revista PARANAENSE DE EDUCAÇÃO MATEMÁTICA

PROFESSIONAL LEARNING IN MOMENTS OF WHOLE-CLASS DISCUSSION IN A LESSON STUDY

DOI: https://doi.org/10.33871/22385800.2023.12.29.40-57

Thuysa Schlichting de Souza[1]
Gorete da Fonseca[2]
João Pedro da Ponte[3]

Abstract: This article aims to present 1st cycle of basic education Portuguese teachers' learning of teaching pupils between 6 and 7 years of age, when they plan, lead and reflect on the moment of whole-class discussion, during a lesson study in mathematics. It is a qualitative research, and data collection was carried out by non-participant observation with the elaboration of a logbook and audio and video recording of sessions. The results show that, when planning the whole-class discussion, the teachers became aware of the importance of planning teaching in detail, paying attention to the objectives that are intended to achieve and the role of the teacher in the whole-class discussion. When they conducted and reflected on the discussion, the teachers understood the need to act during the undertaking of the task, in order to support pupils in terms of mathematical language so that they could better communicate their reasoning to their colleagues.
Keywords: Lesson study. Whole class discussion. Professional knowledge. Elementary school.

APRENDIZAGENS PROFISSIONAIS EM MOMENTOS DE DISCUSSÃO COLETIVA NUM ESTUDO DE AULA

Resumo: Este artigo visa apresentar as aprendizagens de professoras do 1.º ciclo do ensino básico em Portugal a lecionar alunos entre 6 e os 7 anos de idade, quando planeiam, conduzem e refletem sobre o momento da discussão coletiva, durante um estudo de aula em Matemática. É uma investigação qualitativa, tendo a recolha de dados sido feita por observação não participante com a elaboração de um diário de bordo e a gravação áudio e vídeo das sessões. Os resultados mostram que, quando planearam a discussão coletiva, as professoras tomaram consciência da importância do planeamento do ensino de forma pormenorizada, dando atenção aos objetivos que se pretende alcançar e ao papel do professor na discussão coletiva. Quando conduziram e refletiram sobre a discussão coletiva, as professoras compreenderam a necessidade de atuar durante a realização da tarefa, de modo a apoiar os alunos em termos de linguagem matemática para que estes comunicassem melhor seu raciocínio aos colegas.
Palavras-chave: Estudo de aula. Discussão coletiva. Conhecimento profissional. Anos Iniciais.

Introduction

Teaching professional knowledge, especially related to teaching practice, directly underpins and guides teacher practice in the classroom. The basis of professional knowledge is

[1] Doctoral student in Education (Mathematics Education) at Instituto de Educação, Universidade de Lisboa (IE-ULisboa), Lisbon, Portugal. E-mail: thuysa@campus.ul.pt – ORCID: https://orcid.org/0000-0002-3062-7774.
[2] Doctor of Education (Teacher Education) at Instituto de Educação, Universidade de Lisboa (IE-ULisboa), Lisbon, Portugal. Deputy Director at a local School Cluster. Research professor in the UIDEF at IE-ULisboa. Email: mgfonseca@campus.ul.pt – ORCID: https://orcid.org/0000-0001-5652-416X.
[3] Doctor of Education (Mathematics Education), University of Georgia (UGA). Full professor at Instituto de Educação, Universidade de Lisboa (IE-ULisboa), Lisbon, Portugal. E-mail: jpponte@ie.ulisboa.pt – ORCID: https://orcid.org/0000-0001-6203-7616.

experience and reflection on it, carried out both individually and collectively (PONTE; OLIVEIRA, 2002). Lesson study is a teacher education process focused on teaching practice that creates opportunities for teachers to investigate issues of their interest in pupils' learning, work collaboratively, reflect on their practice and share their knowledge (PONTE *et al.*, 2016).

In lesson study, mathematics teaching is associated with the exploratory approach (PONTE, 2005). This approach, unlike direct teaching which is characterized by the one-way transmission of knowledge from teacher to pupil, gives pupils space to carry out a work of discovery and knowledge building (PONTE, 2005; OLIVEIRA; MENEZES; CANAVARRO, 2013). A key moment of a lesson in this perspective is the whole-class discussion, when pupils present their strategies and solutions to the whole class. The teacher takes on "the role of moderator, managing the sequence of interventions and guiding, if necessary, the content of the interventions" (PONTE, 2005, p. 16). However, several studies have shown that organizing and managing whole-class discussions is a complex practice for most teachers (STEIN *et al.*, 2008; OLIVEIRA; MENEZES; CANAVARRO, 2013), requiring specific knowledge and skills that can be developed through their participation in lesson studies (RICHIT; PONTE, 2020).

In this article, based on a lesson study conducted in Portugal, we aim to understand the learning concerning the *didactic knowledge* of a group of 1st cycle basic education teachers in Portugal when they plan, lead and reflect on the moment of whole-class discussion. The research lesson took an exploratory approach, intending to lead pupils to discover the existence of different solutions to the same situation. To this end, we describe and analyze episodes relating to the planning sessions of the research lesson, their enactment and the post-class discussion and reflection sessions, with a focus on whole-class discussion.

