we have in the choice of the Dirac operator (no longer equivariant) so that a different metric can be obtained?
- **Equivariant cyclic homology.** Having an equivariant spectral triple we would expect that it corresponds via certain index formula to an equivariant cyclic cocycle. What is the correct framework of the equivariant cyclic theory and how can this be calculated?

These are just few mathematical problems related to the issue of q-deformed geometry. Learning the answers shall bring closer the possibility of understanding what noncommutative quantum space-time might be.

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Noncommutative spacetimes and their Ghyst

Mario Paschke
(joint work with Rainer Verch)

A.Connes' notion of spectral triples describes the noncommutative generalisation of *compact Riemannian* Spin manifolds (without boundary). However, relativistic quantum field theory resides on globally hyperbolic, and thus in particular *noncompact and Lorentzian* manifolds.

As had first been observed in [1], the only additional ingredient needed for a description of Lorentzian metrics and spin strucures analogously to the language of spectral triples is a "fundamental symmetry" β. Indeed, using ideas from [1] and results given in [3], A.Strohmaier was able to give a description of *"Lorentzian spectral triples" (Losts)*. Such a Lorentzian spectral triple is then given by data $\mathbf{L} = (\mathcal{A}, \mathcal{H}, D, \beta)$, where the operator β on the Hilbert space $\mathcal{H}$ is in the commutant of the unital pre-C^*-algebra $\mathcal{A} \subset \mathcal{B}(\mathcal{H})$ and obeys $\beta^* = -\beta$, $\beta^2 = -1$. The Dirac-operator is β-symmetric, i.e. $D^* = \beta D \beta$ on the domain of D, while the other axioms are essentially the same as those of spectral triples, but with $|D|$ replaced by $\langle D \rangle :== \frac{1}{2}(DD^* + D^*D)$.
It is very important to note that two such Losts $\mathbf{L}_1, \mathbf{L}_2$ are equivalent, and in particular describe the same Lorentzian Spin manifold if $\mathcal{A}$ is commutative if and only if there exists a unitary $V : \mathcal{H}_1 \to \mathcal{H}_2$ such that $X_2 = VX_1V^*$ for $X_k \in \{\beta_k, \mathcal{A}_k, [D, \mathcal{A}_k]\}$. Strohmeier missed this observation, and hence was not able to state the analogue of Connes' reconstruction theorem. Moreover, he had to assume that $\mathcal{A}$ is unital, which is the case only for very few Lorentzian manifolds. However, in [4, 5] a significant progress concerning the generalisation of spectral triples to apply also for nonunital algebras (corresponding to noncompact manifolds) has been achieved. Based on these spadeworks, we first presented in the talk our definition of Losts for nonunital algebras and the analogue of Connes' reconstruction theorem for time-orientable noncompact Lorentzian Spin manifolds (without boundary).
A Lost is called *timelike foliated* if there exists a strongly continuos one-parameter group α_τ of automorphisms of $\mathcal{A}$, represented by unitaries U_τ on $\mathcal{H}$,i.e. $\alpha_\tau(a) = U_\tau a U_\tau^*$ for all $a \in \mathcal{A}$ and if there exists a strongly continuos group of unitaries u^r, $r \in \mathbb{R}$ in a (pregiven) unitalization $\tilde{\mathcal{A}}$ of $\mathcal{A}$ such that $\alpha_\tau(u^r) = e^{i\tau r}u^r$. It is additionally required that $\beta = u^{-1}[D, u]$.

