

## GUIDED ELASTIC INTERFACE WAVES FOR CERAMIC JOINT EVALUATION

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### INTRODUCTION

Because of their excellent thermal and wear properties, structural ceramics are finding increasing use in applications that have traditionally been reserved for metals. Since many ceramics remain stable at temperatures well in excess of the melting points of virtually all of the common structural metals, one such application is in high-temperature engines, where the relatively low weight of ceramics provides an additional advantage over such competitors as refractory metals. Unfortunately, with the relatively low fracture toughness and poor machinability of ceramics, practical designs, at least for the near future, will probably consist of ceramic liners attached to metal substrates, thereby combining the wear and thermal properties of ceramics with the strength of metals.

This configuration introduces new problems, however, since one must now guarantee the integrity of the ceramic-to-metal joint as well as that of the component parts themselves. The presence of nonbonds can easily be detected by high-frequency ultrasonics, but even for joints which exhibit no demonstrable degree of nonbond, it is well known that failure can still occur, sometimes at stresses well below the expected failure load. This suggests that, while the component parts are indeed bonded and local stress is continuous across the interface, the strength of the bond is reduced. One would therefore like to probe the joint region, preferably nondestructively, in a manner such that the local strength of the bond, not just the simple presence or absence of bond, could be assessed. Another aspect of this problem is that the properties of either ceramic or substrate (or both) may vary in the region adjacent to the bond, and the techniques described above are poorly adapted to the determination of this variation. An example of this behavior is afforded by oxide ceramics, which typically exhibit an oxygen depletion region for  $\sim 1$  mm from the bond when brazed with certain braze filler metals. The elastic properties in the depletion region are currently inferred by diamond indentation; this technique is clearly destructive.

In considering possible nondestructive techniques to achieve the goals of bond strength determination and assessment of the elastic properties of the materials in and adjacent to the bond, it is clear that conventional elastic bulk waves will be of limited usefulness. However, if one could propagate an elastic wave along the bond layer, and which was evanescent in the materials on either side of the bond, then both goals could possibly be achieved simultaneously.

In the case of a ceramic joint, we have a three-layer solid, and it appears that the general case has not been previously examined in nondestructive testing. We will therefore briefly describe the theory of elastic wave propagation in the braze layer of such a structure, and the solutions to the equations of motion will be developed numerically from the theory. A more detailed treatment of the problem is given elsewhere [1].

## THEORY

Figure 1 shows the geometry of the problem. The interface wave is assumed to have no y-dependence and to propagate in the x-direction in a braze layer of thickness  $2h$ . We assume that all materials are linear, homogeneous, and isotropic solids. The equation of motion to be satisfied in each medium is [2]

$$\mu \nabla^2 \vec{S} + (\lambda + \mu) \nabla(\nabla \cdot \vec{S}) = \rho \partial^2 \vec{S} / \partial t^2 \quad (1)$$

where  $\vec{S}$  is the particle displacement,  $\mu$  and  $\lambda$  the Lamé elastic constants, and  $\rho$  the density. Now  $\vec{S}$  can be defined in terms of the usual potentials:

$$\vec{S} = \nabla \phi + \nabla \times \vec{\psi} \quad (2)$$

The displacement thus contains an irrotational and a solenoidal component, which give rise to compressional and shear waves, respectively. We seek solutions whose x-dependence is of the form  $\exp(ikx)$  and which are independent of y. The latter condition requires that  $\vec{\psi} = (0, \psi_y, 0) = \psi$ . We also assume that the solutions are time harmonic of the form  $\exp(-i\omega t)$ . Thus the potentials satisfy

$$d^2 \phi / dz^2 = \alpha^2 \phi \quad ; \quad d^2 \psi / dz^2 = \beta^2 \psi \quad (3)$$

where  $\alpha^2 = k^2 - k_1^2$ ,  $\beta^2 = k^2 - k_t^2$ . The solutions to these equations are of the form

$$\begin{aligned} \phi_m &= A_m \exp(\pm \alpha_m z) \exp[i(kx - \omega t)] \\ \psi_m &= B_m \exp(\pm \beta_m z) \exp[i(kx - \omega t)] \end{aligned} \quad (4)$$

