Original Paper

Physical modelling of flow behavior in a stirred glass system

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As stirring efficiency made the use of isothermal models plausible, dimensional analysis was used to extend model laws to include stirrers, gobbing cams and to indicate the conditions required to get data representative of actual or proposed units. Based on full-size modelling, the production department installed stirred glass set-ups and made excellent products. Correlation between modelling and production was excellent and included mechanical variables and temperatures. Furthermore, a new delivery system, including a new gobbing cam designed for a very fluid photosensitive glass, was based on modelling and behaved exactly as predicted.

Upsets in the stirred glass, delivery systems resulted in slow cyclic weight variations of about 1 %. As the weights of the items being made were specified to within ±1.0 %, it was clear that control had to be improved. The weight changes were not caused by compositional changes in the glass but to some instrumentation problems and to some unique, unknown problem(s) associated with stirring. Thus, a better understanding of how temperature and the stirrers controlled flow was needed. Assuming that flow is inversely proportional to viscosity showed that the change in weight should be about 0.9 %/°C. This number is about 10 % too low compared with the empirical value of 1 %/°C quoted by operators.

Next data from a full-size, stirred model were summarized in an equation which relates flow to the viscosity of the fluid and to the rotational speed and vertical position of the stirrer. Partial differentiations and evaluations of this equation at a nominal operating condition showed that the weight should first, vary by 1.02 %/°C, second, decrease by 0.90 % per rpm increase and third, increase by 0.80 % when the stirrer was raised (0.006″) or 1 % when it was at the 5/8″ position.

Physikalisches Modell des Fließverhaltens in einem gerührten Glassystem

Da die Wirksamkeit des Rührens von Glasschmelzen die Verwendung isothermer Modelle plausibel erscheinen läßt, wurde die Dimensionsanalyse zur Erweiterung der Modellgesetze eingesetzt, um sowohl Rührer als auch die Tropfensteuerung einzubeziehen und die Bedingungen festzulegen, die für die Datengewinnung von bestehenden oder geplanten Anlagen erforderlich sind. Auf der Grundlage eines Modells natürlicher Größe installierte die Produktionsabteilung Einrichtungen zum Rühren von Glas und erzielte sehr gute Ergebnisse. Die Übereinstimmung zwischen Modell und Produktion war hervorragend und schloß mechanische Variable und Temperaturen ein. Ferner wurde an Hand des Modells ein neues Rinnensystem, einschließlich einer neuen Tropfensteuerung für ein lichtempfindliches Glas geringer Viskosität, entwickelt, das sich genau wie vorgesehen verhielt.

Stauchungen des gerührten Glases innerhalb der Rinnensysteme führten zu langsamen zyklischen Gewichtsveränderungen von etwa 1 %. Da die Gewichte der herzustellenden Gegenstände mit ±1 % Abweichung vorgeschrieben waren, wurde klar, daß die Kontrolle noch zu verbessern war. Die Gewichtsabweichungen waren nicht durch Zusammensetzungsänderungen des Glases verursacht, sondern durch gewisse Instrumentierungsprobleme sowie einzelne, unbekannte, mit dem Rühren in Zusammenhang stehende Probleme. Daher war ein besseres Verständnis dafür notwendig, wie die Temperatur und die Rührer die Strömung beeinflußten. Unter der Annahme, daß die Strömung der Viskosität umgekehrt proportional ist, konnte gezeigt werden, daß die Gewichtsveränderung bei etwa 0,9 %/°C liegt. Diese Zahl ist um rund 10 % zu niedrig im Vergleich zum empirischen Wert von 1 %/°C, wie er vom Bedienungspersonal angegeben wird.

Danach werden Daten von einem mit Rührern ausgestatteten Modell natürlicher Größe in einer Gleichung zusammengefaßt, die das Fließen mit der Viskosität der Flüssigkeit, der Rotationsgeschwindigkeit und der vertikalen Position des Rührers in Beziehung setzt. Partialdifferentationen und Abschätzungen dieser Gleichung bei nominalen Betriebsbedingungen zeigten, daß das Gewicht, erstens, um 1,02 %/°C variiert; zweitens, um 0,9 %/rpm abnimmt; drittens, um 0,8 % zunimmt, wenn der Rührer um 0,006″ angehoben wurde, und um 1 %, wenn er sich vorher in der 5/8″-Position befunden hatte.

1. Introduction

1.1 Problem and procedures

On occasion, glass-forming machine operators complained that it was sometimes difficult to adjust and maintain flow out of some delivery systems. This problem was not too common when a needle and sleeve [1] were used. However, some delivery systems had bowl well stirrers [2] which were needed in order to reduce cords (off-composition, visual, glass defects) to acceptable levels for high-quality items. In almost all the production lines, the temperature of the glass being delivered was maintained constant and the flow, upwards of 40 000 lbs/day/feeder, was then controlled mechanically. Good flow control was essential as the weights of the items being made were often specified to within ±1 %. To help pinpoint the problem(s), operators were asked to report weight problems while they were occurring. As a result, three instances, involving three stirred delivery systems, were investigated and are discussed in section 1.2 called "three weight runs". A weight run consists of collecting and interpreting all the possible chronological

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Received October 27, 1995, revised manuscript October 1, 1996.

[1] Now retired.

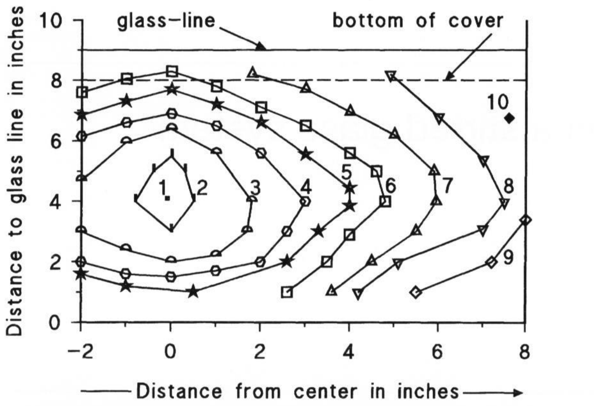


Figure 1. Isotherms in glass exiting a covered forehearth and entering a stirred glass delivery system. 1: 1322 °C; 2: 1320 °C; 3: 1315 °C; 4: 1310 °C; 5: 1305 °C; 6: 1300 °C; 7: 1290 °C; 8: 1280 °C; 9: 1275 °C; 10: 1273 °C.

data associated with selected items of production. From these three runs it was clear that control had to be improved and that it would be practically impossible to get enough data from production lines to relate viscosity and the mechanical variables associated with a gobbing or non-reciprocating stirrer except in very narrow ranges. In order to improve understanding of the problems, two other approaches were tried. One involved assuming a functional relation between flow and viscosity. This approach was good and is treated in section 1.3. However, it did not include any of the mechanical variables associated with gobbing or non-reciprocating stirrers. These variables were included by using dimensional analysis. This approach extended model laws to include stirrers and the design of cams that could be used for gobbing or extruding molten glasses. A model based on these rules yielded the necessary information.

