Mathematisches Forschungsinstitut Oberwolfach

Combinatorial Optimization (hybrid meeting)

Organized by
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ABSTRACT. Combinatorial Optimization deals with optimization problems defined on combinatorial structures such as graphs and networks. Motivated by diverse practical problem setups, the topic has developed into a rich mathematical discipline with many connections to other fields of Mathematics (such as, e.g., Combinatorics, Convex Optimization and Geometry, and Real Algebraic Geometry). It also has strong ties to Theoretical Computer Science and Operations Research. A series of Oberwolfach Workshops have been crucial for establishing and developing the field. The workshop we report about was a particularly exciting event—due to the depth of results that were presented, the spectrum of developments that became apparent from the talks, the breadth of the connections to other mathematical fields that were explored, and last but not least because for many of the particiants it was the first opportunity to exchange ideas and to collaborate during an on-site workshop since almost two years.

Introduction by the Organizers

The workshop has been organized by Karen Aardal (Delft), Ola Svensson (Lausanne), Satoru Iwata (Tokyo), and Volker Kaibel (Magdeburg). It was held in a hybrid format with 31 on-site participants and 17 remote participants. Most participants were affiliated with Mathematics departments and some with Computer Science or Operations Research (Economics) departments. All remote particpiants

were located either in Europe or in the U.S., some of them at the U.S. west coast with a time difference of 9 hours to local time at Oberwolfach. This situation imposed quite a challenge w.r.t. determining a reasanable time schedule for the workshop, which we tried to meet by introducing night sessions at 20:00. In order to nevertheless reserve enough time to discuss and to collaborate, in the mornings we scheduled only one session right after breakfast, followed by a longer break including lunch.

The workshop proved that the field of Combinatorial Optimzation is extremely active and growing, also in terms of establishing more and more connections to other mathematical fields. We provide some insight about examples of topics that played central roles by sketching the topics of the five focus lectures of 60-90 minutes length that were delivered upon invitation of the organizers.

Nathan Klein reported about spectacular progress that has been made by him and his coauthors Anna Karlin and Shayan Oveis Gharan on the traveling salesman problem (TSP). After the question whether there is a better-than 1.5-approximation algorithm for the metric TSP had been open (and urgent) since Christofides came up with his famous algorithm more than 45 years ago, they eventually managed to design an algorithm that indeed beats that barrier (by 10^{-37}). The algorithm is based on constructing from an optimal solution to the subtour relaxation an appropriate probability distribution from which a spanning tree is sampled. While this algorithmic approach had been pursued before, the breakthrough resulted in particular from a deepened understanding of the class of distributions that are relevant here.

The traveling salesman problem is related to survivable network design problems where we wish to find a cheap subgraph that has certain connectivity requirements. In their focus lecture, *Vera Traub and Rico Zenklusen* gave an overview of their recent progress on such problems. This progress has been achieved by combining deep insights into the structure of linear programming relaxations with smart combinatorial techniques. This combination has allowed them to overcome decade-old difficulties and to give improved algorithms for several fundamental problems. In particular, they gave a detailed explanation of their new local search based algorithms for the weighted tree augmentation and steiner tree problems. For the weighted tree augmentation problem they achieve an approximation guarantee of 3/2 (surpassing the old barrier of 2) and for the Steiner tree problem they gave a much cleaner proof of the state-of-the-art guarantee.

The concept of extended formulations has receieved a lot of attention during the last 10-15 years. The approach here is to express polytopes that are naturally associated with combinatorial optimization problems as projections of higher dimensional polytopes that can be described much easier than the original ones. For many concrete optimization problems this has been done very succesfully, for others it has been proved that it is impossible to significantly reduce the complexity of the representation in this way. In any case, the approach raises fundamental extremal questions within the general theory of convex polytopes. In her focus lecture, *Lisa Sauermann* explained results (jointy obtained with Matthew Kwan

and Yufei Zhao) on the extension complexity (the smallest number of inequalities in an extended formulation) of random polytopes in fixed dimensions. They not only obtained remarkably sharp estimates (showing, e.g., that in fixed dimensions the extension complexity of the convex hull of n random points on a sphere grows proportionally to $\sqrt{n}$ with probability tending to one), but they also introduced novel methods for constructing extended formulations that hopefully can be useful in other situations as well.

Nima Anari reported about the solution of a problem that had been posed by Mihail and Vazirani some 30 years ago: The graphs of the base polytopes of matroids have edge expansion at least one. This implies that random walks on those graphs are rapidly mixing, which opens up possibilities for generating random bases of matroids efficiently. For general 0/1-polytopes, the corresponding conjecture remains open. The lecture discussed several special cases and variations of that general conjecture, in particular such ones that seem promosing for implying efficient random generation algorithms as well, thus providing very interesting directions for the future research in this area.

Sebastian Pokutta provided an overview of interplay between machine learning and discrete optimization. Various machine learning tasks can be formulated in terms of discrete optimization. The topics of this direction include best subset selection, optimal classification trees, network verification, and rate-distortion explanation. In the opposite direction, one can think of using machine learning techniques for solving discrete optimization problems. One example is to train a reinforcement learning agent to select good branching variables or cuts for solving integer programs. Another approach is to solve mixed integer programs with neural networks. His explanation on reinforcement learning demonstrated an animation of a game situation, which was very interesting and entertaining as well.

Next to those five focus lectures there were 25 talks of 30 minutes length each, both from on-site as well as from remote participants. You find details on the entire program below, including descriptions of the problems that have been posed and discussed during two open problem sessions.

Workshop (hybrid meeting): Combinatorial Optimization

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Abstracts

Submodular k-Partitioning and Hypergraph k-Cut

CHANDRA CHEKURI

(joint work with Karthik Chandrasekaran)

Submodular k-Partition is the following problem: given a submodular set function $f : 2^V \to \mathbb{R}$ and an integer k, find a partition of V into k non-empty parts $V_1, V_2, \ldots, V_k$ to minimize $\sum_{i=1}^{k} f(V_i)$. Several interesting problems such as Graph k-Cut, Hypergraph k-Cut and Hypergraph k-Partition are special cases. Submodular k-Partition admits a polynomial-time algorithm for $k = 2, 3$ and when f is symmetric also for $k = 4$. The complexity is open for $k = 4$, and when f is symmetric for $k = 5$. Motivated by this problem, we examined the complexity of Hypergraph k-Cut which recently admitted a randomized polynomial-time algorithm for any fixed k. We obtained a deterministic polynomial-time algorithm for Hypergraph k-Cut as well as new insights into Graph k-Cut [1]. The ideas also led to a polynomial-time algorithm for Min-Max Symmetric Submodular k-Partition for any fixed k [2]. The talk will discuss these results with the goal of highlighting the open problem of resolving the complexity of Submodular k-Partition.

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The Two-Stripe Symmetric Circulant TSP is in P

DAVID P. WILLIAMSON

(joint work with Samuel C. Gutekunst, Billy Jin)

The symmetric circulant TSP is a special case of the traveling salesman problem in which edge costs are symmetric and obey circulant symmetry. Despite the substantial symmetry of the input, remarkably little is known about the symmetric circulant TSP. The complexity of the problem has been an often-cited open question. Considerable effort has been made to understand the case in which only edges of two lengths a_1 and a_2 are allowed to have finite cost: the two-stripe symmetric circulant TSP (see Greco and Gerace [2] and Gerace and Greco [1]). In this paper, we resolve the complexity of the two-stripe symmetric circulant TSP, providing the first step toward resolving the polynomial-time solvability of circulant TSP. To do so, we reduce two-stripe symmetric circulant TSP to the problem of finding certain minimum-cost Hamiltonian paths on cylindrical graphs. We then solve this Hamiltonian path problem. Our results show that the two-stripe symmetric

circulant TSP is in P. Note that the input size of a two-stripe symmetric circulant TSP instance is a constant number of numbers (including n, the number of cities), so that a polynomial-time algorithm for the decision problem must run in time polylogarithmic in n, and a polynomial-time algorithm for the optimization problem cannot output the tour. We address this latter difficulty by showing that the optimal tour must fall into one of two parameterized classes of tours, and that we can output the class and the parameters in polynomial time.

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Better ATSP approximation, even on graphs

JENS VYGEN

(joint work with Vera Traub)

In previous work we reduced the approximation ratio for ATSP from 506 [2] to $22 + \epsilon$, for any $\epsilon > 0$ [4]. Now we could improve it further to $17 + \epsilon$. The core is a better approximation algorithm for Graph ATSP, for which we improve the approximation ratio from $13 + \epsilon$ [3] to $8 + \epsilon$. Our two new ingredients are a better potential function for re-intializing Svensson's algorithm [1] and an algorithm for subtour cover with a better local guarantee.

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Sparse PSD approximation of the PSD cone

SANTANU S. DEY

(joint work with Grigoriy Blekherman, Marco Molinaro, Kevin Shu, Shengding Sun)

While semidefinite programming (SDP) problems are polynomially solvable in theory, it is often difficult to solve large SDP instances in practice. One technique to address this issue is to relax the global positive-semidefiniteness (PSD) constraint and only enforce PSD-ness on smaller $k \times k$ principal submatrices – we call this the sparse SDP relaxation. Surprisingly, it has been observed empirically that in

some cases this approach appears to produce bounds that are close to the optimal objective function value of the original SDP. In this talk, we formally attempt to compare the strength of the sparse SDP relaxation vis-'a-vis the original SDP from a theoretical perspective.

In order to simplify the question, we arrive at a data independent version of it, where we compare the sizes of SDP cone and the k-PSD closure, which is the cone of matrices where PSDness is enforced on all $k \times k$ principal submatrices (denoted as $\mathcal{S}^{n,k}$). In particular, we investigate the question of how far a matrix of unit Frobenius norm in the k-PSD closure can be from the SDP cone. Formally, let

$$\overline{\text{dist}}_F(\mathcal{S}^{n,k}, \mathcal{S}^n_+) = \sup_{M \in \mathcal{S}^{n,k}, \|M\|_F \leq 1} \inf_{N \in \mathcal{S}^n_+} \|M - N\|_F.$$

We provide the following two incomparable upper bounds on this farthest distance $\overline{\text{dist}}_F(\mathcal{S}^{n,k}, \mathcal{S}^n_+)$ as a function of k and n:

Theorem 1. [1] *Let $2 \leq k \leq n$ be integers. Then:*

$$\overline{\text{dist}}_F(\mathcal{S}^{n,k}, \mathcal{S}^n_+) \leq \frac{n-k}{n+k-2}.$$

Theorem 2. [2] *Let $2 \leq k \leq n$ be integers. Then:*

$$\overline{\text{dist}}_F(\mathcal{S}^{n,k}, \mathcal{S}^n_+) \leq \frac{(n-k)^{3/2}}{\sqrt{(n-k)^2 + (n-1)k^2}}.$$

The second upper bound is based on the key insight that there is a convex cone $H(e^n_k)$ so that if $X \in \mathcal{S}^{n,k}$, then the vector of eigenvalues of X is contained in $H(e^n_k)$. The cone $H(e^n_k)$ is the hyperbolicity cone of the elementary symmetric polynomial e^n_k (where $e^n_k(x) = \sum_{S \subseteq [n]:|S|=k} \prod_{i \in S} x_i$) with respect to the all ones vector.

We also provide the following matching lower bounds, which show that the upper bounds are tight within a constant in different regimes of k and n.

Theorem 3. [1] *Let $2 \leq k \leq n$ be integers. Then:*

$$\overline{\text{dist}}_F(\mathcal{S}^{n,k}, \mathcal{S}^n_+) \geq \frac{n-k}{\sqrt{(k-1)^2 n + n(n-1)}}.$$

Other than linear algebra techniques, we extensively use probabilistic methods to arrive at these bounds. The second lower bound is obtained by observing a connection between matrices in the k-PSD closure and matrices satisfying the restricted isometry property (RIP).

Finally, we also prove the following result.

Theorem 4. [1] *Let $2 \leq k \leq n$ be integers such that $k = rn$ and $r < \frac{1}{93}$. The:*

$$\overline{\text{dist}}_F(\mathcal{S}^{n,k}, \mathcal{S}^n_+) \geq \frac{\sqrt{r - 93r^2}}{\sqrt{162r + 3}}.$$

We also prove the following result.

Theorem 5. [1] *Let $2 \le k \le n-1$. Consider $\epsilon, \delta > 0$ and let*

$$m = 24 \left(\frac{n^2}{\epsilon^2} \ln \frac{n}{\delta} \right).$$

Let $\mathcal{I} = (I_1, \ldots, I_m)$ be a sequence of random k-sets independently uniformly sampled from $\binom{[n]}{k}$, and define $\mathcal{S}_{\mathcal{I}}$ as the set of matrices satisfying the PSD constraints for the principal submatrices indexed by the I_i's, namely

$$\mathcal{S}_{\mathcal{I}} := \{ M \in \mathbb{R}^{n \times n} \mid M_{I_i} \succeq 0, \ \forall i \in [m] \}.$$

Then with probability at least $1 - \delta$ we have

$$\overline{\mathrm{dist}}_F(\mathcal{S}_{\mathcal{I}}, \mathcal{S}_+^n) \le (1 + \epsilon) \frac{n-k}{n+k-2}.$$

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Uniqueness, continuity and long-term behaviour of Nash flows over time

NEIL OLVER

(joint work with Leon Sering, Laura Vargas Koch)

We consider a dynamic model of traffic that has received a lot of attention in the past few years. Users control infinitesimal flow particles aiming to travel from a source to destination as quickly as possible. Flow patterns vary over time, and congestion effects are modeled via queues, which form whenever the inflow into a link exceeds its capacity. This model was first studied from an algorithmic perspective by Koch and Skutella [3], who uncovered the key structure of equilibria in this model. The precise analytic foundations of the model, as well as a demonstration of the existence of equilibria, followed in work of Cominetti, Correa and Larré [1]. Despite lots of interest, some very basic questions remain open; we resolve a number of them.