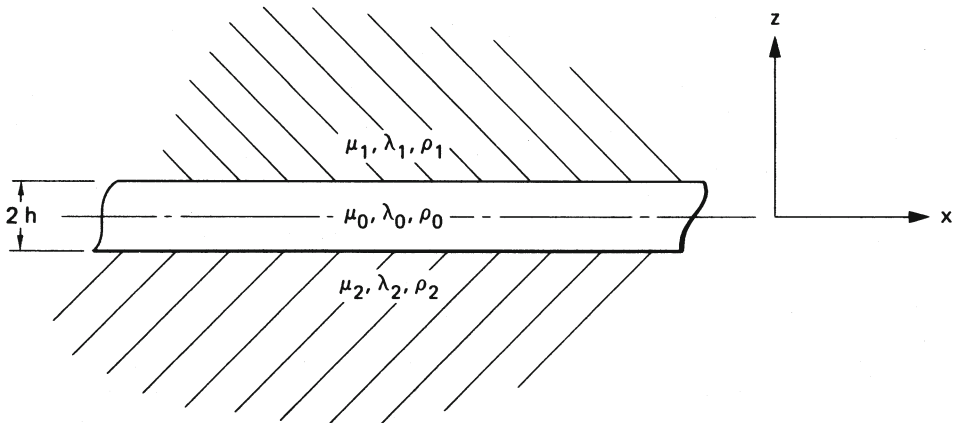


Fig. 1. Geometry for the guided elastic wave problem.

where the subscript  $m$  denotes the region and  $A_m$  and  $B_m$  are arbitrary coefficients, and where the positive sign is taken in the lower half space and the negative in the upper.

The displacements and stresses can be written in terms of the potentials, and with this simplification the solution in each region can be written down straightforwardly. The boundary conditions require that both the normal and tangential displacements and stresses must be continuous at  $z_h$ . Applying these conditions at each interface, the secular equation is obtained from an  $8 \times 8$  determinant. This determinant reduces to that for two solids in perfect contact when the layer thickness goes to zero. Hence, we anticipate that the zero-thickness solution will be a Stoneley wave and that no solution will exist in this limit when the solids are identical.

## NUMERICAL RESULTS

As mentioned previously, we solve the secular equation numerically, using a general FORTRAN program to locate all roots of the secular determinant. In locating these roots, we note that there is a branch point associated with each square root which defines the wave vectors  $\alpha$  and  $\beta$  [see Eq. (3)]. Thus, the roots of the secular equation will lie on several Riemann sheets. In defining the potentials as we have in Eq. (4), however, the real roots (corresponding to waves evanescent in the surrounding solids) should lie on the sheet corresponding to all positive branches of the square root functions. In addition, leaky modes should occur on the sheet corresponding to the negative branch of the wave vector for the potential  $\psi$  in the medium in which the leaky wave propagates. The complicated nature of these roots, including the previously incorrect identification of the limiting cases of some of the roots, has been pointed out for the case of Stoneley waves by Pilant [3]. We emphasize that we did not search each Riemann sheet for all possible roots, although this could certainly be done. Hence, there may be other modes than those we report.

Roots of the secular determinant for the three-layer solid were determined for cases of interest to the ceramic joining program. We began by considering the case of zirconia coupons brazed with a commercial silver-based alloy. The properties of the zirconia are:  $\rho = 5.66 \text{ g/cm}^3$ ,  $C_1 = 7.04 \text{ km/s}$ , and  $C_t = 3.74 \text{ km/s}$ . Unfortunately, the material properties of the braze were not available, and the material itself was only about  $75 \mu\text{m}$  thick. Since the major constituent is silver, however, we simply assumed the properties to be those of silver, viz:  $\rho = 10.5 \text{ g/cm}^3$ ,  $C_1 = 3.6 \text{ km/s}$ , and  $C_t = 1.59 \text{ km/s}$ .

We first searched for guided (undamped) waves by restricting the analysis to the real velocity axis. The search covered the range  $0 < C_r < 4 \text{ km/s}$ . The braze layer half thickness and frequency range were chosen to yield a normalized thickness (layer half thickness divided by the wavelength of a shear wave in the braze) of  $0 < h/\lambda < 2$ . For a total braze layer thickness of  $75 \mu\text{m}$ , the frequency at  $h/\lambda = 1$  would be  $\sim 42 \text{ MHz}$ .

