Short Communication

Stress generation modulus as a counterpart of the stress relaxation modulus

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In order to measure the time dependence of the stress relaxation modulus, E, a stress-strain deformation has to precede which induces a stress within the viscoelastic sample from which the sample relaxes. The generation of stress is characterized by a strain rate-dependent and relaxation rate-dependent portion which exhibits a maximum value, E_{max}, which is called "stress generation modulus". E_{max} was called the "maximum stress relaxation modulus" in earlier papers. Meanwhile, however, it turned out that a better verbal distinction should be made in future by the new term "stress generation modulus" because E_{max} is about one order of magnitude larger than E.

Spannungsaufbaumodul als Gegenpart zum Spannungsrelaxationsmodul

Zur Bestimmung der Zeitabhängigkeit des Spannungsrelaxationsmoduls E muß eine Spannungsdeformation vorangegangen sein, die eine Spannung in der viskoelastischen Probe induziert, von der die Probe relaxiert. Die Spannungserzeugung bzw. der Spannungsaufbau wird durch einen verformungsgeschwindigkeits- und einen relaxationsgeschwindigkeitsabhängigen Anteil charakterisiert, der zu einem maximalen Wert, E_{max}, führt und mit „Spannungsaufbaumodul" bezeichnet wird. E_{max} wurde in früheren Veröffentlichungen „maximaler Spannungsrelaxationsmodul" genannt. Es zeigte sich jedoch inzwischen, daß in Zukunft eine bessere verbale Unterscheidung gemacht werden sollte mit Hilfe des neuen Begriffs „Spannungsaufbaumodul", E_{max}, da sich herausgestellt hat, daß E_{max} etwa eine Zehnerpotenz höher liegt als E.

1. Introduction

The recovery of a viscoelastic body such as glass or glass melt towards or into the isotropic and stress-free state has drawn large attention over decades in glass science and technology which is connected to the terms relaxation and retardation. The stress relaxation modulus is an important quantity to calculate the time-dependent stress after a certain deformation and load, respectively, or after a certain time of an initially injected or generated stress in a glass sample [1]. Concrete measurements of the stress relaxation modulus were performed by Kurkjian [2] with the torsion method in the viscosity range of 10^{15} to 10^{12} Pa s and by Mills and Sievert [3] with the compression method in the range of $6 \cdot 10^{12}$ to 10^{9} Pa s which was extended to 10^{8} Pa s by Larsen et al. [4]. The shear and compressive stress relaxation moduli were given as a function of time at various temperatures. In dynamic vibration experiments these moduli can be determined also as a function of frequency [1, 5 and 6].

Another problem is the injection or generation of stress into a viscoelastic body, i.e., the production of stress during deformation and loading, respectively, and not after loading. While the stress is introduced, the relaxation is immediately acting, and its rate increases as the load is increased up to a certain amount at which the relaxation process overcomes the loading process. Thus, a dynamic modulus is obtained which increases first and decreases to zero after the domination of the relaxation process. This is a situation which is applied very often in the practical glass forming process, particularly in pressing container glass, cutting the glass gob or extrusion and injection moulding.

In the case of the cylinder compression method the process is described by the stress-strain diagram ($\sigma(\varepsilon)$) or stress-deformation diagram (σ-Δh) (figures 1a and b), where Δh is the decrease of the cylinder height and $\varepsilon = \Delta h/h$ the strain. The differentiation of the stress-strain curve leads to the described dynamic modulus, which has a maximum amount at the point of inflexion and turns to zero at the maximum of the stress-strain curve. This maximum value, $E(t)_{max}$, was called the "maximum stress relaxation modulus" in earlier papers [7 to 10] because the authors believed that the relaxation process determined this value. However, this is not exclusively the case because it was found to be by about one order of magnitude larger than the usual stress relaxation modulus measured by Mills et al. [3 and 4]. Therefore, and with respect to not yet published results the authors of the present paper prefer to call this modulus E_{max} the "stress generation modulus". This modulus is a helpful property to characterize the stiffness of a glass melt. It could be shown in [7 to 9] that a large value of this modulus is connected to a large stiffness of

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a glass melt and vice versa. A low stress generation modulus, $E(t)_{max}$, is connected to a large critical deformation rate, $\dot{h}_c$, at which the first crack appears at a given deformation value $\Delta h/h_0$ indicating a good isochomal workability [7 to 10].