If $\mathcal{A} = C_0^\infty(M)$ then one may use the pair (U_τ, u^r) to reconstruct a global time coordinate x_0 , such that $u^r = e^{ix_0}$, and a foliation of $M = \Sigma \times \mathbb{R}$ along x_0. Of course, such a foliation, if it exists, is never unique. We denote by $\mathcal{T}_{\mathbf{L}}$ the set of all admissable pairs (U_τ, u^r) for a given Lost $\mathbf{L}$.
Even if $\mathcal{A}$ is noncommutative, the set $\mathcal{T}_{\mathbf{L}}$ can then be used to construct the analogue $\mathcal{H}_0^\infty$ of the *space of spinors of compact support*, essentially by making use of the observation that for $\psi, \varphi \in \mathcal{H}_0^\infty$ the support in τ of $f_{\psi,\varphi,U}(\tau) := \langle \psi, U_\tau \varphi \rangle$ is compact for all U_τ out of $\mathcal{T}_{\mathbf{L}}$.
Moreover, using similar observations, we also gave completely algebraic conditions under which the "support" of a distribution ξ on $\mathcal{H}_0^\infty$ is contained in the *causal future (respectively past)* of the supprt of another distribution $\eta \in (\mathcal{H}_0^\infty)'$.
This then immediately leads to the definition of *advanced, respectively retarded propagators* $E^\pm : \mathcal{H}_0^\infty \to (\mathcal{H}_0^\infty)'$ for the Dirac-Operator D of the given Lost, i.e.

$$\langle E^\pm \varphi, D\psi \rangle = \langle \varphi, \psi \rangle, \qquad \forall \varphi, \psi \in \mathcal{H}_0^\infty$$

and the support of $E^+\varphi$ is contained in the causal future (resp. past for $E^-\varphi$) of that of φ (viewed as a distribution).
As our working definition, we call a timelike foliated Lost for which there exist uniquely determined advanced and retarded propagators , a *globally hyperbolic spectral triple (Ghyst)*. Ghysts can be thought of as the noncommutative generalization of globally hyperbolic Spin-manifolds. We gave some noncommutative examples, most notably the Moyal-deformed Minkowski space, as well. In the talk we then presented our conjecture that a timelike foliated Lost is a Ghyst if and only if there exists a pair (U_t, u^r) in $\mathcal{T}_{\mathbf{L}}$ such that the corresponding Dirac-Hamiltonian $H = D - \beta\partial_0$, where $U_\tau = e^{i\partial_0 \tau}$, is essentially selfadjoint.
We also speculated that it may be possible and desirable to give an alternative definion of Ghysts that only dealss with the space $\mathcal{H}_0^\infty$ and the maps $E^\pm$ rather than D and $\mathcal{H}$.

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Quantum Gromov Hausdorff Convergence and Scaling Limit Theories

Daniele Guido
(joint work with Tommaso Isola, Rainer Verch)

In this report I describe how scaling limit theories can be described in terms of quantum Gromov Hausdorff convergence, and, more precisely, they constitute an example of a noncommutative Gromov tangent cone.
Let us recall that the Gromov tangent cone of a metric space (X, d) at a point x consists of the cone of metric spaces obtained as pointed Gromov-Hausdorff limits, when $t \to \infty$, of the family $(X, x, t \cdot d)$, hence is a sort of replacement, for metric spaces, of the usual notion of tangent space, one peculiar feature being the non-uniqueness of the tangent space at a point, due to the possible local non-regularity of the original space (cf. [3] for examples). Scaling limit theories of a given net of C*-algebras describing the local observables of a quantum field theory, have been introduced in [1] in order to describe the renormalisation group in the algebraic setting, and possibly to study the phenomenon of confinement. In this case too uniqueness is not guaranteed in general, due to the possible local non-regularity of the original theory.
Let us first observe that it is possible to describe any limit space T in the Gromov tangent cone through the C*-algebra of continuous functions on T, constant at infinity, which in turn is the inductive limit, for $r \to \infty$, of the C*-algebras $\mathcal{C}(B_T(x, r))$ of continuous functions which are constant in the complement of the open ball $B_T(x, r)$.
Moreover, $\mathcal{C}(B_T(x, r))$ can be obtained as the quantum Gromov-Hausdorff limit (on a suitable subsequence $t_n \to \infty$) of $(\mathcal{C}(B_X(x, r/t)), t^{-1} \cdot L)$, where L is the Lip-seminorm associated with the distance d (cf. [7] for the main definitions and properties of quantum Gromov-Hausdorff limits). This suggests the following

