Time-dependent London approach: Dissipation due to out-of-core normal excitations by moving vortices

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The dissipative currents due to normal excitations are included in the London description. The resulting time-dependent London equations are solved for a moving vortex and a moving vortex lattice. It is shown that the field distribution of a moving vortex loses its cylindrical symmetry. It experiences contraction that is stronger in the direction of the motion than in the direction normal to the velocity $\boldsymbol{v}$. The London contribution of normal currents to dissipation is small relative to the Bardeen-Stephen core dissipation at small velocities, but it approaches the latter at high velocities, where this contribution is no longer proportional to v^2. To minimize the London contribution to dissipation, the vortex lattice is oriented so as to have one of the unit cell vectors along the velocity. This effect is seen in experiments and predicted within the time-dependent Ginzburg-Landau theory.

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I. INTRODUCTION

The London (L) equations have proven to be a useful tool in describing the magnetic properties of superconductors. Originally, they were obtained by the London brothers using a heuristic argument to describe the Meissner effect [1]. Later they were derived from the microscopic theory for all temperatures, or, for temperatures near the critical temperature, from the Ginzburg-Landau (GL) current expression,

$$\boldsymbol{J} = -\frac{2e^2 f^2}{mc}\left(\boldsymbol{A} + \frac{\phi_0}{2\pi}\nabla\chi\right), \tag{1}$$

by setting constant the order parameter modulus f. This is the major model shortcoming: the cores of vortices cannot be described by the L theory ($\boldsymbol{A}$ is the vector potential, ϕ_0 is the flux quantum, and χ is the phase). Applying curl to Eq. (1), one obtains L equations,

$$\text{curl curl}\,\boldsymbol{H} + \frac{1}{\lambda^2}\boldsymbol{H} = \frac{\phi_0}{\lambda^2}\hat{\boldsymbol{z}}\delta(\boldsymbol{r} - \boldsymbol{r}_v), \tag{2}$$

where $\boldsymbol{r}_v$ is the position of the phase singularity and $\hat{\boldsymbol{z}}$ is the straight vortex direction. For more than one vortex, the right-hand side here contains a sum of δ functions. Notwithstanding this derivation from GL theory, the L equations hold for all temperatures since they, in fact, express the Meissner effect (under the caveat mentioned). Of course, the temperature and scattering dependences of the penetration depth λ, the only material parameter of the L theory, should be taken from a microscopic consideration or from experiment.

A linear Eq. (2) (or its anisotropic version) is instrumental in studying static intervortex interactions and vortex lattices (VL) for which vortex cores are irrelevant provided the intervortex spacing a exceeds by much the core size ξ.

It this article, the time-dependent L equations are discussed. This approach was employed in the literature for quite some time; see, e.g., Ref. [2]. Here, it is applied to moving vortices and vortex lattices (VLs). Moving VLs have been studied in a number of experiments [3–6]. It was shown theoretically that in the presence of pinning, the vortex system does not have a long-range order at low velocities, but orders to a VL at high velocities with one of the lattice vectors along the velocity [7]. This VL orientation was also proven to be preferable in the absence of pinning near T_c with the help of time-dependent GL equations (TDGL) [8]. As is shown below, the time-dependent London approach (TDL) provides a simpler way to address, e.g., the question of structure of moving VLs, or in general, the time-dependent problems in which vortex cores do not play a role.

In time-dependent situations, the current consists, in general, of normal and superconducting parts:

$$\boldsymbol{J} = \sigma\boldsymbol{E} - \frac{2e^2 f^2}{mc}\left(\boldsymbol{A} + \frac{\phi_0}{2\pi}\nabla\chi\right), \tag{3}$$

where $\boldsymbol{E}$ is the electric field. The conductivity σ for the quasiparticle flow is in general frequency-dependent [9]. If, however, the frequencies are bound by inequality $\omega\tau_n \ll 1$, with τ_n being the scattering time for the normal excitations, one can consider σ as a real ω-independent quantity. Usually, the term with the normal conductivity is small because the density of normal excitations ρ_n is negligible away from T_c for s-wave materials. However, close to T_c or in gapless materials, ρ_n is practically close to that of the normal phase.

