

## Electrical characteristics of horizontal glass melting furnaces and delivery systems

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Leakage currents and their low resistance paths through refractories from the interior of glass melting furnaces to the binding steel are discussed. By definition, a ground current flows from an electrode, through the intervening glass to ground through grounding conductors. All melters have leakage currents and may have ground currents.

Current and voltage characteristics and phasor diagrams have been analyzed for some circuits that can be used in horizontal melters. The furnaces may be heated solely with electricity or in conjunction with fossil fuels. Phasor diagrams may be helpful in the design stage of a melter, may be useful for trouble-shooting and to display electrical distributions for all parts in a precise form. Hence, totally unexpected voltage differences become easily understandable. Phasor diagrams can disclose wiring errors and/or undesirable voltage distributions and may suggest ways to improve electrical characteristics. For symmetrically built and cross-fired furnaces they show side-to-side symmetry and indicate that the central plane of these units can be virtual or phantom grounds. They may also indicate the conditions minimizing voltage differences between adjacent electrodes. Voltage phasor diagrams are even more useful for longitudinally fired furnaces. For these units they also explain why ground currents occur and how to minimize them.

Data from a salt water model with two cross-fired circuits connected to the same phase disclosed linear relations between the electric currents and voltages over a large range. In this range each line current was a linear function of the two line voltages and vice versa and these variables could be treated as scalars. In addition, either line current could be forced to zero by appropriate voltage(s) in the other circuit. Somewhat similar results were obtained when two different phases were used but now neither current could be forced to zero by adjusting the voltage in the other circuit. Again in the linear range each line current was a linear function of the line voltages and vice versa but the variables now had to be treated as phasors and not as scalar quantities. Salt water modelling with a longitudinally fired melt end yielded representative data showing how the applied voltages may be located on a ground current's voltage phasor in both the capped and uncapped condition.

### Elektrische Eigenschaften von horizontalen Glasschmelzöfen und Rinnensystemen

Streuströme und ihre Bahnen niedrigen Widerstandes, die durch das Feuerfestmaterial vom Ofeninneren zur Stahlkonstruktion vagabundieren, werden diskutiert. Erdströme fließen direkt durch die Glasschmelze zu geerdeten Leitern. Alle Schmelzöfen haben Streuströme und können Erdströme haben.

Strom- und Spannungscharakteristiken sowie Zeigerdiagramme wurden für einige Stromkreise analysiert, wie sie in horizontalen Elektroschmelzöfen eingesetzt werden können. Die Öfen werden entweder nur mit elektrischem Strom oder gemischt mit fossilen Brennstoffen beheizt. Zeigerdiagramme können hilfreich sein in der Entwicklungsphase eines Schmelzofens, bei der Fehlersuche und bei der Darstellung der Stromverteilung in allen Teilen. Nun werden gänzlich unerwartete Spannungsunterschiede auf einfache Weise verständlich. Zeigerdiagramme können Verdrahtungsfehler und/oder unerwünschte Spannungsverteilungen sichtbar machen und Wege zur Verbesserung der elektrischen Eigenschaften aufzeigen. Für symmetrische Öfen, bei denen die Elektroden gegenüberliegend zusammengeschaltet sind, zeigen sie eine Achsensymmetrie und lassen erkennen, daß die Mittelebene dieser Einheiten entweder virtuell oder geerdet sein kann. Sie können außerdem die Bedingungen für die Minimierung der Spannungsunterschiede zwischen benachbarten Elektroden aufzeigen. Phasendiagramme sind für längsbeheizte Öfen noch brauchbarer. Für diese Anlagen erklären sie auch, warum Erdströme auftreten und wie sie zu minimieren sind.

Daten eines Salzwassermodells mit zwei Stromkreisen, deren Elektroden einander gegenüberlagen, aber an dieselbe Phase angeschlossen waren, ließen lineare Beziehungen zwischen den elektrischen Strömen und Spannungen über einen weiten Bereich erkennen. In diesem Gebiet war jeder Leitungsstrom eine lineare Funktion von zwei Leitungsspannungen und umgekehrt, und diese Veränderlichen konnten als Skalare behandelt werden. Außerdem konnte einer der Leitungsströme durch entsprechende Spannung(en) im anderen Stromkreis auf Null gefahren werden. Ganz ähnliche Ergebnisse wurden erzielt, wenn zwei verschiedene Phasen angewendet wurden, doch nun konnte keiner der Ströme durch Verstellen der Spannung im anderen Stromkreis auf Null gebracht werden. Wiederum war im linearen Bereich jeder Leitungsstrom eine lineare Funktion der Leitungsspannungen und umgekehrt, aber jetzt müssen die Variablen als Zeigerdiagramme und nicht als Skalargrößen angesehen werden. Das Salzwassermodell mit einer längsbeheizten Schmelzwanne erbrachte repräsentative Werte, die zeigten, wie die angelegten Spannungen auf einen Erdstrom-Spannungszeiger sowohl mit als auch ohne Abdeckung ausgerichtet werden müssen.

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Received October 16, 1995, revised manuscript April 1, 1996.

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## 1. Introduction

### 1.1 Preliminaries

Most horizontal glass melting furnaces have a rear melting zone and the melted glass flows into a front chamber and then out through a delivery system. The delivery system is made up of a forehearth and bowl which has an orifice ring through which the flow of glass is controlled. Many of these furnaces are heated with fossil fuel and batch or fill is added through the back wall by means of fill machines. In addition to fossil fuel, electric power is often used in the rear zone to augment the melter's output and/or quality [1]. Electric heat may or may not be used in the rest of the system. Power is introduced through electrodes which may be made of different materials such as tin oxide [2], graphite [3], molybdenum [4] and which may have different sizes, shapes, immersions [5] and locations [6]. Typical installations include electrodes installed into the glass through the bottom of the unit or through its side walls. With electrodes arranged along the side walls the electric current(s) flow(s) through the molten glass across the width of the melter. This is called cross firing (figure 1) [7]. In a second arrangement the electrodes are parallel to the back wall. Current passes between the adjacent rows. This is called longitudinal firing [1]. Power is provided from 3-phase (3- $\phi$ ) sources but not all the phases need to be used. The voltages are assumed to have sinusoidal wave forms and hence may be represented by three phasors A, B and C [7 to 9]. In this report phasors will always mean voltage phasors unless otherwise specified. Transformer terminals to yield corresponding directions for these three phasors are indicated by large dots on their secondaries. Phasor A will always be drawn horizontal and pointing to the right, phasor B will always make an angle of (+120°) with it and phasor C will always make an angle of (-120°) from it (figure 2a). These phasors must always retain their relative angular positions unless the primaries of the power supplies (not shown) are rewired. Interpretations of phasors are helped by using electrode numbers at each head and tail. Often the applied voltages are not sinusoidal so, strictly speaking, phasor should not be used. However, experience has shown that representing distorted wave forms by phasors often leads to valid conclusions.