1.2 Three weight runs

In each of these runs there was a short period in which the weight first increased and then decreased gradually, about 1 %, without any changes being made by the operators. At first it appeared that these variations might have resulted from changes in viscosity brought about by changes in the composition of the glass. However, this was not a satisfactory explanation as less than 1 h was needed for each "complete excursion". These times were far too short for melters delivering upward of 40 000 lbs of glass per day per feeder. In such large furnaces the distribution of residence times may show a quick result but the decay is slow and smears any variation in composition over several days [3]. Each system had a thermocouple immersed in glass in the front of the bowl and its output was used for control. It is emphasized that in all cases the bowl controllers maintained the temperature of the glass at the orifice to within ±0.7 °C. In one delivery system, this was the only control. Its output correlated with swings in the temperature of the incoming glass. Further study showed small, random malfunctions of some of the other controllers in the other forehearths. The weight changes ranged from 0.8 to slightly more than 1 % and averaged at 1 %/°C. This average was in good agreement with the empirical value of 1 %/°C used by production personnel for the borosilicate glass at the temperatures that were being used.

It was concluded: a) that the operators were doing an excellent job, b) that a single zone of control was inadequate, c) that the flow rate varied in spite of the fact that the bowl controllers were operating inside their specified capabilities, d) that each system needed some upgrading, and finally e) that the problem(s) could be explained by changes in temperature but f) the reason(s) for the apparent cyclic temperature changes were unknown.

1.3 Flow versus viscosity

It was assumed that mass flow rate versus temperature is inversely proportional to viscosity. Mathematically

$$W/t = k/\eta \tag{1}$$

where W is the weight of a glass stream or gob cut in time t, η is the viscosity of this molten glass and k is a constant. With a temperature change, other conditions remaining constant, the viscosity changes and the weight will also change. Two methods of manipulating equation (1) showed that

$$(\mathrm{d}W/W)/\mathrm{d}T = 0.9\,\%/^\circ\mathrm{C} \tag{2}$$

was the percent weight change per °C for the borosilicate glass (sections 13.1 and 13.2). This result was 10 % lower than the factory's number. However, it was evidence supporting equation (1) and its underlying assumption.

This relatively large weight change indicated the desirability of improving flow control as the total weight variation was supposed to be held to substantially less than ±1 %, regardless of causes.

2. Variables

2.1 Isothermal assumption

It was thought that physical models could be used to study flow through stirrers or flows dependent on stirring and give quantitative data on flow behavior depending on stirring, temperature and stirrer position, provided the necessary conditions needed for their construction and operation were known. One way to find the required relations or laws was to use dimensional analysis [4 to 7]. These laws could be used to relate model studies of existing production units and/or to predict performance of proposed stirred sections. For successful dimensional analysis, the correct variables must be chosen.

Temperature distribution of the glass leaving a forehearth and entering a bowl is not uniform. For example, the temperature varied by about 50 K (figure 1) for a borosilicate glass flowing out of an electrically heated, covered forehearth [8]. Both compositional and thermal homogenization of glass by stirring is excellent [9], so temperature variations are unimportant in stirred sections. To a lesser degree the same is true for a rotating sleeve and needle. Assuming thermal homogeneity of temperature was fundamental. It means that all the variables associated with heat transfer can be ignored and the models may be operated isothermally.

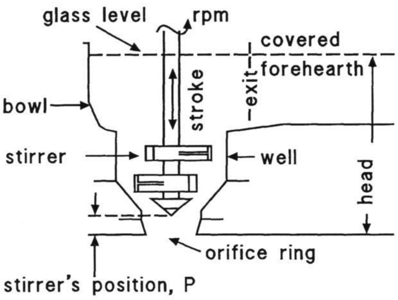


Figure 2. Sketch of a stirred glass delivery system.

2.2 Variables governing the flow of fluid

2.2.1 Internal dimensions

These variables of the delivery system include the size of the orifice ring and other parts as well as the dimensions of the stirrer (figure 2). For a given system only one characteristic, non-varying or fixed length (L) like a diameter needs to be specified. This fixes all the other non-varying dimensions. Thus, all corresponding parts have the same ratio to the characteristic length in the model and in the proposed or actual production counterpart. Nevertheless, some of these other lengths are specified because of their importance in stirred glass delivery systems.

2.2.2 Significant physical properties of the fluid

These are viscosity (η) and density (ϱ). Provided no changes are made to mechanical parts and/or to glass level, viscosity and density are the actual fluid-related variables responsible for flow. Temperature (T) is used to adjust viscosity and density simultaneously. However, when viscosity and density are used explicitly, it is not necessary to include temperature explicity in any flow control equation. For this work the laboratory measured the density and viscosity over a wide temperature range, including temperatures of the glass as it emerged from the orifice ring(s). For meaningful viscosity values, the glass must have the correct chemical composition including changes caused by using cullet and the correct "water" content (also known as the β or OH^- value) as increases in the β value softens the glass or decreases its viscosity.

2.2.3 Dynamics of the delivery system

The internal contour of a model including the diameter of its orifice ring and the shape and size of its stirrer, will be based on an actual or a proposed delivery system. These parameters were kept fixed for this investigation. Therefore, with this stipulation, the dynamics of the delivery system must depend on variables or parameters that involve time (t). These dynamic variables include the actual head or height of glass (H) above the orifice opening or ring, all the velocities (V) and all the accelerations (a) of the fluid as these variables can and do change with changes in temperature (T) and/or flow: In addition, the acceleration due to gravity (g) must be included. Dynamics also requires the vertical position (P) and the centering (c) of the stirrer, its rotational speed (N) and, of course, all accelerations (a) and stroke (S) needed for gobbing. Centering is important. After the initial centering adjustmens, bowl well stirrers are seldom moved horizontally unless this is needed to help "the throw of the gob" or for better alignment. Centering is important as it is related to the performance of a stirrer and is included even though it will not yield a new model law.

3. Dimensional analysis

Each of the pertinent variables required and/or desired for dimensional analysis were now specified and it is clear from the discussion in section 2. that some of them have the same units. For this analysis any valid set of units may be used for mass, length and time. Three more steps were needed:

Step 1: It was postulated that the volumetric flow (Q/t) would be equal to a function of the pertinent variables described above and expressed by:

$$Q/t = f(L,\ H,\ P,\ c,\ S,\ N,\ \varrho,\ \eta,\ V,\ a,\ g)\ . \tag{3}$$

In equation (3) f means a function of the 11 individual variables enclosed in the brackets. Considering both sides, this equation has a total of 13 variables.