Figure 2 shows the dispersion curves obtained for guided waves in the zirconia joint. The permissible modes are thus alternating pure symmetric or antisymmetric waves. Considering first the results for  $h/\lambda > \sim 0.5$ , there are five possible modes. The high-frequency limit of these modes tends toward the phase velocity of a shear wave in the braze material. This is to be expected, since the high-frequency limit corresponds to infinite thickness of the braze layer; hence, the result should approach a bulk wave in the braze. For each mode, there is a low-frequency cutoff below which the mode cannot propagate. For the first three of these

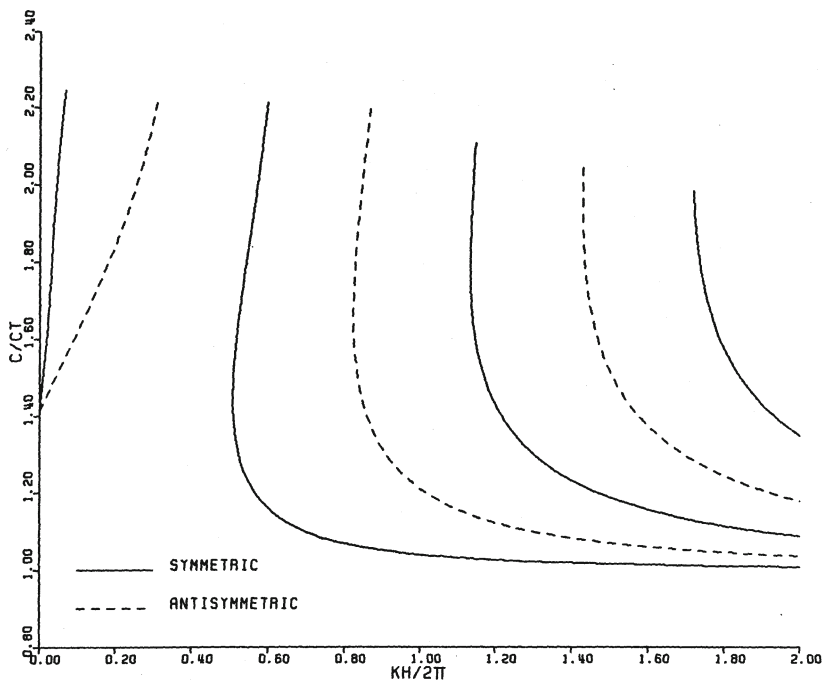


Fig. 2. Dispersion curves for guided waves in a zirconia-zirconia ceramic joint.

modes, however, there is a small range of the normalized thickness (frequency) near cutoff in which the normalized phase velocity is double valued. Thus, for the lowest symmetric mode of the five, the region of normalized thicknesses  $0.5 < h/\lambda < 0.6$  (values approximate) yields two phase velocities for each thickness (frequency). The difference in these waves is not apparent until the group velocities are examined. For the lower part of each curve, the group velocity is positive and less than the phase velocity (normal dispersion). For the upper part of the curve, however, the group velocity is negative - the wave propagates backward (i.e., opposite to the phase velocity). This decidedly strange behavior has been reported for Lamb waves [4], where backward propagating waves were theoretically predicted and experimentally observed.

Below  $h/\lambda = 0.5$ , Fig. 2 indicates that two waves are possible. However, as  $h \rightarrow 0$ , the secular determinant reduces to that for two solids in perfect contact, thus the waves, if any, should approach the Stoneley wave velocity. Since the material on either side of the braze joint is zirconia in Fig. 2, the distinction of an interface is lost in the limit of zero braze thickness, and no wave should exist. This discrepancy is resolved by examining the amplitudes of the two low-frequency waves in Fig. 2. According to the computer program, as  $\omega \rightarrow 0$ , the symmetric and antisymmetric waves both approach a normalized phase velocity of 1.414 ( $=\sqrt{2}$ ), and the amplitude of each wave approaches zero. In the limit, both waves vanish, and no interface waves exist.