- We show *uniqueness* of journey times in equilibria.
- We show *continuity* of equilibria: small perturbations to the instance or to the traffic situation at some moment cannot lead to wildly different equilibrium evolutions.
- We demonstrate that, assuming constant inflow into the network at the source, equilibria always settle down into a "steady state" in which the behavior extends forever in a linear fashion.

One of our main conceptual contributions is to show that the answer to the first two questions, on uniqueness and continuity, are intimately connected to the third. Our result also shows very clearly that resolving uniqueness and continuity, despite initial appearances, cannot be resolved by analytic techniques, but are related to very combinatorial aspects of the model.

To resolve the third question, we substantially extend the approach of [2], who show a steady-state result in the regime where the input flow rate is smaller than the network capacity. In essence, the key is to construct an appropriate potential, but it is far from clear what it should be. Linear programming duality plays a starring role in uncovering the correct choice.

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Better-Than-2 Approximations for Weighted Tree Augmentation and Connections to Steiner Tree
Vera Traub, Rico Zenklusen

Augmentation problems ask about the cheapest way to increase the (edge-)connectivity of a graph by adding edges among a given set of options. One of the most elementary and intensely studied augmentation problems is (Weighted) Tree Augmentation. Here, a spanning tree has to be augmented into a 2-edge-connected graph. We give the first approximation algorithms for Weighted Tree Augmentation that beat the longstanding approximation factor of 2, which can be achieved through many standard techniques. More precisely, we present a relative greedy approach and a local search procedure that can be interpreted as a refinement thereof, leading to a $(1.5 + \epsilon)$-approximation. Moreover, we show how ideas of our local search approach extend to Steiner Tree, leading to an alternative way to obtain the currently best approximation factor of $\ln 4 + \epsilon$. Contrary to prior methods, our approach is purely combinatorial without the need to solve an LP. Nevertheless, the solution value can still be bounded in terms of the well-known hypergraphic LP, leading to an alternative, and arguably simpler, technique to bound its integrality gap by $\ln 4$.

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Dyadic linear programming

Gérard Cornuéjols

(joint work with Ahmad Abdi, Bertrand Guenin, Levent Tuncel)

A rational number is *dyadic* if it is an integer multiple of $\frac{1}{2^k}$ for some nonnegative integer k. Dyadic numbers are important for numerical computations because they have a finite binary representation, and therefore they can be represented exactly on a computer in floating-point arithmetic. When real or rational numbers are approximated by dyadic numbers on a computer, approximation errors may propagate and accumulate throughout the computations. So it is natural to ask when linear programs have dyadic optimal solutions. A vector x is *dyadic* if all its entries are dyadic rational numbers.

A *dyadic linear program* is an optimization problem of the form

$$\sup \left\{ w^\top x : Ax \leq b, x \text{ dyadic} \right\}$$

where A, b, w have integral entries.

Note that we do not restrict ourselves to fixed precision; we just require a finite number of bits in the binary representation. This is an important point as we will see that it makes the problem tractable. On the other hand, if the vector x in the dyadic linear program were restricted to be of the form $\frac{y}{2^k}$ for an integral vector y and a nonnegative integer k bounded above by a given value K, then the problem would be a classical integer linear program. Indeed the problem can then be written as $\max \left\{ w^\top x : Ax \leq b, x = \frac{y}{2^K}, y \text{ integral} \right\}$.

Some natural questions about dyadic linear programs are: When is the problem feasible? Can we check feasibility in polynomial time? If the problem is infeasible, can we provide a certificate of infeasibility? When does a dyadic linear program have an optimal solution? For example, in dimension one, $\sup \{ x : 3x \leq 1, x \text{ dyadic} \}$ does not have an optimal solution. Can dyadic linear programs be solved in polynomial time? When can we guarantee that the dual also has a dyadic optimal solution? This paper addresses these questions. In particular, we show that dyadic linear programs can be solved in polynomial time.

Much of this work can be extended to *p-adic linear programming*. Given a prime integer $p \geq 2$, a rational number is *p-adic* if it is of the form $\frac{q}{p^k}$ for some integer q and nonnegative integer k. A *p-adic linear program* requires the entries of the solution vector x to be *p*-adic. *p*-adic numbers give rise to interesting mathematics; see Gouvêa [1] for example. For ease of exposition, this paper focuses on dyadic linear programming.

We prove the following key lemma. A nonempty rational polyhedron contains a dyadic point if, and only if, its affine hull contains a dyadic point.

It is therefore useful to characterize when a system of linear *equations* $Ax = b$ has a dyadic solution. As earlier, we assume that A, b have integral entries. We show that exactly one of the following statements holds:

(1) $Ax = b$ has a dyadic solution,
(2) there exists a vector $y \in \mathbb{R}^m$ such that $y^\top A$ is integral and $y^\top b$ is non-dyadic.

This is the analog of a classical theorem of the alternative for integer equations (sometimes called the "integer Farkas lemma"), stating that, if A, b have integral entries, exactly one of the following statements holds:

(1) $Ax = b$ has an *integral* solution,
(2) there exists a vector $y \in \mathbb{R}^m$ such that $y^\top A$ is integral and $y^\top b$ is *not integer*.

Putting the two above results together, we get a theorem of the alternative for a polyhedron: Let P be a nonempty rational polyhedron whose affine hull is $\{x : Ax = b\}$, for some A, b with integral entries. Then exactly one of the following statements holds:

(1) P contains a dyadic point,
(2) there exists a vector y such that $y^\top A$ is integral and $y^\top b$ is non-dyadic.

We show that there are four possible outcomes for a dyadic linear program: (i) it is infeasible, (ii) it is unbounded, (iii) it has an optimal solution, (iv) it is not unbounded, has feasible solution(s) and a finite optimal value, but no optimal solution.

In each case, we provide a concise certificate for that outcome. In particular, we give an optimal solution if one exists. We present polynomial algorithms to generate these certificates. In other words, we show how to solve dyadic linear programs in polynomial time.

We also consider dyadic linear programs where the constraints are available through a separation oracle, as well as the situation where the columns of the constraint matrix are accessed through column generation. In each case, we show how to obtain polynomial-time algorithms.

We then introduce the notion of a dyadic polyhedron: A nonempty rational polyhedron is *dyadic* if every nonempty face contains a dyadic point. We prove that a polyhedron P is dyadic if and only if, for every integral vector c for which $\max\{c^\top x : x \in P\}$ has an optimal solution, it has an optimal dyadic value.

A system of inequalities $Ax \leq b$ is *Totally Dual Dyadic* if for every integral c for which $\min\{b^\top z : A^\top z = c, z \geq \mathbf{0}\}$ has an optimal solution, it has an optimal solution that is dyadic. In Section 4, we characterize such systems in terms of dyadic generating sets for subspaces and cones. We prove that Totally Dual Dyadic linear systems give rise to dyadic polyhedra.

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Online Edge Coloring via Tree Recurrences and Correlation Decay

Jakub Tarnawski

(joint work with Janardhan Kulkarni, Yang P. Liu, Ashwin Sah,
Mehtaab Sawhney)

Given a graph $G = (V, E)$ with maximum degree Δ, the edge coloring problem is to assign colors to edges such that any two edges sharing a common vertex get different colors. A well-known theorem by Vizing [7] says that every graph can be edge-colored using $\Delta + 1$ colors.

Bar-Noy, Motwani, and Naor [3] initiated the study of edge coloring in the online setting. Here, the online algorithm has knowledge of the vertex set V of the graph and the maximum degree Δ. However, the edges are revealed one by one, and the online algorithm has to irrevocably assign a color to each newly arriving edge. The goal is to minimize the number of colors used by the online algorithm while maintaining a valid edge coloring of the graph at all time steps. In the original paper, [3] showed that for graphs with maximum degree $O(\log n)$, no online algorithm can maintain a proper coloring using fewer than $2\Delta - 1$ colors — a trivial bound achieved by the greedy algorithm which simply assigns every arriving edge any color that is not used at either endpoint. However, this result only applies to graphs with logarithmic maximum degree. Consequently, the focus has shifted to the much more interesting regime of $\Delta = \omega(\log n)$. In this regime, [3] conjectured that the online algorithm that uses $\Delta + O(\sqrt{\Delta} \log n)$ colors and samples a color for each edge uniformly at random from the set of valid colors succeeds with constant probability. However, we do not know how to analyze this algorithm or give any online algorithm that beats the competitive ratio of 2 achieved by the trivial greedy algorithm; this has been raised as a challenging open problem by all subsequent works.

A competitive ratio of $1+o(1)$ is achievable in important special cases: random-order (instead of adversarial-order) edge arrival [1, 2, 4] and one-sided vertex arrivals (instead of edge arrivals) for bipartite graphs [5]. Very recently, a competitive ratio of 1.9 was obtained by Saberi and Wajc [6] for general *vertex* arrivals on graphs of maximum degree $\omega(\log n)$. Despite these impressive results, no algorithm was known to beat the competitive ratio of 2 in the most general setting of online edge arrivals. The main result of this paper makes the first progress in this direction.

Theorem 1. *There is an online randomized algorithm that on a graph with maximum degree $\Delta = \omega(\log n)$ outputs an $\left(\frac{e}{e-1} + o(1)\right)\Delta$-edge coloring with high probability in the oblivious adversary setting.*

Our proof of the theorem is based on reducing the problem to a matching problem on locally treelike graphs, and then applying a *tree recurrences* based approach for arguing *correlation decay*.

We believe that both our algorithm and its analysis are quite simple.

Correlation decay is a well known and widely used technique in the statistical physics and sampling literature, but was not applied to online problems previously.

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Stable matchings, lattices, and polytopes

Yuri Faenza

(joint work with Xuan Zhang)

Since Gale and Shapley's seminal publication [8], the concept of stability in matching markets has been widely studied by the optimization community. In this work, matching markets have two sides, which we call *firms F* and *workers W*. In the classical *marriage model*, every agent from $F \cup W$ has a *strict preference list* that ranks agents in the opposite side of the market. The goal is to find a *stable matching*, which is a matching where no pair of agents prefer each other to their assigned partners. A stable matching can be found efficiently via the Deferred Acceptance (DA) algorithm [8]. Although successful, the marriage model does not capture important features such as diversity in school cohorts [12, 15].

To model these and other features, instead of ranking individual potential partners, each agent $a \in F \cup W$ is endowed with a *choice function* $\mathcal{C}_a$ that picks a collection of agents she prefers the best from a given set of potential partners. Models with choice functions were first studied in [10, 13]. *Mutatis mutandis*, one can define a concept of stability in this model as well. Two classical assumptions on choices functions are *substitutability* and *consistency*, under which the existence of stable matchings is guaranteed [7, 2]. Clearly, existence results are not enough for applications (and for optimizers). Little is known about algorithms in models with choice functions: only extensions of the classical Deferred Acceptance algorithm for finding the one-side optimal matching have been studied [5, 13].

The goal of our work is to study algorithms for optimizing a linear function w over the set of stable matchings in models with choice functions, where w is defined over firm-worker pairs. Such questions are classical in combinatorial optimization, see, e.g., [14] (and [11] for problems on matching markets). We focus on the QF-MODEL, that assumes that all choice functions are substitutable, consistent, and *quota-filling*. The QF-MODEL generalizes all classical models where agents have strict preference lists, on which results for the question above were known. The stable matchings in this model can be arranged as to form a distributive lattice [1].

Our contributions and techniques. All sets considered in this paper are finite. Let $\mathcal{L} = (\mathcal{X}, \succeq)$ be a distributive lattice, where the elements of $\mathcal{X}$ are distinct subsets of a base set E and $\succeq$ is a partial order on $\mathcal{X}$. We refer to $S \in \mathcal{X}$ as an *element* (of the lattice). Birkhoff's theorem [4] implies that we can associate[1] to $\mathcal{L}$ a poset $\mathcal{B} = (Y, \succeq^\star)$ such that there is a bijection $\psi : \mathcal{X} \to \mathcal{U}(\mathcal{B})$, where $\mathcal{U}(\mathcal{B})$ is the family of *upper sets* of $\mathcal{B}$. $U \subseteq Y$ is an upper set of $\mathcal{B}$ if $y \in U$ and $y' \succeq^\star y$ for some $y' \in Y$ implies $y' \in U$. We say therefore that $\mathcal{B}$ is a *representation poset* for $\mathcal{L}$ with the *representation function* ψ. $\mathcal{B}$ may contain much fewer elements than the lattice $\mathcal{L}$ it represents, thus giving a possibly "compact" description of $\mathcal{L}$. The representation poset $\mathcal{B}$ and the representation function ψ are univocally defined per Birkhoff's theorem. Moreover, the representation function ψ satisfies that for $S, S' \in \mathcal{X}$, $S \succeq S'$ if and only if $\psi(S) \subseteq \psi(S')$. Although $\mathcal{B}$ explains how elements of $\mathcal{X}$ are related to each other with respect to $\succeq$, it does not contain any information on which items from E are contained in each lattice element. We introduce therefore Definition 1. For $S \in \mathcal{X}$ and $U \in \mathcal{U}(\mathcal{B})$, we write $\chi^S \in \{0,1\}^E$ and $\chi^U \in \{0,1\}^Y$ to denote their characteristic vectors, respectively.

Definition 1. *Let $\mathcal{L} = (\mathcal{X}, \succeq)$ be a distributive lattice on a base set E and $\mathcal{B} = (Y, \succeq^\star)$ be a representation poset for $\mathcal{L}$ with representation function ψ. $\mathcal{B}$ is an affine representation of $\mathcal{L}$ if there exists an affine function $g : \mathbb{R}^Y \to \mathbb{R}^E$ such that $g(\chi^U) = \chi^{\psi^{-1}(U)}$, for all $U \in \mathcal{U}(\mathcal{B})$. In this case, we also say that $\mathcal{B}$ affinely represents $\mathcal{L}$ via function g and that $\mathcal{L}$ is affinely representable.*

As we show next, affine representability allows one to efficiently solve linear optimization problems over elements of a distributive lattice. In particular, it generalizes properties that are at the backbone of algorithms for optimizing a linear function over the set of stable matchings in the marriage model and its one-to-many and many-to-many generalizations (see, e.g., [9, 3]). For instance, in the marriage model, the base set E is the set of potential pairs of agents from two sides of the market, $\mathcal{X}$ is the set of stable matchings, and for $S, S' \in \mathcal{X}$, we have $S \succeq S'$ if every firm prefers its partner in S to its partner in S'.