All voltages used in this report have been chosen arbitrarily. In addition, all circuits use separate secondaries. For simplicity transformers for the forehearth including the bowl are not shown but their polarities and phases are indicated in the appropriate figures.

### 1.2 Resistance to ground and leakage currents

#### 1.2.1 Resistance to ground

When a furnace is cold and dry the resistances of any of its installed electrodes and internal metallic parts to its binding steel should be extremely high. If this is not true something is wrong and should be corrected. Generally, as the furnace is heated and filled, all the binding steel,

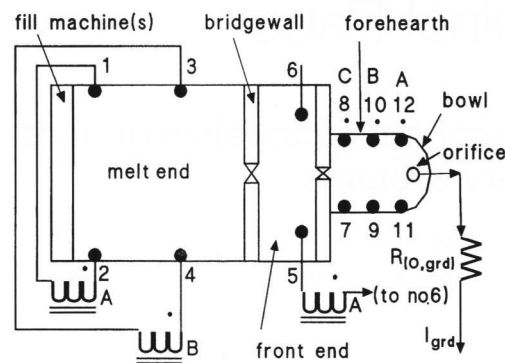
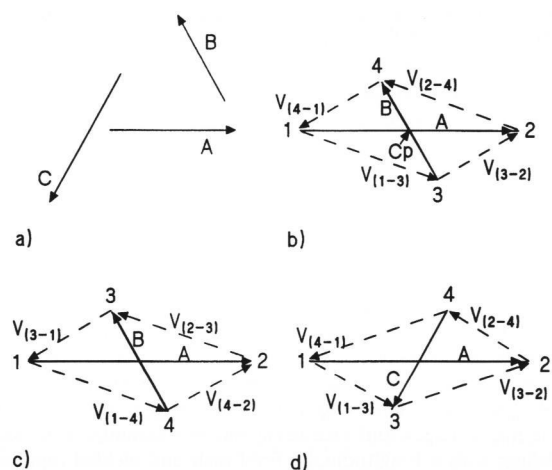


Figure 1. Sketch, top view of a horizontal glass-melting furnace with a cross-fired melt end, a front end and a delivery system.



Figures 2a to d. Uncoupled and coupled voltage phasors for the cross-fired connections used for the melt end of the furnace shown in figure 1; a) fixed directions for A, B and C phasors used exclusively in this report; b) phasor diagram with  $V_A$  connected from electrode no. 1 to no. 2 and  $V_B$  wired from electrode no. 3 to no. 4 (figure 1); c) like figure b with leads to electrodes no. 3 and 4 reversed, thus decreasing the voltage between the adjacent side wall electrodes; d) like figures b or c except phasor C is wired from electrode no. 4 to no. 3, thus maintaining the lower voltage between the adjacent electrodes.

that is not electrically isolated from hot refractories or is not grounded, becomes electrically hot. Simultaneously, the resistances from the electrodes and inside surfaces to its binding steel will decrease and at operating temperature this resistance may be as low as 5 to 10  $\Omega$ . This resistance may be measured with a high-frequency ohmmeter that rejects the 60 Hz voltage used for power<sup>2)</sup>. To understand this low resistance consider any square inch of glass-contacting refractory extending from the inside surface of the furnace to its external surface and finally to its contacting binding steel. The refractory's inside temperature may be at 1400 °C while its outside temperature, for example, may be 300 °C. With

<sup>2)</sup> This instrument is manufactured by GSC, Inc., 141 Railroad Drive, Ivyland, PA 18974 (USA), and is described in their Bulletin 9050 A.

this temperature distribution each elemental piece, which extends from the inside surface to the outside surface, still has a high resistance but there are thousands of them all of which can conduct a small but finite amount of current out of the furnace simultaneously. These small currents are in-parallel and they constitute a part of the leakage current. In addition with hot crown operation, the hot gases and vapors above the molten glass are partially ionized and are in contact with hot refractories in the melter and checkers and on through the stack. All these paths contribute to the leakage current and thus also help reduce the resistance to ground. As their resistances are also in-parallel, they help lower the total resistance. The net result of these leakage currents is that for a symmetrically built and operated cross-fired furnace the voltage from an electrode to the binding steel will be equal to one half of the applied voltage. This effectively puts the central plane of totally symmetrical cross-fired units at zero potential relative to the binding steel. If this steel is also grounded, then the centerline (plane) of the cross-fired unit is also at ground potential.

### 1.2.2 Leakage current with cross firing

Consider a symmetrical, cross-fired melt end, with two single-phase circuits (figure 1). Suppose each circuit has 100 V with 50 V across the resistance to ground which has been found to be  $5 \Omega$ . For this case  $I_{\text{leakage}} = 50/5 = 10 \text{ A}$ . This current is small compared to the current used for heating which could be 2000 A. Note that the leakage and heating currents are in-parallel and their ratio is equal to the inverse ratio of their resistances. Even though the leakage current is small, it does increase the temperatures of the refractories through which it passes and thus increases conductivity. If the increasing temperature (conductivity) of a refractory part is not stabilized, failure will occur. Unfortunately, in the early years, electrodes were often installed with no design changes to make the units satisfactory for electric boosting. In many cases leakage currents went out of control. The results included erratic currents, hot spots, melting of refractories due to local arcing, and catastrophic failures including 100% glass drain. In the modern boosted furnaces these types of failures do not occur as refractories with high electrical resistivities are used. The importance of high electrical resistivities of refractories is shown by the work of Smith [10].

### 1.2.3 Leakage current in a longitudinally fired melt end

Siebold [1] discusses a regenerative, electrically boosted melter with longitudinal firing between three rows of electrodes inserted through the bottom blocks. He estimated that for one of the two similar rear sets, the voltage would be 140 V with 700 A. Again if the resistance to ground was  $5 \Omega$ , the leakage current would be  $I_{\text{leakage}} = 70/5 = 14 \text{ A}$  or about  $(14/700) 100\% = 2\%$  for this circuit.

## 1.3 Ground currents

In contradistinction to leakage currents which pass directly through the refractories and the ionized gases for each electrode pair of a circuit, other unwanted, electric currents may flow in the unit and are called ground currents and identified as  $I_{\text{grd}}$  (figure 1). Ground currents may be quite large. They pass directly from molten glass through grounding electrode(s), or improperly electrically isolated thermocouple sheaths or shears used for gobbing or other highly conductive materials which contact the molten glass, etc.

### 1.3.1 Ground current in a cross-fired melter

Let the melter be completely symmetrical. A grounding electrode installed from the binding steel into the glass on the melter's centerline would contact a zero equipotential plane and no current would flow. However, if the melter was not totally symmetrical, the contact would be on a nonzero equipotential plane and consequently would force a shift in the equipotential planes and currents. Concomitantly part of the leakage current would be diverted back into the glass through the grounding electrode. This ground current would flow in-parallel with the rest of the leakage current on its way toward the other electrode and the ratio of these two currents would be inversely equal to the ratio of their resistances. The ground current would be approximately the same as the leakage current calculated above in section 1.2.2.