Definition 2. *Given a net $B(y, r) \to \mathcal{A}(B(y, r))$ of unital C*-algebras associated with the open balls of a metric space (X, d), the net being endowed with a compatible Lip-seminorm L, a tangent net at $x \in X$ (on a tangent set T) is defined as follows: the Lip-normed C*-algebra $\mathcal{A}(B_T(y, r))$ associated with the ball of radius r and center y of the tangent set T is the quantum Gromov-Hausdorff limit (on a suitable subsequence $t_n \to \infty$) of $(\mathcal{A}(B_X(y_t, r/t)), t^{-1} \cdot L)$, where $y_t \in X$ converges to $y \in T$ in a suitable sense.*

Let us remark that, when passing to noncommutative C*-algebras, quantum Gromov-Hausdorff distance has to be replaced with some stronger distance, such as the matricial distance introduced by Kerr [5].
When the space X has a natural action of dilations, as is the case for the Minkowski space, we may define, for the tangent net at the origin, the algebra $\mathcal{A}_T(\mathcal{O})$ associated with a bounded open region $\mathcal{O}$ as the quantum Gromov-Hausdorff limit (on a suitable subsequence $t_n \to \infty$) of $(\mathcal{A}(t^{-1}\mathcal{O}), t^{-1} \cdot L)$.
Under a uniform compactness assumption, namely the existence, for any bounded open region $\mathcal{O}$, of a constant $C_\mathcal{O}$ such that $n_r(\mathcal{S}(\mathcal{A}(r\mathcal{O}))) \leq C_\mathcal{O}$, where n_r denotes

the minimum number of balls of radius r which are needed to cover the given set, and the distance on the state space is the one induced by the Lip-seminorm L, the local algebras of the tangent net may be identified with the Lip-ultraproducts (on a suitable ultrafilter) of $(\mathcal{A}(t^{-1}\mathcal{O}), t^{-1} \cdot L)$ [4].
Making use of this result, and assuming that the net $\mathcal{O} \to \mathcal{A}(\mathcal{O})$ describing a quantum field theory satisfies the uniform compactness assumption above, we may prove that the Scaling Limit Theories described in [1] can be identified with the vacuum representation of the tangent nets, where the compatible Lip-seminorm on the original net is given by the norm of the commutator with the generators of translations.
We observe that, even though a limit of C*-algebras under the matricial quantum Gromov-Hausdorff distance is not a C*-algebra in general, this is the case when the Lip-seminorm is induced by a commutator (compare with the notion of f-Leibniz Lip-seminorm in [6]).

Let us remark in conclusion that the validity of the uniform compactness assumption described above has still to be checked in concrete models, and possibly related with other compactness assumptions, such as Buchholz-Wichmann nuclearity [2].

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Scaling limit of superselection charges: a class of models

Gerardo Morsella
(joint work with Claudio D'Antoni)

Scaling algebras have been introduced in [1] as a version of the Renormalization Group adapted to the algebraic approach [2] to Quantum Field Theory. As such, a natural application is the intrinsic characterization of the ultraviolet properties of QFT, and in particular of the confinement phenomenon: a confined charge of the theory described by the net $O \to \mathcal{A}(O)$ of the C*-algebras of local observables is a charge of its scaling limit net $O \to \mathcal{A}_0(O)$ which is not also a charge of $\mathcal{A}$. In order to compare the superselection (i.e. charge) structures of the theories $\mathcal{A}$

and $\mathcal{A}_0$, it is useful to have a criterion of "charge preservation", identifying those sectors of $\mathcal{A}$ which are also sectors of $\mathcal{A}_0$. Such a criterion was formulated in [3] by first generalizing the scaling algebra construction to the canonical Doplicher-Roberts field net describing the superselection structure of $\mathcal{A}$ [4], and then imposing appropriate, physically motivated phase-space restrictions to the multiplets of orthogonal isometries implementing the DHR endomorphisms associated to the preserved charge.
In the seminar I reported about a follow-up work [5], where, with the purpose of illustrating the general framework of [3], we construct a class of QFT models possessing both preserved and non-preserved sectors. More in detail, our main result is the following:

Theorem 3. *For each pair (G, N) with G a compact Lie group and $N \subset G$ a closed normal subgroup, there exists a net of local observables $\mathcal{A}$ which has DHR sectors in 1-1 correspondence with classes of irreducible representations of G, and such that only the sectors corresponding to representations which factor through G/N are preserved in the scaling limit theory $\mathcal{A}_0$.*

Such a net is obtained as the fixed point net $\mathcal{A} = \mathcal{F}^G$ of a field net $\mathcal{F}$ which is in turn a tensor product $\mathcal{F} = \mathcal{F}_1 \otimes \mathcal{F}_2$, and the nets $\mathcal{F}_i$ are constructed, similarly as in [6], as follows. We fix a finite, generating and symmetric set Δ of irreducible representations of G, and consider for each $v \in \Delta$ which is nontrivial on N a v-multiplet ϕ_k^v of generalized free scalar fields with mass measure $d\rho(m) = dm$. We also fix a non-increasing function $\lambda \in \mathbb{R}_+ \to n(\lambda) \in \mathbb{N}$ which diverges as $\lambda \to 0$. Then $\mathcal{F}_1(O)$ is the von Neumann algebra generated by $\phi_k^v(f)$ with $f \in \Box^{n(\mathrm{diam} O)}\mathcal{D}(O)$, i.e. as O shrinks to a point, the algebra $\mathcal{F}_1(O)$ contains only elements with rapidly worsening ultraviolet properties, and it is therefore not surprising that the corresponding scaling limit theory is trivial: $\mathcal{F}_{1,0} = \mathbb{C}1$. On the other hand $\mathcal{F}_2(O)$ is the von Neumann algebra generated by $\varphi_k^v(f)$ with $f \in \mathcal{D}(O)$, where φ_k^v is a v-multiplet of free scalar fields of mass m_v for each $v \in \Delta$ which is trivial on N (i.e. which factors through G/N). Using results in [7], we show that $\mathcal{F}_{2,0}$ is the net generated by the same multiplets of free fields with zero masses, and that the corresponding sectors of $\mathcal{F}_2^{G/N}$ are all preserved in the scaling limit in the sense of [3].

In order to determine the scaling limit theory $\mathcal{F}_0$, we need conditions under which the operations of scaling limit and of forming the tensor product of two theories can be interchanged. We obtain sufficient conditions for this in terms of nuclearity properties of the nets $\mathcal{F}_i$ (we refer the reader to [2] for a discussion of nuclearity properties in QFT). In particular, if $\Theta^{(i)}_{\beta,O} : \mathcal{F}_i(O) \to \mathcal{H}_i$ is the map $\Theta^{(i)}_{\beta,O}(F) = e^{-\beta H_i} F \Omega_i$, we have the following:

Theorem 4. *If the maps $\Theta^{(i)}_{\beta,O}$ are p-nuclear for $p \in (0, 1/6)$ and the nuclear p-norms satisfy $\limsup_{\lambda \to 0} \|\Theta^{(i)}_{\lambda\beta,\lambda O}\|_p < +\infty$, and if $\mathcal{F}_{i,0}$ satisfies (twisted) Haag duality, then $\mathcal{F}_0 \cong \mathcal{F}_{1,0} \otimes \mathcal{F}_{2,0}$.*

Applying this theorem to the net $\mathcal{F} = \mathcal{F}_1 \otimes \mathcal{F}_2$ above, we get that $\mathcal{F}_0 \cong \mathcal{F}_{2,0}$, and therefore the sectors of $\mathcal{A}$ associated to G-representations non-trivial on N are non-preserved in the scaling limit.
Since it is built using generalized free fields, the net $\mathcal{A}$ does not satisfy Haag duality, but only essential Haag duality, and this therefore leaves the possibility open that requiring Haag duality rules out the appearence of non-preserved sectors.
Another interesting open problem is the one of finding more general conditions under which the scaling limit and tensor product operations can be interchanged. Although we don't have examples of theories which don't satisfy our hypoteses and for which the two operations don't commute, it seems quite natural that some kind of phase space condition has to play a role, also in view of the fact that, if a specific such condition holds, scaling limits are limits with respect to a suitable metric [8], and therefore enjoy good functorial properties.