Within the London model, $f^2 = \text{const}$ and Eq. (3) becomes

$$\frac{4\pi}{c}\boldsymbol{J} = \frac{4\pi\sigma}{c}\boldsymbol{E} - \frac{1}{\lambda^2}\left(\boldsymbol{A} + \frac{\phi_0}{2\pi}\nabla\chi\right). \tag{4}$$

Operating by curl, one obtains

$$\text{curl curl}\,\boldsymbol{H} + \frac{1}{\lambda^2}\boldsymbol{H} + \frac{4\pi\sigma}{c^2}\frac{\partial\boldsymbol{H}}{\partial t} = \frac{\phi_0}{\lambda^2}z\sum_{\boldsymbol{r}_v}\delta(\boldsymbol{r} - \boldsymbol{r}_v), \tag{5}$$

where $\boldsymbol{r}_v(t)$ are positions of the phase singularities, which might change in time.

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II. MOVING VORTEX

Equation (5) can be considered as a general form of the time-dependent London equation. For a straight vortex along z moving with a constant velocity $\boldsymbol{v}$ in the xy plane, this equation reads

$$-\lambda^2\nabla^2 H + H + \tau\,\frac{\partial H}{\partial t} = \phi_0\delta(\boldsymbol{r}-\boldsymbol{v}t), \tag{6}$$

where $H(\boldsymbol{r},t)$ is the z component of the magnetic field, $\boldsymbol{r} = (x,y)$, and

$$\tau = \frac{4\pi\sigma\lambda^2}{c^2} \tag{7}$$

is the "current relaxation time," the term used in the literature on TDGL [10,11].

Clearly, the field distribution described by Eq. (6) differs from the solution that would have existed in the absence of the term $\tau\,\partial_t H$,

$$H_0(\boldsymbol{r},t) = \frac{\phi_0}{2\pi\lambda^2}\,K_0\!\left(\frac{|\boldsymbol{r}-\boldsymbol{v}t|}{\lambda}\right), \tag{8}$$

which corresponds to translation of the static field distribution with velocity $\boldsymbol{v}$; K_0 is the modified Bessel function.

As is done for the diffusion equation [12], Eq. (6) can be solved by first finding the time dependence of the Fourier transform $H_{\boldsymbol{k}}$, $\boldsymbol{k} = (k_x,k_y)$:

$$\tau\,\partial_t H_{\boldsymbol{k}} + (1+\lambda^2k^2)H_{\boldsymbol{k}} = \phi_0\,e^{-i\boldsymbol{k}\boldsymbol{v}t}. \tag{9}$$

The general solution of this equation is

$$H_{\boldsymbol{k}} = \frac{\phi_0\,e^{-i\boldsymbol{k}\boldsymbol{v}t}}{1+\lambda^2k^2-i\boldsymbol{k}\boldsymbol{v}\tau} + Ce^{-t(1+\lambda^2k^2)/\tau}. \tag{10}$$

For a stationary solution, the arbitrary constant C is zero.

To find the field distribution in real space for a constant $\boldsymbol{v}$, it suffices to consider $t=0$:

$$H(\boldsymbol{r},t=0) = \frac{\phi_0}{4\pi^2}\int\frac{d\boldsymbol{k}\,e^{i\boldsymbol{k}\boldsymbol{r}}}{1+\lambda^2k^2-ik_xv\tau}, \tag{11}$$

where x is chosen along the velocity.

