ANALYSIS OF GUIDED WAVES IN A BILAYERED PLATE

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INTRODUCTION

Inspection of coated material is vital in order to ensure the integrity of the protective barrier. In some cases, the inspection process is complicated by the fact that the surface of the protective coating may not be accessible, thus the inspection must proceed with only access to the surface of the opposite side (this will be referred to as the inner surface). One method which can be applied in such a situation is the excitation of guided waves or Lamb waves in the coated material. Lamb waves excited from the inner surface will sense the variation in the coating conditions as well as flaws in the steel plate. Therefore for a correct and unambiguous interpretation of Lamb wave data for corrosion-related flaws, the effects of the coating and how they differ from the effects of corrosion-related flaws must be understood. To this end this paper will concentrate on the effects of the coating and its influence on dispersive characteristics of a soft layer bonded to a steel plate. By calculating the dispersion relations for a bare and coated steel plate and comparing the calculated results, unique new modes are seen to emerge in the coated plate. In addition, the coated plate is also modeled as a thin layer on a halfspace allowing a comparison between the problem of a coated halfspace and a coated plate. The comparison demonstrates that only limited information can be obtained by treating the problem as a layered halfspace. More detailed information can be obtained by treating the problem as a coated plate of finite thickness.

The dispersion relation for Lamb waves in a single isotropic and homogeneous medium is found in a few monographs[1,2] and those for layered substrate media were recently given by several authors[3-5]. There has been some prior investigations concerning soft layers on stiff plates. The work of Jones[6] presents phase velocities vs wavenumber curves for various coating thicknesses. Farnell and Adler[7] presented similar curves with additional information on mode shapes for two particluar modes which nearly intersect. In this paper we give a detailed discussion of guided waves in a bilayered plate when both layers are homogeneous and isotropic.

DISPERSION RELATIONS

The formulation of the two-layered medium is straightforward. We follow the notation of Viktorov[1]. The positive x-axis extends to the right along the interface and the positive z-axis extends upwards perpendicular to the interface. The upper layer is the coating

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with a thickness of h_1, density ρ_1, and Lame's constants μ_1 and λ_1. Similarly, the bottom layer is the steel plate with the constants $-h_2$, density ρ_2, and Lame's constants μ_2 and λ_2. The indices 1 and 2 refer to the coating and steel media, respectively. The waves propagating in the x-z plane can be described by two scalar potentials ϕ and ψ as follows:

$$\phi_1 = [A \cosh q_1 (z-h_1) + B \sinh q_1 (z-h_1)]\, e^{i(kx-\omega t)} \quad (0 \le z \le h_1) \tag{1}$$

$$\psi_1 = [C \cosh s_1 (z-h_1) + D \sinh s_1 (z-h_1)]\, e^{i(kx-\omega t)} \quad (0 \le z \le h_1) \tag{2}$$

in the upper coating layer and

$$\phi_2 = [E \cosh q_2 (z+h_2) + F \sinh q_2 (z+h_2)]\, e^{i(kx-\omega t)} \quad (-h_2 \le z \le 0) \tag{3}$$

$$\psi_2 = [G \cosh s_2 (z+h_2) + H \sinh s_2 (z+h_2)]\, e^{i(kx-\omega t)} \quad (-h_2 \le z \le 0) \tag{4}$$

in the lower steel medium. A,B,C,D,E,G, and H are eight undetermined coefficients. Additional intermediate constants and quantities are defined as:

ω = circular frequency

k = wavenumber

$C_{Li} = [(\lambda_i + 2\mu_i)/\rho]^{1/2}$

$C_{Si} = [\mu_i/\rho]^{1/2}$

$k_{Li} = \omega/C_{Li}$

$k_{Si} = \omega/C_{Si}$

$q_i = (k^2 - k_{Li}^2)^{1/2}$

$s_i = (k^2 - k_{Si}^2)^{1/2}$

The longitudinal and transverse displacements components U and W and the normal tractions components σ_{xz} and σ_{zz} in each layer can be written in terms of the potentials for that layer,

$$U_i = \frac{\partial \phi_i}{\partial x} - \frac{\partial \psi_i}{\partial z}$$

$$W_i = \frac{\partial \phi_i}{\partial z} + \frac{\partial \psi_i}{\partial x}$$

$$\sigma_{zx_i} = \mu_i \left[2 \frac{\partial^2 \phi_i}{\partial x \partial z} + \frac{\partial^2 \psi_i}{\partial x^2} - \frac{\partial^2 \psi_i}{\partial z^2} \right] \tag{5}$$

$$\sigma_{zz_i} = (\lambda_i + 2\mu_i) \frac{\partial^2 \phi_i}{\partial z^2} + \lambda_i \frac{\partial^2 \phi_i}{\partial x^2} + 2\mu_i \frac{\partial^2 \psi_i}{\partial x \partial z}$$