Theoretical framework

In this section, we discuss relevant knowledge for teaching, with the teaching of mathematics as a reference and we present lesson study associated with exploratory teaching and discuss its potential for the development of teachers' professional knowledge, especially when they are involved in the planning, enactment and reflection on whole-class discussions.

a) Teacher's professional knowledge

Professional knowledge is knowledge mobilized to successfully perform a professional activity. In the field of teaching, it involves knowledge related to teaching practice, other professional roles (*e.g.*, pupil tutoring, participation in school projects and interaction with

community members), and vision of their professional development (PONTE; OLIVEIRA, 2002). According to Shulman (1987), there is an important category of teacher knowledge, called *pedagogical content knowledge*, which is characterized as a body of knowledge related to teaching, owned by the teacher, that distinguishes teaching knowledge from that of a content expert. It concerns the teacher's ability to transform his/her content knowledge into pedagogically powerful forms (SHULMAN, 1987).

As regards the knowledge of the teacher who teaches mathematics, Ponte (2012) emphasizes that it is oriented by a practical activity, combining elements of theoretical, social and experiential nature. It includes a fundamental dimension directly related to teaching practice, called didactic knowledge, which is divided into four domains: knowledge of mathematics for teaching, knowledge of the curriculum, knowledge of pupils and their learning processes, and knowledge of teaching practice, meaning the classroom work processes. Due to its orientation to practical situations, didactic knowledge is also closely related to aspects of everyday life, such as knowledge of the school context and the teacher's knowledge of him/herself.

In this article, we are particularly interested in knowledge of pupils and their learning processes and knowledge of teaching practice. The first relates to pupils' interests, their aptitudes, habitual forms of behavior, difficulties, and the way they learn. In a practical situation, when the teacher needs to select an exploratory task (PONTE, 2005) and analyze its potential to achieve teaching goals, he/she needs to know the interests and skills of pupils and anticipate their strategies and difficulties. According to Fujii (2018), anticipating solutions deepens the teacher's understanding of the knowledge and skills of his/her pupils. This knowledge can then be used in later phases, such as in defining the presentation order of the pupils' solutions and managing pupil participation during the discussion.

Knowledge of teaching practice, a fundamental core of teaching knowledge, refers to lesson planning, task preparation, conducting activities in the classroom, regulation of communication, and assessment of pupils' learning (PONTE; OLIVEIRA, 2002; PONTE, 2012). In the discussion of solving a task with the whole class, knowledge of teaching practice is mobilized when teachers need to regulate pupil interactions in the discussion, encouraging questioning to clarify the ideas presented and the confrontation and comparison between solutions (OLIVEIRA; MENEZES; CANAVARRO, 2013).

Researching classroom experiences and reflecting on the action of teaching in collaborative contexts are keyways in which teachers may develop their professional knowledge (PONTE, 2012). Lesson study is a practice-based professional development process

carried out in a collaborative environment that allows deepening the mathematical, teaching and organizational domains (PONTE *et al.*, 2016). It is, therefore, a formative process with the potential to develop teachers' didactic knowledge.

b) Whole-class discussions in the context of lesson study and teachers' knowledge

Lesson study is a teacher professional development process that provides teachers with an opportunity to reflect on pupils' learning and on their own practice in their workplace (MURATA, 2011; PONTE *et al.*, 2016). This teacher education process is carried out by a group of teachers in an eminently collaborative environment and consists of the following steps: identification of a relevant problem in pupil learning, curriculum study and detailed planning of a research lesson, teaching and observation of the lesson, and post-lesson reflection (PONTE *et al.*, 2016).

The lesson plan produced during the planning phase is a primary resource for teachers, as they constantly operate, problematize and report on the plan throughout the process (FUJII, 2018). Teachers plan the research lesson in detail, preparing tasks and creating strategies for overcoming classroom difficulties. They also anticipate pupils' thinking, predict their difficulties, detail important points of the lesson, and develop data collection tools for the research lesson (FUJII, 2018; PONTE *et al*., 2016).

The research lesson is taught by one of the teachers while the others observe it by focusing on the pupils' work. Real-time observation is a fundamental feature of lesson study and provides the teacher with the opportunity to develop his/her professional knowledge (LIM; KOR; CHIA, 2016; MURATA, 2011). During the research lesson, teachers can observe different ways of teaching and understand how each choice affects pupils' learning. The post-lesson discussion, based on observations of pupils' activities, allows them to share and reflect on that work, seeking to make inferences about pupils' learning.

In Japan, a fundamental aspect of teaching mathematics in lesson studies is the problem-solving approach (FUJII, 2018). Lesson studies in Portugal are also often associated with a Japanese-like teaching approach called "exploratory teaching" (*e.g.*, FONSECA; PONTE, 2022; QUARESMA; PONTE, 2015; PONTE *et al*., 2016). In this approach, pupils work on open tasks that allow them to build their own solution strategies (PONTE, 2005). An exploratory lesson is usually developed in three phases: (i) introduction of the task, (ii) pupils' autonomous work, usually in pairs or small groups, and (iii) whole-class discussion of solutions and a summary of the learning from the work carried out (GUERREIRO *et al*., 2016).