Lemma 2. *Suppose we are given a poset $\mathcal{B} = (Y, \succeq^\star)$ that affinely represents a lattice $\mathcal{L} = (\mathcal{X}, \succeq)$ with representation function ψ. Let $w : E \to \mathbb{R}$ be a linear function over the base set E of $\mathcal{L}$. Then $\max\{w^\intercal \chi^S : S \in \mathcal{X}\}$ can be solved in*

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[1]The result proved by Birkhoff is actually a bijection between the families of lattices and posets, but we shall not need it in full generality.

time `min-cut`$(|Y| + 2)$, where `min-cut`(k) is the time complexity required to solve a minimum cut problem with nonnegative weights in a digraph with k nodes.

We want to apply Lemma 2 to the QF-MODEL. As a choice function may be defined on all the (exponentially many) subsets of agents from the opposite side, we model access to choice functions via an oracle model. We let `oracle-call` be time required to compute the choice function $\mathcal{C}_a(X)$ of any agent $a \in F \cup W$ for any set X in the domain of $\mathcal{C}_a$.

Theorem 3. *The distributive lattice $(\mathcal{S}, \succeq)$ of stable matchings in the QF-MODEL is affinely representable. Its representation poset $(\Pi, \succeq^\star)$ has $O(|F||W|)$ elements. This representation poset, as well as its representation function ψ and affine function $g(u) = Au + x^0$, can be computed in time $O(|F|^3|W|^3 \texttt{oracle-call})$. Moreover, matrix A has full column rank.*

Theorem 3 is the union of two statements. First, the distributive lattice of stable matchings in the QF-MODEL is affinely representable. Second, this representation and the corresponding functions ψ and g can be found efficiently. From Theorem 3, Lemma 2 and algorithms for min-cut (see, e.g., [14]), we obtain the following.

Corollary 4. *The problem of optimizing a linear function over the set of stable matchings in the QF-MODEL can be solved in time $O(|F|^3|W|^3 \texttt{oracle-call})$.*

As a consequence of studying a distributive lattice via the poset that affinely represents it, one immediately obtains a linear description of the convex hull of the characteristic vectors of elements of the lattice.

Theorem 5. *Let $\mathcal{L} = (\mathcal{X}, \succeq)$ be a distributive lattice and $\mathcal{B} = (Y, \succeq^\star)$ be a poset that affinely represents it via function $g(u) = Au + x^0$. Then the extension complexity of $\mathrm{conv}(\mathcal{X}) := \mathrm{conv}\{\chi^S : S \in \mathcal{X}\}$ is $O(|Y|^2)$. If moreover A has full column rank, then $\mathrm{conv}(\mathcal{X})$ has $O(|Y|^2)$ facets.*

Theorem 3 and Theorem 5 imply the following description of the stable matching polytope, i.e., the convex hull of the characteristic vectors of stable matchings.

Corollary 6. $\mathrm{conv}(\mathcal{S})$ *has $O(|F|^2|W|^2)$ facets in the QF-MODEL.*

For examples and extended discussions, we refer to the arXiv version of the paper [6] and to the Ph.D. thesis of the second author [16].

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Integer programs with bounded subdeterminants and two nonzeros per row

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(joint work with Gwenaël Joret, Yelena Yuditsky)

Consider integer programs $\max\{c^\intercal x : Ax \leq b, x \in \mathbb{Z}^n\}$ with integer matrices A. It is an open question whether such integer programs can be solved in polynomial time if the absolute value of the determinant of every square submatrix of A is bounded by some constant Δ. In the case $\Delta = 1$, the matrix A is totally unimodular and the integer program can be replaced by its linear programming relaxation. Recently, Artmann, Weismantel & Zenklusen [1] gave a strongly polynomial-time algorithm for the case $\Delta = 2$. However, the question is open in the case $\Delta \geq 3$.

In this work, we give a strongly polynomial-time algorithm for general constant Δ under the further requirement that A contains at most two nonzero entries in each row. The core of our approach is the first polynomial-time algorithm for the weighted stable set problem on graphs that do not contain more than k vertex-disjoint odd cycles, where k is any constant. Previously, polynomial-time algorithms were only known for $k = 0$ (bipartite graphs) and for $k = 1$ (implied by [1]).

We also observe that the case of two nonzeros per column can be also solved in strongly polynomial-time, using a reduction to b-matching.

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On complete classes of valuated matroids

László Végh

(joint work with Edin Husić, Georg Loho, Ben Smith)

Valuated matroids were introduced by Dress and Wenzel in 1992 [1]. They are a central object in discrete convex analysis, and play important roles in other areas such as mathematical economics and tropical geometry. Finding a constructive characterization, i.e., showing that all valuated matroids can be derived from a simple class by some basic operations has been a natural question proposed in various contexts.

Motivated by this, we study the class of R-minor valuated matroids, that includes the indicator functions of matroids, and is closed under operations such as taking minors, duality, and induction by network. Our main result exhibits valuated matroids that are not R-minor.

Valuated matroids are inherently related to gross substitute valuations in mathematical economics. By the same token we refute the Matroid Based Valuation Conjecture by Ostrovsky and Paes Leme from 2015 [2], asserting that every gross substitute valuation arises from weighted matroid rank functions by repeated applications of merge and endowment operations. Our result also has implications in the context of Lorentzian polynomials: it reveals the limitations of known construction operations.

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On the Integrality Gap of Binary Integer Programs with Gaussian Data

Daniel Dadush

(joint work with Sander Borst, Sophie Huiberts, Samarth Tiwari)

For a binary integer program (IP) $\max c^\mathsf{T} x, Ax \leq b, x \in \{0,1\}^n$, where $A \in \mathbb{R}^{m \times n}$ and $c \in \mathbb{R}^n$ have independent Gaussian entries and the right-hand side $b \in \mathbb{R}^m$ satisfies that its negative coordinates have ℓ_2 norm at most $n/10$, we prove that the gap between the value of the linear programming relaxation and the IP is upper bounded by $\text{poly}(m)(\log n)^2/n$ with probability at least $1 - 2/n^7 - 2^{-\text{poly}(m)}$. Our

results give a Gaussian analogue of the classical integrality gap result of Dyer and Frieze (Math. of O.R., 1989) in the case of random packing IPs. In constrast to the packing case, our integrality gap depends only polynomially on m instead of exponentially. Building upon recent breakthrough work of Dey, Dubey and Molinaro (SODA, 2021), we show that the integrality gap implies that branch-and-bound requires $n^{\text{poly}(m)}$ time on random Gaussian IPs with good probability, which is polynomial when the number of constraints m is fixed. We derive this result via a novel meta-theorem, which relates the size of branch-and-bound trees and the integrality gap for random *logconcave* IPs.

On finding exact solutions of linear programs in the oracle model

Giacomo Zambelli

(joint work with Daniel Dadush, László A. Végh)

We consider the linear programming in the oracle model, that is, the problem $\min\{c^\top x \,:\, x \in P\}$ where the polyhedron $P = \{x \in \mathbb{R}^n \,:\, Ax \leq b\}$ is given by a separation oracle that returns violated inequalities from the system $Ax \leq b$. We present an algorithm that finds exact primal and dual solutions using $O(n^2 \log(n/\delta))$ oracle calls and $O(n^4 \log(n/\delta) + n^5 \log\log(1/\delta))$ arithmetic operations, where $\delta := \delta_{(A,b)}$ is a geometric condition number associated with the matrix (A, b). Specifically, for a matrix V with rows $v_i \in \mathbb{R}^n$, $i \in [m]$, δ_V is the largest value such that, for any set $I \subseteq [m]$ such that the vectors $\{v_i \,:\, i \in I\}$ are linearly independent and for every $\lambda \in \mathbb{R}^I$,

$$\left\| \sum_{i \in I} \lambda_i v_i \right\| \geq \delta_V \max_{i \in I} |\lambda_i| \cdot \|v_i\| .$$

This condition number was previously studied in the context of the shadow simplex algorithm by various authors [1, 3, 5]. The running time bounds above do not depend on the cost vector c. Furthermore, the algorithm does not require any knowledge of the problem instance, such as the condition number δ.

The algorithm works in a black box manner, requiring a subroutine for approximate primal and dual solutions; the above running times are achieved when using the cutting plane method of Jiang, Lee, Song, and Wong [8] for this subroutine. Other methods can be used, such as the ellipsoid method [7], geometric rescaling methods [4, 9] or Vaidya's cutting plane method [10], albeit with worse running time bounds. Whereas approximate solvers may return primal solutions only, we develop a general framework for extracting dual certificates based on the work of Burrell and Todd [2].

Our algorithm works in the real model of computation, and extends results by Grötschel, Lovász, and Schrijver [7], and by Frank and Tardos [6] on solving LPs in the bit-complexity model.In particular, our method does not rely on simultaneous Diophantine approximation.

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Approximate CVP in time $2^{0.802n}$ - now in any norm!

Thomas Rothvoss

(joint work with Moritz Venzin)

A *(full rank) lattice* is the set $\mathcal{L} := \{Bx \mid x \in \mathbb{Z}^n\}$, where $B \in \mathbb{R}^{n \times n}$ is a regular matrix. The two best studied computational problems on lattices are the following: For the *Shortest Vector problem* we are given a lattice $\mathcal{L}$ (described by a matrix B) and a norm $\| \cdot \|_K$ and the goal is to find the shortest non-zero vector in the lattice, i.e. $\min\{\|x\|_K : x \in \mathcal{L} \setminus \{\mathbf{0}\}\}$. For the *Closest Vector problem* we are given a lattice $\mathcal{L}$, a norm $\| \cdot \|_K$ and a *target vector* $t \in \mathbb{R}^n$. The goal is to find the lattice vector closest to t, i.e. $\min\{\|x - t\|_K \mid x \in \mathcal{L}\}$. We abbreviate SVP_p and CVP_p for the case of the norms $\|x\|_p := (\sum_{i=1}^{n} |x_i|^p)^{1/p}$.

Both SVP and CVP and their respective (approximation) algorithms have found considerable applications. These include Integer Programming [6, 5], factoring polynomials over the rationals [7] and cryptanalysis [9]. The problem SVP_K can be solved in time $2^{O(n)}$ regardless of the norm [1]. The Euclidean norm $\| \cdot \|_2$ allows a faster algorithm with time 2^n [2]. In fact, for SVP_2 one can find a constant factor approximation in time $2^{0.802n}$; then this result can be transfered to any $\| \cdot \|_p$-norm [4].

For the harder CVP problem, only the case CVP_2 can be solved exactly in single exponential time; the best exact algorithm for any other norm takes time $n^{O(n)}$. However $(1 + \varepsilon)$-approximations can be obtained for any norm $\| \cdot \|_K$ in time $(1/\varepsilon)^{O(n)}$ [3]. Moreover, if one is satisfied with a large constant factor approximation, then for CVP_p and any $p \geq 1$, running time $2^{0.802n}$ suffices [4].

The main contribution of our work is the following:

Theorem 1. *For any lattice $\mathcal{L} \subseteq \mathbb{R}^n$ and any norm $\|\cdot\|_K$ on $\mathbb{R}^n$ one can find an $O(1)$-approximate solution to CVP_K and SVP_K in time $2^{0.802n}$ and space $2^{0.401n}$.*

The core algorithmic ingredient is the well known *sieving procedure* due to Ajtai, Kumar, Sivakumar [1] which can be paraphrased as follows:

Theorem 2. *Given $\epsilon > 0$, $R > 0$, $N \in \mathbb{N}$ and a lattice $\mathcal{L} \subseteq \mathbb{R}^d$ of rank n, there is a randomized procedure that produces independent samples $v_1, \cdots, v_N \sim \mathcal{D}$, where the distribution $\mathcal{D}$ satisfies the following two properties:*

> *(1) Every sample $v \sim \mathcal{D}$ has $v \in \mathcal{L}$ and $\|v\|_2 \leq a_\epsilon \cdot R$, where a_ϵ is a constant only depending on ϵ.*
> *(2) For any $s \in \mathcal{L}$ with $\|s\|_2 \leq R$, there are distributions $\mathcal{D}_0^s$ and $\mathcal{D}_1^s$ and some parameter ρ_s with $2^{-\epsilon n} \leq \rho_s \leq 1$ such that the distribution $\mathcal{D}$ is equivalent to the following process:*
>> *(a) With probability ρ_s, sample $u \sim \mathcal{D}_0^s$. Then, flip a fair coin and with probability $1/2$, return u, otherwise return $u + s$.*
>> *(b) With probability $1 - \rho_s$, sample $u \sim \mathcal{D}_1^s$.*

This procedure takes expected time $2^{(0.802+\epsilon)n} + N \cdot 2^{(0.401+\epsilon)n}$ and requires $N + 2^{(0.401+\epsilon)n}$ space.

In short: we can generate short lattice vectors whose length is within a constant factor of a target vector s (most importantly, that vector s does not need to be known to the algorithm). Moreover a substantial number of returned samples will be a uniform choice from $\{u, u + s\}$ for some lattice vector u.

Note that this procedure works in the specified running time only for the Euclidean norm. Now, given an arbitrary norm $\|\cdot\|_K$, the key obstacle is to approximate the symmetric convex body K by balls (or ellipsoids). For two convex bodies $A, B \subseteq \mathbb{R}^n$ we define the *covering number* $N(A, B)$ as the minimum number of translates of B necessary to cover A. Then we rely on the following deep result from convex geometry:

Theorem 3 (Milman, Pisier). *For any symmetric convex body, after applying a linear transformation one has*

$$N(K, tB_2^n), N(B_2^n, tK) \leq \exp\left(O(1) \cdot \frac{n}{t^{1.99}}\right) \quad \forall t \geq 1$$

(in fact, also $N(K^\circ, tB_2^n), N(B_2^n, tK^\circ) \leq \exp\left(O(1) \cdot \frac{n}{t^{1.99}}\right)$).