The boundary conditions which must be satisfied are:

(1) Traction free surfaces at $z= +h_1$ and $-h_2$, and

(2) Continuity of displacements and tractions at the interface.

Application of these boundary conditions initially yields eight equations with eight unknowns. Furthermore, the first boundary condition allows four of the eight unknowns to be written in terms of the other four, thus reducing the system of eight equations to only four equations and four unknowns. This remaining set of equations has nontrival solutions when the determinant of the coefficient matrix is zero. The elements of this coefficient matrix are found in Appendix A. The dispersion relation for the layered-halfspace geometry can be found in previously cited references for multilayered semi-infinite medium. The coefficient matrix of the half-space geometry can equally well be expressed as a 4 by 4 matrix. The solution to the transcendental equations provides the dispersion relations between the circular frequency and the wavenumber.

DISCUSSION OF RESULTS

The material constants used for these calculations are given in Table 1. The steel plate thickness was kept constant at 10mm, while the coating thickness was varied between 0 and 1mm. Figure 1 presents the results for the dispersion relation for a bare and coated plate with coating to plate thickness ratios of 0.05, and 0.1. In all these figures the ordinate represents the normalized frequency defined as the circular frequency multiplied by the thickness of the steel plate and divided by the shear velocity in the steel. The abscissa stands for the normalized wavenumber which is the wavenumber multiplied by the thickness of the steel plate. In Fig. 1 the first six modes are shown for the bare steel plate and the first seven modes of the two coated plates. The higher order modes have been omitted for clarity. It is interesting to note how the modes for the coated plates match the modes for the bare plate near the cutoff frequencies. The coated plate modes then follow the bare plate modes for a small distance then migrate to the next lower mode. In effect this means that the coated plate modes alternate between symmetric and antisymmetric motion of the bare plate. This will be seen later in plots of the modes shapes of certain modes. Additionally, at a cutoff frequency of 4.41 a new mode is seen for the 1mm coated plate. Similarly, a new mode is seen at 9.85 for the 0.5mm coated plate. The presence of the coating lowers the cutoff frequecies. Thus the cutoff frequency at 8.82 corresponds to a mode of the 0.5mm coated plate that would have been analogous to the higher bare plate mode. These new modes adhere to the same pattern of following the symmetric and antisymmetric modes of the bare plate. New modes are also found at higher frequencies. In Fig. 1, the increase in the thickness of the coating from 0.5m to 1.0mm led to a lower cutoff frequency of the first new mode for the 1.0mm coated plate.

Table 1

Elastic Properties of the Layers

	Thickness (cm)	Density (g/cc)	C_L (km/s)	C_S (km/s)
Steel	1.00	7.86	5.96	3.23
Coating	0-0.1	1.20	2.5	1.00