In a whole-class discussion, the teacher organizes the presentations of the mathematical

work carried out by the groups of pupils during autonomous work to be communicated to and discussed with the whole class (GUERREIRO *et al.*, 2016). It is assumed that the pairs or groups of pupils will present various solutions and strategies. The teacher will then focus on comparing and discussing them (FUJII, 2018). The whole-class discussion, as in exploratory teaching, assumes an organizational structure that is different from the usual teaching practice (PONTE, 2005). The teacher must empower the pupils, invite them to present their reasoning, make choices, and stand by them with arguments to justify their solution strategies, in addition to listening and analyzing the strategies of their colleagues (INOUE, 2011; GUERREIRO *et al.*, 2016; STEIN *et al.*, 2008).

Leading whole-class discussions poses several challenges to teachers (STEIN *et al.*, 2008) and requires them to have substantial mathematical knowledge, understand how pupils learn, and know the best ways to develop this learning (GUERREIRO *et al.*, 2016). Stein and collaborators (2018) propose five practices for teachers to foster a mathematically productive discussion. During lesson preparation, the teacher anticipates possible strategies for solving the task and pupils' difficulties. During autonomous work, the teacher monitors the work of the pairs or groups in order to know the different strategies and the difficulties felt by pupils. This knowledge, supported by the teacher's prior preparation, allows the selection and sequencing of the solutions for the discussion. Finally, the teacher relates the answers to each other and to key mathematical ideas.

Due to its dynamics, lesson study enables teachers to develop knowledge about conducting discussions with the entire class. For example, Fonseca and Ponte (2022) report that the 1st cycle elementary school teachers participating in a lesson study have shown to be more aware of the importance of preparing in advance for the moments of whole-class discussion for the communication of strategies and institutionalization of knowledge by the influence they have on the understanding and appropriation of concepts by the pupils. In the lesson study with secondary school teachers presented in Gomes, Quaresma and Ponte (2022), the teachers' re-examination of their teaching practice provided them with a new perspective on conducting discussions in which pupils have an active role in mathematics learning. Similarly, Inoue's (2011) research found that teachers took on the role of discussion facilitators, overcoming the old habit of mastering pupil discussions. According to the author, the change process was difficult, requiring courage and focus on pupil reasoning, but resulted in enriched pupil understanding, higher levels of pupil participation and motivation in classroom activities.

Methodology

Context and participants. The lesson study took place between November 2022 and March 2023 in a grouping of schools[4] located in the Northwest of Lisbon, Portugal. The participants were six 1st cycle basic education teachers with more than 15 years of teaching experience. Four teachers (which we name with the pseudonyms Camila, Linda, Marta and Clara) taught at grade 1 and had never participated in a lesson study. Laura and Gorete (the second author of the article) had already experienced lesson study developed in previous years in the same grouping. Both teachers had no class assigned because they had other professional roles in the grouping. Gorete (hereinafter referred to as the facilitator) was responsible for organizing and conducting the lesson study sessions. The teachers' participation was voluntary, and they accepted to participate in the lesson study after the facilitator's invitation.

Lesson study structure and features. The lesson study had 15 sessions (Table 1), of which eight were for planning, four corresponded to research lessons, two to post-lesson reflections and the last to post-lesson reflection and final assessment. Immediately after each research lesson, the teachers met for 20 to 40 minutes for a brief discussion (D*n*) of their perceptions of the work done by pupils, some constraints and general impressions. These meetings were not part of the initial planning outlined by the facilitator but were spontaneous conversations that did not follow a pre-established structure as occurred in the reflection sessions. However, these were relevant moments because they allowed the teachers to share their experiences with each other.

Table 1: Lesson study structure

Stages	Session	Month/Year	Date	Duration
Planning Sessions	1 (S1)	November 22	16	1h 45 min
	2 (S2)		30	1h 10 min
	3 (S3)	December 22	14	1h 40 min
	4 (S4)	January 23	04	1h 55 min
	5 (S5)		18	2h 05 min
	6 (S6)		25	2h 10 min
	7 (S7)	February 23	15	1h 35 min
	8 (S8)		24	1h 45 min
1st Research lesson (Camila)	9 (RL1)		27	60 min
Post-lesson reflection	10 (R1)		27	1h 10 min
2nd Research lesson (Linda)	11 (RL2)		28	60 min
3rd Research lesson (Marta)	12 (RL3)	March 23	01	60 min
Post-lesson reflection	13 (R2)		01	1h 20 min

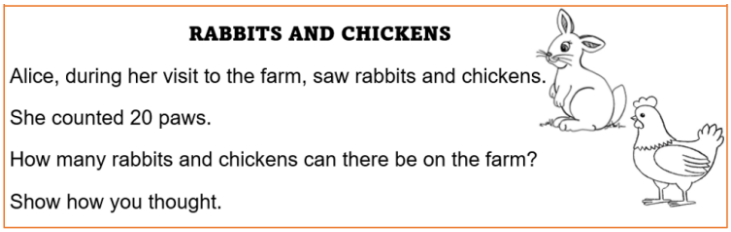
[4] A grouping of schools is an administrative unit composed usually by a main school with 2nd and 3rd cycle pupils plus several 1st cycle and early childhood education schools.