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On the local index theorem in noncommutative geometry

Denis Perrot

In [1], Connes and Moscovici give a formula computing the Chern character of a regular spectral triple (A, H, D) in terms of residues of certain zeta-functions. This allows to give a local expression for the index map $K_*(A) \to \mathbb{Z}$ from the K-theory of the algebra A induced by the K-homology class of the spectral triple, as in the classical Atiyah-Singer index theorem. In this talk we propose to obtain such a local index formula by considering the chiral anomaly of an adequate noncommutative quantum field theory. In fact noncommutative index theory and anomalies are in a sense equivalent, the link being provided by Bott periodicity. The advantage of considering anomalies, however, stems from the fact that the expressions found are automatically local. We recall that given an associative algebra A, a spectral triple (A, H, D) is given by:

i) a homomorphism from A to the algebra $B(H)$ of bounded operators on a separable Hilbert space H;
ii) an unbounded selfadjoint operator D with compact resolvent on H;
iii) the commutator $[D, a]$ extends to a bounded operator on H for any $a \in A$.

In addition, the spectral triple has even degree if there is an involutive operator $\gamma \in B(H)$, $\gamma^2 = 1$, which anticommutes with D and commutes with any $a \in A$. It is p-summable for a real number $p \geq 1$ if $(1 + D^2)^{-p/2}$ lies in the Schatten p-class $\ell^p(H)$. We also impose the following *regularity* condition on (A, H, D):

iv) A and $[D, A]$ belong to the domains of all powers of the derivation $\delta = [|D|,\ ]$;
v) Let $\Psi_0(A)$ denote the algebra of operators generated by the derivatives $\delta^n(A)$ and $\delta^n([D, A])$. Then for any $b \in \Psi_0(A)$, the zeta-function

$$\zeta_b(z) = \mathrm{Tr}(b|D|^{-z}) \qquad z \in \mathbb{C}$$

extends to a meromorphic function with poles contained in a discrete set $Sd \subset \mathbb{C}$ (the dimension spectrum).
The local index formula of Connes-Moscovici precisely works for regular p-summable spectral triples. It is interesting to note that the same conditions allow to build a noncommutative gauge theory of fermions, as follows. From now on we restrict ourselves to spectral triples of even degree, and assume for simplicity that the Dirac operator D is invertible. Let $\psi \in H$ be in the domain of D and $\overline{\psi} \in H^*$. We consider the classical action functional

$$S(\psi, \overline{\psi}, V) = \langle \overline{\psi}, (D + V)\psi \rangle$$

where V is a chiral potential constructed from the elements of A and $[D, A]$. One wishes to give a sense to the functional integral (ψ and $\overline{\psi}$ are treated as fermions)

$$Z(V) = \int d\psi\, d\overline{\psi}\ e^{-S(\psi, \overline{\psi}, V)}$$

According to the general principles of perturbative quantum field theory, it amounts to compute the free energy $W(V) = \ln Z(V)$ as a *formal* power series of the potential V:

$$W(V) = -\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \mathrm{Tr}((D^{-1}V)^n)$$

Actually the first few terms of this series are divergent because the operator $(D^{-1}V)^n$ is not of trace-class when n is less than the summability degree of the spectral triple. Hence these terms need to be renormalized, for example by replacing Tr by any linear extension $\tau : \Psi(A) \to \mathbb{C}$ of the operator trace on a suitable algebra of operators $\Psi(A)$. For example, one can choose a zeta-function renormalization

$$\tau(b) = \mathop{\mathrm{Res}}_{z=0} \frac{1}{z} \mathrm{Tr}(b|D|^{-z}) \qquad \forall b \in \Psi(A)$$