2. Stress relaxation and stress generation modulus

The stress relaxation modulus, which was determined intensively by Mills et al. [3 and 4] was evaluated in the following principle way. A glass cylinder is compressed between two pistons by a certain deformation $\Delta h/h_0$ with a certain deformation rate (figure 2b). The resulting stress shows a maximum versus time (figure 2a). The decay of the stress with time $\sigma(t)$ was determined on the right side of the maximum. Divided by $\Delta h/h_0$, the time-dependent stress relaxation modulus is given by:

$$\sigma(t)\, h_0/\Delta h = E(t)\,. \qquad (1)$$

Various values of $\Delta h/h_0$ (between 0.5 and 2.3 %) showed that $E(t)$ is independent of the degree of deformation.

The stress generation modulus, $E(t)_{max} \mathrel{\widehat{=}} E_{max}$, which was determined in [7 to 11], was evaluated not on the right but on the left side of the stress maximum (figure 1a) from the point of inflexion [7, 8 and 11]:

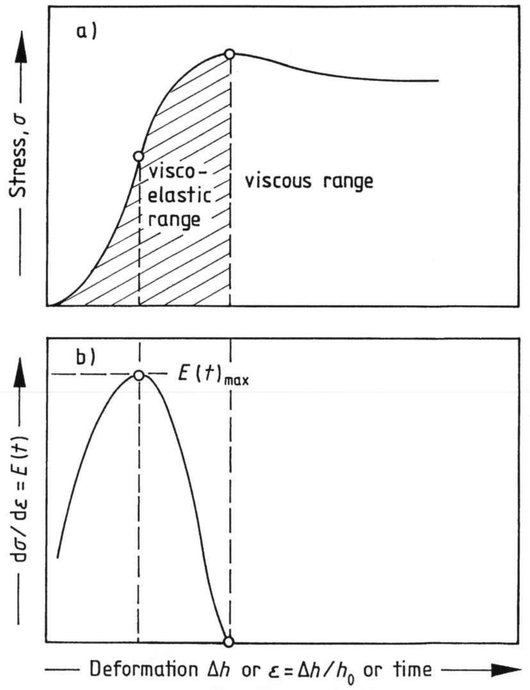
$$\begin{aligned} E(t)_{max} &\mathrel{\widehat{=}} E_{max} \equiv [\dot{\sigma}(t)/\dot{\varepsilon}(t)]_{max} \equiv \\ &\equiv [\mathrm{d}\sigma(t)/\mathrm{d}\varepsilon(t)]_{max} \approx \frac{h_0}{A_0}\left[\frac{\mathrm{d}F(t)}{\mathrm{d}\Delta h(t)}\right]_{max} \end{aligned} \qquad (2)$$

where $\dot{\sigma}(t) = \mathrm{d}\sigma(t)/\mathrm{d}t$, $\dot{\varepsilon}(t) = \mathrm{d}\varepsilon(t)/\mathrm{d}t$, $\mathrm{d}\sigma(t) \approx \mathrm{d}F(t)/A_0$ and $\mathrm{d}\varepsilon(t) \approx \mathrm{d}\Delta h(t)/h_0$; h_0 is the initial cylinder height of the sample at $t = 0$, A_0 the area of the initial cross-section at $t = 0$, $F(t)$ the force after time, t, $\Delta h(t)$ the axial deformation of the sample at time, t, the time during pressing.

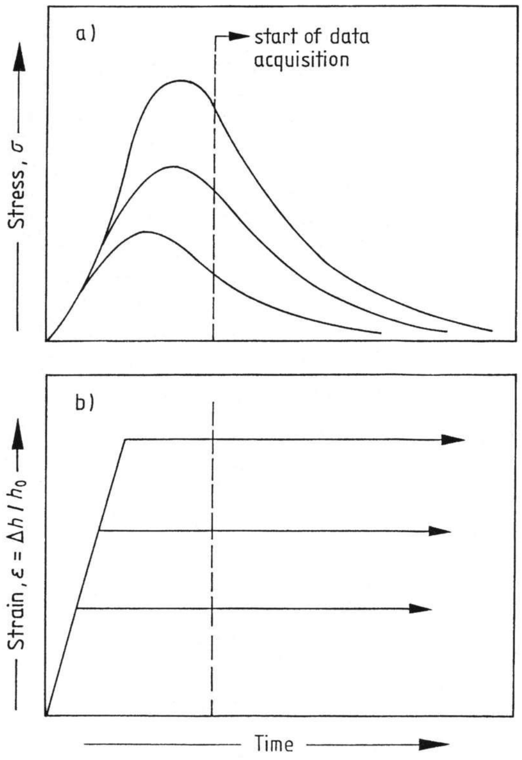
Figure 3 shows the results of Mills et al. [3 and 4] as a plot of the logarithm of the stress relaxation modulus, $E(t)$, versus lg time at various temperatures or (Newtonian) viscosities (table 1), for a container glass with a composition listed in table 2. Introduced into figure 3 are also the results of the stress generation modulus, $E(t)_{max}$, obtained from float glass (table 2).