Milman proved this statement for $t = 1$ [8]. The later proof of Pisier for general $t \geq 1$ is non-constructive. However one can modify Milman's *Isomorphic symmetrization* scheme [8] to make it work for any fixed t. We prove:

Theorem 4. *Let $\varepsilon > 0$. In time $n^{O(\log n)}$ one can bring K in a position so that*

$$N(K, O_\varepsilon(1) \cdot B_2^n), N(B_2^n, O_\varepsilon(1) \cdot K) \leq 2^{\varepsilon n}$$

Then the algorithm to solve SVP_K and CVP_K works by transforming the lattice accordingly; then cover K with $2^{\varepsilon n}$ many balls and apply the AKS sieving argument with some care.

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Computing buyer-optimal Walrasian prices in multi-unit matching markets via a sequence of max flow computations

Britta Peis

(joint work with Katharina Eickhoff, S. Thomas McCormick, Niklas Rieken, Laura Vargas Koch)

We consider a multi-unit auction market where discrete indivisible items of n different types, denoted by $\Omega = \{i_1, \ldots, i_n\}$, are sold to a set of m buyers $B = \{j_1, \ldots, j_m\}$. We assume that each product type (or object) $i \in \Omega$ is available in a quantity $b_i \in \mathbb{Z}_+$. Each buyer $j \in B$ is interested in buying at most $d_j \in \mathbb{Z}_+$ items in total. The goal of the auctioneer is to find a per-unit price $p(i)$ for each object i, together with an allocation (or assignment) $x \in \mathbb{Z}_+^{\Omega \times B}$ of items to buyers such that the prices p and the allocation x satisfy certain desirable properties. Certainly, the allocation x should be feasible in the sense that at most b_i units are sold of each object $i \in \Omega$, and each buyer j buys at most d_j items. That is, $x \in \mathbb{Z}_+^{\Omega \times B}$ needs to satisfy the feasibility constraints

$$(1) \qquad \sum_{j \in B} x_{ij} \le b_i \quad \text{for all } i \in \Omega, \quad \text{and} \quad \sum_{i \in \Omega} x_{ij} \le d_j \quad \text{for all } j \in B.$$

The full version of this this extended abstract, which can be found on *https://rwth-aachen.de/oms*) is mainly concerned with the case where the buyers have *linear* valuations of the objects, i.e., where buyer $j \in B$ has a value of $v_{ij} \in \mathbb{Z}_+$

for each copy of object $i \in \Omega$. This value is reduced by the price $p(i)$ that the auctioneer charges, so the net value (or payoff) of one unit of object i to buyer j is $v_{ij} - p(i)$. For an assignment $x \in \mathbb{Z}^{\Omega \times B}$ we denote the items assigned to buyer j by $x_{\bullet j}$. Then, in addition to the feasibility constraints (1), each buyer $j \in B$ should be happy with her allocation $x_{\bullet j}$, i.e., she should be assigned one of her *preferred bundles* under the current prices. That is, $x_{\bullet j}$ should be an optimal integral solution of the linear program

$$(LP_j) \quad \max_{x_{\bullet j}} \left\{ \sum_{i \in \Omega} (v_{ij} - p(i)) \cdot x_{ij} \;\middle|\; \sum_{i \in \Omega} x_{ij} \le d_j, 0 \le x_{ij} \le b_i \text{ for all } i \in \Omega \right\}.$$

A feasible assignment where each buyer achieves a preferred bundle is called *stable*. Prices which admit a stable allocation are called *competitive*. Note that competitive prices can easily be achieved. For example, if each price $p(i)$ exceeds the maximum valuation $\max_{j \in B} v_{ij}$ for this item, the prices are competitive since each buyer achieves a preferred bundle under the assignment where nothing is sold. Thus, the goal of an auctioneer lies in finding competitive prices which are *market-clearing* in the sense that the maximum possible amount $D := \min\{\sum_{i \in \Omega} b_i, \sum_{j \in B} d_j\}$ of items is sold. Such market-clearing competitive prices are also known as *Walrasian prices*. A tuple (p^*, x^*) consisting of Walrasian prices p^* and an associated stable allocation x^* is called *Walrasian equilibrium* [1].

Special case: housing market. The model we consider generalizes the classical matching market (a.k.a. housing market) model, which corresponds to the special case where $b_i = 1$ for all $i \in S$ and $d_j = 1$ for all $j \in B$. For such housing markets, Demange et al. [2] describe an ascending auction which starts at the minimal possible selling prices (e.g. $p_i = 0$ for all $i \in \Omega$) and iteratively raises the prices on some overdemanded set ("Hall set") until the prices are market-clearing. By always raising the prices on an inclusion-wise minimal overdemanded set, Demange et al. guarantee that the prices are the (unique) component-wise minimal competitive prices, and that they are market-clearing, thus buyer-optimal Walrasian prices. A naïve approach to reduce our more general multi-unit auction to a single-unit auction is via the following *copy method*: we replace the b_i items of object i by b_i copies of a unit object, and replace the d_j items demanded by buyer j by d_j unit-item buyers, with the same valuations. Certainly, an ascending auction of the single-unit instance will return market-clearing prices, but these prices are in general not buyer-optimal: Consider, for example, one buyer with a demand of two and two different items with a supply of one which are valued differently by her, say $v = (5, 1)$. If we copy the buyer, both copies will prefer object 1 until the price increased to 4. Now both copies of the sole buyer are indifferent between the objects and thus $p = (4, 0)$ and $x = (1, 1)$ is a Walrasian equilibrium. However, considering the original situation, since the buyer is alone $p = (0, 0)$ and $x = (1, 1)$ is the buyer-optimal Walrasian equilibrium. Thus, the prices computed by the copy method are not buyer-optimal.

More general case: multi-unit auctions with strong substitute valuations. The model we consider fits into the more general model of multi-unit

auctions on heterogenous indivisible goods with strong substitute valuations functions, as studied by e.g. Ausubel [4] and Murota, Shioura and Yang [3]. It is known (see, e.g., [4]) that the minimal minimizer of the Lyapunov function $L(p) = \sum_{j \in B} V_j(p) + \sum_{i \in \Omega} b_i p(i)$, where $V_j(p)$ denotes the maximum payoff each buyer $j \in B$ can achieve under prices p, corresponds exactly to the unique buyer-optimal Walrasian price vector p^* of such multi-unit auctions. Moreover, the ascending auction, which starts at all-zero prices and iteratively raises the prices uniformly by one on the unique minimal minimizer of the submodular function $X \to L(p + \chi_X)$ for $X \subseteq \Omega$, is known to terminate with the unique buyer-optimal Walrasian price vector (cf. [4]). Using tools from discrete convexity and the fact that the definitions of strong substitute and $M^\natural$-concavity are essentially equivalent, Murota, Shioura and Yang [3] show that each price-raising step can be done in strongly polynomial time. Additionally, [3] provides tight bounds on the number of iterations of the auction.

Our contributions. In this paper, we focus on multi-unit auctions with linear valuation functions and demands. For this special case we provide a flow based ascending auction. We prove our results independent from the literature on auctions with strong substitute valuations and discrete convex analysis by using network flow properties. This enables us to show sensitivity regarding changes in supply and demand. More concretely, we present an ascending auction which iteratively raises the prices on the objects in the left-most min cut in an associated auxiliary flow network, and prove that the algorithm terminates with component-wise minimal Walrasian prices. We show how to construct the corresponding stable allocation where as much as possible is sold, and where every object with positive price is completely sold. We furthermore show that structural insights obtained from our flow-based approach lead to several insights regarding the sensitivity analysis of our ascending auction.

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[Focus] A (Slightly) Improved Approximation Algorithm for Metric TSP

Nathan Klein

(joint work with Anna Karlin, Shayan Oveis Gharan)

In an instance of TSP we are given a set of n cities V along with their pairwise symmetric distances, $c : V \times V \to \mathbb{R}_{\geq 0}$. The goal is to find a Hamiltonian cycle of minimum cost. In the metric TSP problem, which we study here, the distances satisfy the triangle inequality. Therefore, the problem is equivalent to finding a closed Eulerian connected walk of minimum cost. An algorithm of Christofides-Serdyukov from four decades ago gives a $\frac{3}{2}$-approximation for TSP. This remained the best known approximation algorithm for the general case of the problem despite significant work.

In [1, 2] we prove the following theorem:

Theorem 1. *For some absolute constant $\varepsilon > 10^{-36}$, there is a randomized algorithm that outputs a tour with expected cost at most $(\frac{3}{2} - \varepsilon) \cdot c(x)$, where x is an optimal solution to the subtour elimination LP for TSP.*

The algorithm we analyze is (a slight variant of) the so-called *max entropy* algorithm for TSP, introduced by Oveis Gharan, Saberi, and Singh in 2010 [3]. In particular, we first solve the subtour elimination relaxation of TSP to obtain a point x. Then, we find a distribution of maximum entropy over spanning trees with marginals x and sample a tree T from this distribution. Finally, similar to Christofides' algorithm, we add the minimum cost perfect matching on the odd degree vertices of the sampled tree.

This theorem also immediately implies that the integrality gap of the subtour polytope is at most $\frac{3}{2} - \epsilon$. We remark that [1] only showed that the max entropy algorithm has expected cost at most $(\frac{3}{2} - \varepsilon) \cdot OPT$; [2] extended this to compare against the cost of the LP.

Here we highlight three techniques which are crucial to the analysis of the algorithm. The analysis is solely concerned with bounding the expected cost of the matching, and uses the dominant of the O-Join polytope, where O is the set of odd vertices of the sampled tree T.

The Polygon Structure for Near Minimum Cuts Crossed on one Side. Let $G = (V, E, x)$ be an undirected graph equipped with a weight function $x : E \to \mathbb{R}_{\geq 0}$ such that for any cut $(S, \overline{S})$ such that $u_0, v_0 \notin S$, $x(\delta(S)) \geq 2$. For some (small) $\eta \geq 0$, consider the family of η-near min cuts of G, i.e. sets S such that $x(\delta(S)) \leq 2 + \eta$. Let $\mathcal{C}$ be a connected component of crossing η-near min cuts. Given $\mathcal{C}$ we can partition vertices of G into sets $a_0, \ldots, a_{m-1}$ (called atoms); this is the coarsest partition such that for each a_i, and each $(S, \overline{S}) \in \mathcal{C}$, we have $a_i \subseteq S$ or $a_i \subseteq \overline{S}$.

Benczúr and Goemans [5, 4] studied the case when $\eta \leq 6/5$ and introduced the notion of *polygon representation*, in which case atoms can be placed on the sides of

an equilateral polygon and some atoms placed inside the polygon, such that every cut in $\mathcal{C}$ can be represented by a diagonal of this polygon.

In this pair of papers, we show that (after a reduction using the polygon representation) it suffices to study the structure of edges in a special family of polygon representations. Suppose we have a polygon representation for a connected component $\mathcal{C}$ of η-near min cuts of G such that no atom is mapped inside, and if we identify each cut $(S, \overline{S}) \in \mathcal{C}$ with the interval along the polygon that does not contain a_0, then any interval is only crossed on one side (only on the left or only on the right). Then, we have that for any atom a_i, $x(\delta(a_i)) \leq 2 + O(\eta)$ and for any pair of atoms a_i, a_{i+1}, $x(E(a_i, a_{i+1})) \geq 1 - \Omega(\eta)$.

Generalized Gurvits' Lemma. For a real stable polynomial $p \in \mathbb{R}_{\geq 0}[z_1, \ldots, z_n]$ (with non-negative coefficients), Gurvits proved the following inequality:

$$\frac{n!}{n^n} \inf_{z>0} \frac{p(z_1, \ldots, z_n)}{z_1 \ldots z_n} \leq \partial_{z_1} \ldots \partial_{z_n} p|_{z=0} \leq \inf_{z>0} \frac{p(z_1, \ldots, z_n)}{z_1 \ldots z_n}.$$

As an immediate consequence, one can prove the following theorem about strongly Rayleigh distributions, a class of distribution which includes max entropy trees.

Theorem 2. *Let* $\mu : 2^{[n]} \rightarrow \mathbb{R}_{\geq 0}$ *be SR and* $A_1, \ldots, A_m$ *be random variables corresponding to the number of elements sampled in m disjoint subsets of $[n]$ such that* $\mathbb{E}A_i = n_i$ *for all i. If $n_i = 1$ for all $1 \leq i \leq n$, then* $\mathbb{P}[\forall i, A_i = 1] \geq \frac{m!}{m^m}$.

One can ask what happens if the vector $\vec{n} = (n_1, \ldots, n_m)$ in the above theorem is not equal but close to the all ones vector. We show that as long as $||\vec{n} - \mathbf{1}||_1 < 1 - \varepsilon$ then $\mathbb{P}[\forall i, A_i = 1] \geq f(\varepsilon, m)$ where $f(\varepsilon, m)$ has no dependence on n, the number of underlying elements in the support of μ.

Theorem 3. *Let* $\mu : 2^{[n]} \rightarrow \mathbb{R}_{\geq 0}$ *be SR and let* $A_1, \ldots, A_m$ *be random variables corresponding to the number of elements sampled in m disjoint subsets of $[n]$. Suppose that there are integers $n_1, \ldots, n_m$ such that for any set $S \subseteq [m]$,* $\mathcal{P}\sum_{i \in S} A_i = \sum_{i \in S} n_i \geq \varepsilon$. *Then,*

$$\mathbb{P}[\forall i, A_i = n_i] \geq f(\varepsilon, m).$$

Conditioning while Preserving Marginals. Consider a strongly Rayleigh distribution $\mu : 2^{[n]} \rightarrow \mathbb{R}_{\geq 0}$ and let $x : [n] \rightarrow \mathbb{R}_{\geq 0}$, where for all i, $x_i = \mathbb{P}_{T \sim \mu}[i \in T]$, be the marginals.