The renormalized free energy $W_\tau(V)$ is therefore well-defined as a formal power series. It can be shown that its variation $dW_\tau(V) = \Delta(\omega, V)$ under an infinitesimal chiral gauge transformation of parameter ω, i.e. the anomaly, is a *finite sum of local terms*. The integration of the anomaly along nontrivial loops of gauge transformations establishes the link with noncommutative index theory:

Theorem 5. *Let (A, H, D) be a regular p-summable spectral triple of even degree. Let W_τ be the free energy of the corresponding fermionic quantum field theory, renormalized by means of any linear extension $\tau : \Psi(A) \to \mathbb{C}$ of the operator trace. Let $e \in A$ be an idempotent representing an element $[e] \in K_0(A)$, and $g = 1 + (\beta - 1)e$ be the corresponding idempotent loop, with β the Bott generator of the circle. Then*
i) The integral of the anomaly $\Delta(\omega, V) = dW_\tau(V)$ along this loop is the index of the Dirac operator D against the K-theory class $[e]$:

$$\langle [D], [e] \rangle = \frac{1}{2\pi i} \oint \Delta(\omega, V)$$

ii) The anomaly $\Delta(\omega, V)$ is cohomologous to the finite sum of residues of zeta-functions

$$\operatorname*{Res}_{z=0} \frac{1}{z} \mathrm{Tr}(\gamma\omega|D|^{-2z}) + \sum_{n\geq 1, k\geq 0} (-1)^{n+k} c(k) \times$$
$$\times \operatorname*{Res}_{z=0} \left(\frac{\Gamma(z+n+k)}{z\Gamma(z)} \mathrm{Tr}((q\omega V^{(k_1)} D V^{(k_2)} \dots D V^{(k_n)} |D|^{-2(z+n+k)}) \right) ,$$

where $k = (k_1, \dots, k_n)$ is a multi-index, $q\omega = \frac{1+\gamma}{2}[\omega, D]$, $V^{(k_i)}$ denotes the k_i-th power of the derivation $[D^2, \ ]$ on V, and

$$c(k)^{-1} = (k_1! \dots k_n!)(k_1 + 1)(k_1 + k_2 + 2) \dots (k_1 + \dots + k_n + n) .$$

In particular all the coefficients involved are rational.

This theorem is an alternative to the local index formula of Connes-Moscovici. It is worth mentioning also that it admits a generalization to the bivariant case.

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Reporter: Mario Paschke

Participants

Dr. Paolo Aschieri

DiSTA
Universita del Piemonte Orientale,
and INFN
Via Bellini 25/G
I-15100 Alessandria

Dr. Dorothea Bahns

MPI für Gravitationsphysik
Albert-Einstein-Institut
Am Mühlenberg 1
14476 Golm

Prof. Dr. Sergio Doplicher

Dipartimento di Matematica
Universita di Roma "La Sapienza"
Istituto "Guido Castelnuovo"
Piazzale Aldo Moro, 2
I-00185 Roma

Prof. Dr. Klaus Fredenhagen

II. Institut für Theoretische
Physik
Universität Hamburg
Luruper Chaussee 149
22761 Hamburg

Dr. Stefan Fredenhagen

Institut für Theoretische Physik
ETH Zürich
Hönggerberg
CH-8093 Zürich

Dr. Victor Gayral

Mathematical Institute
University of Copenhagen
Universitetsparken 5
DK-2100 Copenhagen

Prof. Dr. Jose M. Gracia-Bondia

Departamento de Fisica Teorica I
Universidad Complutense
E-28040 Madrid

Dr. Jesper M. Grimstrup

Nordita
Blegdamsvej 17
DK-2100 Kobenhavn

Prof. Dr. Harald Grosse

Institut für Theoretische Physik
Universität Wien
Boltzmanngasse 5
A-1090 Wien

Prof. Dr. Daniele Guido

Dipartimento di Matematica
Universita di Roma "Tor Vergata"
V.della Ricerca Scientifica, 1
I-00133 Roma