Step 2: Each variable in equation (3) was manipulated so only dimensionless groups remained. As an example, to convert the ratio Q/t to a dimensionless group, it was multiplied by $(\varrho/\eta)^2$ and by V. Units, in order, are

$$(\mathrm{in}^3/\mathrm{s})\,[(\mathrm{lb}/\mathrm{in}^3)/(\mathrm{lb\ in}^{-1}\,\mathrm{s}^{-1})]^2\,(\mathrm{in}/\mathrm{s}) = 1$$

which is dimensionless. Three more examples are trivial. They are L/L, H/L and a/g and the ratio of units in each is clearly equal to one. In practice usually no attempt is made to control viscosity and density independently. Usually composition of the fluid is not changed so these variables are controlled simultaneously by temperature. So their ratio $\eta/\varrho = \nu$ (called the kinematic viscosity) is used.

By trial and error each variable in equation (3) has been multiplied by the other variables until a dimensionless group resulted. In other words all the 13 variables are still present but they are now combined into 9 dimensionless groups. (Note that Q and t are always combined as the ratio Q/t so the number of variables are reduced from 13 to 12. Also the ratio of η to ϱ will be replaced by v, thus, reducing the variables from 12 to 11. The characteristic length, L, will always occur with another linear quantity, so the variables go from 11 to 10. Finally, a/g is used and the number of variables agrees with the number 9 of dimensionless groups.)

Thus, equation (3) has been manipulated to

$$(Q/t)\,(V/v^2) = f'(H/L,\ P/L,\ c/L,\ S/L,\ a/g,\ V^2/(L\,g),\ L\,V/v,\ (N\,v)/V^2)\,. \tag{4}$$

Here f' means a function of the 9 dimensionless groups enclosed in the brackets. Equation (4) is valid for any model of a corresponding, actual stirred section or proposed unit including channels and bowls with or without gobbing. It indicates that if two similar systems have identical values for all the corresponding dimensionless groups, they will behave alike.

Step 3: The model laws were derived by equating the corresponding dimensionless groups of the two systems, the model and the production situation. The subscripts M and P were added to indicate model or analogues for production. L_M/L_P = (SF) was defined as the scale factor.

4. Laws for any size model

1st law: From the first four terms on the right-hand side of equation (4)

$$H_M \equiv H_P \times (\mathrm{SF}),\ P_M \equiv P_P \times (\mathrm{SF}),\ c_M \equiv c_P \times (\mathrm{SF}),\ S_M \equiv S_P \times (\mathrm{SF}) \tag{5}$$

are obtained. These four equations relate the corresponding linear dimensions of the two systems.
2nd law: From the next term of equation (4)

$$(a/g)_M \equiv (a/g)_P \tag{6}$$

is obtained. As long as the model and its counterpart are subjected to the same value of gravity, $g_M \equiv g_P$ (as assumed from now on) and these terms cancel so

$$a_M \equiv a_P\,. \tag{7}$$

This equation requires some interpretation. Substituting L/t^2 for acceleration for both systems, converts this equation to

$$L_M/t_M^2 = L_P/t_P^2 \tag{8a}$$

or

$$t_M/t_P \equiv [L_M/L_P]^{1/2} = (\mathrm{SF})^{1/2}\,. \tag{8b}$$

Equation (8b) states that the ratio of times for corresponding events in the two systems (subscripts M and P for the model and the production situation, respectively) is equal to the square root of the ratio of the linear sizes.

The next term is the Froude relation, or the 3rd law:

$$V_M^2/(L_M\,g_M) = V_P^2/(L_P\,g_P)\,. \tag{9}$$

This equation is used when relative velocities or speeds are to be compared. Substituting the ratio L/t for the values of V and canceling those of g yields

$$[L_M/t_M]^2/L_M \equiv [L_P/t_P]^2/L_P \tag{10a}$$

or

$$t_M/t_P = [L_M/L_P]^{1/2} = (\mathrm{SF})^{1/2}\,. \tag{10b}$$

This agrees with equation (8b). For example, it states that the time required to cut a gob on the model is equal to the time required to cut the corresponding gob on the production or proposed unit multiplied by the square root of the scale factor.

The next term is Reynold's number and yields the fourth law:
4th law:

$$(V\,L/v)_M \equiv (V\,L/v)_P \tag{11a}$$

or

$$v_M \equiv v_P\,(V_M/V_P)\,(L_M/L_P) \tag{11b}$$

which may be converted to

$$v_M \equiv v_P\,(L_M/L_P)^{-1/2}\,. \tag{11c}$$

This is the fundamental model law governing kinematic viscosities.

The next term is the stirrer number:
5th law:

$$\{(N\,v)/V^2\}_M = \{(N\,v)/V^2\}_P\,. \tag{12a}$$

It can be written as

$$N_M/N_P \equiv (v_P/v_M)\,(V_M/V_P)^2 \tag{12b}$$

or as

$$N_M/N_P \equiv (v_P/v_M)\,(L_M/L_P) \tag{12c}$$

or as

$$N_M/N_P \equiv (L_M/L_P)^{-1/2}\,. \tag{12d}$$

From the next term the
6th law:

$$(Q/t)_M/(Q/t)_P = (L_M/L_P)^{5/2} \tag{13}$$

is obtained. This equation relates volumetric flows. Using density, it can be converted to mass flow, yielding the
7th law:

$$(W/t)_M/(W/t)_P = (\varrho_M/\varrho_P)\,(L_M/L_P)^{5/2}\,. \tag{14}$$

Either equations (13 or 14) can be used to determine the flow rate for the model in terms of what is desired for production, or vice versa.

Reiterating these laws are valid for any stirred system in which heat transfer through its surface can be ignored. The model laws involve the relative linear lengths. Their application was simplified by choosing a full-size model. In addition, this 1 : 1 size allowed the very big advantage of using production parts for the model. For example, bowls, orifice rings, needles, cams and associated feeder mechanisms, shears, etc.

Thus, for the model the values for S, H, L etc. are the same as in the production situation. The dependency of W or Q on rotational speed, stirrer position, and temperature or viscosity and density of the liquid, can be determined by the model and translated to the production case.