Integration over k_x is straightforward since the poles of the integrand are easily found:

$$H\,\frac{2\pi\lambda^2}{\phi_0} = e^{-xs/\lambda^2}\int_0^\infty\frac{e^{ik_yy-|x|\eta/\lambda}}{\eta}\lambda\,dk_y,$$
$$\eta = \sqrt{1+\lambda^2k_y^2+s^2/\lambda^2},\quad s = v\tau/2. \tag{12}$$

Although difficult in general, analytic integration over k_y can be done for $x=0$ or $y=0$:

$$H(0,y) = \frac{\phi_0}{2\pi\lambda^2}\,K_0\!\left(\frac{|y|}{\lambda^*}\right), \tag{13}$$

$$H(x,0) = \frac{\phi_0}{2\pi\lambda^2}\,\exp\!\left(-\frac{xs}{\lambda^2}\right)K_0\!\left(\frac{|x|}{\lambda^*}\right), \tag{14}$$

$$\lambda^* = \frac{\lambda}{\sqrt{1+s^2/\lambda^2}}. \tag{15}$$

If $v=0$, this reduces to the standard static London solution.

Hence, the field distribution of a moving vortex (a) is not symmetric with respect to $x\to -x$, (b) is contracted more strongly along the velocity (x) than across it (y) (not only is the argument of the Bessel function K_0 scaled by $\lambda^*<\lambda$, there is an extra factor $e^{-xv\tau/\lambda^2}$ breaking the symmetry $x\to -x$), and (c) the intervortex interaction in moving system of vortices differs from that in the static case.

Physically, the distortion of the field distribution is due to the contribution of the out-of-core normal excitations to currents of the moving vortex. At small velocities, the distortion can be disregarded. Indeed, the ratio

$$\frac{s}{\lambda} = \frac{2\pi\sigma\lambda v}{c^2} = \frac{v}{v_c},\quad v_c = \frac{c^2}{2\pi\sigma\lambda} = \frac{2\lambda}{\tau}, \tag{16}$$

where v_c is a crossover value for "low" and "high" velocities. At low temperatures T, the quasiparticles are nearly absent (for the s-wave symmetry) and $\sigma\approx 0$, whereas λ is finite, therefore the ratio v/v_c along with the vortex field distortion are small. Hence, the distortion may have an effect at high T's where the conductivity is close to that of the normal phase. Gapless superconductors are an exception to this rule, since the normal excitation density of states is close to normal even at low T's.

The integral (12) can be evaluated numerically. The upper panel of Fig. 1 shows contours $h(x,y) = 2\pi\lambda^2H/\phi_0 = \text{const}$ for $v/v_c = 0.4$. The lower panel gives $h(x,y)$ for $v/v_c = 2$; although unrealistic, this example is given to show clearly the distortion of the field distribution of a moving vortex.

A. Extra dissipation by a moving vortex

It should be stressed that the dissipation considered here is due to the moving nonuniform distribution of the vortex magnetic field out of the vortex core. This dissipation is usually small relative to Bardeen-Stephen core dissipation [13]. The dissipation of interest here is σE^2 and the electric field $\boldsymbol{E}$ is obtained with the help of Maxwell equations $i(\boldsymbol{k}\times\boldsymbol{E}_{\boldsymbol{k}})_z = -\partial_t H_{z\boldsymbol{k}}/c$ and $\boldsymbol{k}\cdot\boldsymbol{E}_{\boldsymbol{k}} = \boldsymbol{0}$ [14]:

$$E_{x\boldsymbol{k}} = -\frac{\phi_0vk_xk_y\,e^{-i\boldsymbol{k}\boldsymbol{v}t}}{ck^2(1+\lambda^2k^2-i\boldsymbol{k}\boldsymbol{v}\tau)}, \tag{17}$$

$$E_{y\boldsymbol{k}} = \frac{\phi_0vk_x^2\,e^{-i\boldsymbol{k}\boldsymbol{v}t}}{ck^2(1+\lambda^2k^2-i\boldsymbol{k}\boldsymbol{v}\tau)}. \tag{18}$$

It is worth noting that $H_{z\boldsymbol{k}} = \phi_0$ at $\boldsymbol{k}=\boldsymbol{0}$ (the flux quantization), i.e., $H_{z,\boldsymbol{k}=\boldsymbol{0}}$ does not change in time; therefore, $\boldsymbol{E}_{\boldsymbol{k}=\boldsymbol{0}}$ should be set to zero.

