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# Some Problems in the Representation Theory of Simple Modular Lie Algebras

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CONTEMPORARY MATHEMATICS

652

# Lie Algebras and Related Topics

Workshop on Lie Algebras
in Honor of Helmut Strade's 70th Birthday
May 22–24, 2013
Università degli Studi di Milano-Bicocca, Milano, Italy

Marina Avitabile
Jörg Feldvoss
Thomas Weigel
Editors

American Mathematical Society

# Lie Algebras and Related Topics

Courtesy of Helmut Strade

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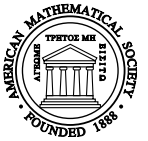


**American Mathematical Society**
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# Preface

The study of Lie groups goes back to the end of the nineteenth century when Sophus Lie investigated symmetry groups of differential equations. The latter can be studied locally by means of the Lie algebra structure on the tangent space in the identity element of the group. Later, an interest in algebraic groups emerged from studying problems in number theory and algebraic geometry. According to the algebraic nature of these objects, it was quite natural (and often also useful) to consider algebraic groups defined over fields of arbitrary characteristic, or even more generally, over arbitrary commutative rings. It turned out quite early that many of the classical results over the real or complex numbers and the correspondence between the group and its associated Lie algebra, which is essential in many of the proofs, are no longer valid over fields of non-zero characteristic. So modular Lie algebras (that is, Lie algebras over fields of prime characteristic) were studied in its own right and very soon several applications (for example, to purely inseparable field extensions or to the restricted Burnside problem) appeared.

This volume originates from a Workshop on Lie Algebras which took place at the Università degli Studi di Milano-Bicocca from May 22$^{\text{nd}}$ to May 24$^{\text{th}}$, 2013. The workshop featured invited talks by 13 mathematicians from around the world and was attended by 39 participants from 12 countries, including several graduate students. The aim of the workshop was to bring together some of the leading experts in the field of Lie (super)algebras (simple, locally finite, modular, or real), their gradings, the ideal structure of their universal enveloping algebras, as well as some computational aspects, and several related topics such as representation theory, $W$-algebras, algebraic groups, quantum groups, Weyl algebras, and the Lie structure of group algebras. Established researchers were joined by younger colleagues with the goal of promoting the exchange of ideas and establishing new collaborations.

Lie algebras are at the core of several areas of mathematics, for example, Lie groups, algebraic groups, quantum groups, representation theory, homogeneous spaces, integrable systems, and algebraic topology. The talks given at the workshop coverered a broad spectrum of these topics. The first part of these proceedings combines research papers with survey papers, all written by the invited speakers (alone or with co-authors). The second part consists of several collections of problems on modular Lie algebras, their representations, and the conjugacy of their nilpotent elements as well as the Koszulity of (restricted) Lie algebras and Lie properties of group algebras or restricted universal enveloping algebras.

The classification of finite-dimensional simple Lie algebras over an algebraically closed field of prime characteristic $p > 3$ has been one of the milestones in Lie theory. The workshop was dedicated to Helmut Strade, one of the main architects of

this extraordinary mathematical achievement. This volume will represent a snapshot of the state of the art of the post-classification period. Many applications concerning the structure of an arbitrary finite-dimensional modular Lie algebra require a detailed knowledge of the structure of the simple Lie algebras and their representations. In the proof of the classification many of the properties of modular simple Lie algebras were established. Since many details of the classification have only been published recently in the three books written by Helmut Strade, it is now possible to attack many open problems in the structure and representation theory of Lie algebras in prime characteristic. The same phenomenon has been observed after the classification of finite simple groups was completed.

This volume exhibits some of the progress in this vibrant area of mathematics. We especially hope that the open problems in the second part will inspire future research in Lie algebras and also in some related topics.

The organizers wish to thank the Dipartimento di Matematica e Applicazioni at the Università degli Studi di Milano-Bicocca for the hospitality, the logistical support and—last but not least—the financial funding. We are also very grateful to the editorial staff of *Contemporary Mathematics*, and in particular to Christine Thivierge for her patience and help in editing these proceedings. Our special thanks go to the anonymous referees for their insight and the high standards employed in writing their reports. Finally, we would like to take this opportunity to express our appreciation to the participants for making the workshop such a success, to the speakers for their very interesting talks, and especially to the authors for contributing to this volume.

Marina Avitabile, Jörg Feldvoss, and Thomas Weigel

## List of Participants

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*Group Gradings on Simple Lie Algebras of Cartan Type and Locally Finite Lie Algebras*

Alexander Baranov
*Simple Locally Finite Lie Algebras*

Georgia Benkart
*Algebras on the Weyl Side*

Willem de Graaf
*Computation with Semisimple Real Lie Algebras*

Alberto Elduque
*Some Simple Modular Lie Superalgebras*

Lutz Hille
*Parabolic Group Actions and Representation Theory*

Naihong Hu
*Loewy Filtration and Quantum de Rham Cohomology*

Sandro Mattarei
*Grading Switching*

Alexander Premet
*Multiplicity-Free Primitive Ideals and Finite $W$-Algebras*

Salvatore Siciliano
*Lie Solvable Enveloping Algebras*

Serge Skryabin
*Simple Reduced Enveloping Algebras*

Ernesto Spinelli
*On the Lie Subalgebra of Skew Elements of a Group Algebra*

Alexandre E. Zalesskii
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