5. Rules for a full-size model

These rules are simple as corresponding lengths, kinematic viscosities, times, rotational speeds and volumes are equal. This also means that any change in these variables is the same for the model or its counterpart. So by differentiation

$$\begin{gathered} \mathrm{d}L_M = \mathrm{d}L_P;\ \mathrm{d}P_M = \mathrm{d}P_P;\ \mathrm{d}S_M = \mathrm{d}S_P;\ \mathrm{d}\nu_M = \mathrm{d}\nu_P; \\ t_M = t_P;\ N_M = N_P;\ Q_M = Q_P\,. \end{gathered} \tag{15}$$

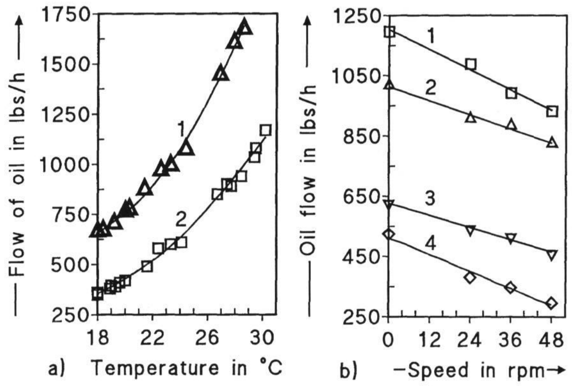
Equation (14) reduces to

$$(W/t)_M = (\varrho_M/\varrho_P)\,(W/t)_P \tag{16}$$

and by logarithmic differentiation with densities and times constant

$$(\mathrm{d}W/W)_M = (\mathrm{d}W/W)_P\,. \tag{17}$$

It states that the fractional or percent weight variations for the full-size model and its counterpart will be equal. This equation will be used in the rest of this paper and the data are restricted to flow from a stirred bowl with one-size orifice. The dependency of glass flow on stirrer action and temperature were not known but data from a model could now be used quantitatively to find these relations.



Figures 3a and b. Flow rate of the model's fluid at different temperatures and rotational speeds; a) flow versus the fluid's temperature at a stirrer speed N = 24 rpm and at two positions of the stirrer. 1: P = 1 1/4″; 2: P = 5/8″, b) flow versus the rotational speed with four averaged values of ϱ^2/η (in (g/cm^3)^2/P) and at P = 5/8″. 1: $\{\varrho^2/\eta\} \cdot 10^4 = 7.60$; 2: $\{\varrho^2/\eta\} \cdot 10^4 = 6.59$; 3: $\{\varrho^2/\eta\} \cdot 10^4 = 4.27$; 4: $\{\varrho^2/\eta\} \cdot 10^4 = 3.71$.

6. Weight variation from model data

A full-size model of a typical bowl was constructed, filled with a temperature-controlled viscous oil. The flow from the orifice was continuous and controlled by adjusting a bowl well stirrer (figure 2). Gobbing was considered but was rejected. It was not considered feasible as the flow was to be varied over a very large range. Flow (in lb/h) was calculated by weighing the fluid delivered from the orifice in a time period that gave a representative sample, yet was of a size that could be handled. The errors in these data could easily be up to about ± 5 %.

The flow of oil versus oil temperature was drawn for two different positions of the stirrer and with other conditions remaining unchanged (figure 3a). The flow increased rapidly with increasing temperatures and the increase was more pronounced at the higher position. These curves were used to find variation in weight versus the stirrer's position and temperature of the oil flowing out of the orifice.

a) $(\mathrm{d}W/W)$ versus $\mathrm{d}P$: A sample calculation is shown for N = 24 rpm at an operational temperature of 20 °C and at P = 5/8″. Here the difference between the two curves was equal to the change in the stirrer's position which was (1 1/4″ − 5/8″) = +5/8″. The corresponding change in flow for this change in the stirrer's position was approximated by

$$[\Delta(W/t)]/\Delta P_1 = [(W/t)_2 - (W/t)_1]/[P_2 - P_1] \tag{18a}$$

yielding

$$[\Delta(W/t)]/\Delta P_1 = [760 - 425]/(5/8) = 536\,\mathrm{lb/(h\,in)}\,. \tag{18b}$$

Dividing by $(W/t)_1$ = 425 lb/h, converts this equation to the fractional change in weight corresponding to a change of stirrer position equal to ΔP. With the time cancelled and converting to percent

$$(\Delta W/W)_M = 127\,\mathrm{d}P_M\,\%/\mathrm{in}\,. \tag{18c}$$

For $P = 5/8''$ a 1% change in stirrer position would be 0.006″. This minor difference would result in a relatively large $(127 \cdot 0.006)\,\% = 0.8\,\%$ change in weight. As the height was adjusted to ±1/32″ (±0.03″), the stirrer's vertical maladjustments could have caused a large part of the error in weight as $(127\,\%/\mathrm{in}) \cdot (0.03\,\mathrm{in}) = 4\,\%$. Thus, for low positions of the stirrer, the seemingly good adjustments in elevation were inadequate for the desired accuracy (±1 %) in the flow rate.

b) $(\mathrm{d}W/W)$ versus $(\mathrm{d}T\,°\mathrm{C})$: Either of the mentioned curves can be used to estimate a percent weight change per °C. The result clearly depends on the temperature and to a some extent on the position of the stirrer. For the same point just used, this required finding the slope at 20 °C at the 5/8″ position. Now W/t is a function of temperature. Its slope may be found by differentiation with all the other variables constant and approximated by

$$\mathrm{d}(W/t)/\mathrm{d}T = [(W/t)_2 - (W/t)_1]/[T_2 - T_1] \tag{19a}$$

$$= (790 - 305)/(30 - 17) = 37.3\,\mathrm{lb/(h\,°C)}\,. \tag{19b}$$

To convert to percent multiply by 100% and divide by 425 lb/h (oil flow at 20 °C).

So

$$\mathrm{d}(W/T)/(W/T) = \{[37.3/425][100\,\%]\}\;\mathrm{d}T/°\mathrm{C}$$

or

$$(\mathrm{d}W/W)/\mathrm{d}T = 8.78\,\%/°\mathrm{C}\,. \tag{19c}$$

Thus, the percent weight change was 8.78 % for a change of 1 °C in the temperature of the oil. The change in weight divided by weight is valid for either the model or the production unit (equation (17)) but the change in temperature in equation (19c) refers to the oil in the model. To convert equation (19c) to represent glass, the equivalent temperatures obtained from equating kinematic viscosities had to be used (equation (11c)). A graph of the model fluid's temperature versus the corresponding temperature of a borosilicate glass at equal kinematic viscosities showed that the 13 °C spread for the model was equivalent to a 116 °C spread for glass (see figure 6). Therefore, for glass, the ordinate of the slope had to be multiplied by the ratio of densities while the abscissa had to be multiplied by 13 °C and divided by 116 °C. This conversion resulted in the percent change in weight for the borosilicate glass (note that the conversion for weights canceled). Thus

$$[\mathrm{d}W/W]_P = (8.78)\,(13/116) = 0.98\,\%\;\mathrm{d}T_P/°\mathrm{C}\,. \tag{19d}$$

This variation was in excellent agreement with the empirical value of 1 %/°C used by production personnel.