Note that if the zero equipotential plane is not centered in the delivery system, contacting the glass at the orifice will result in an electric shock. A grounding electrode effectively shields personnel working on the delivery system from electric shocks [11].

### 1.3.2 Ground current in a longitudinally fired melt end

Again Siebold [1] discusses a regenerative, electrically boosted melter with longitudinal firing between three rows of electrodes. He estimated that for one of the two similar rear sets, the average resistance of the glass would be about  $0.2 \Omega$  and require about 140 V and approximately 700 A. For the forward set he estimated the resistance would be about  $0.08 \Omega$  for a current of about 1340 A and with a voltage of about 100 V. For this type of melter, a grounding electrode may be installed on the centerline near the throat between the front end and the delivery system. Again note that such an installation effectively shields personnel working on the delivery system from electric shocks. The current from this grounding electrode could be approximately equal to the driving voltage divided by the assumed resistance of  $5 \Omega$  for the ground path. The driving voltage,  $V_{\text{drive}}$ , (as will be explained in section 4.) is the phasor sum of the two applied voltages. Thus,

$$V_{\text{drive}} = 140 \text{ V } (\delta = 0^\circ) + 100 \text{ V } (\delta = 120^\circ)$$

or

$$V_{\text{drive}} = 125 \text{ V } (\delta = 44^\circ).$$

Again if the resistance to ground is  $5 \Omega$ , the magnitude of the ground current would be  $I_{\text{grd}} = 125/5 = 25 \text{ A}$  which also is small compared to the total heating current (upward of 2000 A).

This subject will be discussed further in section 8.1.

## 2. Cross-fired, symmetrically built and operated horizontal melters

### 2.1 With phases A and B

Figure 1 represents a boosted furnace with side-to-side symmetry including both structure and operating conditions. It has two sets of side wall electrodes in the melt end. It may be powered using any combination of a 3- $\phi$  power supply. In this report the three voltage phasors A, B and C always have the directions shown in figure 2a and, with clockwise rotation, the angles between their positive directions must always be  $120^\circ$  while their magnitudes and couplings depend entirely on the application. For the first case electrodes no. 1 and 2 are connected to phase A and the voltage between them has been adjusted to  $V_A$ . From symmetry the voltages from electrodes no. 1 and 2 to the central plane of the melter are equal to  $1/2 V_A$ . Phasor A represents voltage  $V_A$  and is horizontal with a magnitude equal to  $V_A$ . As connected the phasing requires that the tail of this phasor is identified with electrode no. 1 while its head is associated with electrode no. 2 (figure 2b). Electrodes no. 3 and 4 are connected to phase B with a voltage difference adjusted to  $V_B$ . This voltage is represented by phasor B whose tail is associated with electrode no. 3 while its head has electrode no. 4 attached to it. Phasor B has a magnitude equal to  $V_B$  and must retain its angular position, so phasors A and B must remain  $120^\circ$  apart. Again from symmetry it follows that the voltages from electrodes no. 3 and 4 to the centerline must be equal to  $1/2 V_B$ . Thus, the voltage at the central plane of the melter corresponds to the mid-points (CP) of phasors A and B and the central plane is a phantom or virtual ground. Electrodes no. 1 and 3 influence each other in the same way as electrodes no. 2 and 4. This means that the voltage differences between the adjacent electrodes of the two sets must be equal. Thus, it follows that phasors A and B must bisect each other. Phasor diagram (figure 2b) displays the electrical symmetry of this cross-fired, totally symmetrical unit. If the actual electrical distribution from a production unit does not agree with the predicted values, the phasor diagram may be wrong, the physical structure or operating conditions may be asymmetric or by accident or misunderstanding the wiring is incorrect.

In figure 2b the dashed line from point 1 to point 3 represents the voltage difference between electrodes no. 1 and 3. A similar statement holds true for points 2 and 4. These two lines are labeled  $V_{(1-3)}$  and  $V_{(2-4)}$ , have the same magnitude and direction but have different spacial locations. They are two different phasors representing

the two voltages between the adjacent wall electrodes on the opposite sides of the tank. These two voltages,  $V_{(1-3)}$  and  $V_{(2-4)}$ , along with their associated resistances control the local heating between the adjacent electrodes. To minimize the corrosion at the walls between the electrodes, it is desirable to have these voltages as low as possible. If figure 2b has been drawn to scale, these voltages can be determined by direct measurement; otherwise they can be calculated using the trigonometric cosine law. Thus,

$$V_{(1-3)}^2 = V_{(2-4)}^2 = (1/2 V_A)^2 + (1/2 V_B)^2 - 2 [1/2 V_A] [1/2 V_B] \cos 120^\circ. \quad (1)$$

Assuming  $V_A = 220 \text{ V}$  and  $V_B = 100 \text{ V}$ , this equation reduces to 132 V. Incredible as it may seem, this means that the voltage between the adjacent wall electrodes is greater than the voltage applied between electrodes no. 3 and 4.

This phasor diagram also displays the two phasors (dashed arrows) associated with the "cross voltage" from electrode no. 4 to no. 1 and from no. 3 to no. 2. For these "cross voltages" the cosine law is

$$V_{(1-4)}^2 = V_{(3-2)}^2 = (1/2 V_A)^2 + (1/2 V_B)^2 - 2 [1/2 V_A] [1/2 V_B] \cos 60^\circ \quad (2)$$

and with the previous values these voltages are 87 V which is significantly smaller than the voltage between the adjacent electrodes.

Further examination of this figure suggests that the voltage between adjacent electrodes can be reduced.

### 2.2 Leads to one phase reversed

Interchanging the leads in one set can not change the magnitude or direction of the voltage phasors as these are controlled by the power supplies. As corresponding electrodes influence each other similarly, the phasors must still bisect each other. This can be inferred from figure 2b, but the effects of interchanging leads of phase B to electrodes no. 3 and 4 are shown in figure 2c, in which the positions of the electrodes no. 3 and 4 have been interchanged from those shown in figure 2b. From an examination of this new figure or by calculation with the cosine law, it is clear that the side wall voltages have been decreased from 132 to 87 V. Thus, this wiring will result in a longer tank life and is preferable.

### 2.3 With phases A and C

For this melter, using phase C instead of phase B changes only the relative phase angles and does not result in different voltages. This is apparent as phase C only rotates the effects of phase B through  $120^\circ$ . Another way of arriving at the same conclusion is to draw a new diagram (figure 2d) or to consider phase C to be the mirror image of phase B across the phase A direction.

The substitution of phases is an important consideration when loads are to be distributed in a 3- $\phi$  system.

## 2.4 With single-phase circuits

(For simplicity from this section through section 2.5.2 the rear circuit is identified as no. 1 while the other is called circuit no. 2.) Further examination of these phasor diagrams indicates that, if single-phase heating is used, the two applied phasors (not shown) become collinear and still "bisect" each other. For this single-phase case let  $V_1$  be the voltage for the rear circuit and use  $V_2$  for the voltage for the other or forward circuit.