The group velocities of the two low-frequency waves in Fig. 2 are positive and greater than the phase velocities (anomalous dispersion).

The simple zirconia joint thus appears to harbor an astonishing panoply of possible interface waves.

Since we would expect the zero-thickness solutions of the three-layer problem to reduce to that of Stoneley waves, we next chose a material combination which is known to support such waves. We therefore chose the case of titanium bonded to iron by a thin braze layer. The existence of a Stoneley wave at a titanium-iron interface is known [5], and this material combination also occurs in the zirconia transition joint. We assume that the braze layer is silver, and the material properties of the titanium are  $\rho = 4.44 \text{ g/cm}^3$ ,  $C_l = 6.11 \text{ km/s}$ ,  $C_t = 3.27 \text{ km/s}$ , while those of iron are  $\rho = 7.86 \text{ g/cm}^3$ ,  $C_l = 5.89 \text{ km/s}$ , and  $C_t = 3.21 \text{ km/s}$ . We again restrict the search for roots to the real velocity axis in order to determine if guided waves may exist. The range for the search was  $0 < C_r < 4$  as before, and the frequency range was chosen to provide a normalized thickness range of  $0 < h/\lambda < 2$ . Figure 3 shows the results, which are quite similar to the case of zirconia bonded to zirconia. The two vanishing waves are again present, and the zero-thickness phase velocities of these waves is again the square root of 2. The curves are different from the zirconia joint, however, for finite layer thicknesses.

The major difference between Figs. 2 and 3 is shown in the inset of the latter figure. For the titanium-silver-iron joint, there is an additional wave which exists at small values of the normalized thickness and whose amplitude remains finite as the layer thickness approaches zero. The phase velocity approaches the value 3.20865 km/s as the thickness is reduced, which is the Stoneley wave velocity for the titanium-iron structure. The wave is now dispersive, however, and survives only in the range  $0 < h/\lambda < \sim 0.04$ . Above this value, the root leaves the Riemann sheet.

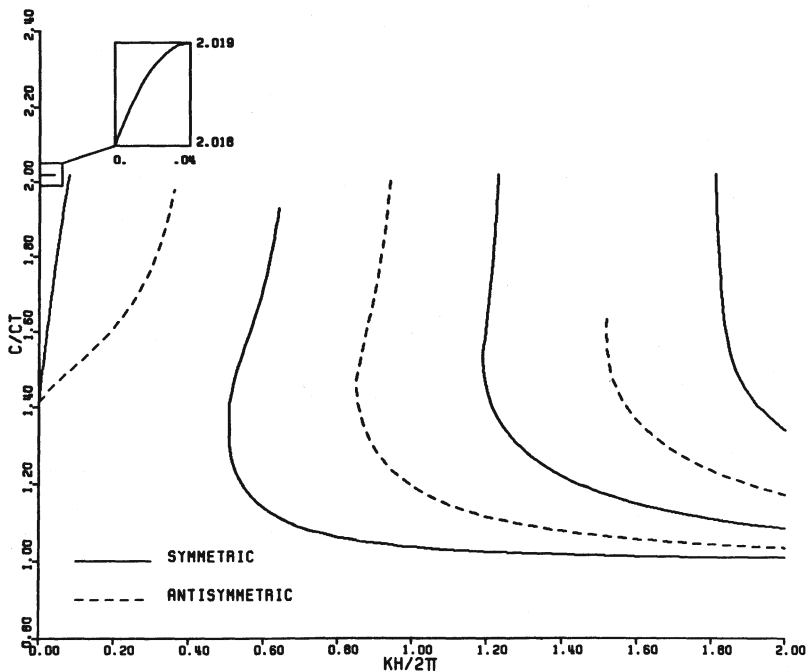


Fig. 3. Dispersion curves for guided waves in a titanium-iron joint.

We next examined the possibility of leaky waves in the braze layer of a ceramic joint. We initially chose the zirconia-silver-zirconia joint, although we did not expect any leaky modes for this structure. The search was limited to the range  $0 < C_r < 4$  and  $-0.3 < C_i < 0$ . As expected, no complex roots were found.