The dissipation power per unit length of the vortex is

$$w = \sigma\int d^2\boldsymbol{r}\,E^2 = \sigma\int\frac{d\boldsymbol{k}}{4\pi^2}(|E_{x\boldsymbol{k}}|^2+|E_{y\boldsymbol{k}}|^2)$$
$$= \frac{\phi_0^2\sigma v^2}{4\pi^2c^2}\int\frac{d\boldsymbol{k}\,k_x^2}{k^2|1+\lambda^2k^2-i\boldsymbol{k}\boldsymbol{v}\tau|^2}. \tag{19}$$

Note that the contribution to the dissipation due to the changing in time order parameter modulus [10,13] is left out of the London approximation. After integration over $\boldsymbol{k}$ directions, the integral takes the form

$$\int_0^\infty\frac{dk}{k}\left[1-\frac{1+\lambda^2k^2}{\sqrt{(1+\lambda^2k^2)^2+4k^2s^2}}\right] = \frac{\pi\lambda^2}{2s^2}\ln\!\left(1+\frac{s^2}{\lambda^2}\right). \tag{20}$$

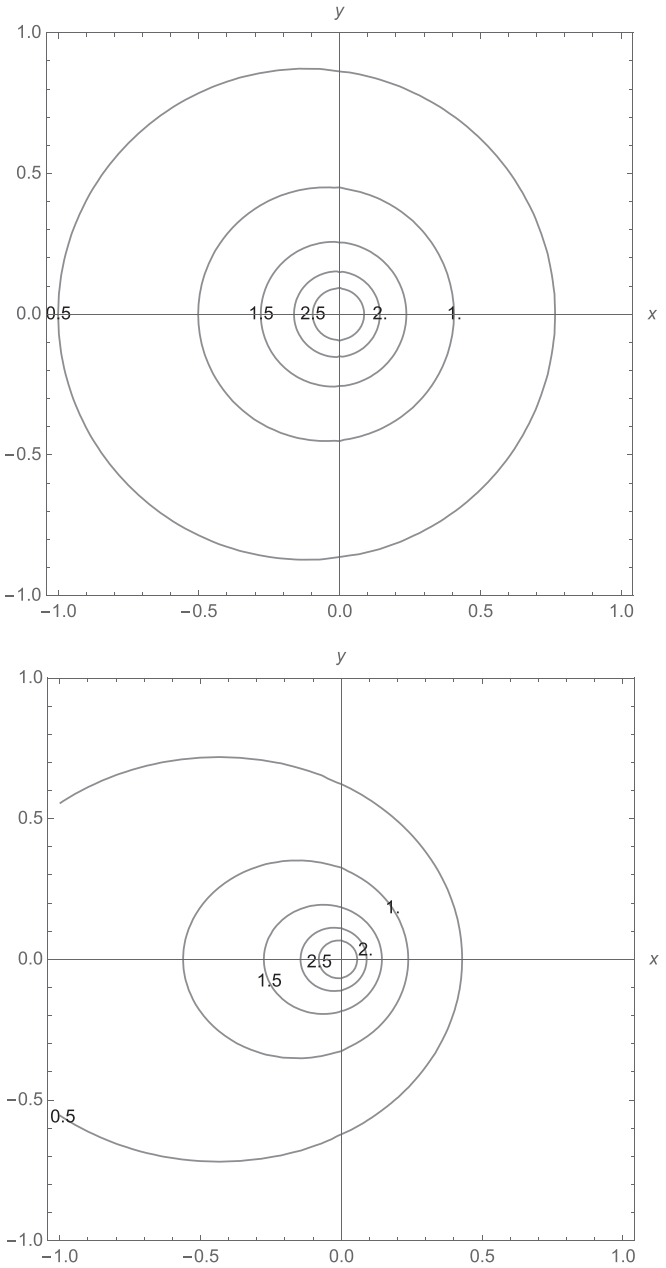

FIG. 1. The upper panel: contours of $h(x,y) = \text{const}$ for the parameter $s/\lambda = v/v_c = 0.4$. The lower panel: $v/v_c = 2$. x and y are in units of λ.