Jan Hendryk Jureit

CPT UPR 7061
Case 907
163 Ave. de Luminy
F-13288 Marseille Cedex 9

Prof. Dr. Tomas Kopf

Institute of Mathematics
Silesian University
74601 Opava
CZECH REPUBLIC

Prof. Dr. Nicolaas P. Landsman

Dept. of Mathematics
Radboud Universiteit Nijmegen
Postbus 9010
NL-6500 GL Nijmegen

Prof. Dr. Edwin Langmann

Mathematical Physics
Dept. of Physics, KTH, SCFAB
S-10691 Stockholm

Gandalf Lechner

Institut für Theoretische Physik
Universität Göttingen
Tammannstr. 1
37077 Göttingen

Prof. Dr. Fedele Lizzi

Dipartimento di Scienze Fisiche
Complesso Univ. Monte Sant'Angelo
Via Cintia
I-80126 Napoli

Prof. Dr. Roberto Longo

Dipartimento di Matematica
Universita di Roma "Tor Vergata"
V.della Ricerca Scientifica, 1
I-00133 Roma

Prof. Dr. John Madore

Laboratoire de Physique Theorique
Universite de Paris XI
Batiment 211
F-91405 Orsay Cedex

Dr. Gerardo Morsella

Istituto Nazionale di Alta
Matematica "F. Severi"
Universita di Roma "La Sapienza"
Piazzale Aldo Moro, 2
I-00185 Roma

Dr. Michael Müger

Dept. of Mathematics
Radboud Universiteit Nijmegen
Postbus 9010
NL-6500 GL Nijmegen

Prof. Dr. Mario Paschke

Max-Planck-Institut für Mathematik
in den Naturwissenschaften
Inselstr. 22 - 26
04103 Leipzig

Dr. Denis Perrot

Institut Camille Jordan
UFR de Mathematiques
Univ. Lyon 1; Bat. Braconnier
21, Avenue Claude Bernard
F-69622 Villeurbanne Cedex

Dr. Gherardo Piacitelli

Via G. Gallina 3
I-34122 Trieste

Prof. Dr. Adam Rennie

Department of Mathematics
Copenhagen University
Universitetsparken 5
DK-2100 Copenhagen O

Yurii Savchuk

Max-Planck-Institut für Mathematik
in den Naturwissenschaften
Inselstr. 22 - 26
04103 Leipzig

Prof. Dr. Florian Scheck

Fachbereich 18 - Physik
Universität Mainz
Staudinger Weg 7
55128 Mainz

Prof. Dr. Andrzej Sitarz

Institute of Physics
Jagiellonian University
ul. Reymonta 4
30-059 Krakow
POLAND

Dr. Walter D. van Suijlekom

SISSA
International School for Advanced
Studies
Via Beirut n. 2-4
I-34014 Trieste

Dr. Rainer Verch

Max-Planck-Institut für Mathematik
in den Naturwissenschaften
Inselstr. 22 - 26
04103 Leipzig

Prof. Dr. Fabien Vignes-Tourneret

Laboratoire de Physique Theorique
Batiment 210
Universite Paris XI
F-91405 Orsay Cedex

Dr. Mihaly Weiner

Dipartimento di Matematica
Universita degli Studi di Roma II
Tor Vergata
Via della Ricerca Scientifica
I-00133 Roma

Prof. Dr. Stanislaw L. Woronowicz

Department of Physics
Warsaw University
Hoza 69
00-681 Warsaw
Poland

Dipl.-Phys. Jochen Zahn

II. Institut für Theoretische
Physik
Universität Hamburg
Luruper Chaussee 149
22761 Hamburg

Prof. Dr. Eberhard Zeidler

Max-Planck-Institut für Mathematik
in den Naturwissenschaften
Inselstr. 22 - 26
04103 Leipzig