Let $A, B \subseteq [n]$ be two disjoint sets such that $\mathbb{E}[A_T], \mathbb{E}[B_T] \approx 1$. It follows from the above that $\mathbb{P}[A_T = B_T = 1] \geq \Omega(1)$. Here, however, we are interested in a stronger event; let $\nu = \mu|A_T = B_T = 1$ and let $y_i = \mathbb{P}_{T \sim \mu}[i \in T]$. It turns out that the y vector can be very different from the x vector, in particular, for some i's we can have $|y_i - x_i|$ bounded away from 0. We show that there is an event of non-negligible probability that is a subset of $A_T = B_T = 1$ under which the marginals of elements in A, B are almost preserved.

Theorem 4. *Let $\mu : 2^{[n]} \to \mathbb{R}_{\geq 0}$ be a SR distribution and let $A, B \subseteq [n]$ be two disjoint subsets such that $\mathbb{E}[A_T], \mathbb{E}[B_T] \approx 1$. For any $\alpha \ll 1$ there is an event $\mathcal{E}_{A,B}$ such that $\mathcal{P}\mathcal{E}_{A,B} \geq \Omega(\alpha^2)$ and*

- $\mathcal{P} A_T = B_T = 1 | \mathcal{E}_{A,B} = 1,$
- $\sum_{i \in A} |\mathbb{P}[i] - \mathbb{P}[i \mid \mathcal{E}]| \leq \alpha,$
- $\sum_{i \in B} |\mathbb{P}[i] - \mathbb{P}[i \mid \mathcal{E}]| \leq \alpha.$

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Node Connectivity Augmentation via Iterative Randomized Rounding
LAURA SANITÀ

(joint work with Haris Angelidakis, Dylan Hyatt-Denesik)

Many network design problems deal with the design of low-cost networks that are resilient to the failure of their elements (such as nodes or links). One such problem is Connectivity Augmentation, with the goal of cheaply increasing the (edge- or node-)connectivity of a given network from a value k to $k + 1$. The problem is NP-hard for $k \geq 1$, and the most studied setting focuses on the case of edge-connectivity with $k = 1$.

In this work, we give a 1.892-approximation algorithm for the NP-hard problem of augmenting the node-connectivity of any given graph from 1 to 2, which improve upon the the state-of-the-art approximation previously developed in the literature. The starting point of our work is a known reduction from Connectivity Augmentation to some specific instances of the Node-Steiner Tree problem [1, 2], and our result is obtained by developing a new and simple analysis of the iterative randomized rounding technique [3] when applied to such Steiner Tree instances. Our results also imply a 1.892-approximation algorithm for the problem of augmenting the edge-connectivity of a given graph from any value k to $k + 1$. While this does not beat the best approximation factor known for this problem [4], a key point of our work is that the analysis of our approximation factor is less involved when compared to previous results in the literature. In addition, our work

gives new insights on the iterative randomized rounding method, that might be of independent interest.

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A QPTAS for Stabbing Rectangles

FRIEDRICH EISENBRAND

(joint work with Martina Gallato, Ola Svensson and Moritz Venzin)

We consider the following problem, calles *stabbing*: Given a set $\mathcal{R} = \{R_1, \cdots, R_n\}$ of axis-aligned rectangles in the plane, the task is to find a set of horizontal line segments of minimal total length such that all rectangles are stabbed. A rectangle is stabbed if a line-segment in the solution intersects both its left and right edge. This natural geometric optimization problem was introduced by Chan et al. [4]. Stabbing can be understood as a geometric interpretation of various combinatorial optimization problems such as *message scheduling* with time-windows on a directed path, *frequency assignment*, or problems in network design, see, for example, [1, 4, 7].

To understand stabbing as a message scheduling problem, consider the case of a directed path $\{v_1, \ldots, v_k\}$ with weighted edges, along which we want to send some messages. Each message has a release node v_i and an arrival node v_j, for some $i < j \in [k]$, and it needs to be sent during a specific time window $[t_i, t_j]$. Opening an edge at a certain time to transmit messages along it has a cost equal to its weight. Furthermore, edges are uncapacitated, meaning that a single edge can transmit an arbitrary number of messages simultaneously. Therefore, messages which are sharing some edges in the path and whose time intervals overlap, can aggregate along the common edges and be transmitted together to decrease the total cost of the transmission. The goal is to transmit all messages and minimize the total cost of activated edges over time.

Each message request can be seen as a rectangle $[v_i, v_j] \times [t_i, t_j] \subseteq \mathbb{R}^2$. A segment $[v_i, v_j] \times \{t\}$ stabbing this rectangle, corresponds to a transmission of the message from node v_i to node v_j at time t, with $t_1 \leq t \leq t_2$.

We can think of the cost of transmitting a message as the width of the corresponding rectangle, that is to say, we can define rectangles such that $|v_{i+1} - v_i| = w(v_i, v_{i+1})$. Messages can aggregate exactly where their corresponding rectangles

overlap and can share portions of the same segment to be stabbed. Finding an optimal stabbing solution exactly corresponds to finding an optimal schedule.

We note that this generalises a special case studied in [1], where all messages have the same arrival node. The frequency assignment problem can be modeled analogously, see [4].

stabbing in turn can be interpreted as a *geometric set cover problem*, in which the rectangles are the elements and the line-segments are the sets. An element (rectangle) is contained in a set (line segment), if it is stabbed by the line segment. This immediately implies that there is a $O(\log n)$-approximation algorithm for stabbing [5, 10].

Improving upon the $\log n$-approximation for set cover in geometric settings has been an important area of research in computational geometry. One successful approach [2] for *unweighted* geometric set cover is via ϵ-*nets* [8] in range-spaces of bounded *VC-dimension*. Since ϵ-nets are of linear size in certain geometric settings [6, 11, 12], constant factor approximation algorithms can be obtained via linear programming in these cases. Another very successful approach to tackle geometric set cover problems is the *quasi-uniform sampling* technique of Varadarajan [13]. It gives rise to a sub-logarithmic approximation algorithms for geometric set cover problems of small union complexity. The technique was improved by Chan et al. [3] which then yields constant factor approximation algorithms for weighted geometric set cover problems of small *shallow cell complexity*. This is the case in the *weighted disk cover* problem for example.

The state-of-the-art for stabbing is as follows. Chan et al. [4] provide a *constant-factor* approximation algorithm for stabbing that is based on a decomposition technique and the framework of quasi-uniform sampling. More precisely, they show how to decompose stabbing into two set cover instances of small shallow cell complexity for which the technique of Varadarajan [13] and its improvement by Chan et al. [3] yields a constant factor approximation. The authors show furthermore that stabbing is NP-hard via a reduction from *planar vertex cover*.

Our contribution. In this paper, we provide the following results.

(1) We show that there is a PTAS for instances of stabbing for which the ratio between the widths of the rectangles is bounded by a constant.

(2) This technique can then be recursively applied to yield a quasi-polynomial time approximation scheme (QPTAS) for stabbing in general. The running time of our algorithm is $n^{O(\log^3(n)/\epsilon^2)}$. This shows that stabbing is not APX-hard unless NP $\subseteq$ DTIME($2^{\text{polylog}(n)}$).

(3) We provide a simple 8-approximation algorithm for stabbing. First, we round the instance to a *laminar* instance, i.e., an instance in which the projections of the rectangles to the x-axis yields a laminar family of intervals. This laminar instance is then solved optimally via dynamic programming.

The contributions 2 and 3 settle two open problems raised in [4].

To put our work into perspective, we also rely on a decomposition technique to obtain our QPTAS. However, we do not rely on the balanced cut framework. Instead we use a variation of the *shifted grid* technique by Hochbaum and Maass [9] that we combine with a simple charging scheme and careful guessing.

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Determinant Maximization: Approximation and Estimation Algorithms

Mohit Singh

(joint work with Vivek Madan, Sasho Nikolov, Uthaipon Tantipongpipat, Weijun Xie)

In the determinant maximization problem, given a collection of vectors, we aim to pick a subset to maximize the determinant of a natural matrix associated with these vectors. The abstract problem captures problems in multiple areas including statistics, convex geometry, allocation problems and network design. We will survey the known results and techniques for the problem when the picked subset must satisfy natural combinatorial constraints. The results vary from arbitrary good approximations [3, 4, 5] to only estimation algorithms [1, 2]. The techniques vary from stable and log-concave polynomials to sparsity properties of solutions of convex programs.

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On the Integrality Gap of Binary Integer Programs with Gaussian Data

Daniel Dadush

(joint work with Sander Borst, Sophie Huiberts, Samarth Tiwari)

For a binary integer program (IP) $\max c^\mathsf{T} x, Ax \leq b, x \in \{0, 1\}^n$, where $A \in \mathbb{R}^{m \times n}$ and $c \in \mathbb{R}^n$ have independent Gaussian entries and the right-hand side $b \in \mathbb{R}^m$ satisfies that its negative coordinates have ℓ_2 norm at most $n/10$, we prove that the gap between the value of the linear programming relaxation and the IP is upper bounded by $\mathrm{poly}(m)(\log n)^2/n$ with probability at least $1 - 2/n^7 - 2^{-\mathrm{poly}(m)}$. Our results give a Gaussian analogue of the classical integrality gap result of Dyer and Frieze (Math. of O.R., 1989) in the case of random packing IPs. In constrast to the packing case, our integrality gap depends only polynomially on m instead of exponentially. Building upon recent breakthrough work of Dey, Dubey and

Molinaro (SODA, 2021), we show that the integrality gap implies that branch-and-bound requires $n^{\mathrm{poly}(m)}$ time on random Gaussian IPs with good probability, which is polynomial when the number of constraints m is fixed. We derive this result via a novel meta-theorem, which relates the size of branch-and-bound trees and the integrality gap for random *logconcave* IPs.

A Deterministic Parallel Algorithm for Maximum Coverage via Steiner Systems

Eric Balkanski

In this paper, we study the problem of maximizing a coverage function in the adaptive complexity model. Adaptivity was recently introduced in the context of submodular optimization as an information-theoretic measure for the parallel runtime of an algorithm [1]. A recent line of work has designed algorithms that achieve constant factor approximations for maximizing submodular functions, as well as other classes of functions, under various constraints in a logarithmic (or poly-logarithmic) number of adaptive rounds of function evaluations. These parallel algorithms all heavily rely on randomization and a fundamental question is whether randomness is necessary to obtain parallel algorithms in the adaptive complexity model.

We present a deterministic algorithm for maximizing coverage functions, an important subclass of submodular functions, under a cardinality constraint that is $\mathcal{O}(\epsilon^{-2} \log n)$ adaptive and achieves a nearly optimal $1 - 1/e - \epsilon$ approximation. This algorithm is the first deterministic algorithm for maximum coverage with sublinear adaptivity that achieves a constant factor approximation. When the algorithm is given as input the explicit representation of a coverage function, instead of a value oracle, we obtain the first NC approximation algorithm for maximum coverage. The algorithm relies on a novel connection between combinatorial optimization and Steiner systems from combinatorial design theory that is of independent interest.

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[Focus] On the extension complexity of low-dimensional polytopes

Lisa Sauermann

(joint work with Matthew Kwan, Yufei Zhao)

The *extension complexity* $\mathrm{xc}(P)$ of a d-dimensional polytope P is defined to be the minimum number of facets in a (possibly higher-dimensional) polytope P' such that one can obtain P as the image of P' under a linear projection. This notion is motivated by its relevance in combinatorial optimization, and has been studied intensively for various specific polytopes associated with important optimization

problems. In contrast, this talk discusses some (more theoretical) questions about extension complexity for some more general classes of (low-dimensional) polytopes.

A natural extremal question about extension complexity is the following (essentially asking how much the extension complexity of a polytope is controlled by its dimension): For a fixed dimension d, what is the maximum possible extension complexity of a d-dimensional polytope with n vertices (or with n facets)? It is equivalent to ask this question for polytopes with n vertices or for polytopes with n facets (this can be seen by passing between a polytope and its polar dual polytope). This question is still wide open, and most likely very difficult.

For $d = 2$, the best known upper bound is that every 2-dimensional n-vertex polytope (i.e. every n-gon) has extension complexity $O(n^{2/3})$ [4], while for $d \geq 3$ no non-trivial upper bounds are known (meaning that the best known upper bound is n). The best known lower bounds are of the form $\Omega(\sqrt{n})$ for every fixed d (see [2] and also the earlier work [1] for $d = 2$). These lower bounds were obtained by considering random d-dimensional polytopes and proving that their extension complexity is typically at least $\Omega(\sqrt{n})$. Heuristically, it makes sense to expect that random d-dimensional polytopes should exhibit high extension complexity, so it is natural to consider random polytopes in order to prove lower bounds for the original extremal question.

This approach raises the question of actually finding the typical extension complexity of a random d-dimensional polytope (this may, of course, depend on the chosen model of random d-dimensional polytopes). In [1] and [2], the authors only prove lower bounds for the typical extension complexity of the random polytopes they consider, and it is a priori possible that the actual extension complexity of these random polytopes is much higher.

We answer this question up to constant factors (depending on d) for two different models of random d-dimensional polytopes, namely polytopes obtained as the convex hull of n independent uniformly random points on the unit sphere or of m independent uniformly random points in the unit ball (the latter model is a very well-studied model for random polytopes and it is known that in this model the number of vertices is asymptotically almost surely of the form $\Theta(m^{(d-1)/(d+1)})$, see for example [3]). More precisely, we prove the following two theorems.