4th Research lesson (Clara)	14 (RL4)		02	60 min
Post-lesson reflection and the wrap-up	15 (R3)		06	1h 25 min

Source: Created by the authors

In the first planning session, the facilitator presented the lesson study to the teachers, the objectives of the work and a proposal for a timetable which the group discussed and approved with small adjustments. In the following planning sessions, the group analyzed the guiding documents of the mathematics curriculum for grade 1, decided on the research lesson objective, content and teaching approach. As at the time of the research lesson, the teachers would be working on "counting up to twenty objects", they analyzed and discussed different tasks on the topic, and selected a task from the pupil's textbook. The selection resulted from the combination of two criteria: that it was an open task that appealed to the development of reasoning and it presented several possible solutions. This was a situation that the pupils did not experience before, enabling the promotion of a rich whole-class discussion regarding the possibility of collective construction of mathematical ideas. After the task was selected, the group made some adjustments, resulting in the final statement (Figure 1).

Figure 1: Statement of the task



Source: Research archives.

The task was to be solved in pairs, and each pair would have access to manipulative materials if pupils wanted to use them. Each teacher was responsible for choosing the material that she found most suitable for her class, such as straws, bottle caps, or fitting cubes. The participants opted for the exploratory approach to underpin the research lesson, with discussions throughout the sessions focused on planning of the three lesson moments: presentation, pupils' autonomous work, and whole-class discussion and final synthesis. It was decided that the research lesson would have 60 minutes: 5 minutes devoted to presenting the task, 20 minutes following the autonomous work, and the last 35 minutes to discussing the task and doing the final synthesis.

Data collection and analysis. This investigation is qualitative and interpretative

(ERICKSON, 1986). Data collection was carried out by non-participating observation with the preparation of a logbook (LB) and audio (AR) and video (VR) recording of the sessions. The parts of then recordings related to the whole-class discussions were transcribed. We analyzed the data through content analysis (BARDIN, 2002), from which we sought to discuss the learning demonstrated during the planning, conducting and reflection of the whole-class discussion carried out in the lesson study. For the analysis of the learning related to the didactic knowledge of participants we considered the framework proposed by Ponte (2012), namely the knowledge of teaching practice and the knowledge of pupils and their learning processes.

Results

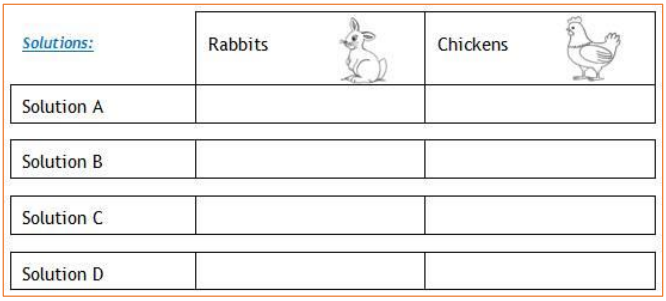
The teaching of the research lesson by the four teachers (Camila, Linda, Marta and Clara) in their classes led to different whole-class discussions. In this section, we describe episodes relating to the planning of the whole-class discussions and the teaching and reflection of the whole-class discussions that took place in the four research lessons.

a) Planning the whole-class discussion

In the sessions leading up to the first research lesson, the participants constructed a script with detailed planning of the sequencing of the various moments of the class. They anticipated difficulties, outlining strategies to overcome them and orchestrating how to conduct the whole-class discussion, always referring to the intended goal in terms of pupils' learning.

Specifically, in relation to the whole-class discussion, they decided that they would display a PowerPoint grid that would serve to record the solutions found by the pupils' pairs during the discussion, and they planned how to begin conducting the discussion. Under the guidance of the teacher, the pairs that had the correct answers would be called, if possible, in sequence: one rabbit and eight chickens, two rabbits and six chickens, three rabbits and four chickens, and four rabbits and two chickens, registering them in the projected grid.

Figure 2: The grid projected on the whiteboard for recording pupils' solutions



Source: Research archives.

The teacher could only call pairs with the same solution if they had followed different strategies. If there were pairs with an incorrect solution, they would be called and led to understand why it was not a correct solution. If some solutions were not indicated, the teacher would ask, "Are there more solutions?", and would rearrange the data from the grid on the whiteboard, leading the pupils to identify possible solutions for the situation. Once the four solutions were discovered, pupils would register on the worksheet, completing the sentence corresponding to the final synthesis. The teacher would close the session by drawing attention to the existence of problems with more than one solution, a situation that the pupils had not encountered before, as well as to the existence of different strategies followed.

b) Teaching and reflecting on the whole-class discussion

First research lesson. In this lesson, led by Camila, the first student pair called by the teacher presented orally two solutions (4 rabbits and 2 chickens, 1 rabbit and 8 chickens) explaining the strategies used: in the first solution, they resorted to successive sums, and, in the second, they made sets of 2 and 4 dots. Camila recorded on the grid the solutions as follows: 4r and 2c, 1r and 8c. The second pair presented the solution with 2 rabbits and 2 chickens, explaining how they solved the problem using straws. The third and fifth pairs presented incorrect answers. The fourth pair repeated a solution and used the straws as a resource. So, by then, the pupils had found three different solutions using three different strategies: successive sums, drawings, and the use of straws. Since the fourth solution was still missing, the teacher urged the pupils to think about whether there was another possibility, and they were able to come up with the missing solution: 3 rabbits and 4 chickens. It is worth noting that all groups were called to show their solutions, even if they repeated the answers and/or strategies, a situation that had not been previously planned.