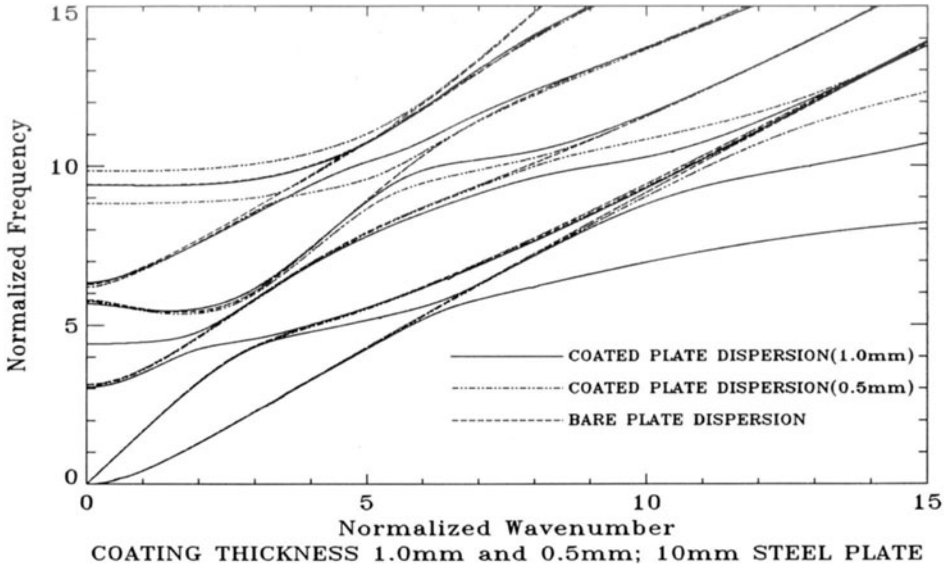

Figure 1 Comparison of the dispersion for the bare steel and the 1.0mm and 0.5mm coated plates

In Figure 2 the phase velocity in km/sec is plotted for the 1mm coated plate(represented by the □ symbol). The phase velocity as a function of frequency for the coated halfspace (represented by the asterisks) is superimposed on the plot of the phase velocity for the layered plate geometry. Note the good agreement between the two geometries for the lower modes. The agreement is expected to deteriorate with increasing order of the mode. But also note the wealth of information above the shear velocity of the steel that the plate analysis provides. The behavior of the modes above the steel's shear velocity should provide more beneficial information for nondestructive evaluation and testing. Further inspection of Fig. 2 revals the asymptotical behavior of the modes. The first two modes show similar behavior to that of a bare steel plate but quickly asymptote to the free surface Rayleigh velocity of the coating(0.94 km/s). The remaining modes first asymptote to the free surface Rayleigh velocity of the steel plate(2.98 km/s) and then evenutally asymptoting to the bulk shear velocity of the coating(1.0 km/s).

Turning to the intersecting modes along the Rayleigh velocity of steel plate. The first instance of a possible intersection is seen approximately at the frequency of 0.7 MHz and is plotted in Figure 3. This figure show the third and fourth modes plotted on an expanded scale. Clearly, from Fig. 3 these modes do not cross. This behavior was also seen by Farnell and Adler[7]. The phase velocity of the third mode continues to decrease to the shear velocity of the coating. The fourth mode on the other hand continues to be asymptotic to the Rayleigh velocity of the steel until it intersects with the fifth mode at approximately 1.19 MHz. This is shown in Figure 4 on an expanded scale. In this figure the modes shapes for the longitudinal displacements are shown for the two modes on either side of the intersection. The points B and C represent the fourth mode. The two points A and D represent the fifth mode. Clearly, the mode shpaes for the fifth mode are the same on either side of the intersection. The modes shapes for the fourth mode have similar behavior except for the sign

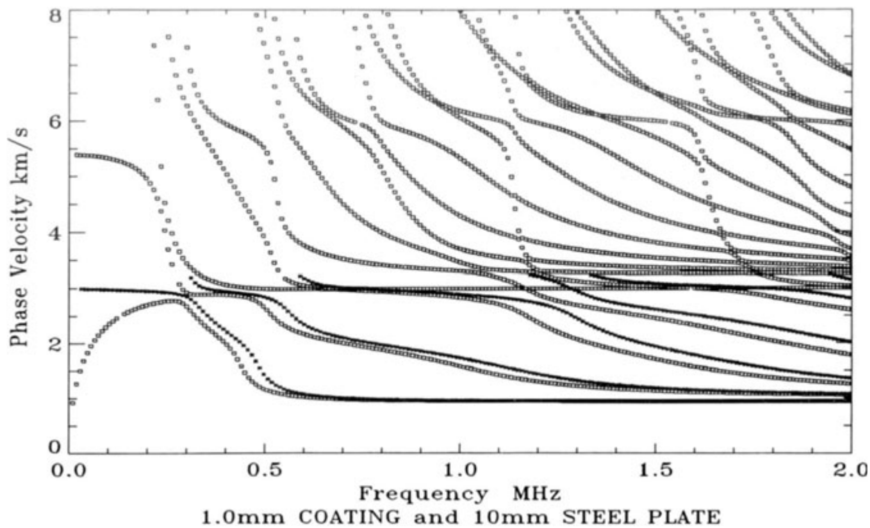

Figure 2 Comparison of dispersion relations for the 1mm coated steel plate and coated half-space