The next configuration chosen was that of a titanium-silver-iron joint. The search was carried out in the range described above, and waves were found which leaked into the iron, the boundary solid with the slower shear velocity. The calculations have not been completed and would require a three-dimensional plot for proper display, but the existence of leaky waves in a practical attenuation range appears established. We are planning to repeat the calculations for various ceramic-ceramic joints.

## BONDING MODEL

The *raison d'être* for the guided wave studies was the possible establishment of a technique to assess bond strength directly for typical ceramic joints. A model which appears to achieve this result for two solids with varying degrees of bonding at the interface has been presented [6,7]. The results were achieved by introducing a thin viscoelastic layer between the two solids and computing the interface wave velocity as a function of the limiting value of the ratio of viscosity to layer thickness as the thickness goes to zero. If this ratio is unbounded as the layer thickness goes to zero, the problem reduces to two solids in welded contact, and the interface wave velocity is the Stoneley wave velocity. If the ratio goes to zero in the limit of vanishing thickness, the interface wave velocity is just the appropriate Rayleigh velocity. For intermediate values of the ratio, corresponding to a loosely bonded interface, the interface wave velocity varies smoothly between the two limiting values. The implication is thus that the degree of bonding can be inferred from measurement of the interface wave velocity. In addition, Murty's results [7] indicate that for the loosely bonded case, the interface wave is leaky; thus, the phase velocity can be determined by measuring the critical angle necessary to excite the wave.

In the present case, we introduced a thin viscoelastic layer between the ceramic and the braze layer, thus giving two finite layers between semi-infinite half spaces. The secular determinant for this case is thus  $12 \times 12$  and has been determined. Our intention is to study the effect of this layer on the previously computed modes, depicted in Figs. 2 and 3, as a function of the bonding parameter defined by Murty, but this work has just begun.

## EXPERIMENT

In order to confirm the results presented in Figs. 2 and 3, which are trapped modes confined to the braze layer of the ceramic joint, we designed the sample shown in Fig. 4. The ceramic materials are zirconia, and the geometry of the sample was chosen to permit surface waves to be generated on one of the ceramic coupons, which would then excite guided waves in the braze layer, and be detected by a similar surface wave transducer on the other side of the joint. Before attempting to excite guided waves, however, the braze layer was first interrogated with high-frequency compressional waves focused at the interface in order to determine whether gross nonbonds were present. The results are shown in Fig. 5, where the



dark regions are essentially completely unbonded. The sample was thus totally unsuitable. Destructive analysis of the joint indicated that the problem was due to extreme porosity in the braze layer.