In the post-lesson reflection and from the notes collected by the observers, much of the dialogue focused on how the whole-class discussion was conducted and on the difficulties some pupils had in understanding concepts by confusing the number of paws with the number of sets and failing to associate the animal with the number of paws. The fact that Camila called all pairs was also the subject of reflection, with the facilitator questioning the contribution brought to the whole-class discussion since the diminishing attention from the pupils was visible. The reflection generated allowed the teachers to become aware of the importance of efficient management of the order in which pairs should be called to present the solutions, enabling a more accessible discussion and the institutionalization of mathematical ideas. The reflection also focused on: (i) how to support each pair during their autonomous work, to help them associate groups or clusters of paws with the number of rabbits and chickens, and (ii) how to record solutions and strategies in the whiteboard. The moments of discussion about the orchestration of the discussion were particularly important due to the reflection generated, allowing changes to be made to the plan:

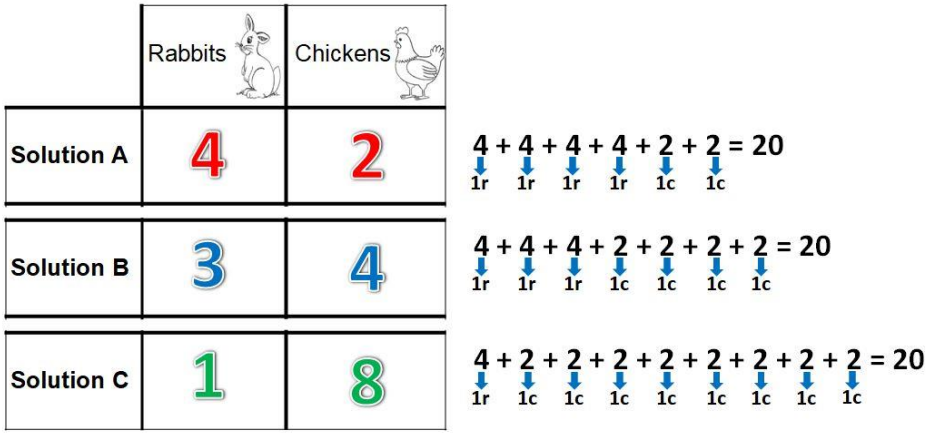
> Linda: Let's start by the most clarifying strategy, shall we? The clearest one from the perspective of associating the animal and the number of paws. And, later, let's also record right into the grid, no need to sort the data by the number of rabbits, correct?
> Facilitator: As entries show up, the record can be done right away.
> Linda: Exactly, as they [pupils] present, correct? And at the end, after the 4 solutions are there, we take them to reason of there is a way to classify [the data] and sort [by the number of rabbits] on the side. [VR – R1].

As proposals for improvement, it was decided that: (a) during the autonomous work, the pupils would have bottle caps as manipulative material instead of straws; (b) when the teacher passed the pairs, she should question and guide them to make clear the association between the number of paws and the animal, ensuring that they realized that two caps represented a chicken and four caps a rabbit; (c) at the time of the whole-class discussion, only the pre-defined pairs would be called; and (d) after presenting the four solutions, the pupils would be asked if there would be any way to organize the data, for example, by the minimum number of rabbits. Once the solutions were arranged, the teacher would ask the pupils about the possibility that Alice had seen five rabbits, leading the pupils to see why there could be no more than four solutions to the task.

Second research lesson. This lesson was led by Linda, who integrated the suggestions of previous reflection into her planning. While the pupils were doing the task, the teacher went through all the pairs making sure they understood the association of the set of paws with the

number of animals, including making circles on the table with a pencil delimiting the sets of 4 and 2 pads. In the whole-class discussion, a trio presented the three solutions they found: 4 rabbits and 2 chickens; 3 rabbits and 4 chickens; and 1 rabbit and 8 chickens, drawing sets of two and four dots representing the chickens and the rabbits' paws, respectively, on the board to explain the strategy that they followed. In the presentation, the association made was very clear: "We did not count the bottle caps; we counted the circles [sets] to know how many chickens and rabbits there could be" (Matilde, LB, S11). The teacher recorded each solution directly on the grid in the order they appeared. In the space next to it, she wrote down the successive sums of the 4 and 2 rabbit and chicken paws, respectively, and every time the number 4 appeared in the sum, she added "1r" below the number, indicating a rabbit. Similarly, she did the same with the chickens (Figure 3).