c) $(\mathrm{d}W/W)$ versus $(\mathrm{d}N)$: Data with the stirrer's rotational speed showed that W/t was a linear function of the rotational speed for different values of ϱ^2/η (figure 3b). It was also observed that clockwise rotation of the stirrer, viewed from above, increased this stirrer's impedance to flow as the flow decreased as the rotational speed increased[2]. In figure 3b the slope of the lowest straight line was used to find the percent change in weight at a rotational speed of 24 rpm and at a flow rate of 380 lb of model fluid per hour. Thus, $\mathrm{d}(W/t)/\mathrm{d}N$ was approximated by

$$\{(W/t)_2 - (W/t)_1\}/\{(N_2 - N_1\}$$

or

$$\Delta(W/t)/\Delta N = \{494 - 296\}/\{0 - 48\} = \\ = -4.125\,\mathrm{lb/(h \cdot rpm)}\,. \tag{20a}$$

So

$$\Delta W/W = [-4.125/380]\;100\,\%\;\Delta N = 1.09\,\%\;\Delta N\,. \tag{20b}$$

So the percent weight change per rpm was about 1.1 %. At 24 rpm, a 1 % change in speed is 0.24 rpm and would result in a percent weight variation of about 0.3 %. This could be a significant part of the total variation except after the speed is set it remains very constant.

These examples illustrate ways to find percent weight variations. Actually for production much less data are required. For example as a minimum only two weights at two different positions with constant temperature and constant rotational speed (in rpm) will suffice to give an approximate weight change for a change in stirrer position, etc. Numbers so obtained may be adequate for production but this procedure can not explain what is occurring. So a more general method was desired.

7. Flow control equations

For a more general approach, it was again assumed that the volumetric flow (Q) was inversely proportional to viscosity (η), directly proportional to time (t) and to density (ϱ) as increases in density should increase flow. Mathematically

$$Q/t = K'\,(\varrho/\eta) \tag{21a}$$

with K' equal to a proportional constant. To convert to weight this equation was multiplied by density yielding

$$W/t = K'\,(\varrho^2/\eta) \tag{21b}$$

(This equation has the square of density but density variations for any glass are so small that the first power would also be satisfactory and may even be preferable.)

Graphs of extensive data of W/t versus ϱ^2/η (not shown) revealed that the relationship was not quite linear. So the data did not satisfy equation (21b). Thus, assuming that flow was inversely proportional to viscosity, as was done in the first part of this report, was an over-simplification in that higher powers of ϱ^2/η were neglected.

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[2] Reversing the stirrer's rotational direction caused it to pump downward.

Equation (21b) was modified to include a second-order term and still satisfy the condition that flow would be equal to zero for infinite viscosity. Equation (22) met these conditions:

$$(W/t)/(\varrho^2/\eta) = A(P,N) + [\varrho^2/\eta]\,B(P,N)\,. \tag{22}$$

It includes two functions $A(P,N)$ and $B(P,N)$ which are constant unless the rotation and/or position of the stirrer is changed. (As written these functions assume a fixed orifice size, a constant glass level, no change in gobbing stroke and no recentering of the stirrer. Data for the model and its production counterpart were recorded after these conditions were met.) Validity of this equation with N or P constant was verified as graphs of $(W/t)/(\varrho^2/\eta)$ versus (ϱ^2/η) were straight lines within the accuracy of the data. These data covered several stirrer positions but only one rotational speed, with $N = 24$ rpm (figure 4a). In addition, graphs of $(W/t)/(\varrho^2/\eta)$ versus (ϱ^2/η) were straight lines for different stirrer speeds at $P = 5/8''$ (figure 4b).

The slopes and intercepts of the best straight lines from figure 4a, as found using least squares, are plotted in figure 5a. As indicated by this figure, with $N = 24$ rpm, $A(P, 24)$ and $B(P, 24)$ are different linear functions of P. Their values are

$$A(P,24) = 1.575 \cdot 10^6\,P + 0.143 \cdot 10^6 \tag{23a}$$

and

$$B(P,24) = -0.224 \cdot 10^9\,P + 0.553 \cdot 10^9\,. \tag{23b}$$

The slopes and intercepts of the best straight lines from figure 4b, as found using least squares, are plotted versus rotational speed in figure 5b and yielded straight lines with the following values

$$A(5/8,N) = 1.492 \cdot 10^6 - 0.0143 \cdot 10^6\,N \tag{23c}$$

and

$$B(5/8,N) = 0.07457 \cdot 10^9 + 0.01057 \cdot 10^9\,N\,. \tag{23d}$$

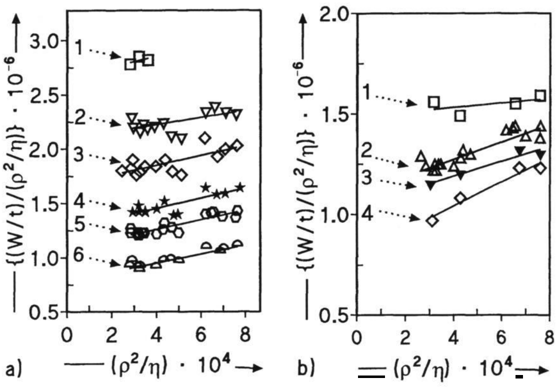
$A(P,N)$ and $B(P,N)$ were generalized to

$$A(P,N) = A_0 + P\,A_1 + N\,A_2 + P\,N\,A_3 \tag{23e}$$

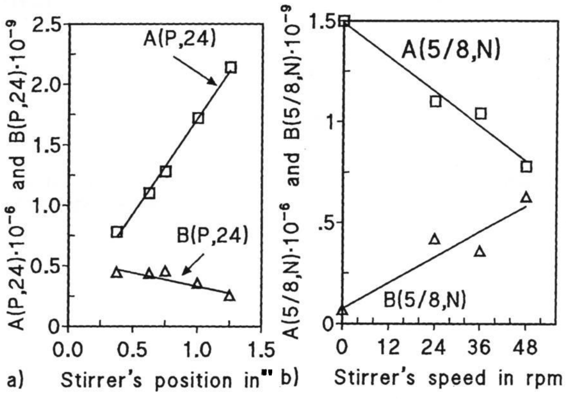
and

$$B(P,N) = B_0 + P\,B_1 + N\,B_2 + P\,N\,B_3 \tag{23f}$$

where A_i and B_i are constants. These relations showed that equation (22) is complicated and that the flow depends not only on P, N and (ϱ^2/η) but also on their products.



Figures 4a and b. Least square fits of the model values for $(W/t)/(\varrho^2/\eta)$ versus (ϱ^2/η); a) at a stirrer speed $N = 24$ rpm with various stirrer positions. 1: $P = 1\ 3/4''$; 2: $P = 1\ 1/4''$; 3: $P = 1''$; 4: $P = 3/4''$; 5: $P = 5/8''$; 6: $P = 3/8''$, b) at a stirrer position $P = 5/8''$ with various stirrer speeds. 1: $N = 0$ rpm; 2: $N = 24$ rpm; 3: $N = 36$ rpm; 4: $N = 48$ rpm.