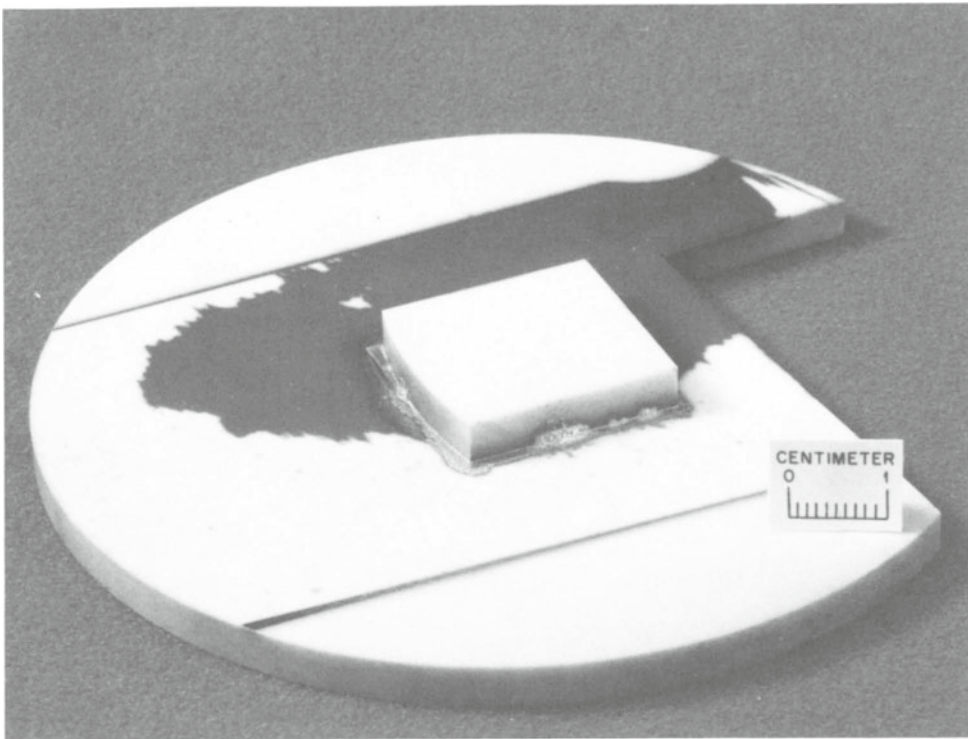


Fig. 4. Zirconia-zirconia ceramic joint for guided wave studies.

In the available time, two additional samples of similar design were procured. Both were found to have similar problems of porosity and one coupon was severely cracked as well. These results are not typical of the joints which have been produced on this program, and we do not know why there was difficulty in fabricating a suitable specimen. We were thus unable to verify experimentally the results given in Figs. 2 and 3.

#### SUMMARY

The secular determinant for elastic waves propagating in the center layer of a general three-layer solid was obtained. A FORTRAN program was written to find the roots of the secular equation, and both guided and leaky waves were found for the case of titanium bonded to iron with a thin silver layer. In the case of identical solids bonded by a thin layer, no interface waves were found in the limit of vanishing layer thickness. For different solids known to have a Stoneley wave at the welded interface, a dispersive Stoneley wave was found. A possible bonding model was examined by introducing a thin viscoelastic layer between one of the bounding solids and the center layer. The secular determinant was obtained for this case, but the analysis has not yet been completed.

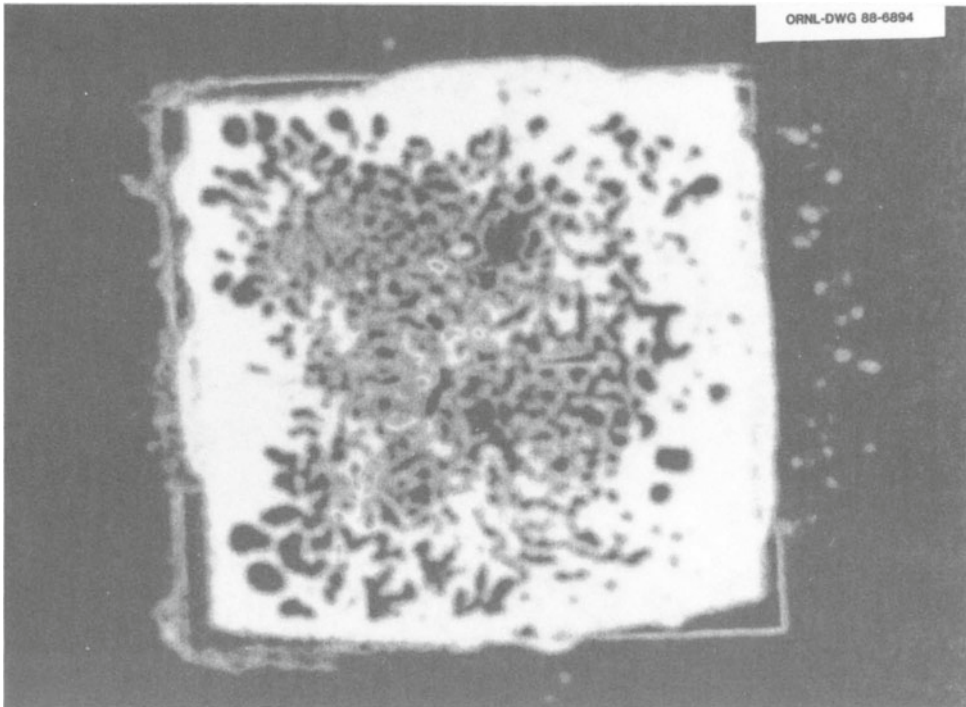


Fig. 5. High-frequency ultrasonic scan of the joint region in the sample of Fig. 4.

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