Theorem 1. *For every fixed $d \geq 2$, there exist constants c_d and C_d sch that the following holds. Let P be the convex hull of n random points on the unit sphere in $\mathbb{R}^d$. Then, we have $c_d \cdot \sqrt{n} \leq \mathrm{xc}(P) \leq C_d \cdot \sqrt{n}$ with probability tending to 1 as $n \to \infty$.*

Theorem 2. *For every fixed $d \geq 2$, there exist constants c_d and C_d sch that the following holds. Let P be the convex hull of m random points on the unit ball in $\mathbb{R}^d$, and let $n = m^{(d-1)/(d+1)}$. Then, we have $c_d \cdot \sqrt{n} \leq \mathrm{xc}(P) \leq C_d \cdot \sqrt{n}$ with probability tending to 1 as $m \to \infty$.*

In short, for fixed dimension $d \geq 2$, for both of the two models of random polytopes that we considered, we proved that the extension complexity is asymptotically almost surely on the order of the square root of the number of vertices.

While the lower bounds were known (or follow from known techniques), the new contributions of our results are the upper bounds for the extension complexity in the two theorems above.

In light of these results, and the heuristic that random polytopes should exhibit high extension complexity, it would be tempting to conjecture that for every fixed $d \geq 2$, every d-dimensional polytope with n vertices has extension complexity at most $O(\sqrt{n})$. Since showing such a bound would most likely be extremely difficult, it might also make sense to consider some special classes of d-dimensional polytopes, for example polytopes with all vertices on a common sphere.

Is it true that for every fixed $d \geq 2$, every d-dimensional n-vertex polytope with all vertices on a common sphere has extension complexity at most $O(\sqrt{n})$? This question is also open, but we prove that the answer is "Yes" for $d = 2$:

Theorem 3. *Let P be a (2-dimensional) n-gon with all vertices on a common circle. Then P has extension complexity at most $24\sqrt{n}$.*

This bound is tight up to the constant factor 24.

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Algorithmic Tools for US Congressional Districting: Fairness via Analytics

DAVID SHMOYS

(joint work with Nikhil Garg, Wes Gurnee, David Rothschild)

The American winner-take-all congressional district system empowers politicians to engineer electoral outcomes by manipulating district boundaries. To date, computational solutions mostly focus on drawing unbiased maps by ignoring political and demographic input, and instead simply optimize for compactness and other related metrics. However, we maintain that this is a flawed approach because compactness and fairness are orthogonal qualities; to achieve a meaningful notion of fairness, one needs to model political and demographic considerations, using historical data.

We will discuss two papers that explore and develop this perspective.

- In the first paper [1] (joint with Wes Gurnee), we present a scalable approach to explicitly optimize for arbitrary piecewise-linear definitions of fairness; this employs a stochastic hierarchical decomposition approach to

produce an exponential number of distinct district plans that can be optimized via a standard set partitioning integer programming formulation. This enables the largest-ever ensemble study of congressional districts, providing insights into the range of possible expected outcomes and the implications of this range on potential definitions of fairness.

• In the second paper [2] (joint with Nikhil Garg, Wes Gurnee, and David Rothschild), we study the design of multi-member districts (MMDs) in which each district elects multiple representatives, potentially through a non-winner-takes-all voting rule (as currently proposed in H.R. 4000). We carry out large-scale analyses for the U.S. House of Representatives under MMDs with different social choice functions, under algorithmically generated maps optimized for either partisan benefit or proportionality. We find that with three-member districts using Single Transferable Vote, fairness-minded independent commissions can achieve proportional outcomes in every state (up to rounding), and this would significantly curtail the power of advantage-seeking partisans to gerrymander. We believe that this work opens up a rich research agenda at the intersection of social choice and computational redistricting.

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[Focus] The High-Dimesnional-Expander Perspective on Combinatorial Distributions

Nima Anari

Over the past few years, a new tool, namely high-dimensional expanders, has emerged as a powerful method for analyzing local Markov chains that sample from combinatorial distributions. I will survey some of the breakthroughs in the field of approximate sampling and counting obtained using this framework.

As the main motivation, I will introduce an open conjecture posed by Mihail and Vazirani: in every polytope whose vertices are a subset of $\{0,1\}^n$, the skeleton, i.e., the graph formed by vertices and edges, has expansion ≥ 1. The original target of this conjecture were base polytopes of matroids, where the conjecture would imply fast mixing of random walks on the so-called basis-exchange graph. I will briefly talk about how high-dimensional expanders were used in resolving this conjecture for matroids.

Beyond matroids, the conjecture of Mihail and Vazirani remains open. However, unlike the matroid case, for general 0/1 polytopes, a resolution of the conjecture does not imply fast Markov chains or sampling algorithms. I will introduce new conjectures related to the one made by Mihail and Vazirani, that if true, would imply fast mixing of random walks for 0/1 polytopes that have short edges. In

fact, if true, these conjectures would nail the precise mixing time of natural random walks that can sample vertices of these polytopes.

Providing evidence for these conjectures, I will demonstrate fast mixing of random walks for several classes of (not necessarily uniform) distributions whose supports are 0/1 polytopes with short edges. These distributions include, amongst others, monomers in monomer-dimer systems, and nonsymmetric determinantal point processes. As an application of fast mixing of random walks for these distributions, we resolve a question of Jerrum about sampling from monomer-dimer systems in planar graphs.

I will talk about the main two tools from the study of high-dimensional expanders that we use in analyzing random walks: spectral independence and entropic independence. These tools can respectively yield polynomial (but lossy) mixing time and optimal mixing of random walks associated to the aforementioned combinatorial distributions. I will connect these tools to the geometry of polynomials, revealing new connections between spectral and entropic analysis of Markov chains.

Based on several joint works with Yeganeh Alimohmmadi, Vishesh Jain, Frederic Koehler, Kuikui Liu, Shayan OveisGharan, Huy Tuan Pham, Kiran Shiragur, Cynthia Vinzant, and June Vuong.

Symmetric Rendezvous on the Line
Martin Skutella
(joint work with Max Klimm, Guillaume Sagnol, Khai Van Tran)

In the Symmetric Rendezvous Search on the Line with Unknown Initial Distance, two identical agents are placed on the real line with their distance, the other's location, and their orientation unknown to them. Moving along the line at unit speed and executing the same randomized search strategy, the agents' goal is to meet up as early as possible. The expected meeting time obviously depends on the unknown initial distance and orientations. The quality of a randomized search strategy is thus measured by its competitive ratio, that is, the ratio of the expected meeting time and the earliest possible meeting time (half the initial distance).

We present a class of successively refined randomized search strategies together with a rigorous mathematical analysis of their continuously improved competitive ratios. These strategies all rely on the basic idea of performing an infinite sequence of steps of geometrically increasing size in random directions, always returning to the agent's initial position before starting the next step. In addition, our more refined strategies use two novel ideas. First, remembering their past random choices, the agents randomly choose the direction of the next step in a Markov-chain-like manner. Second, choosing the next few random directions in advance, each agent may combine consecutive steps in the same direction into one longer step. As our main result, we show that this combination of looking into the past as well as into the future leads to a substantially improved competitive ratio of 13.93 compared to the previously best known bound of 24.85 (Ozsoyeller et al. 2013).

[Focus] Discrete Optimization in Machine Learning – an (informal) overview

Sebastian Pokutta

Machine Learning problems are often linked to empirical risk minimization problems that in turn are typically solved with continuous optimization methods. More recently however, it has been recognized that certain problems in machine learning naturally lead to discrete optimization problems. In this talk I will give an overview of the role of discrete optimization methods in machine learning and discuss some associated tradeoffs. Problems of interest include

(1) Best Subset Selection
(2) Optimal Classification Trees
(3) Feature Selection
(4) Architecture Search for NNs
(5) Boosting for Classification
(6) Network Verification
(7) Sinkhorn Networks
(8) Rate Distortion Explanations

as well as many related problems. Currently many important questions, for example when the discrete solution to a learning problem outperforms the continuous solution in learning tasks are wide open. The references below provide a sample of works (a) using discrete optimization in machine learning, (b) using machine learning in discrete optimization, and (c) combining machine learning with discrete optimization.

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Single source unsplittable flows with arc-wise lower and upper bounds

Sarah Morell

(joint work with Martin Skutella)

In a digraph with a source node and several destination nodes with associated demands, an unsplittable flow routes each demand along a single path from the common source to its destination. Given some flow x that is not necessarily unsplittable but satisfies all demands, we ask for an unsplittable flow y that does not deviate from x by too much, i.e., $y_a \approx x_a$ for all arcs a.

Twenty years ago, in a landmark paper, Dinitz, Garg, and Goemans [1] proved that, given some flow x, there exists an unsplittable flow y such that

$$y_a \ \leq \ x_a + d_{\max} \quad \text{for all arcs } a,$$

where $d_{\max}$ denotes the maximum demand value. Unsplittable flows with arc-wise lower bounds have, to the best of the authors' knowledge, not been considered yet. Based upon an entirely new approach, we prove the following result: Given some flow x, there exists an unsplittable flow y such that

$$y_a \ \geq \ x_a - d_{\max} \quad \text{for all arcs } a.$$

Secondly, building upon an iterative rounding technique previously introduced by Kolliopoulos and Stein [2] and Skutella [4], we prove existence of an unsplittable flow that simultaneously satisfies the upper and lower bounds for the special case when demands are integer multiples of each other. For arbitrary demand values, we prove the slightly weaker simultaneous bounds

$$x_a/2 - d_{\max} \ \leq \ y_a \ \leq \ 2x_a + d_{\max} \quad \text{for all arcs } a.$$

This talk is based on the paper [3].

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The Geometry of All Pivot Rules for the Simplex Method

JESÚS A. DE LOERA

(joint work with Alexander Black, Niklas Lutjeharms, Raman Sanyal)

For a matrix $A \in \mathbb{R}^{n \times d}$, and vectors $b \in \mathbb{R}^n, c \in \mathbb{R}^d$ we consider the linear program (LP)

$$\max \quad c^t x$$
$$\text{s.t.} \quad Ax \leq b$$

The linear inequalities give rise to a polyhedron P that we will throughout assume to be bounded. That is, we assume that $P := \{x \in \mathbb{R}^d : Ax \leq b\}$ is a **polytope**. We denote by $G(P)$ the vertex-edge graph of P. We further assume that the objective function c is generic and hence yields an acyclic orientation of $G(P)$. We will refer to (P, c) as the **linear program**.

The *simplex method* is one of the two most popular algorithms for solving linear programs. Geometrically the simplex method finds a c-monotone path from any initial vertex to the optimal vertex. The algorithm starts at some vertex v of P and proceeds along directed edges to the unique sink which is the optimum vertex v_{opt}. At any non-sink v, a pivot rule chooses a neighboring vertex u of v with $c^t u > c^t v$. Since the inception of the simplex algorithm, many different pivot rules have been proposed and analyzed. To this day, no pivot rule is known to produce only polynomially many steps on every LP. In this paper we provide polyhedral structure that organizes the pivot rules on LPs.

In 1972 Klee and Minty first showed that Dantzig's original pivot rule may require exponentially many steps. The algorithm could be tricked into visiting all 2^d vertices of a deformed cube to find a path between two nodes which are only one step apart in the skeleton of the cube. Since 1972 researchers have shown many of the popular pivot rules are known to require an exponential number of steps to solve some concrete "twisted" linear programs.

We will not try to give a precise definition of what constitutes a pivot rule. A first reason is pivot rules are all about decisions of switching pairs of variables, one variable enters another leaves, but several authors arrived to the conclusion that pivot rules can be used to encode hard problems, hard in the sense of complexity theory. E.g., Fearnley and Savani showed that it is PSPACE-complete to decide whether Dantzig's pivot rule ever chooses a specific variable to enter the basis. In
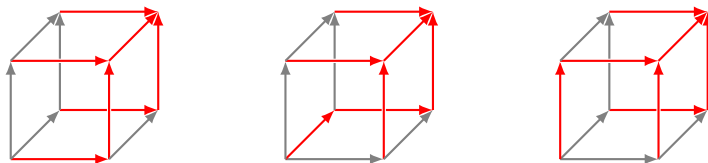
FIGURE 1. Three arborescences on an cube. The middle arborescence does not come from a NW-rule.

contrast, Adler et al. showed the simplex method with the shadow vertex pivot rule can decide this query in polynomial time. A second reason is that we do not need to formally defined what a pivot rule is because our taxonomy of pivot rules uses only a polyhedral geometry perspective.

Definition 1. *The **footprint** of a pivot rule Γ on an LP (P, c) is the directed subgraph obtained as the union of all monotone paths produced by Γ, starting with any vertex of P.*

Definition 2. *A **memory-less** pivot rule is one where its footprint for every LP is an **arborescence**, i.e., a directed tree with root at optimum.*

An equivalent way to say this is a pivot rule is **memory-less** if it chooses the neighbor of $v \neq v_{opt}$ using only *local* information provided by the set of neighbors $\mathrm{Nb}_P(v)$ of v. Many rules that are used in practice, including *greatest improvement* and *steepest edge*, are memory-less. Pivot rules not in this class include Zadeh's least visited facet rule as well as the original the shadow vertex rule. Similarly, the celebrated random-edge pivot rule may or may not have an arborescence as a footprint, so it cannot be considered memory-less.

Memory-less pivot-rules are interesting because for every pivot rule Γ there is always a Memory-less pivot rule applied to the LP, whose footprint is contained in the footprint of Γ and the lengths of monotone paths are less or equal than the paths produced by Γ. This follows immediately from the computation of the shortest path arborescence of any rooted directed graph. Therefore if we wish to show there is a pivot rule that gives polynomial-size paths for all LPs, then one can find such a rule within the family of memory-less pivot rules.

Now, the essence of a memory pivot rule to (P, c) is captured by an **arborescence** induced by the union of edges in the collection of simplex-paths obtained by varying the starting vertex. Note also that the knowledge of the arborescence is sufficient to give a pivot rule for (P, c). Figure 1 shows three such arborescences on a 3-cube.

The two main questions that we address in this paper are

(1) How do the arborescences vary for fixed objective function c and varying pivot rule?
(2) How do the arborescences vary for fixed pivot rule and varying objective function c?