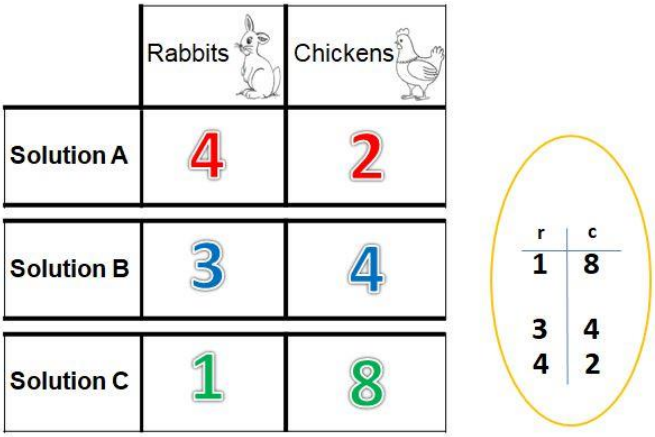
Figure 3: Record of the successive sums together with the number of chickens and rabbits



Source: Created by the authors.

Since no group came up with the fourth solution, the teacher followed the plan and reorganized the three previous solutions in increasing order of rabbits next to the designed grid, leading the pupils to discover the fourth solution. This arrangement of solutions helped some pupils to make other discoveries: "in the rabbit column we count one by one"; "in the chicken column we count two by two", "if the rabbit number increases, the chicken number decreases" (LB, S11), allowing important connections that were not made in the previous class (Figure 4).

Figure 4: Solutions sorted by descending number of rabbits



Source: Created by the authors.

After the research lesson, a brief reflection was made on the solutions and difficulties observed in the student pairs and the results obtained with the changes made in autonomous work and the leading of the discussion. The teachers agreed that Linda's support during autonomous work helped pupils to better understand the relationship between the set of paws and the number of animals, which facilitated the communication among pupils during the whole-class discussion. There were still some suggestions regarding the way to show the strategies used by the pupils when they are called to justify the solutions:

> Marta: Yes, when Lisa was "four plus…", to me...
> Facilitator: It continues to be abstract [to the pupils, regarding the relation].
> Marta: Therefore, if there were magnets, "OK, four!", instead of writing the number four, we would have four magnets.
> Facilitator: To link with the quantity.
> Linda: And circle the four together...
> Marta: Circle four and that is a rabbit, two magnets are a chicken. I believe… it can help. The same way that it helped when the caps were circled, I believe this will also support when it is in the whiteboard [when the pupils are explaining to others how they thought]. (AR – D2)

Third research lesson. In this lesson, led by Marta, magnets were introduced as a new support material for the whole-class discussion so that when pupils went to the board, they could also use manipulative material to explain their strategy, similar to what they had used at their desk. Like Linda, Marta also called a student pair with three different solutions to share with their colleagues: 2 rabbits and 6 chickens; 1 rabbit and 8 chickens; and 3 rabbits and 4 chickens. The pupils handled the magnets in the board using the same strategy that they followed at autonomous work when they had the cubes available. To explore the last solution, the teacher called another pair who had identified a wrong solution, four rabbits and three

chickens, and asked them to work with the magnets, starting with the four-paw groups, leading them to realize that the only possibility was 4 rabbits and 2 chickens.

In the post-lesson reflection, the introduction of magnets representing the paws was considered an added-value to support the whole-class discussion. The teachers agreed that it was essential to make circles around the magnets to represent each animal, as it made possible for pupils to make the association between the paws and the animals, as well as it would facilitate the explanation of the strategies by the pupils. They also found that the pupils were more attentive in the discussion than in previous classes when the teachers resorted to successive sums.

Another topic addressed was the difficulty of the pupils in understanding why a solution had two related quantities instead of just one number, as they were used to. The following dialogue took place:

> Laura: And from this we can even learn that they did not have same the perception as a former group I had: they wrote that three plus four equals seven but didn't know what the seven is.
> Marta: Yes, they do not know what the seven is.
> Laura: Because they think they have to run the algorithm and the and the result is the answer of the problem.
> Facilitator: Exactly. This is what I see.
> Marta: This fixation with the calculation is driving me crazy.
> Facilitator: Is it in all [the grade 1 classes].
> Marta: All. All classes until now needed a formula.
> Linda: We had a problem and, to them [the pupils] the problems are solved with formulas. Because they are used to it. [AR – R2]

The teachers noticed that some pupils were looking for a solution with a single number, so they added up the number of rabbits and chickens found. This discussion also highlighted the observation that many pupils resorted to using calculations to record strategies when they used drawings or manipulative material to find their solutions. It was suggested to Clara that at the end of the whole-class discussion, she would restate the question of the task, emphasizing that each solution was associated with a number of rabbits and a number of chickens.

Fourth research lesson. Finally, Clara taught the last research lesson. While the pupils were solving the task, the teacher went to the pairs questioning them to ensure they were making the right associations while also noting on paper the solutions and strategies that they used, thus preparing for the next pair to call in the whole-class discussion. It is important to say that Clara, in the post-lesson reflection the day before, expressed the difficulty that she felt in knowing how to select and sequence pairs to begin the whole-class discussion while observing the pupils' work. In presenting the solutions, the first pair presented two solutions (4 rabbits and 2 chickens,

and 3 rabbits and 4 chickens), the second pair showed the third solution (2 rabbits and 6 chickens), and another pair explained the fourth solution (1 rabbit and 8 chickens), all of which resorted to magnets to form groups of 4 and 2 paws. In the final synthesis, Clara recalled the question to be answered, as suggested in the previous discussion, leading the pupils to realize why there were four solutions and not only one.