As these two voltages are in-phase, the voltage between the adjacent electrodes becomes  $1/2 (V_{(1-2)} - V_{(3-4)})$ . For this case with 200 and 100 V applied to the two sets, this voltage would be 50 V. Reversing a set of leads would change the voltages to  $1/2 (V_{(1-2)} + V_{(3-4)})$  or with the previous values to 150 V. Therefore, properly connected single-phase heating results in the lowest voltage between adjacent electrodes. Single-phase heating may be distributed between several large melters and is usually used for small units and in delivery systems.

## 2.5 Current and voltage relations

A salt water model for the melt end of the furnace shown in figure 1 was used. For simplicity the rear circuit is still identified as no. 1 while the other is called circuit no. 2. The voltages and currents are  $V_1$ ,  $V_2$ , and  $I_1$ ,  $I_2$ , respectively.

### 2.5.1 Model with single phase connections

For the two cross-fired circuits in the melt end, corresponding currents and voltages remained in-phase throughout an extended normal operating range. Keeping the voltage of circuit no. 1 constant at  $V_1$  and reducing the second circuit's voltage ( $V_2$ ) caused the currents in both circuits to decrease very linearly until the second circuit's current ( $I_2$ ) reached zero. With further decreases in the  $V_2$  voltage, this current then went through a 180° phase reversal and both currents now were non-linear. Thus, both currents were linear and non-linear simultaneously. Figure 3a shows some typical data. For this model, voltages could be adjusted to force either current to zero and each line current could be expressed as a linear combination of the line voltages and vice versa. A typical equation of this set is

$$I = N_1 V_1 + N_2 V_2. \quad (3)$$

In this equation  $N_1$  and  $N_2$  are constants depending on the melter, including its operating conditions, and scalar addition is valid.

Similar results were obtained when  $V_2$  was kept constant and  $V_1$  was changed.

### 2.5.2 Model with phases A and B

For this case the second circuit was reconnected to a different phase. The voltage and current relations now were somewhat different as neither current could be forced to zero. Again both currents were linear in the normal operating range and then became non-linear simultaneously (figure 3b). Simultaneous linearity of both currents indicates that changes in the internal impedances of the transformers were unimportant in that region. In addition, the linearity indicates that the superposition principle holds and that each line current phasor may be assumed to be a linear combination of the line voltage phasors and vice versa. Thus, equation (3) and its related or associated equations are also valid for this case provided the voltages and currents are in the linear region and are treated as phasors. Again the roles of the voltages were interchangeable. Other comments on the electrical characteristic for figures 3 a and b may be found in section 8.2.

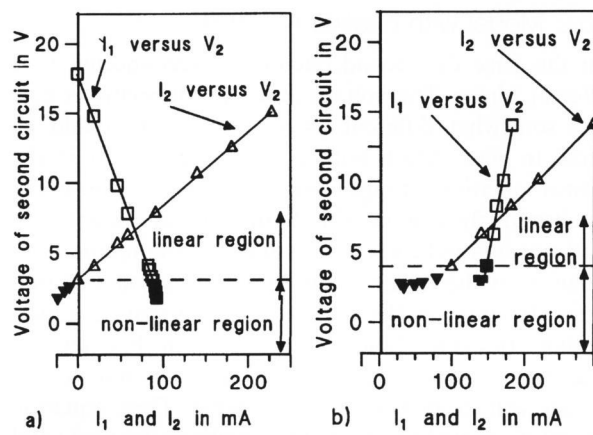
## 3. Cross-fired melter including a delivery system

If a delivery system heated with fossil fuel and/or cross-fired is added to this furnace (figure 1), the phasor diagrams prepared for the melt end remain unchanged. Similar diagrams are also valid for the corresponding circuits used in the delivery systems. The preceding two statements will now be discussed.

Assume that all electric power is disconnected and then (with arbitrarily chosen voltages) connect electrodes no. 5 and 6 to the secondary of a transformer for cross firing with phase A. As the furnace is symmetric the voltages from its centerline to electrodes no. 5 and 6 are equal. Thus, the centerline (plane) of the melter is associated with the center point of the voltage phasor for this circuit. This phasor is horizontal, has the correct magnitude and has electrode no. 5 at its tail and electrode no. 6 at its head (figure 4). Next connect phase C from another secondary to forehearth electrodes no. 7 and 8. The phasor for this circuit will have electrode no. 7 at its tail and electrode no. 8 at its head to conform to the relative polarities shown in figure 1. Again, because of symmetry and since these four electrodes influence each other in the same way, these two phasors must bisect one another. Using the same reasoning allows to complete figure 4.

Now with all the circuits of this melter energized, all corresponding phasors must remain collinear and the complete or final voltage phasor diagram(s) (not shown) must have a common central point as each part is associated with the neutral, central line of the melter. The voltages between all the electrodes may be found using the equations discussed earlier even though the melt end and delivery end sets of electrodes are at opposite ends of the melter.

With this setup the central point of the orifice has no voltage to ground and if any ground current is present it is negligible. In general, adding additional pairs of uni-



Figures 3a and b. Cross-fired, electric currents versus voltages of the second circuit with the voltage of the first circuit kept fixed at 15.0 V; a) voltage of second circuit versus single phase, cross-fired currents of both circuits with  $V_{1,\delta_1} = 15.0$  V ( $\delta_1 = 0^\circ$ ); b) same as figure a except a two-phase supply was used with  $V_{1,\delta_1} = 15.0$  V ( $\delta_1 = 120^\circ$ ).

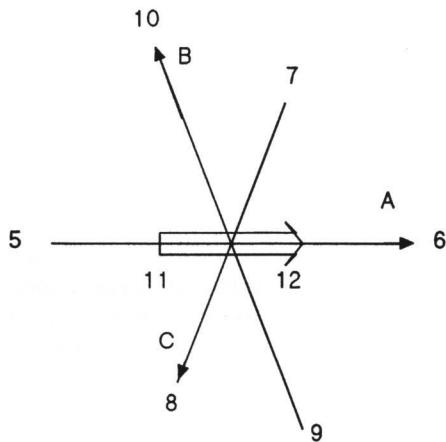


Figure 4. Voltage phasor diagram for a cross-fired front end and delivery system with melt end power disconnected.

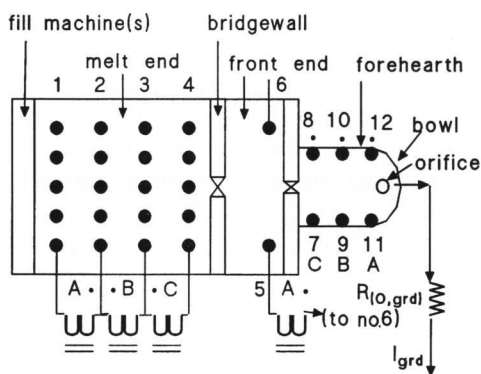


Figure 5. Horizontal melter with melt end fired longitudinally: cross-fired front end and delivery system.

formly displaced side wall electrodes to the melt end and/or to the delivery system will result in similar conclusions. In a symmetrically built and operated unit,

with the cross-fired circuits already discussed, no ground current will flow even if a grounding electrode or any conductor is inserted anywhere along the bottom centerline of the furnace.