To be able to change the pivot rules in a controlled and continuous manner, we restrict to the following setup: For given $P \subset \mathbb{R}^d$ and $c \in \mathbb{R}^d$, choose a **normalization** $\eta : \mathbb{R}^d \to \mathbb{R}$ and a **weight** $w \in \mathbb{R}^d$. For $v \neq v_{opt}$, the next vertex on the simplex-path from v is

$$(1) \qquad u_* = \operatorname{argmax} \left\{ \frac{w^t(u-v)}{\eta(u-v)} : u \text{ adjacent to } v \text{ and } c^t u > c^t v \right\}.$$

A mechanism to choose w and η given (P, c) is called a **normalized-weight** pivot rule, or **NW-rule** for short. If R is a normalized-weight pivot rule, we sometimes write $\eta^R(P, c)$ and $w^R(P, c)$ to stress the dependence of η and w on the LP (P, c). As we explain in the next section, the main pivot rules (greatest-improvement, steepest-edge), as well as our generalization of the shadow-vertex rule, max-slope, belong to that class.

Although NW-rules are a strict subclass of memory-less pivot rules, we first show the following universality result.

Theorem 3. *For every simple polytope P there is a perturbation P' such that for any memory-less pivot rule there is a NW-rule that produces the same arborescence for (P', c) for every c.*

The simplicity of P means that (P, c) is a non-degenerate LP for every c and guarantees that the combinatorics of P and P' are isomorphic.

Now for a fixed LP (P, c) and a fixed normalization η, (1) determines an arborescence $\mathcal{A}$, that is, a map on the vertices of P with $\mathcal{A}(v_{opt}) = v_{opt}$ and $\mathcal{A}(v) = u_*$ otherwise. We can continuously change the pivot rule by varying the weight w. We call an arborescence that arises via (1) for a fixed weight w a **coherent** arborescence and write $\mathcal{A} = \mathcal{A}^\eta_{P,c}(w)$. This terminology underlines the proximity to the theory of coherent monotone paths started in the 1990's by Billera and Sturmfels.

An answer to question (A) is provided by the following theorem.

For a polytope $Q \subseteq \mathbb{R}^d$, we write $Q^w = \{x \in Q : w^t x \geq w^t y, y \in Q\}$ for the face that maximizes $x \mapsto w^t x$.

Theorem 4. *Let (P, c) be a linear program and η a normalization function. There is a polytope $\Pi^\eta_{P,c} \subset \mathbb{R}^d$ called the **pivot polytope** such that the following holds: For any generic weights w, w'*

$$\mathcal{A}^\eta_{P,c}(w) = \mathcal{A}^\eta_{P,c}(w') \iff (\Pi^\eta_{P,c})^w = (\Pi^\eta_{P,c})^{w'}.$$

Now coherent arborescences are special and we compare their number the total number of arborescences. We are able to completely determine the face lattice of the pivot rule polytopes in terms of arborescences and show that how they relate to the *monotone path polytope*.

Question (B) is strongly related to parametric linear programming. Whereas the basic question there is roughly which objective functions yield the same optimum, we will address the more subtle question which objective functions yield the same arborescence. We make two assumptions on the NW-rule R, namely that $\eta^R(P, c)$ is independent of c and that $w^R(P, c) = c$. Thus, for a fixed normalization function η, we will write $\mathcal{B}^\eta_P(c) := \mathcal{A}^\eta_{P,c}(c)$ for the arborescence of (P, c) obtained from (1)

with respect to η and weight $w = c$. We show that the that the collection of arborescences $\mathcal{B}_P^\eta(c)$ is governed by another polytope.

Theorem 5. *Let $P \subset \mathbb{R}^d$ be a polytope and R a NW-rule as defined above. There is a polytope $\Gamma_P^\eta \subset \mathbb{R}^d$ called the **neighbotope** of P and η such that the following holds: For any generic objective functions $c, c' \in \mathbb{R}^d$*

$$\mathcal{B}_P^\eta(c) \ = \ \mathcal{B}_P^\eta(c') \quad \Longleftrightarrow \quad (\Gamma_P^\eta)^c \ = \ (\Gamma_P^\eta)^{c'}.$$

How many matchings cover the nodes of a graph?
András Sebő
(joint work with Dehia Ait Ferhat, Zoltán Király, Gautier Stauffer)

Given an undirected graph, are there k matchings whose union covers all of its nodes, that is, a *matching-k-cover*? When $k = 1$, the problem is equivalent to the existence of a perfect matching for which Tutte's celebrated matching theorem [1] provides a 'good' characterization. We prove here, when k is greater than one, a 'good' characterization *à la Kőnig*: *for $k \geq 2$, there exist k matchings covering every node if and only if for every stable set S, we have $|S| \leq k \cdot |N(S)|$.* Moreover, somewhat surprisingly, we use only techniques from bipartite matching in the proof, through a simple, polynomial algorithm. A different approach to matching-k-covers has been previously suggested by Wang, Song and Yuan [2], relying on general matching and using matroid union for matching-matroids, or the Edmonds-Gallai structure theorem. Our approach provides a simpler polynomial algorithm together with an elegant certificate of non-existence when appropriate.

Further results, generalizations and interconnections between several problems are then deduced as consequences of the new minimax theorem, with surprisingly simple proofs (again using only the level of difficulty of bipartite matchings). One of the equivalent formulations leads to a solution of weighted minimization for non-negative edge-weights, while the edge-cardinality maximization of matching-2-covers turns out to be already NP-hard.

We have arrived at this problem as the line graph special case of a model arising for manufacturing integrated circuits with the technology called 'Directed Self Assembly'.

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Complexity of optimizing over the integers
Amitabh Basu

We will consider problems of the form

(1) $$\inf\{f(x,y) : (x,y) \in C, (x,y) \in \mathbb{Z}^n \times \mathbb{R}^d\}.$$

where $f : \mathbb{R}^n \times \mathbb{R}^d \to \mathbb{R}$ is a convex (possibly nonsmooth) function and $C \subseteq \mathbb{R}^n \times \mathbb{R}^d$ is a closed, convex set. We wish to design algorithms that returns a point in $C \cap (\mathbb{Z}^n \times \mathbb{R}^d)$ with value at most ϵ more than the optimal value. Since we want to allow general nonlinear objective functions f and convex sets C, we will assume the algorithm will have access to a first order oracle for f, i.e., for any point $z \in \mathbb{R}^n \times \mathbb{R}^d$, the oracle returns $f(z)$ and a subgradient in $\partial f(z)$, and assume access to a separation oracle for C.

To analyze the complexity of such problems, we have to introduce a way to parameterize the problems in a reasonable way and investigate the complexity as a function of these parameters. The parameters are meant to label problems to mimic their "difficulty levels". In other words, the "harder" instances should have "higher" values of the parameter. The most common choice of parameters for this class of optimization problems are the following five: $n, d \in \mathbb{N}$ and $R, M, \rho \in \mathbb{R}$. $\mathcal{I}_{n,d,R,M,\rho}$ are those instances of (1) such that

(1) The domain of f and C are both subsets of $\mathbb{R}^n \times \mathbb{R}^d$.
(2) C is contained in the box $\{z \in \mathbb{R}^n \times \mathbb{R}^d : \|z\|_\infty \leq R\}$, and
(3) f is Lipschitz continuous with respect to the $\|\cdot\|_\infty$-norm with Lipschitz constant M on any set of the form $\{x\} \times [-R, R]^d$ with $x \in [-R, R]^n \cap \mathbb{Z}^n$, i.e., for any $(x, y), (x, y')$ with $\|y - y'\|_\infty \leq R$, $|f(x, y) - f(x, y')| \leq M\|y - y'\|_\infty$.
(4) If $(x^\star, y^\star)$ is the optimum solution, then there exists $\widehat{y} \in \mathbb{R}^d$ and $\rho > 0$ such that $\{(x^\star, y) : \|y - \widehat{y}\|_\infty \leq \rho\} \subseteq C$, i.e., there is a "strictly feasible" point $(x^\star, \widehat{y})$ in the same fiber as the optimum $(x^\star, y^\star)$ with a fiber box of width ρ in $\mathbb{R}^d$ (the continuous space) around $(x^\star, \widehat{y})$ contained in C. Note that if $d = 0$ (the *pure integer* case), then this requirement becomes vacuous.

The ϵ-*information complexity* of the class of optimization problems (1), parameterized by n, d, R, M, ρ, is defined as the minimum number of oracle queries needed by any algorithm to return an ϵ-approximate solution to a problem in $\mathcal{I}_{n,d,R,M,\rho}$. This will be denoted by $\mathrm{icomp}_\epsilon(n, d, R, M, \rho)$. Thus, $\mathrm{icomp}_\epsilon(n, d, R, M, \rho)$ is an unconditional lower bound on the overall complexity of any ϵ-approximation algorithm for (1).

One can prove the following bounds on ϵ-information complexity.
Lower bounds.

- If $n, d \geq 1$,

$$\mathrm{icomp}_\epsilon(n, d, R, M, \rho) \in \Omega\left(d2^n \log\left(\frac{R}{\rho}\right)\right).$$

- If $d = 0$,
$$\text{icomp}_\epsilon(n, d, R, M, \rho) \in \Omega\left(2^n \log(R)\right).$$

- If $n = 0$,
$$\text{icomp}_\epsilon(n, d, R, M, \rho) \in \Omega\left(d \log\left(\frac{MR}{\rho\epsilon}\right)\right).$$

Upper bounds.

- If $n, d \geq 1$
$$\text{icomp}_\epsilon(n, d, R, M, \rho) \in O\left((n+d)d2^n \log\left(\frac{MR}{\rho\epsilon}\right)\right).$$

- If $d = 0$
$$\text{icomp}_\epsilon(n, d, R, M, \rho) \in O\left(n2^n \log(R)\right).$$

- If $n = 0$
$$\text{icomp}_\epsilon(n, d, R, M, \rho) \in O\left(d \log\left(\frac{MR}{\rho\epsilon}\right)\right).$$

Note that when $n = 0$, i.e., we consider continuous convex optimization with no integer variables, we have $\text{icomp}_\epsilon(n, d, R, M, \rho) = \Theta\left(d \log\left(\frac{MR}{\rho\epsilon}\right)\right)$, giving a tight characterization of the complexity. In fact, these results can be obtained for a much broader class of oracles that include first-order/separation oracles as special cases; see [2, 3, 1, 4].

For pure integer optimization with $d = 0$, our upper and lower bounds are off by a linear factor in the dimension, which is of much lower order compared to the dominating term of $2^n \log(R)$. Put another way, both bounds are $2^{O(n)} \log(R)$. Since the strict feasibility assumption is vacuous and for small enough $\epsilon > 0$, ϵ-approximate solutions are the same as exact optimum solutions, M, ϵ and ρ do not play a role in the upper and lower bounds.

There seems to be scope for nontrivial improvement in the bounds presented for the mixed-integer case with $n, d \geq 1$:

(1) It would be nice to unify the lower bound for $n = 0$ (the continuous case) and $n \geq 1$ (the truly mixed-integer case). The proof for $n, d \geq 1$ is based on the feasibility question, which is why M and ϵ do not appear in the lower bound.

(2) When one plugs in $n = 0$ in the mixed-integer upper bound ($n, d \geq 1$), one does not recover the tight upper bound for $n = 0$; instead, the bound is off by a factor of d. We believe this can likely be improved.

We do not have upper bounds on algorithmic complexity that are close to the bounds established on information complexity when integer variables are involved. In the case with no integer variables, $n = 0$, the classical *ellipsoid algorithm* has complexity $O\left(d^2 \log\left(\frac{MR}{\rho\epsilon}\right)\right)$. Combining this with techniques from algorithmic geometry of numbers and enumeration based on branching on general split disjunctions, one can give an algorithm for the general mixed-integer case with

complexity $O\left(2^{n\log(n+d)}\log\left(\frac{MR}{\rho\epsilon}\right)\right)$. It is a major open question to design an algorithm with improved running time closer to the information complexity bound of $O\left((n+d)2^n\log\left(\frac{MR}{\rho\epsilon}\right)\right)$.

However, the algorithms that achieve $O\left(2^{n\log(n+d)}\log\left(\frac{MR}{\rho\epsilon}\right)\right)$ complexity do not explicitly utilize non-trivial cutting planes, i.e., halfspaces H such that $C \not\subseteq H$ but $C \cap (\mathbb{Z}^n \times \mathbb{R}^d) \subseteq H$. While it has not been established theoretically that using such cutting planes can produce algorithms with better overall complexity, it has been observed in practice that mixed-integer solvers rely heavily on such cutting planes. Recently, two results have been established that give some theoretical basis to this empirical fact. The first result establishes very general conditions under which combining non-trivial cutting planes with enumeration schemes gives an exponential advantage over using cutting planes alone or branching schemes alone. See [5]. If we hold n to be a fixed constant, then the complexity $O\left(2^{n\log(n+d)}\log\left(\frac{MR}{\rho\epsilon}\right)\right)$ is a polynomial in the remaining parameters, and is achieved by algorithms that use branching schemes without cutting planes. It is an intriguing question to see if one can design a pure cutting plane algorithm that is polynomial time if the number n of integer variables is a fixed constant. Such a result has been recently established for pure integer problems ($d = 0$) in the plane, i.e., $n = 2$; see [6]. Even the $n = 3$ case remains open.

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Open Problem: Approximating Symmetric Submodular Functions by Hypergraphs

CHANDRA CHEKURI

Devanur et al. [1] raised the question of approximating submodular functions and symmetric submodular functions by simpler classes of functions. In particular they showed that any non-negative symmetric submodular function $f : 2^N \to \mathbb{R}_+$ can be approximated to a relative factor of $(n - 1)$ by graph cut functions; the approximator is simply the cut function of the Gomory-Hu tree of f. They also

observed that this bound is tight for a very simple function, namely f where $f(S) = 0$ for $S = \varnothing$ and $S = N$ and $f(S) = 1$ otherwise. They raised the question of approximating symmetric submodular functions via hypergraphs and also observed that rank r hypergraphs cannot approximate better than a factor of $\Omega(n/r)$.