Thus, we obtain

$$w = \frac{\phi_0^2 \sigma v_c^2}{8\pi c^2 \lambda^2} \ln\left(1 + \frac{v^2}{v_c^2}\right). \tag{21}$$

For $v^2 \ll v_c^2$, we have

$$w = \frac{\phi_0^2 \sigma v^2}{8\pi c^2 \lambda^2}, \tag{22}$$

so that the quantity

$$\eta_L = \frac{\phi_0^2 \sigma}{8\pi c^2 \lambda^2} \tag{23}$$

is the London contribution to the drag coefficient (for the unit length of a single vortex). Bardeen-Stephen dissipation related to vortex cores corresponds to drag coefficient $\eta_{\rm BS} = \phi_0^2 \sigma_n / 2\pi c^2 \xi^2$, where σ_n is the normal state conductivity. Hence, $\eta_L/\eta_{\rm BS} \sim \sigma/\sigma_n \kappa^2$. Again, this ratio is not necessarily small in gapless materials; see also Ref. [15].

The opposite limit, $v^2/v_c^2 \gg 1$, is hardly realistic at low T's. However, at high T's both λ and the normal excitation conductivity increase when approaching T_c, while v_c drops; see Eq. (16). Hence, the ratio v^2/v_c^2 may become large. One sees that in this case the dissipation is not proportional to v^2:

$$w \approx \frac{\phi_0^2 \sigma v_c^2}{4\pi c^2 \lambda^2} \ln \frac{v}{v_c}. \tag{24}$$

Therefore, at high temperatures the London dissipation is not analogous to that of the viscous flow, $w \propto \ln(v/v_c)$.

It is worth noting that there are situations when the vortex velocities are very high. An example is flux avalanches observed in thin YBCO films with velocities up to 5×10^6 cm/s [16]. Also, very high vortex velocities were recently recorded in Pb films [17].

III. DISSIPATION BY THE MOVING LATTICE

It was mentioned above that the interaction of moving vortices differs from the static situation. Finding the VL structure of moving VLs is in general complicated, one of the reasons being that the energy is no longer a thermodynamic potential with a minimum at the structure of the moving lattice even at a constant velocity. Instead one has to consider the dissipation and use the principle of minimum entropy production [8].

The dissipation power per unit volume is given by

$$W = \sigma \frac{B}{\phi_0} \int_{\rm cell} d\boldsymbol{r} |E|^2 = \sigma \sum_{\boldsymbol{G}\neq 0} |E(\boldsymbol{G})|^2, \tag{25}$$

where the sum is over the reciprocal lattice $\boldsymbol{G}$. The Fourier components of $\boldsymbol{E}$ are given by Eqs. (17) and (18) in which ϕ_0 should be replaced with B and $\boldsymbol{k} \to \boldsymbol{G}$ (see, e.g., Ref. [18]):

$$W = \frac{\sigma B^2 v^2}{c^2} \sum_{\boldsymbol{G}\neq 0} \frac{G_x^2}{G^2[(1+\lambda^2 G^2)^2 + G_x^2 v^2 \tau^2]}. \tag{26}$$

In intermediate fields $H_{c1} \ll H \ll H_{c2}$ one disregards 1 relative to a large $\lambda^2 G^2$. Besides, even for $v \sim v_c$, the last term in the denominator is small. Indeed, $v_c \tau/\lambda^2 G \sim 1/\lambda G \ll 1$. However, the evaluation of W is still a problem because the VL is affected by motion. Therefore, one has to figure out first what structure the moving lattice adopts and then evaluate the dissipation W.