This statement will be true even when metallic stirrers [12] are used to control the molten glass flow from a centrally located orifice such as that shown in figure 1. However, if for any reason, the grounding electrode or a conductor does not contact the glass at a zero equipotential plane or line, then an electric current will flow through it.

#### 4. Phasors for a longitudinally fired furnace with delivery system

##### 4.1 Phases A, B and C

Consider a boosted melter with four rows of electrodes arranged parallel to the rear wall. These electrodes can be fired longitudinally. This unit also has a front end and delivery system composed of a forehearth and bowl whose parts can be cross-fired with any desired voltages (figure 5). The phasor diagram for this delivery system when it is powered alone has been discussed in section 3. and shown in figure 4. With this setup the center point of the phasor diagram is associated with the centerline of the delivery system.

Now disconnect these circuits and connect rows no. 1 and 2 in the melter to phase A of a separate secondary adjusted to  $V_A$ . The phasor A for the electrodes of row no. 1 to no. 2 will be labeled and drawn horizontal pointing to the right. Next connect rows no. 2 and 3 to phase B as shown in figure 5 and adjust to  $V_B$ , Phasor B for rows no. 2 to 3 makes an angle of  $120^\circ$  with phasor A. The phasing (polarity connections) shown in figure 5 requires that the head of phasor B be attached to the head of phasor A. This must be true as their secondaries are tied together. Thus, phasor B's tail corresponds to the voltage at the third row of electrodes. Now connect phase C, adjusted  $V_C$ , to rows no. 3 and 4. Again from figure 5 the head of phasor C corresponds to the voltage at the third row and must be attached to the tail of phasor B and point in the proper direction. The tail of phasor C corresponds to the voltage at the fourth row of electrodes. This information is summarized in figure 6 by the phasors A, B and C.

Line current at the second row of electrodes results from the phasor sum of phase currents from two different transformer secondaries and thus is not in-phase with either voltage. A similar statement holds true for the third row of electrodes.

In figure 6 the dashed line drawn from the tail of phasor A to the tail of phasor C, indicates the magnitude of the voltage between the first and the fourth row of electrodes. Its phasor,  $V_{(1-4)}$ , is the resultant of the three applied voltages and makes an angle of  $\theta$  with phasor A and it will vary as the applied voltages in the melting zone are changed. This resultant voltage can be found directly from a scaled drawing, otherwise it can

be calculated. For example let the magnitudes of the phasors A or  $V_{(1-2)}$ , B or  $V_{(2-3)}$  and C or  $V_{(3-4)}$  be 100, 50 and 100 V, respectively. Their projections on the X axis are +100, +25 and +50 V, respectively, with an algebraic sum equal to 175 V. This sum is the X component of  $V_{(1-4)}$  and is called  $R_X$ . In a similar fashion  $R_Y$ , the Y component of  $V_{(1-4)}$ , results from the algebraic sum of the projections of the three applied voltage phasors onto the Y axis. These projections are 0,  $-50 \sin 60$ , and  $+100 \sin 60$  whose sum is equal to 43.3 V. From these projections the voltage between electrodes no. 1 and 4 equals  $[175^2 + 43.3^2]^{1/2} = 180.3$  V and the phase angle is  $\theta = \arctan(43.3/175.0) = 13.9^\circ$ . These numbers can be found from figure 6, provided it is drawn to scale.

The voltage of phasor  $V_{(1-4)}$  drives a ground current,  $I_{\text{grd}}$ . This is assumed to flow from the fourth row of electrodes through the intervening glass to the delivery end of the furnace, to ground, then, for the return path, from ground through the fill machine(s) and the rear end of the furnace and finally back through the molten glass to the first row of electrodes. This ground current is independent of the leakage current(s) and the main electric currents flowing between the electrodes. It presumably spreads and contracts crosswise as it moves longitudinally through the glass and its effective path length is lengthwise – not crosswise – through the system. There is a voltage drop between the midpoint of the fourth row of electrodes to the centerline between electrodes no. 5 and 6; call it  $V_{4,F}$ . Call the effective resistance for this path  $R_{4,F}$ . Though molten glass is an electrolytic conductor, it behaves like an ohmic resistor. This means a voltage and its resulting current are in-phase and Ohm's law holds. Thus,  $V_{4,F} = I_{\text{grd}} R_{4,F}$ . Next let  $R_{F,1}$  be the effective resistances from the center point between electrodes no. 5 and 6 to the center point between the first pair of electrodes no. 7–8; from pair no. 7–8 to pair no. 9–10 use  $R_{F,2}$ ; from pair no. 9–10 to pair no. 11–12 use  $R_{F,3}$ ; from pair no. 11–12 to the orifice use  $R_{F,4}$ ; call the resistance between the orifice and ground  $R_{O,\text{grd}}$ . So the voltage between the orifice and ground is by Ohm's law  $V_{O,\text{grd}} = I_{\text{grd}} R_{O,\text{grd}}$ .  $V_{O,\text{grd}}$  is undesirable and may be exceedingly high and dangerous. For the rest of the assumed return path the resistances from ground to fill machine(s), use  $R_{\text{grd},M}$ ; from the fill machine to the hot conductive material at the rear of the melter, use  $R_{M,E}$ ; and then from the rear end to the first row of electrodes, use  $R_{E,1}$ . As  $V_{(1-4)}$  is in-phase (collinear) with  $I_{\text{grd}}$  and with Kirchhoff's voltage law,

$$V_{(1-4)} = I_{\text{grd}} \{R_{4,F} + R_{F,1} + R_{F,2} + R_{F,3} + R_{F,4} + R_{O,\text{grd}} + R_{\text{grd},M} + R_{M,E} + R_{E,1}\} \quad (4)$$

is obtained.

Under steady operating conditions  $V_{(1-4)}$  is constant but  $R_{M,E}$  varies as arcing at the fill machine(s), including blanket feeders, to the hot material in the melter changes. In addition,  $R_{O,\text{grd}}$  is highly variable when gobs are being cut with grounded shears. Of course these

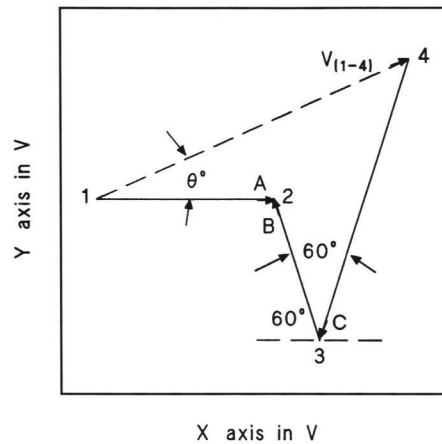


Figure 6. Voltage phasors for melt end of the furnace shown in figure 5, including resultant voltage phasor  $V_{(1-4)}$ , with front end and delivery system power disconnected.

changes cause  $I_{\text{grd}}$  and all the voltage drops in equation (4) to vary. Equation (4) is represented graphically in figure 6 where the line  $V_{(1-4)}$  represents the driving voltage and also the scalar sum of the pertinent voltage drops.