reversal in lower porition of the mode shape in the steel. This reversal of sign is characteristic of the symmetric and antisymmetric mode shapes in a bare steel plate at the Rayleigh velocity as discussed by Viktorov[1] and Farnell and Adler[7]. Apparently at the intersection, the fourth mode is following the symmetric and antisymmetric motions of the bare plate by following the different modes of the bare plate as previously seen in Fig. 1. As another example of the modes intersecting can be seen in Figure 5. This figure shows the intersection of two higher modes at the phase velocity of approximately 6.12 km/s and a frequency of 1.35 MHz. This intersection can be seen in upper left hand corner in Figure 2. Obviously from the matching of modes shapes on either side of the intersection, these modes cross.

CONCLUSIONS

The dispersion characteristics of a coated plate have demonstrated the emergence of new modes. Increasing the thickness of the coating causes these new modes to appear at lower frequencies. Overall, the modes of the coated plate follow those of a bare plate with some of the modes crossing one another. Treating the layered plate as a layered substrate shows the limited information that can be obtained. The rich information that is contained in Fig. 2 can be helpful when trying to interpret the excited Lamb waves in the layered plate. Being able to interpret the Lamb wave data can be helpful in the detection of flaws in the steel plate. More investigations into the detection and sizing of the flaws in both coating and the plate are needed so that Lamb waves can be used for the inspection of coated material.

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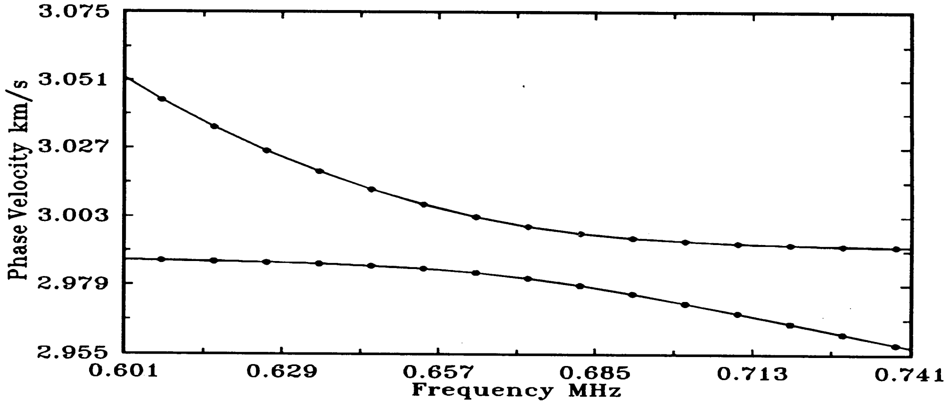

Figure 3 Phase veolcity versus frequency for the 1mm coated plate showing the close proximity of the third and fourth modes

APPENDIX A

These sixteen equations comprise the four by four determinant for the layered plate dispersion relation.

$$a11 = [(2ikq_1s_1)/(k^2+s_1^2)]\sinh s_1h_1 - ik\sinh q_1h_1$$
$$a12 = [(2k^2s_1)/(k^2+s_1^2)]\cosh q_1h_1 - s_1\cosh s_1h_1$$
$$a13 = [(2ikq_2s_2)/(k^2+s_2^2)]\sinh s_2h_2 - ik\sinh q_2h_2$$
$$a14 = s_2\cosh s_2h_2 - [(2k^2s_2)/(k^2+s_2^2)]\cosh q_2h_2$$
$$a21 = q_1\cosh q_1h_1 - [(2k^2q_1)/(k^2+s_1^2)]\cosh s_1h_1$$
$$a22 = [(2iks_1q_1)/(k^2+s_1^2)]\sinh q_1h_1 - ik\sinh s_1h_1$$
$$a23 = [(2k^2q_2)/(k^2+s_2^2)]\cosh s_2h_2 - q_2\cosh q_2h_2$$
$$a24 = [(2iks_2q_2)/(k^2+s_2^2)]\sinh s_2h_2 - ik\sinh s_2h_2$$