Since the energy can no longer be used as a thermodynamic potential with a minimum at the moving VL structure, the only solution is to use the principle of "minimum entropy production," i.e., to minimize the dissipation power W. The problem of W minimization with respect to different VL structures is challenging: the VL cell is determined by two lattice vectors under the restriction of flux quantization. In other words, one has to minimize the sum (26) with respect to variation of three parameters. Unfortunately, this sum has

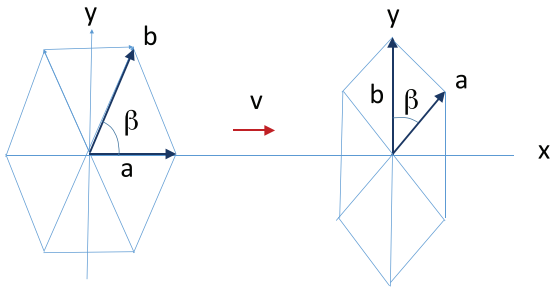

FIG. 2. Structures A at the left and A' at the right.

many local minima, results may depend on initial guesses, and the numerical problem becomes massive.

One can pose a less ambitious question. The calculations based on TDGL near $H_{c2}(T)$ for clean materials have shown that when the VL moves fast enough, it has one of the unit cell vectors along the velocity $\boldsymbol{v}$ [8]. This result holds in the presence of disorder as well, where VL adopts this structure at large velocities of the flux flow [7]. The TDL has an advantage of applicability at all temperatures and in fields well under H_{c2}. Hence, it is of interest to see whether a relatively simple TDL gives results compatible with previous work.

Below, two structures of moving VLs are compared: the structure A, with one of the cell vectors parallel to $\boldsymbol{v}$, and the structure A', where one of the cell vectors is normal to $\boldsymbol{v}$. Both structures are assumed to consist of isosceles triangles, as shown in Fig. 2; this is imposed for A' by necessity in order to have the x axis as a symmetry plane; for A the y axis is assumed to be a symmetry plane. One readily obtains the reciprocal lattice for A:

$$G_x = \frac{2\pi}{\lambda}\sqrt{\frac{h\tan\beta}{2}}\,m, \quad G_y = \frac{2\pi}{\lambda}\sqrt{\frac{2h}{\tan\beta}}\left(n-\frac{m}{2}\right), \tag{27}$$

where m,n are integers, $h = B\lambda^2/\phi_0$, and β is the angle between unit cell vectors; see Fig. 2. The reciprocal lattice for the structure A' is

$$G'_x = \frac{2\pi}{\lambda}\sqrt{\frac{2h}{\tan\beta}}\left(m-\frac{n}{2}\right), \quad G'_y = \frac{2\pi}{\lambda}\sqrt{\frac{h\tan\beta}{2}}\,n. \tag{28}$$

These structures are determined by two parameters, h and β (the cell area and the angle between cell vectors).

To compare the dissipation for these two VLs, one has to evaluate the sum (26). This sum is slowly convergent, so that, when calculated numerically, it will depend on a summation domain chosen for m and n. On the other hand, within the London approach, there is no sense in extending the summation to $G > 1/\xi$. To make this truncation smooth, one adds to the summand a factor $e^{-G^2\xi^2}$ and calculates the dimensionless sum

$$S = \sum_{\boldsymbol{g}\neq\boldsymbol{0}} \frac{g_x^2\, e^{-g^2/\kappa^2}}{g^2\left[(1+g^2)^2+4g_x^2u^2\right]}, \tag{29}$$

where $\boldsymbol{g} = \lambda\boldsymbol{G}$, $u = v/v_c$, and $\kappa = \lambda/\xi$ is the GL parameter. It turns out that the quantity $S(\beta,h,u)$ is nearly velocity-independent in the range $0 < u < 2$. Numerically evaluated $S(\beta,u)$ for a fixed h is shown in Fig. 3. Hence, the London