In the final session, the teachers expressed satisfaction with the outcome of the last research lesson, as they noted that the changes made over the four classes led to significant improvements in the quality of pupils' learning.

Discussion

The episodes regarding the whole-class discussion described above show professional learning related to didactic knowledge during lesson study. Although teachers have extensive teaching experience, conducting the whole-class discussion in a way that takes advantage of pupils' solutions and strategies has proved to be both a challenge and an opportunity to reflect on teaching practice.

Planning the whole-class discussion. In the moments when they planned the whole-class discussion, the teachers anticipated pupils' strategies and their possible mistakes, thought about the types of representation that would be used during the presentation on the board, discussed how they would sequence pupils' solutions to the whole-class discussion, what answers they would consider and how they should use wrong answers, focusing on the sought pupils' learning. This effort of anticipation and planning supported conversations on ways to support pupils' communication during the whole-class discussion without reducing the degree of difficulty of the task (FUJII, 2018). These discussions were of great relevance to the development of teachers' knowledge about the strategies that pupils could follow (*knowledge of pupils and their learning processes*) and how to conduct and organize the whole-class discussion (*knowledge of teaching practice*). Therefore, like other investigations (RICHIT; PONTE, 2020; PONTE *et al*., 2016), the teachers become aware of the importance of planning in detail, paying attention to the objectives to be achieved and the teacher's roles in a whole-class discussion.

Teaching and reflecting on the whole-class discussion. Observing the activities of pupils during the research lesson proved to be an important learning mechanism for teachers, such as in Lim, Kor and Chia (2016), mainly related to the knowledge of pupils' communication and their strategies and difficulties. In exploratory teaching, the pupils should have autonomous

working time (PONTE, 2005), allowing them the opportunity to discuss the task with their colleagues in the group. When monitoring the activities of pupils with a focus on preparing the whole-class discussion, the teachers were able to identify their different solutions as well as conceptual or interpretational misunderstandings of the task, giving them the opportunity to develop their *knowledge of pupils and their learning processes*.

During the reflections held after the research lessons, a recurring theme was the difficulty of pupils communicating verbally and in writing, especially when they needed to record their strategies on the sheet and explain them to the entire class on the board. By becoming aware of these difficulties, the teachers discussed what actions they could take, deciding to make supporting materials available to help pupils during the whole-class discussion. They also became more attentive to the fact that pupils felt the need to use numbers and mathematical symbols to register their strategies on the sheet when they used drawings and the support of the bottle caps as solution strategies. The teachers expressed surprise at realizing that such young pupils were already conditioned to use traditional mathematical representations, such as numbers and signals for operations. Therefore, they became more attentive to the pupils' reasoning processes as well as their ability to communicate their reasoning, as evidenced in other studies (FONSECA; PONTE, 2022; QUARESMA; PONTE, 2015).

As regards *knowledge of teaching practice*, the episodes highlight learning linked to the teacher's actions in the whole-class discussion, the promotion of the communication of mathematical ideas in the classroom, and the management of pupil relationships.

Regarding the teachers' action, they reflected on how they could act to promote discussions with more mathematical quality and to guide the interactions between pupils. In the first research lesson, Camila used numerals and initials of the animals' names ("r" for rabbits and "c" for chickens) to record the solutions found by the pupils and resorted to successive sums with factors 4 and 2, representing the numbers of rabbit and chicken paws. When they discussed the forms of representation and their effectiveness for the understanding of the pupils, they realized that the pupils immediately became distracted and only the group that was explaining the solutions in the whiteboard was effectively involved in the discussion. Thus, they sought to prepare other ways of registering the solutions and strategies in the board, as well as to provide pupils with the opportunity to note down their strategies with the support of the magnets. Therefore, discussing the role of the teacher in conducting the whole-class discussion led them to rethink ways of engaging pupils and keeping the discussion focused on the objectives of the lesson, similar to what was observed in the studies of Inoue (2011) and of

Gomes, Quaresma and Ponte (2022).

In terms of promoting the communication of mathematical ideas, after the first research lesson, the teachers realized the need to act during the autonomous work to support the use of mathematical language so that pupils could better communicate their reasoning to colleagues during the whole-class discussion. Therefore, the teachers who led the classes began to suggest ideas and give clues to pupils (GUERREIRO *et al*., 2016), as well as ask for further clarification of questions, in order to develop this communication ability in pupils.

Along the lesson study cycle, the teachers recognized the importance of managing pupil participation and defining the order of presentations. However, during different moments of the lesson, they had difficulty choosing and sequencing the pairs to show their solution and strategies, even though these issues had been addressed in the planning phase. For example, Camila chose to invite all pairs to present solutions and strategies, thus avoiding the task of selecting them. In the discussion before the fourth research lesson, Clara demonstrated difficulty in knowing how to select and sequence pairs to begin the whole-class discussion, despite having observed the three previous ones. In fact, several studies (GUERREIRO *et al*., 2016; STEIN *et al*., 2008) point to the leading of whole-class discussions as one of the challenges faced by teachers in the exploratory approach. However, this question has appeared at different times during discussions and post-lesson reflections, indicating that teachers paid attention to the relevance of managing the relationship between pupils to achieve the intended learning objectives.