Now the phasors for the front end and the delivery system are located when the melt end circuits are also powered. As long as the capped-up condition does not allow a ground current, the delivery end phasors remain undispersed and lie centered on phasor  $V_{(1-4)}$  but closer to its head than tail.

When glass flows, it is clear that the phasors in figure 4 must now be dispersed by the ground current as successive phasors are separated by the voltage drop between their respective circuits. Now on line  $V_{(1-4)}$  in figure 6 (redrawn in figure 7 for clarity) move from point 4 towards point 1 a distance equal to  $V_{4,F}$ . This is the location for phasor  $A_{(5-6)}$  which is drawn centered and horizontal at this point. Continue in the same way until all the phasors associated with the return path for the ground current are located. By Kirchhoff's law these voltage drops completely fill line  $V_{(1-4)}$  in figure 7. For this figure the fill machine has arbitrarily been chosen to be closer to ground than any other point. However, this is not always true. If several points in a furnace have low resistance paths to ground then each will have a ground current. These flow in-parallel. However, only one ground point can exist on a phasor diagram. It will indicate the common point at which the separate ground currents flowing in-parallel must meet and from which they may diverge.

Ground currents are real. In production units their presence has been disclosed by sparks at grounded shears when gobs of glass were being cut and/or by arcs from the fill machines to the conducting material in the rear of the melter. From these observations it is clear that the resistance to ground may be very variable and so getting reliable data to construct a phasor diagram

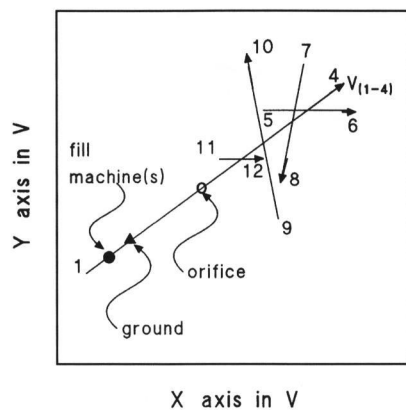


Figure 7. Resultant voltage phasor  $V_{(1-4)}$  with front end and delivery system phasors distributed along its length according to the voltage drops.

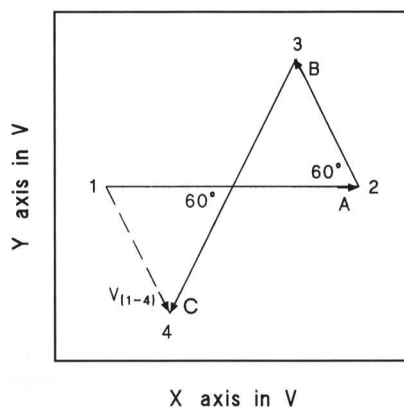


Figure 8. Revised melt end voltage phasor diagram with reduced  $V_{(1-4)}$  resulting from a revised wiring of the melt end, other power supplies disconnected.

similar to that shown in figure 7 may be very difficult. However, ground current influence can be illustrated with a simple salt water model (section 8.1).

#### 4.2 Revised phase connection, melt end

Ground currents can be reduced or even eliminated by rephasing the melter. Suppose the leads for phase B between rows no. 2 and 3 and also for phase C between rows no. 3 and 4 are interchanged. With the same voltages the new phasor diagram (figure 8) has a much smaller resultant  $V_{(1-4)}$ . Proceeding as above the new values for the X and Y projections are  $R_X = 25$  V and  $R_Y = -43.3$  V, respectively. Therefore, the resultant voltage for  $V_{(1-4)}$  is  $[25^2 + 43.3^2]^{1/2} = 50.0$  V and  $\theta = \arctan(-43.3/25) = -60.0^\circ$ . Thus, for this particular example, there has been a significant change in the phase angle but more importantly the voltage for  $V_{(1-4)}$  has been reduced from 180.3 to 50 V so the ground current is significantly smaller and the resulting voltage at the orifice will be low enough to be considered acceptable. (Other ways to achieve the same result include interchanging the leads for phase A in the melt end.)

#### 4.3 Voltage from the orifice to ground, eliminated

In practice the hazardous voltage in the delivery system and the current from orifice to ground are often eliminated by inserting a grounding electrode at or near the throat(s) at the bridgewall of the melter or near the entrance to the forehearth. This grounding electrode does not eliminate the ground current from the fill end. In fact it should increase it.

Various other possible phasor diagrams for the melt end can also be used to design circuits for ground current reduction or elimination.

### 5. Summary, comments and conclusions

In general, leakage currents flow from the hot, interior surface of a melter through its refractories and ionized vapors to its binding steel so that the resistance between them is low.

Phasor diagrams for vertical glass melters, exclusive of delivery systems were treated in an earlier report [8] but some of the results reported therein are also valid for horizontal melters. For example, the horizontal melters can be widened, lengthened, deepened and different electrode spacing and immersions may be used without causing any changes in the voltage phasor diagrams. On the other hand, the work on delivery systems described herein is also applicable to vertical glass melters<sup>3)</sup>.

Phasor diagrams have been constructed for some circuits that can be used with horizontal melters. These diagrams present an easily understandable summary of the voltages that occur between all the electrodes. Some of these diagrams may disclose wiring errors and/or undesirable voltage distributions and may suggest ways to improve the electric characteristics. One may conclude that reversing a phase in a cross-fired system or properly substituting a new phase may cause a significant reduction in the voltage between the adjacent wall electrodes. The phasor diagram was quite powerful in leading to these conclusions. Phasor diagrams can also explain why ground currents occur and how to minimize them.

Salt water modelling of a melter with two single-phase, cross-fired circuits has shown that the electric currents are linear over a large range and each can be forced to zero with different voltage combinations. In addition, the currents can become non-linear simultaneously. In the linear range and for single-phase circuits, each line current is a scalar function of the applied voltages and vice versa. With two-phase circuits and again in the linear range, the same equations are true provided the variables are treated as phasor quantities.

<sup>3)</sup> In one large vertical electric melter, on "come-in" the initial voltage from an orifice to ground was 800 V. The first reaction of the construction engineers was that this voltage was due to an accidental ground. A phasor diagram disproved this and led to "rephasing" the circuits so that the resultant voltage was acceptable.



A ground current was shown with a single-phase, longitudinally fired melt end, salt water model. Data from it were also used to show how the phasors for its melt end were related to those for its cross-fired delivery system.

In practice, the single electrodes shown for the cross-fired circuits are often replaced by sets of parallel connected electrodes and the voltage phasor diagrams remain unchanged. For all cases when parallel connected electrodes are used, current balancers [13] may be used to help in equalizing the currents in a set.