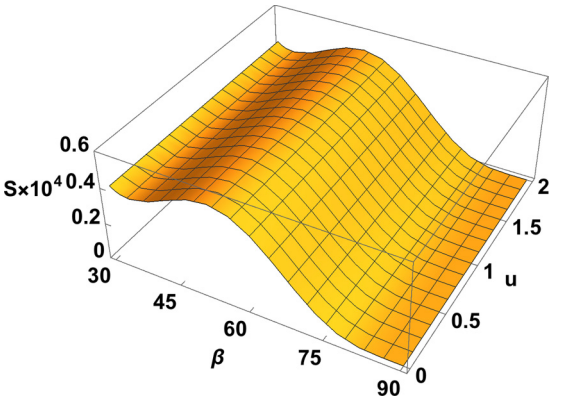

FIG. 3. The sum $S \times 10^4$ vs angle β in degrees and velocity $u = v/v_c$ for $h = 3$ and $\kappa = 10$. S is nearly u-independent in the interval of velocities chosen.

contribution to the dissipation power,

$$W = \frac{\sigma B^2}{c^2} S v^2, \tag{30}$$

is proportional to v^2 as in a viscous flow. The drag coefficient, however, depends on S, i.e., on the VL structure (the angle β and the field h).

The quantity S is plotted in Fig. 4 for $h = 3$, $\kappa = 10$, and $u = 0.5$ for structure A (solid line) and A' (dashed line). For $\beta > 60^\circ$, the dissipation is clearly the lowest when one of the unit cell vectors is parallel to the velocity, the structure A; this result was obtained in Ref. [8] with time-dependent GL theory. This demonstrates that the time-dependent London model works qualitatively well, shortcomings (the cores are out) of the London approach notwithstanding, and with the added bonus of arbitrary temperatures and simplicity.

It is worth noting that, as shown in Fig. 4, the London drag vanishes when $\beta \to 90^\circ$ for the structure A. This means that the principle of minimum entropy production pushes the system to a structure made of isosceles triangles with a shrinking base a and a large distance $\approx b$ between rows parallel to $\boldsymbol{v}$; see the left panel of Fig. 2. Such a structure would look like a system of vortex chains parallel to $\boldsymbol{v}$ with interchain separation $b\sin\beta \gg a$. Physically, dissipation for this structure is low

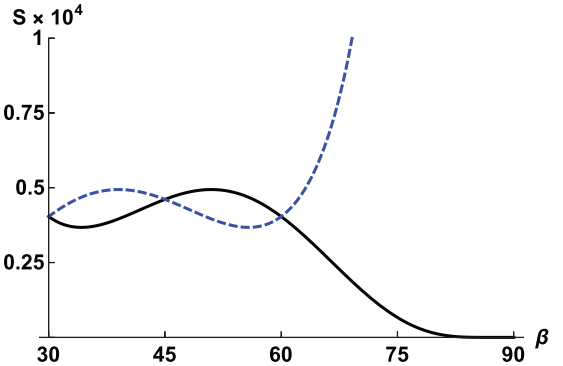

FIG. 4. The sum S vs angle β for $h = 3$ and $\kappa = 10$. The solid curve is for the structure A, the dashed one is for A'. Clearly, for $\beta > 60^\circ$ the dissipation of the structure A is less than that of A'.

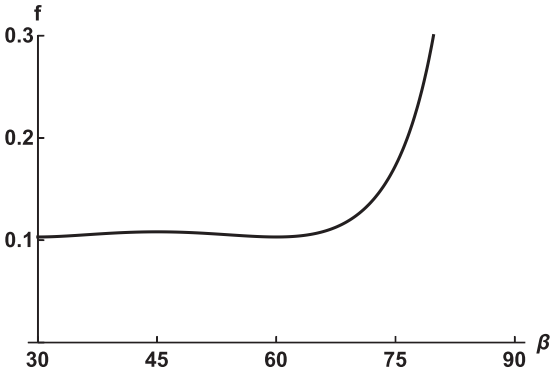

FIG. 5. Dimensionless energy $f = (8\pi\lambda^4/\phi_0^2)\,F$ of the structure A vs angle β for $h = 3$, $\kappa = 10$.

because in dense chains the x dependence of the magnetic-field distribution is weak. When such a structure moves along x, the time derivative of the field in the laboratory frame $\partial_t h = -v\,\partial_x h \to 0$, i.e., the induced electric field would also be small. One can say that the system has a tendency to move along the channels (chains) with low dissipation.