Figures 5a and b. Slopes and intercepts for the model's equation $(W/t)/(\varrho^2/\eta) = A(P,N) + (\varrho^2/\eta)\,B(P,N)$, a) at a stirrer speed N=24 rpm with changes in the stirrer's position; b) at a stirrer position $P = 5/8''$ with changes in stirrer speed.

These equations are of some help in understanding and designing stirred glass melt delivery systems but otherwise they are not used.

8. Weight variation equations

Logarithmic partial differentiation of equation (22) yielded three equations to calculate weight variations,

a) for the first one, only position was varied;

b) for the second, only rotational speed changed;

c) the third included only temperature as the variable.

Each of these equations was evaluated with a mixed set of units that were convenient and familiar to the technicians doing the work and were actually used by production personnel. This was very practical. The mixed units avoided confusion and eliminated unneeded conversions from laboratory results. Evaluations were performed with $P = 5/8''$, $N = 24$ rpm, $(\varrho^2/\eta) = 3.3 \cdot 10^{-4}\,(\mathrm{g/cm^3})^2/P$ and

$$(W/t)/(\varrho^2/\eta) \equiv 1.2 \cdot 10^6\,(\mathrm{lb/h})/[(\mathrm{g/cm^3})^2/\mathrm{P}].$$

8.1 For variable position

$$\frac{\mathrm{d}W}{W} = \frac{\varrho^2/\eta}{W/t}\left[\frac{\partial A}{\partial P} + (\varrho^2/\eta)\frac{\partial B}{\partial P}\right]\mathrm{d}P\,. \tag{24a}$$

With equations (23 a and b), the result is

$$\begin{aligned}\frac{\mathrm{d}W}{W} &= \frac{1}{1.2\cdot 10^6}[1.575\cdot 10^6 - (3.3\cdot 10^{-4})(0.224\cdot 10^9)]\mathrm{d}P\\ &= \frac{1}{1.2\cdot 10^6}[1.575\cdot 10^6 - 0.739\cdot 10^5]\cdot 100\,\%\,\mathrm{d}P\\ &= \frac{1}{1.2\cdot 10^6}1.50\cdot 10^6\cdot 100\,\%\,\mathrm{d}P\,;\end{aligned}$$

$$\frac{\mathrm{d}W}{W} = 125\,\%\,\mathrm{dP}\,. \tag{24b}$$

So for a 1 % change in position, $\mathrm{d}P = 0.00625$ in and

$$\mathrm{d}W/W = (125\,\%)\,(0.00625) = 0.8\,\%\,. \tag{24c}$$

This number agrees with the earlier result found in section 6.

8.2 For variable rotational speed

$$\frac{\mathrm{d}W}{W} = \frac{\varrho^2/\eta}{W/T}\left[\frac{\partial A}{\partial N} + (\varrho^2/\eta)\frac{\partial B}{\partial N}\right]\mathrm{d}N \tag{25a}$$

and with equations (23c and d)

$$\begin{aligned}\frac{\mathrm{d}W}{W} &\equiv \frac{1}{1.2\cdot 10^6}[-0.0143\cdot 10^6 + (3.3\cdot 10^{-4})\,\cdot\\ &\qquad\cdot(0.01057\cdot 10^9)]\,100\,\%\,\mathrm{d}N\\ &= \frac{1}{1.2\cdot 10^6}[-0.0143\cdot 10^6 + 0.003488\cdot 10^6]\,100\,\%\,\mathrm{d}N\\ &= \frac{1}{1.2\cdot 10^6}[-0.01081\cdot 10^6]\,100\,\%\,\mathrm{d}N\,;\end{aligned}$$

$$\frac{\mathrm{d}W}{W} = -0.90\,\%\,\mathrm{d}N/\mathrm{rpm}\,. \tag{25b}$$

So $\mathrm{d}W/W = -0.2\,\%$ for a 1 % change in rpm ($\mathrm{d}N = 0.24$ rpm).

This result is in good agreement with the result found earlier in section 6.

8.3 For variable temperature

It was more difficult to calculate this change. For small temperature changes, variations in density are negligible, so density was considered as a constant. Partial logarithmic differentiation of equation (16) with η variable gave

$$\frac{\mathrm{d}W}{W} = -\left[1 + \frac{(\varrho^2/\eta)\,B}{[A + (\varrho^2/\eta)\,B]}\right]\frac{\mathrm{d}\eta}{\eta}\,. \tag{26a}$$

Evaluation at the selected point (with equations (23a to d)) yielded

$$\frac{\mathrm{d}W}{W} = -\left[1 + \frac{(3.3\cdot 10^{-4})\,(0.371\cdot 10^9)}{1.26\cdot 10^6}\right]\frac{\mathrm{d}\eta}{\eta}\,. \tag{26b}$$

So

$$\frac{\mathrm{d}W}{W} = -\,[1.097]\,\frac{\mathrm{d}\eta}{\eta}\,. \tag{26c}$$

In equations (26a to c) $\mathrm{d}\eta/\eta$ refers to the model's fluid. To convert to glass $\mathrm{d}\eta/\eta$ must be converted. As ϱ is essentially constant for this small change in temperature, the ratio $\mathrm{d}\eta/\eta$ can be converted to $(\mathrm{d}\eta/\varrho)/(\eta/\varrho)$ without changing its value. In terms of kinematic viscosity it is $(\mathrm{d}\nu/\nu)_\mathrm{M}$ in which the subscript M has been added to indicate it refers to the model. From equation (15)

$$(\mathrm{d}\nu/\nu)_\mathrm{M} \equiv (\mathrm{d}\nu/\nu)_\mathrm{P}\,. \tag{26d}$$

The derivative of the Fulcher equation (as shown in section 13.1, equation (31) is

$$\frac{1}{\eta}\frac{\mathrm{d}\eta}{\mathrm{d}T} = -\frac{B'}{0.4343\,(T - T_0)^2}$$

and evaluated with $B' = 5130\,°\mathrm{C}$, $T = 1350\,°\mathrm{C}$, $T_0 = 213\,°\mathrm{C}$ yields

$$(\mathrm{d}\eta/\eta)_\mathrm{P} = -0.00914\,\mathrm{d}T_\mathrm{P}\,. \tag{26e}$$

Division of both the numerator and denominator by ϱ yields

$$(\mathrm{d}\eta/\varrho)\,(\eta/\varrho) = (\mathrm{d}\nu/\nu)_\mathrm{P} = -0.00914\,\mathrm{d}T_\mathrm{P}\,. \tag{26f}$$

Substitution into equation (26c) resulted in

$$\mathrm{d}W/W = 1.0\,\%\,\mathrm{d}T/°\mathrm{C} \tag{27}$$

in good agreement with the empirical value used in the factory.