All conclusions for longitudinally fired melt ends are also valid for longitudinally fired, horizontal or vertical channels.

## 6. Nomenclature

### 6.1. Symbols

A, B, C	phasors
CP	center point
grd	ground
$I$	current
$N$	constant
$R$	resistance
$V$	voltage
$Z$	impedance

$\delta, \theta$  phase angles

### 6.2 Subscripts

A, B, C	phases
E,1	from conducting material at rear end of melter to the first row of electrodes
F,1	from midpoint between electrodes no. 5 and 6 to the midpoint between electrodes no. 7 and 8
F,2	from midpoint between electrodes no. 7 and 8 to the midpoint between electrodes no. 9 and 10
F,3	from midpoint between electrodes no. 9 and 10 to the midpoint between electrodes no. 11 and 12
F,4	from midpoint between electrodes no. 11 and 12 to the center of the orifice
grd	ground
grd,M	from ground to the fill machine(s)
(i-j)	from electrode i to electrode j (= 1 through 12)
M	fill machine
M,E	from fill machine(s) to conducting material at rear end of melter
O	orifice
O,grd	from ground to the fill machine(s)
1, 2	circuit number
4,F	from midpoint of the fourth row of electrodes to the midpoint between electrodes no. 5 and 6

$\delta, \theta$  phase angles

\*

The author wishes to thank Corning Inc., Corning, NY (USA), for permission to publish this material. Thanks are also extended to R. W. Palmquist, W. E. Siebold, R. E. Miller, R. V. Shafer, Myron A. Smith, J. H. Bruns, R. A. Dreifuss and to G. B. Collins for their helpful discussions on leakage and ground currents.

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## 8. Appendices

### 8.1 Ground current and voltage distributions from a longitudinally fired melt end model

A ground current was produced with a very simple salt water model which was based on figure 5. The model's melt end had two rows of electrodes labeled no. 1 and 2. In its delivery end only electrodes no. 11 and 12 were installed. A resistor,  $R_{O,grd}$ , contacted the fluid between these two bowl electrodes and ground and another resistor,  $R_{grd,M}$ , was used to simulate the resistance from ground through the fill machine and into the fluid at the melt end. A single phase was adequate to demonstrate the ground current and avoided the complications of multiphase power. The structure was non-conductive and its lack of leakage currents was considered unimportant. To study the voltage distribution a sensitive, high-impedance voltmeter was connected between the leads of the first set or ground and a probe which could be immersed in the fluid.

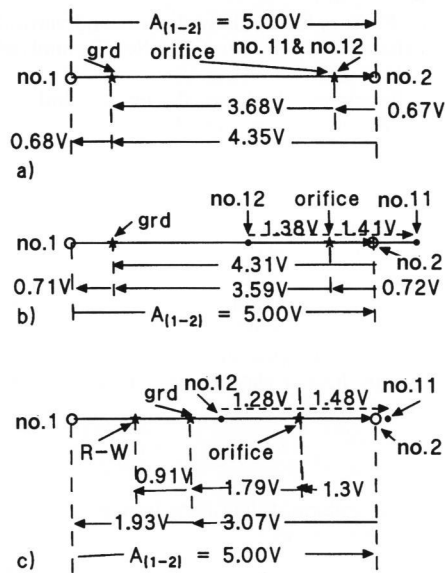
To simulate a capped-up condition, the connection from the orifice to ground was removed. In the first experiment the bowl electrodes were disconnected from the power supply. The probe in contact with any of the first row electrodes read zero. It was moved from the center rear of an electrode towards the rear wall, then left to a line midway between this electrode and its neighbor, then along this line back between these two first row electrodes and on to and past the corresponding two electrodes of the next set. During this motion the probe's voltage increased steadily. When further forward and/or sideways motion would produced practically no further changes, the probe was moved to the right and back to the center of the second row electrode. During this motion the probe's voltage continued to increase until it became equal to that impressed on the two rows of electrodes. This motion followed a tube of electric current between the two sets of electrodes. As there was no ground current, the

Table 1. Electrical relations for the linear regions of figures 3a and b

for figure 3a, $V_{1,\delta_1}$ : 15.0 V with $\delta_1 = 0^\circ$	for figure 3b, $V_{1,\delta_1}$ : 15.0 V with $\delta_1 = 120^\circ$
(1) $V_1/I_1 = 15.0/0.067 = 223.9 \Omega$	(1) $V_1/I_1 = 15.0/0.188 = 79.8 \Omega$
(1) $V_2/I_2 = 15.0/0.198 = 75.8 \Omega$	(1) $V_2/I_2 = 15.0/0.219 = 47.8 \Omega$
(2) $V_1/I_1 = 15.0/0.0948 = 158.2 \Omega$	(2) $V_1/I_1 = 15.0/0.170 = 88.8 \Omega$
(2) $V_2/I_2 = 10.0/0.103 = 97.5 \Omega$	(2) $V_2/I_2 = 10.0/0.219 = 45.7 \Omega$
(3) $\partial I_2/\partial V_2 = 18.8 \text{ mA/V}$	(5) $\partial I_2/\partial V_2 = 19.96 \text{ mA}$
(4) $\partial I_1/\partial V_2 = -5.6 \text{ mA/V}$	(6) $\partial I_1/\partial V_2 = 3.61 \text{ mA}$

Explanations:

- (1):  $V_1 = V_2 = 15.0 \text{ V}$ ; (2):  $V_1 = 15.0 \text{ V}$ ,  $V_2 = 10.0 \text{ V}$ ;
- (3): By partial differentiation of  $V_2 = 0.053225(I_2) + 4.5415$ ;
- (4): By partial differentiation of  $V_2 = 0.277061(I_1) - 37.1797$ ;
- (5): By partial differentiation of  $V_2 = 0.050112(I_2) - 0.0981058$ ;
- (6): By partial differentiation of  $V_2 = 0.277061(I_2) - 37.1797$ ;



Figures 9a to c. Voltage distributions from a longitudinally fired, melt end salt water model. Electrodes of row no. 1 and 2 are indicated by circles with their phasor  $A_{(1-2)}$  between them. Bowl electrodes no. 11 and 12 are shown as dots and the phasors for them are collinear with  $A_{(1-2)}$  but are displaced for clarity; a) location of ground and orifice with only melt end voltage. Unit is in a capped-up condition and no power is on in the delivery system; b) same conditions except bowl electrodes are powered; c) same conditions as figure b except  $R_{O,grd}$  was connected to represent an uncapped furnace.

probe's voltage at the rear wall was the same as if it had been connected to ground. A representative phasor diagram is shown in figure 9a. Note that both the ground and orifice positions are between but not at the ends of the phasor,  $A_{(1-2)}$ .

Next electrodes no. 11 and 12 were powered but  $R_{O,grd}$  was left unconnected, so a capped-up condition was simulated. A representative diagram (figure 9b) shows that the orifice's position has changed only by a small amount and that the voltage phasors for the bowl electrodes are centered on it and are collinear with  $A_{(1-2)}$ .