The trend to transform VL to a chain structure, i.e., to larger β, will, however, be opposed by an increase of the VL energy for strong deviations from the static hexagonal arrangement. The energy of interacting vortices within the structure A can be estimated for intermediate fields [19]:

$$F = \frac{B^2}{8\pi}\sum_{\boldsymbol{G}\neq\boldsymbol{0}}\frac{1}{1+\lambda^2 G^2} \approx \frac{\phi_0^2}{8\pi\lambda^4}h^2\sum_{\boldsymbol{g}\neq\boldsymbol{0}}\frac{e^{-g^2/\kappa^2}}{g^2}. \tag{31}$$

The quantity $f = (8\pi\lambda^4/\phi_0^2)\,F$ evaluated numerically is shown in Fig. 5 for $h = 3$, $\kappa = 10$. A sharp increase of $f(\beta)$ for $\beta > 60^\circ$ and divergence at $\beta \to \pi/2$ suggest that β cannot approach $\pi/2$ since the VL energy there will exceed the condensation energy. For a clean material at low temperatures and intermediate fields, one can roughly estimate the energy of a hexagonal VL as

$$F_0 \approx \frac{\phi_0 B}{32\pi^2\lambda^2}\ln\frac{H_{c2}}{B}, \quad \lambda^{-2} = \frac{8\pi e^2 N(0) v_F^2}{3c^2}, \tag{32}$$

where e is the electron charge, v_F is the Fermi velocity, and $N(0)$ is the density of states per spin. The condensation energy in this case is $F_c = N(0)\Delta^2(0)/2$. One easily obtains that the ratio $F_0/F_c \sim B/H_{c2}$ is not a very small number. That means that formally the energy of a deformed VL can easily approach the condensation energy.

As an example, equating the ratio of the condensation energy $H_c^2/8\pi = \phi_0^2/64\pi^3\lambda^2\xi^2$ at high temperatures to the energy (31), one obtains $f(\beta) = \kappa^2/8\pi^2 \approx 1.27$ for the parameters of Fig. 5. One readily finds numerically that this corresponds to the maximum possible $\beta \approx 86^\circ$, which corresponds to the separation between vortices within the chain $a \approx 0.216\lambda$, whereas the interchain distance $b \approx 1.54\lambda$.

IV. SUMMARY

Thus, it is demonstrated that the formalism of time-dependent London equations can be employed to consider dynamic problems of type-II superconductivity provided the order-parameter modulus can be considered constant. The linear TDL approach is much simpler than, e.g., the nonlinear time-dependent GL. As in the static case, the TDL provides a simple all-temperatures tool to address such problems as moving vortices and vortex lattices.

The TDL is based on the notion that in t-dependent situations, the current consists of normal and superconducting parts; see Eq. (4). What follows is a diffusion-type equation (5) for the magnetic field. It is shown that the field distribution of a moving vortex is distorted. It contracts and loses cylindrical symmetry; see Eqs. (13)–(15) and Fig. 1.

The t-dependent field distribution of a moving vortex gives rise to an extra dissipation not included in the Bardeen-Stephen core dissipation. The London dissipation is usually small but it may constitute a considerable part of the core dissipation in gapless materials. It is also worth noting that as $T \to T_c$, the velocity separating slow and fast motion $v_c = c^2/2\pi\sigma\lambda$ becomes small. Then if $v > v_c$, the London dissipation is $\propto \ln(v/v_c)$, i.e., the analogy with slow viscous flow is lost.

Distortions of the field distribution of single moving vortices lead to distorted intervortex interactions and therefore to a change in the vortex lattice structure. The effects of disorder were considered in Ref. [7]; it turned out that at large velocities the moving VL adopts the structure with one of the lattice vectors along the velocity, the same result as in clean systems studied within TDGL in Ref. [8]. This effect was seen in a few experiments [3–6]. Employing the principle of minimal dissipation in a stationary state, it is shown that TDL can reproduce this result without the temperature restrictions of TDGL, see also remark [20].

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