Conclusion

The dynamics of the lesson study, with four research lessons combined with an exploratory approach, enabled the teachers to discuss and implement changes in the next research lesson, which gave an opportunity to understand the need to establish well-defined pedagogical intentions to directly address pupils' needs. Focusing on the moments of whole-class discussion, this study highlights the potential of lesson study in promoting teachers' learning in relation to various aspects of their didactic knowledge, in particular the development of knowledge of the pupil and their learning processes and of teaching practice, especially regarding preparing and leading whole-class discussions.

References

BARDIN, L. **Análise de conteúdo**. Lisboa: Edições 70, 1977.

ERICKSON, F. Qualitative methods in research on teaching. In: WITTROCK, M. C. (ed.). **Handbook of research on teaching**. 3. ed. New York: Macmillan, 1986, p. 119-161.

FONSECA, G.; PONTE, J. P. da. O estudo de aula no desenvolvimento do conhecimento sobre o ensino da matemática de professores do 1.º ciclo. **RELIME**, v. 25, n. 2, p. 223-246, jun. 2022.

FUJII, T. Lesson study and teaching mathematics through problem solving: The two wheels of a car. In: QUARESMA, M.; WINSLØW, C.; CLIVAZ, S.; PONTE, J. P. da; SHÚILLEABHÁIN, A. N.; TAKAHASHI, A. (ed.). **Mathematics lesson study around the world:** theoretical and methodological issues. Switzerland: Springer, 2018, p. 1-19.

GOMES, P.; QUARESMA, M.; PONTE, J. P. da. Leading whole-class discussions: from participating in a lesson study to teaching practice. **International Journal for Lesson & Learning Studies**, v. 12, n. 2, p. 139-151, 17 nov. 2022.

GUERRERO, A.; FERREIRA, R. A. T.; MENEZES, L.; MARTINHO, M H. Comunicação na sala de aula: a perspetiva do ensino exploratório da matemática. **Zetetike**, v. 23, n. 2, p. 279, 18 mar. 2016.

INOUE, N. Zen and the art of neriage: Facilitating consensus building in mathematics inquiry lessons through lesson study. **Journal of Mathematics Teacher Education**, [S.L.], v. 14, n. 1, p. 5-23, maio 2010.

LIM, C. S.; KOR, L. K.; CHIA, H. M. Revitalising mathematics classroom teaching through Lesson Study (LS): a Malaysian case study. **ZDM Mathematics Education**, v. 48, n. 4, p. 485-499, 2 abr. 2016. Springer Science and Business Media LLC.

MURATA, A. Introduction: conceptual overview of lesson study. In: HART, L. C.; ALSTON, A. S.; MURATA, A. (ed.). **Lesson study research and practice in mathematics education**: learning together. New York, NY: Springer, 2011. p. 1-12.

OLIVEIRA, H.; MENEZES, L.; CANAVARRO, A. P. Conceptualizando o ensino exploratório da Matemática: Contributos da prática de uma professora do 3.º ciclo para a elaboração de um quadro de referência. **Quadrante**, v. 22, n. 2, p. 29-53, out. 2013.

PONTE, J. P. da. Gestão curricular em Matemática. In: GTI (ed.). **O professor e o desenvolvimento curricular**. Lisboa: APM, 2005. p. 11-34.

PONTE, J. P. da. Estudiando el conocimiento y el desarrollo profesional del profesorado de matemáticas. In: PLANAS, Núria (Ed.). **Teoría, crítica y práctica de la educación matemática**. [S. L.]: Graó, 2012, p. 83-98.

PONTE, J. P. da; OLIVEIRA, H. Remar contra a maré: a construção do conhecimento e da identidade profissional na formação inicial. **Revista de Educação**, Lisboa, v. 2, n. 11, p. 145-163, 2002.

PONTE, J. P. da.; QUARESMA, M.; MATA-PEREIRA, J.; BAPTISTA, M. O Estudo de Aula como processo de desenvolvimento profissional de Professores de Matemática. **Bolema**, Rio Claro (Sp), v. 30, n. 56, p. 868-891, dez. 2016.

QUARESMA, M.; PONTE, J. P. da. Comunicação, tarefas e raciocínio: aprendizagens profissionais proporcionadas por um estudo de aula. **Zetetike**, v. 23, n. 44, p. 297-310, jul/dez. 2015.

RICHIT, A.; PONTE, J. P. da. Conhecimentos profissionais evidenciados em estudos de aula na perspectiva de professores participantes. **Educação em Revista**, Belo Horizonte, v. 36, n. 12, p. 1-29, dez. 2020.

SHULMAN, L. S. Those who understand: Knowledge growth in teaching. **Educational Researcher**, Washington, v. 15, n. 2, p. 4-14, 1986.

STEIN, M. K.; ENGLE, R. A.; SMITH, M. S.; HUGHES, E. K. Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. **Mathematical Thinking And Learning**, v. 10, n. 4, p. 313-340, 21 out. 2008.