Installing  $R_{O,grd}$  represents a flowing glass condition. In figure 9c ground is close to the rear wall (R-W), as  $R_{M,E} = 50 \Omega$ , while  $R_{O,grd}$  was  $= 190 \Omega$ . In this figure R-W represents the hot, inside rear wall which no longer is at ground potential. Further note that the orifice's location is now closer to ground and that the bowl electrode phasors are still centered on it.

As all the voltages in the fluid are shown in the phasor diagram it is clear that all the glass in horizontal or boosted electric melters has a high voltage to ground and suitable precautions must be taken when it is contacted.

### 8.2 Additional comments on the electrical characteristics for figures 3a and b

#### 8.2.1 Comments for figure 3a

The experiment was started with both voltages,  $V_{1,\delta_1}$  and  $V_{1,\delta_2}$  adjusted to 15.0 V and with  $\delta_1 = \delta_2 = 0^\circ$ . Then  $V_{1,\delta_1}$  was kept constant at 15.0 V with  $\delta_1 = 0^\circ$  while  $V_2$  was decreased. The scalar values of the line currents,  $I_1$  and  $I_2$  were measured using 1  $\Omega$  precision resistors in each supply line. For figure 3a  $V_1$ ,  $V_2$ ,  $I_1$  and  $I_2$  remained in-phase in the linear region. With each circuit at 15.0 V, the applied voltage divided by the measured line current is the impedance and in this case the circuit (cct) resistance. Thus,

$$(V_1/I_1)_{V_1, V_2=15V} = R'_{cct,1} = 15.0/0.067 = 224 \Omega \quad (5)$$

while that of the second circuit was

$$(V_2/I_2)_{V_1, V_2=15V} = R'_{cct,2} = 15.0/0.198 = 75.8 \Omega. \quad (6)$$

(These values and others are listed in table 1.)

The ratio of these two resistances is  $(224/75.8) = 3.0$ . This ratio shows that the circuit no. 1 was greatly constricted by the circuit no. 2.

Next  $V_2$  was reduced to 10.0 V, and the line current,  $I_1$ , changed to 0.0948 A. The resistance of the circuit no. 1 decreased to

$$(V_1/I_1)_{V_1=15; V_2=10V} = R_{cct,1} = 15.0/0.0948 = 158.2 \Omega \quad (7)$$

while that of the circuit no. 2 increased to

$$(V_2/I_2)_{V_1=15; V_2=10V} = R_{cct,2} = 10.0/0.103 = 97.5 \Omega \quad (8)$$

and the ratio of the circuit resistances decreased from 3.0 to  $(158.2/97.5) = 1.6$ . These data illustrate that the circuit resistances are very voltage-sensitive and that the voltage ratios allocate the space for each circuit.

The least square fit of the data for  $I_2$  versus  $V_2$  in the linear range was

$$V_2 = 0.053225(I_2) + 4.5415 \quad (9)$$

with a correlation coefficient of 0.999644 indicating an almost perfect fit of the data to a straight line. The rate of change of  $I_2$  versus  $V_2$  with  $V_1$  kept constant was obtained by the partial differentiation of equation (9). Thus, with a little manipulation

$$(\partial I_2/\partial V_2)_{V_1} = 18.8 \text{ mA/V}. \quad (10)$$

The least square fit of the data for  $I_1$  versus  $V_2$  in the linear range was

$$V_2 = -0.177356(I_1) + 26.8060 \quad (11)$$

with a correlation coefficient of 0.999613 and by partial differentiation

$$(\partial I_1 / \partial V_2)_{V_1} = -5.64 \text{ mA/V}. \quad (12)$$

This interaction  $(-5.64/18.8) = -30\%$  was surprisingly large.

### 8.2.2 Comments for figures 3b

The experiment was continued with both  $V_{1,\delta_1}$  and  $V_{2,\delta_2}$  again adjusted to 15.0 V but with  $\delta_1 = 120^\circ$  and  $\delta_2 \equiv 0^\circ$ . The current phasors were again bilinear functions of the line voltage phasors and in this case equation (3) shows the functional relationship in a clearer fashion when rewritten as

$$I_\theta = N_1 V_{1,\delta_1=120^\circ} + N_2 V_{2,\delta_2=0^\circ}. \quad (13)$$

From this equation it is clear that the current phasors can not be collinear with either voltage phasor as long as both power supplies are being used with different phases.

For the remaining steps,  $V_{1,\delta_1}$  was kept constant at 15.0 V with  $\delta_1 = 120^\circ$  while  $V_2$  was decreased. Even though the voltage and current phasors are not collinear, the ratios of their magnitudes are of interest and will be denoted by  $Z_{\text{cct},m}$  which is the impedance (apparent circuit resistance) of circuit  $m$ . With each circuit at 15.0 V, the applied voltage divided by the measured line currents were

$$(V_1/I_1)_{V_1,V_2=15V} = Z_{\text{cct},1} = 15.0/0.1883 = 79.8 \Omega \quad (14)$$

while that of the second circuit was

$$(V_2/I_2)_{V_1,V_2=15V} = Z_{\text{cct},2} = 15.0/0.3189 = 47.0 \Omega. \quad (15)$$

(These values and others are tabulated in table 1.)

The ratio of these two "resistances" is  $(79.8/47.0) = 1.7$ . This ratio shows that the circuit no. 1 was greatly constricted by the circuit no. 2.

Next  $V_2$  was reduced to 10.0 V, and the line current,  $I_1$ , changed to 0.170 A. So the  $V/I$  ratio of the circuit no. 1 decreased to

$$(V_1/I_1)_{V_1=15;V_2=10V} = Z_{\text{cct},1} = 15.0/0.170 = 88.2 \Omega \quad (16)$$

while that of the circuit no. 2 scarcely changed to  $56.2 \Omega$

$$(V_2/I_2)_{V_1=15;V_2=10V} = Z_{\text{cct},2} = 10.0/0.178 = 56.2 \Omega \quad (17)$$

and the ratio remained essentially unchanged from 1.7 to  $(88/56) = 1.6$ . These data illustrate that the impedances are less sensitive to voltages when different phases are used.

The least square fit of the data for  $I_2$  versus  $V_2$  in the linear range was

$$V_2 = 0.050112(I_2) - 0.981058 \quad (18)$$

with a correlation coefficient of 0.999690 indicating an almost perfect fit of the data to a straight line. Partial differentiation of this equation yielded

$$\partial I_2 / \partial V_2 = 19.96 \text{ mA/V}. \quad (19)$$

The least square fit of the data for  $I_1$  versus  $V_2$  in the linear range was

$$V_2 = 0.277061(I_1) - 37.1797 \quad (20)$$

with a correlation coefficient of 0.994547. By partial differentiation

$$\partial I_1 / \partial V_2 = 3.6 \text{ mA/V}. \quad (21)$$

The interaction is  $(3.6 \text{ mA}/20.0 \text{ mA}) = 18\%$  is also surprisingly large but it is less than that for the single-phase case.

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