Is there a fixed constant c such that every symmetric submodular function can be approximated by a hypergraph cut function within a factor of c? This is open. We know that $c \geq 2$. Experiments have shown that all functions on upto 10 elements can be approximated to within a factor of 2. Via relatively simple arguments, Calvin Beideman, Karthik Chandrasekaran, Chao Xu, and myself have shown that hypergraph cut functions can approximate, to within a constant factor, symmetrizations of functions of the form $f(S) = g(|S|)$ where g is a concave function. Thus, hypergraph cut functions are able to overcome the strong lower bound that holds for graph cut functions on these classes of simple functions. Natural examples to try are symmetrizations of rank functions of simple matroids. Laminar matroids are a natural example to try to generalize the case of concave functions and these already pose various challenges.

On the pessimistic side we observed the following. Suppose we can construct a hypergraph cut function that approximates f in an efficient and compact fashion to within a factor of α. Then we can approximate the sparsest cut, defined as $\min_{|S| \leq |N|/2} f(S)/|S|$, to within an $O(\alpha \log n)$ factor since we can approximate sparsest cut for hypergraphs to within a factor of $O(\log n)$. The results of Svitkina and Fleischer [2] show that one needs exponential number of value queries to f to approximate $f(S)/|S|$ to better than a polynomial factor in n.

Is it likely that f can be approximated via hypergraph cut functions to within a small factor but it is computationally difficult to find such a representation? Note that sparsification results for hypergraphs show that $O(\frac{1}{\epsilon^2} n \log n)$ weighted hyperedges suffice to approximate the cut function of any hypergraph on n vertices to within a $(1+\epsilon)$-factor [3]. Thus the size of the hypergraph is not the bottleneck.

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Open Problem: Computing the diameter of the bipartite perfect matching polytope

Laura Sanità

The diameter of a polytope $\mathcal{P}$ is the maximum value of a shortest path between any two vertices on the 1-skeleton of $\mathcal{P}$, where the 1-skeleton of $\mathcal{P}$ can be regarded as the graph where vertices (resp. edges) correspond to 0-dimensional (resp. 1-dimensional) faces of $\mathcal{P}$. We are here interested in the complexity of computing the diameter of the polytope given by perfect matchings of a bipartite graph.

Formally, for a given bipartite graph $G = (V, E)$, the perfect matching polytope $\mathcal{PM}(G)$ associated to G is:

$$\mathcal{PM}(G) := \{x \in \mathbb{R}^E : \sum_{e \in \delta(v)} x_e = 1 \ \forall v \in V, \ x_e \geq 0 \ \forall e \in E\}$$

where $\delta(v)$ indicates the edges of G with one endpoint being v. Consider the following problem:

- **Input:** Bipartite graph $G = (V, E)$.
- **Output:** Diameter of $\mathcal{PM}(G)$.

Is the above problem NP-hard?

Open Problem: Integer infeasible subsystems

Robert Weismantel

(joint work with Marcel Celaya)

Let A be an integral $m \times n$ matrix and b an integral vector with m components. Let Δ be the largest magnitude of any subdeterminant of A. We define $P = \{x \mid Ax \leq b\}$ and make the following assumptions.

- P is full dimensional,
- P does not contain any integer point, but any proper subsystem of $Ax \leq b$ contains integer points.

Can we upper bound m as a linear function of n, provided that Δ is constant? More formally, do there exist one-dimensional functions f, g such that $m \leq nf(\Delta) + g(\Delta)$?

From a theorem of Doignon it follows that $m \leq 2^n$ [1]. In the special case when $\Delta \in \{1, 2\}$ it follows from LP theory, theory of TU matrices and [2] that $m \leq n + 1$. For $\Delta = 3$ an upper bound of the desired form is not known. Note that the number of distinct rows of A can always be upper bounded by $O(n^2 h(\Delta))$ where h is a one-dimensional function only depending on Δ, see [3] for details.

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Open Problem: Iterative rounding for feedback vertex set
Samuel Fiorini

The (undirected) feedback vertex set problem is to find, given a vertex-weighted graph (G, c) a minimum cost set X such that $G - X$ is a forest. This seems to be a central problem. However, there is basically just one 2-approximation algorithm to solve it, due to Bafna, Berman and Fujito (1995) and Becker and Geiger (1994). Chudak, Hochbaum, Goemans and Williamson (1998) gave a primal-dual interpretation of this algorithm, based on the following LP:

$$
\begin{array}{lll}
\min & \sum_{v \in V(G)} c(v) x(v) & \\
\text{s.t.} & \sum_{v \in S} (d_S(v) - 1) x(v) \geq |E(S)| - |S| + 1 & \forall S \subseteq V(G) : E(S) \neq \varnothing \\
& x(v) \geq 0 & \forall v \in V(G) .
\end{array}
$$

Is it true that for every extreme solution x^* to the LP, there is a vertex v with $x^*(v) \geq 1/2$? This would immediately imply a new 2-approximation algorithm based on iterative rounding. I can prove that when the LP is weakened by changing the right-hand sides of the first block of constraints to $|E(S)| - |S|$, there is always a vertex v such that $x^*(v) \geq 1/3$. Stefan Weltge can show that both LPs can be solved in polynomial time, since each has a polynomial size extended formulation.

Open Problem: Maximizing Subdeterminants in Nonsymmetric Matrices
Nima Anari

Given a matrix $L \in \mathbb{R}^{n \times n}$ with $L + L^{\mathsf{T}} \succeq 0$, and an integer $k \in \{1, 2, \ldots, n\}$, find the $k \times k$ principal submatrix of L that has the maximum determinant:

$$
\max \left\{ \det(L_{S,S}) \ \middle| \ S \in \binom{[n]}{k} \right\} .
$$

This is NP-hard to even approximately solve within a factor of c^k for some universal constant c (Di Summa-Eisenbrand-Faenza-Moldenhauer'14), so the ultimate goal is a $2^{O(k)}$ approximation. We currently know how to find a $k^{O(k)}$-approximation by local search (Anari-Vuong'21). In the case where L is symmetric (and PSD), the approximation factor of $2^{O(k)}$ is achievable by a clever continuous relaxation (Nikolov'14).

An important application of the conjectured $2^{O(k)}$ approximation would be a $O(\log n)$-factor approximation for computing the hereditary discrepancy of an $n \times n$

matrix (see Reis-Jiang'21), beating the current best $O(\log^{3/2} n)$. For this particular application, it's enough to solve the subdeterminant maximization problem for skew-symmetric L: $L = -L^\intercal$. It's even enough to assume L has the following form:

$$L = \begin{bmatrix} 0 & A \\ -A^\intercal & 0 \end{bmatrix},$$

where A is some arbitrary rectangular matrix. In this special case the problem becomes equivalent to finding the $k/2 \times k/2$ (not necessarily principal) submatrix of A with the largest determinant in magnitude.

Open Problem: α-Multiplicative Connectivity Augmentation and Polygon Augmentation
Nathan Klein

Let $\alpha > 1$ be a constant, let $G = (V, E)$ be a k-edge-connected graph, and let L be a weighted multiset of edges that may be added to the graph G called *links* (to distinguish them from the edges of G). Then, the α-multiplicative connectivity augmentation problem is to add the minimum weight set of links to G from L such that $E \cup L$ is at least $\lceil \alpha k \rceil$-edge-connected.

For $\alpha \le 6/5$, the cuts of G of size at most αk admit a compact *polygon representation* [4, 3]. Therefore, we propose first studying α-multiplicative connectivity augmentation for $\alpha \le 6/5$ which may alternately be called polygon augmentation, following the naming convention of the problems tree augmentation and cactus augmentation in which the goal is to increase the connectivity of a graph by 1.

The main open problem is to design a better-than-2 approximation for this problem for any constant $\alpha > 1$. Due to recent exciting progress on the tree augmentation problem [1, 2], we believe this is a good time to study its generalizations. This is a natural generalization since for many networks, increasing connectivity by 1 may have a negligible effect (for example, if the edge connectivity of the graph is already very large).

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Open problem: Approximating submodular functions by matroid rank functions

LÁSZLÓ VÉGH

Let $f : 2^V \to \mathbb{R}_+$ be a nonnegative and monotone increasing submodular function with $f(\varnothing) = 0$. Further, assume that for each $i \in V$, $f(\{i\}) = 1$. If we further assume that f is integer-valued, then f is a matroid rank function. The question is whether a non-integer function f can still be well-approximated by a matroid rank?

That is, does there exists a constant $\alpha > 0$ such that for any function f as above, there exists a matroid rank function $f : 2^V \to \mathbb{Z}_+$ such that

$$\frac{r(S)}{\alpha} \le f(S) \le \alpha r(S) \quad \forall S \subseteq V?$$

Open problem: Signings in the subtour polytope

MICHEL GOEMANS

Consider the subtour polytope P of the TSP on $G = (V, E)$ (where we have dropped the degree constraints, without loss of generality, for the question below):

$$P = \{x \in \mathbb{R}^E : x(\delta(S)) \ge 2 \text{ for all } S \subseteq V, x \ge 0\}.$$

For any $T \subseteq V$ of even cardinality, we know that $x/2$ belongs to the dominant of the T-join polytope for G [2]. But is the following true:

Open question 1. *For any $x \in P$ and for any $T \subseteq V$ of even cardinality, does there exist a signing $s : T \to \{-1, +1\}$ (with $s(T) := \sum_{v \in T} s(v) = 0$) such that there exists a transshipment (flow) f in G with capacities $\{u_e : e \in E\}$ and whose excess at vertex v is $s(v)$ for $v \in T$ and 0 otherwise.*

By Gale's theorem (see Corollary 11.2g in [1]), this transshipment exists (s is a *valid* signing) if and only if s satisfies that, for all $S \subset V$, one has $s(S) \le x(\delta(S))$ (and this can be tested efficiently and the transshipment can be found efficiently). Using splitting-off (à la Lovász), it is sufficient to consider the case in which $|V|$ is even and $T = V$.

The signing clearly exists if x is the incidence vector of an Hamiltonian cycle (and in this case there is an exponential number of valid signings). If x is half-integral, then a valid signing also exists; one can for example take an Eulerian walk in $2x$, consider the *first* visit of every vertex, and alternate $+1$ and -1 sign assignments. Proving the existence of the transshipment in the graph with capacities αx for some small $\alpha \ge 1$ would also be interesting ($\alpha < 1$ does not work for many x). In fact, this would be implied if one could show the existence of *thin* trees (as one could then alternate sign in a traversal of the tree).

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Open problem: Representation for 4/3-near-mincuts
Michel Goemans

In the recent result of Anna Karlin, Nathan Klein and Shayan Oveis Gharan on the subtour polytope [3] presented at this workshop, the polygon representation of approximately mincuts played an important role (among other tools). This polygon representation (Benczúr Ph.D. thesis and [1]) generalizes the cactus representation of minimum cuts in a weighted undirected graph, and allows to represent $\frac{6}{5}$-mincuts (in a simple geometric way); these are the cuts whose capacity is strictly less than $6/5$ times the minimum cut value. Finding a compact representation of all 4/3-mincuts is an important open problem. The ratio 4/3 is the limit α where the number of α-mincuts is always at most $\binom{n}{2}$ in a graph with n vertices (and when other properties start breaking as well, like approximate splitting-off [2]).

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Open problem: Unrelated graph balancing
Nicole Megow

Let $G = (V, E)$ be an undirected graph. Each edge $e = \{u, v\} \in E$ has two possible weights, $w(u, v) \in \mathbb{R}_+$ and $w(v, u) \in \mathbb{R}_+$, depending on how the edge gets oriented. An *orientation* $\gamma : E \to V$ is a function that maps every edge to one of its endpoints, we say, it orients the edges. Given an orientation, the *weighted indegree* of a node v is the total weight of edges oriented towards v, that is, $\sum_{\{u,v\} \in \gamma^{-1}(v)} w(u, v)$. The *unrelated graph orientation problem* is to find an orientation γ for G that minimizes the maximum weighted indegree over all nodes in G. The natural generalization of this problem to multigraphs is called *unrelated graph balancing*.

Lenstra, Shmoys, and Tardos [2] provide a 2-approximation via LP rounding. In fact, they consider the more general unrelated machine scheduling problem $R||C_{\max}$. Graph balancing is a special case thereof, in which each job has a finite processing time only on two machines; one could formulate the scheduling problem

as unrelated graph balancing in hypergraphs. Improving upon the approximation factor of 2 for $R||C_{\max}$ is considered as one of the major open problems in scheduling [3].

Unrelated graph balancing seems to capture already the major difficulties and no better approximation factor than 2 is known, not even in simple graphs. Verschae and Wiese [4] show that the configuration LP has an integrality gap of 2 for this problem. The instance in their gap construction has edge weights only in $\{1, k\}$ for some arbitrary but large k. We are interested in beating the factor of 2 for this particular class of instances via a strengthened LP or some other approach. We can show (with my PhD students Alexander Lindermayr and Jens Schlöter) that there is a 7/4-approximation if the optimum value is k.

Note that Ebenlendr, Krcál and Sgall [1] give a 7/4-approximation via LP rounding for the special case in which each edge has the same weight independently of its orientation.

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Open problem: Perfect matching that maximizes the k-heaviest edges
OLA SVENSSON

Given a bipartite graph $G = (V, E)$ with edge-weights $w : E \to \mathbb{R}$, efficient algorithms for finding a perfect matching of maximum weight can be traced back to the work of Jacobi over a century ago [2].

However, can you efficiently (in polynomial time) find a perfect matching that maximizes the weight of the k-heaviest edges? This problem can be equivalently stated as the following problem: find k edges of maximum weight that can be extended to a perfect matching.

If the weights are polynomially bounded, then the randomized technique by Mulmuley, Vazirani, & Vazirani [1] gives such an algorithm (even a parallel one). It remains open if we can get a *deterministic* polynomial-time algorithm for this problem. In addition, the complexity of the problem (without restrictions on the edge-weights) remains open.

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