The two compositions are very similar, thus, a rough comparison of the results for $E(t)$ and $E(t)_{max}$ appears to be possible. The conversion from the results, $E(t)_{max}$ versus $\dot{\varepsilon}$ or $\dot{h} = \mathrm{d}h/\mathrm{d}t$, to that versus Δt, the stress generation time, was done by the relations

$$\dot{\varepsilon} = \frac{1}{h}\,\frac{\Delta h}{\Delta t} = \dot{h}/h$$



Figures 1a and b. Principle of the stress-strain curve (figure a) at a certain deformation rate and its differentiated curve (figure b) $\mathrm{d}\sigma(t)/\mathrm{d}\varepsilon(t) = E(t)$ with the maximum value $E(t)_{max}$ at the point of inflexion of the stress-strain curve.



Figures 2a and b. Principle of the stress-time curve (figure a) produced by the deformation-time curve (figure b) from which $\sigma(t)\, h_0/\Delta h = \sigma(t)/\varepsilon = E(t)$ is obtained by Mills et al. [3, 4 and pers. commun.].

with

$$\Delta t = \frac{1}{h}\,\frac{\Delta h}{\dot{\varepsilon}} = \Delta h/\dot{h} \approx 0.1/\dot{h}\,,$$

Table 1. Viscosity values for the temperatures given in figure 3 for the container glass used by Mills et al. [3 and 4]

temperature in °C:	540	556	580	600	620	640	660	680
η in Pa s:	$1.38 \cdot 10^{12}$	$2.51 \cdot 10^{11}$	$2.40 \cdot 10^{10}$	$5.21 \cdot 10^{9}$	$1.10 \cdot 10^{9}$	$2.75 \cdot 10^{8}$	$7.94 \cdot 10^{7}$	$3.24 \cdot 10^{7}$

Table 2. Compositions (in wt%) of two industrial glasses

	SiO_2	Al_2O_3	CaO	MgO	Na_2O	K_2O	Fe_2O_3	SO_3
container glass used by Mills et al. [3 and 4]	71	1.5	8.0	4.5	15	–	–	–
float glass used by Brückner et al.	71.6	0.6	9.4	4.0	13.9	0.16	0.10	0.24

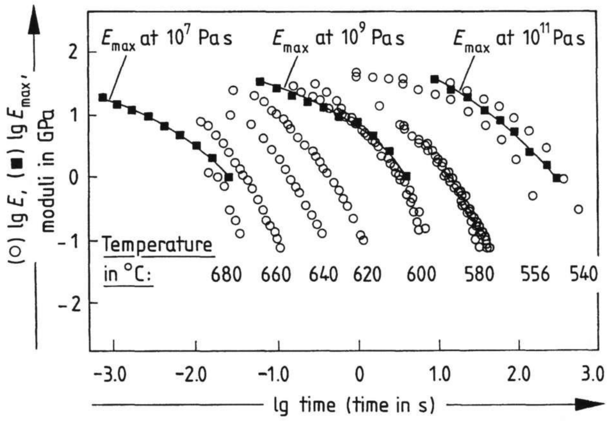


Figure 3. Stress relaxation modulus, $E(t)$ (○), versus time on logarithmic scales after Mills et al. [3 and 4] for a container glass melt and the stress generation modulus, $E(t)_{max}$ (■), versus time for a float glass melt at viscosities of 10^7, 10^9 and 10^{11} Pa s. The corresponding viscosity values for 8 temperatures of the container glass melt used by Mills et al. are given in table 1 and the compositions of both glasses in table 2.

because the deformation, Δh, at which $E(t)_{max}$ is obtained, is usually constant $\approx$ 0.1 mm and nearly independent of deformation rate and viscosity.

It is seen from figure 3 that the curvature of lg E and lg E_{max} versus lg t plots is the same to a great extent, but the curves of lg E_{max} are shifted by a factor of about 6.3 at $E = E_{max} \approx 18$ GPa to larger t values at comparable viscosities; i.e. E_{max} is about one order of magnitude larger than E at comparable t values. This means that E_{max} does not depend only on relaxation but also on the generated mechanical input of stress and therefore also on the strain rate (not on the strain itself) in contrast to E which is independent not only of the strain but also of the strain rate (which is kept constant in figure 3). Therefore, E_{max} is a velocity-dependent differential quantity (equation 2) in contrast to E as a time-dependent ratio (equation 1). This is the reason why E_{max} is now called "stress generation modulus" because stress is generated by the strain rate in the more or less relaxing sample. The direct dependence of E_{max} on the strain rate will be given in [11].

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3. References

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