9. Verification

Stirring was extended to large commercial feeders. Stirred molten glass streams were used for turret-chain blown ware and gobs were used for pressed as well as blown items [9]. Some of these commercial applications were simulated with the full-size model in a serious effort to see how well model and production results correlated. Extreme care was used to make exact 1 : 1 geometric correspondence. Kinematic viscosities in the models were varied by temperature until the volumetric flow from the model matched that from the stirred delivery system as closely as possible. All these data were obtained during stable production periods.

Table 1. Data from five stirred glass delivery systems taken during normal production conditions (figure 6)

experiment[3]	delivery system			stirrer system			gob variables		
	system no.[4]	stirring-well type	orifice[5]	stirrer no.[6]	stirrer rotation speed in rpm	stirrer position[7] in in	gob weight in oz	gobs/min	gob temperature in °C
Expt A	52	straight	3 3/8	1Sp	25	5/8	12.5	45	1364
Expt B	52	straight	3 3/8	1	25	3/8	13	37	1328
Expt C	42	tapered	3 3/8	11A	38.6	11/16	13.5	40	1317
Expt D	52	straight	3 5/8	1	25	2 to 5/6	23	30	1288
Expt E	42	tapered	3 3/8	11A	38.6	13/16	10.5	38.6	1350

[3] Five simulations (Expt A to E), each was a separate experiment.

[4] Identification number of the locations of the stirred delivery systems.

[5] Diameter of the orifice ring in inch.

[6] Identification number of the stirrers.

[7] Position of the stirrer above the orifice in inch.

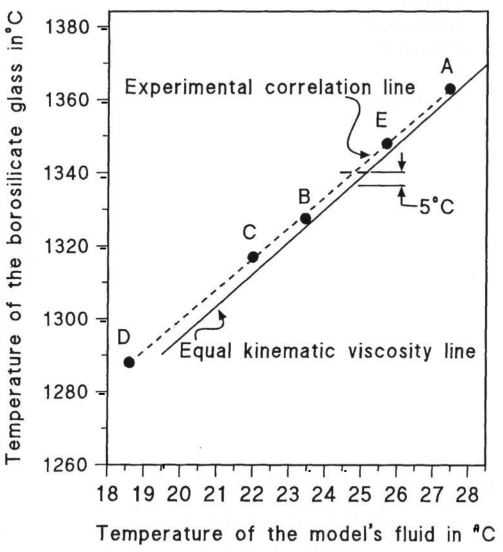


Figure 6. Equal kinematic viscosity curve and locations of the corresponding kinematic viscosity points resulting from simulating five, widely different production systems.

Five different set-ups were checked. They included two furnaces melting a borosilicate glass (table 1). From this table, it will be noted that the two locations had different stirring wells (figure 1 shows a straight well) with two orifice rings with different diameters. In addition, three different stirrers were operated at two different rotational speeds and at five different stirrer positions *P*. They delivered five different weight gobs at five different rates (gobs per minute) and at five different temperatures varying from 1288 to 1364 °C. The experiments resulted in five sets of data, including first, the temperature of the glass as it was extruded, second, the kinematic viscosity of the glass at this temperature, this number was also the kinematic viscosity of the model's fluid, and third, the temperature of the model's fluid corresponding to the glass temperature. These two temperatures will be used in figure 6 as it is developed.

The laboratory measured the viscosity and density of the glass samples taken during the five experiments. These data were used to calculate the kinematic viscosity curve of the borosilicate glass samples and to construct figure 6. In this figure the temperature of the molten glass is plotted on the vertical axis while that of the model's fluid is shown on the horizontal axis. The curve for matching kinematic viscosity was constructed as follows. From a temperature on the vertical axis draw a horizontal line (not shown) to the right. For this temperature look up the kinematic viscosity of the glass. Now look up the temperature of the model's fluid that has this identical kinematic viscosity. Mark it on the horizontal axis and draw a vertical line (not shown) upwards from it. The intersection of this vertical line and the horizontal line is a point with matching kinematic viscosities. Repeat this process until enough points have been found to draw the equal kinematic viscosity curve. This is the solid straight line, fitted by least squares, which is drawn in figure 6.

Figure 6 was then completed as follows. A horizontal line (not shown) was drawn at the temperature of the glass for experiment A and a vertical one at the experimental, corresponding temperature found for the model. The intersection of these two lines is shown as the solid circle and labeled "A". The other four data points were entered in a similar manner. A dashed straight line was fitted using least squares. It was a close fit as its correlation coefficient (r) was 0.999936 [10]. The fit would be perfect if $r = 1$ and then all the five points would be exactly on this line. Correlation between the actual temperature of the glass from that predicted by the modelling seemed to be off by about 5 K. This was no problem as both the production set-ups and the models had sufficient leeway to accommodate this small discrepancy.

The modelling group was asked for a stirrer to gob a new photosensitive glass at 500 P [9]. A stirrer was designed to do this, but it was impossible to form a gob with any known cam. After analyzing the contours of the cam that appeared to be the most promising, a new cam was designed. It gave very satisfactory results in the model.

At the start of production with this new glass, the forming specialist tried all his cams without success. The modelling group then installed the model cam and had acceptable gobs within three strokes. So the isothermal model predicted the exact, correct conditions for the proposed delivery system including the gobbing cam.

10. Summary and conclusions

As stirrers are ectremely efficient in smoothing out inhomogeneities, models for stirred glass may be operated isothermally.

Dimensional analysis was used to extend model laws to gobbing stirrers and indicated that a carefully adjusted and operated full-size model of a bowl delivery system could yield quantitative data representative of an actual or a proposed unit. This meant that the volumetric flows from both units would be identical provided the kinematic viscosity of the fluid for the model was equal to that of the molten glass at its operating temperature., Reiterating, if a full-size model was operated properly, the stream or gobs from it would have shapes and sizes identical to those from the production counterpart or for a proposed analog. This and the fact that models may be used to design gobbing cams was proven beyond any doubt.

Data were used to find how weight varied with changes in the temperature of the material and with the position and rotational speed of the stirrer controlling the flow. As flow was varied over a very large range, samples from a stream of the model's fluid were used to determine the flow rate. The data were taken with several positions and different rotational speeds of the stirrer. The experimental values were summarized in a general equation showing that $(W/t)/(\varrho^2/\eta)$ was a linear function of (ϱ^2/η) for the system being investigated. Some of the data were re-examined and it was evident that using kinematic viscosity instead of ϱ^2/η as the variable would also have resulted in a straight line. This general equation was expanded to show how the variables controlling flow were actually interrelated. The relations showed that flow varied with the position and rotational speed of the stirrer and with viscosity and with the products of these variables. These relations may change if the configuration of the delivery system and/or stirrer are altered. Percent weight changes calculated from the general equation were in good agreement with data generated specifically in production for these numbers. In practice weight changes caused by small changes in the temperature of the glass, position and rotational speed of the stirrer are used to determine percent weight variations because of their simplicity.

Data from the model showed that assuming flow was inversely proportional to viscosity was a good first approximation as the correction introduced by using a second order term was about 10 %.

11. Nomenclature

11.1 Symbols

A	constant
A_i	($i = 0, 1, 2, 3$) constant
A'	constant in the Fulcher equation
$A(P, N)$	function of P and N
a	acceleration in m^2/s
B	constant
B_i	($i = 0, 1, 2, 3$) constant
B'	constant in the Fulcher equation
$B(P, N)$	function of P and N
c	centering in inch
d	derivative
g	acceleration due to gravity in in/s^2
H	head of glass in inch
K	constant
K'	constant
L	length (or characteristic length for system) in inch
N	rotational speed in rpm
P	position in inch
Q	volume in $inch^3$
S	stroke in inch
T	temperature in °C
t	time in h
V	velocity in in/s
W	weight in lb
η	viscosity in P
κ	constant
ϱ	density in g/cm^3
ν	kinematic viscosity in $P/(g\,cm^{-3})$

11.2 Subscripts

i	index number 0 to 3
M	model
m	index number 1 or 2
P	production or prototype

12. References

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13. Appendix: Weight variation of glass stream or gob

13.1 Using the Fulcher equation

Mathematically it was assumed

$$W/t = \kappa/\eta \tag{28}$$

where W is the weight of the glass stream or gob cut in time t, η is its viscosity and κ is a constant which depends on all the other variables which are kept constant. For a given time, the fractional change in weight for a change in temperature, was obtained by logarithmic differentiation of the above equation with respect to temperature and is expressed by

$$\frac{\mathrm{d}W/W}{\mathrm{d}T} = -\frac{\mathrm{d}\eta/\eta}{\mathrm{d}T}\,. \tag{29}$$

To evaluate the right-hand side of this equation required using the Fulcher equation [11],

$$0.4343 \ln\eta \equiv \lg_{10}\eta \equiv -A' + B'/(T-T_0)\,. \tag{30}$$

This equation expresses exactly how the viscosity of a molten glass or the model's fluid changes with temperature (T). A', B' and T_0 are experimentally determined constants which vary for different materials. Differentiation of this equation gave

$$\frac{1}{\eta}\frac{\mathrm{d}\eta}{\mathrm{d}T} = -\frac{B'}{0.4343\,(T-T_0)^2} \tag{31}$$

or using equation (29)

$$\frac{\mathrm{d}W/W}{\mathrm{d}T} = \frac{B'}{0.4343\,(T-T_0)^2}\,; \tag{32}$$

Typical values for A', B' and T_0 for a borosilicate glass are 1.03, 5130 °C and 213 °C, respectively. Therefore, at $T = 1350$ °C

$$(\mathrm{d}W/W)/\mathrm{d}T = 0.92\,\%/°\mathrm{C}\,. \tag{33}$$

This result is about 10 % lower than the known actual value.

13.2 Using the viscosity–temperature curve

The percent weight change per °C can also be calculated using viscosity–temperature curves (figure 7). These curves are available to production personnel and are normally plotted on semi-log cross-section paper and show $\lg_{10}\eta$ as a function of temperature, thus

$$0.4343 \ln_e\eta = \lg_{10}\eta = f(T)\,. \tag{34}$$

This equation includes the relation between logarithms to the base e and to the base 10. Its derivative is

$$0.4343\,\mathrm{d}\eta/\eta = \mathrm{d}(\lg_{10}\eta) = [\mathrm{d}f(T)/\mathrm{d}T]\,\mathrm{d}T \tag{35}$$

where $\mathrm{d}f(T)/\mathrm{d}T$ is the slope of this curve.

This slope may also be found by drawing the tangent to the $\lg_{10}(\eta - T)$ curve at the delivery temperature of the glass which, for example, has been chosen to be 1350 °C (figure 7). This tangent line is evaluated over one cycle of the semi-log cross-section paper for which $\eta_1 = 10\,\eta_2$. Its slope is

$$\{\lg\eta_2 - \lg\eta_1\}/(T_2 - T_1\} = -1/\{T_2 - T_1\} \tag{36}$$

or $1/(1210-1473) = -0.003802$. Substituting into equation (35) yields

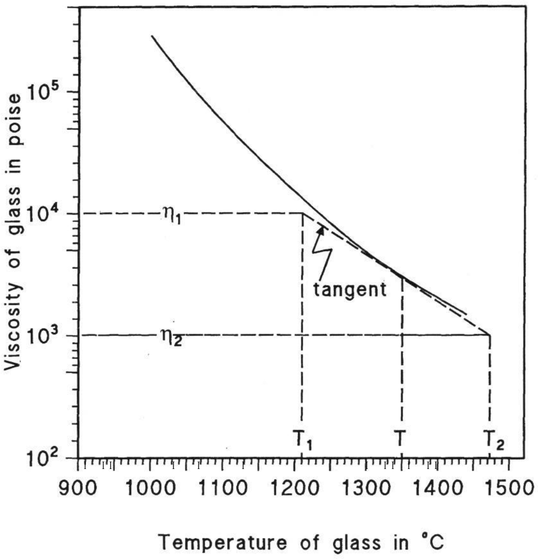

Figure 7. Viscosity–temperature curve of the borosilicate glass and the tangent at 1350 °C.

$$0.4343\,\mathrm{d}\eta/\eta = -0.003802\,\mathrm{d}T$$

or

$(\mathrm{d}\eta/\eta)/\mathrm{d}T = -0.00876$ and finally with equation (29)

$$(\mathrm{d}W/W)/\mathrm{d}T = 0.88\,\%/°\mathrm{C}\,. \tag{37}$$

By this method at 1350 °C for a borosilicate glass (for any size gob or stream sample) the percent weight change is 0.88 %/°C. This value is also about 10 % too low.

*

Notice

The work for this paper was done in a production complex for two practical reasons: 1. to help production personnel understand how stirred glass-feeders behave, and 2. to furnish basic ideas to engineers so that improved stirred glass delivery systems could be designed. In industrial practice in the USA, the use of British rather than SI units is still common to a great extent. A change to SI units in this paper would only have changed the coordinates for some figures and the numerical values of the constants in the equations. These constants, in part, depend on the size of the delivery system. As no one will ever have an exact duplicate of the delivery system used for this investigation, no need will arise to use these constants. It was decided, therefore, not to change the British into SI units in this paper.

Acknowledgement

The author wishes to thank Corning Inc. for permission to publish this material. Thanks are also extended to Mr. R. A. Dreifuss from the Technical Information Center, to Mr. J. H. Bruns and to Dr. R. W. Palmquist from Furnace Development and to Dr. R. R. Thomas from Process Development for their help.

0297P002

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