$$a31 = \mu_1 2ikq_1(\cosh q_1h_1 - \cosh s_1h_1)$$
$$a32 = \mu_1\left\{(k^2+s_1^2)\sinh s_1h_1 - [(4k^2s_1q_1)/(k^2+s_1^2)]\sinh q_1h_1\right\}$$
$$a33 = \mu_2 2ikq_1(\cosh q_2h_2 - \cosh s_2h_2)$$
$$a34 = \mu_2\left\{(k^2+s_2^2)\sinh s_2h_2 - [(4k^2s_2q_2)/(k^2+s_2^2)]\sinh q_2h_2\right\}$$
$$a41 = \mu_1\left\{[(4k^2s_1q_1)/(k^2+s_1^2)]\sinh s_1h_1 - (k^2+s_1^2)\sinh q_1h_1\right\}$$
$$a42 = \mu_1 2iks_1(\cosh s_1h_1 - \cosh q_1h_1)$$
$$a43 = \mu_2\left\{(k^2+s_2^2)\sinh q_2h_2 - [(4k^2q_2s_2)/(k^2+s_2^2)\sinh s_2h_2\right\}$$
$$a44 = \mu_1 2iks_2(\cosh q_2h_2 - \cosh s_2h_2)$$

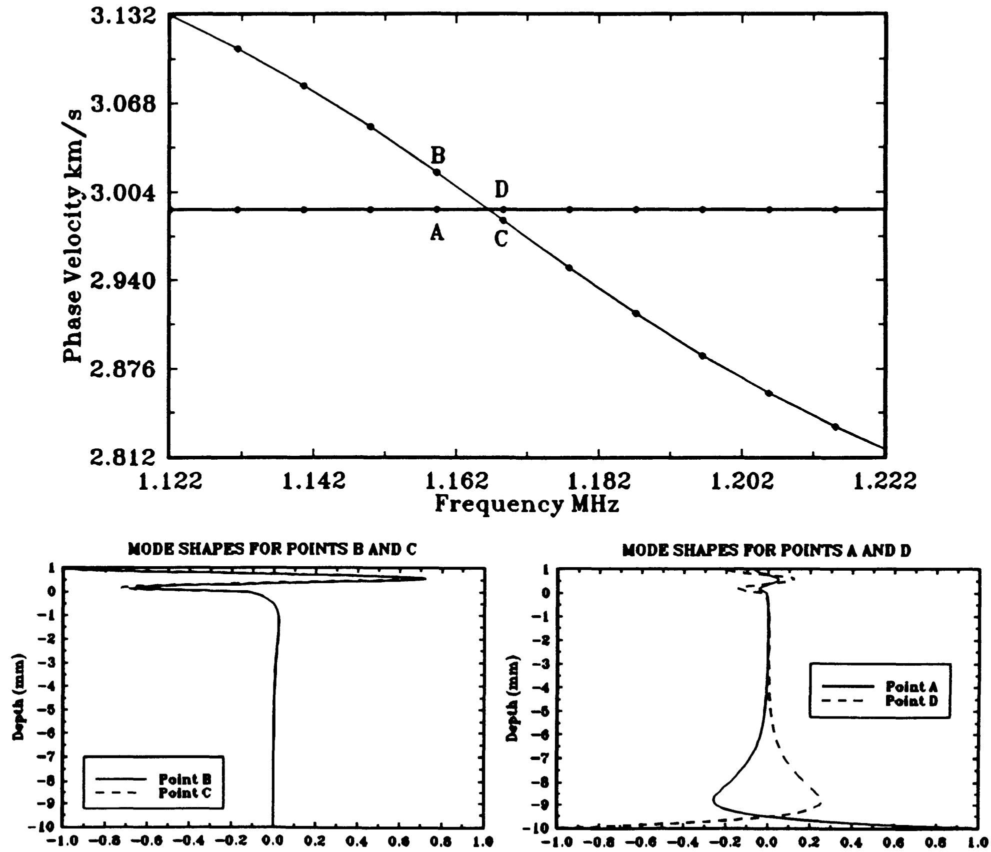

Figure 4 Diagrams showing the intersection of two modes and their x-displacement components on either side of the intersection for a 1mm coated steel plate.

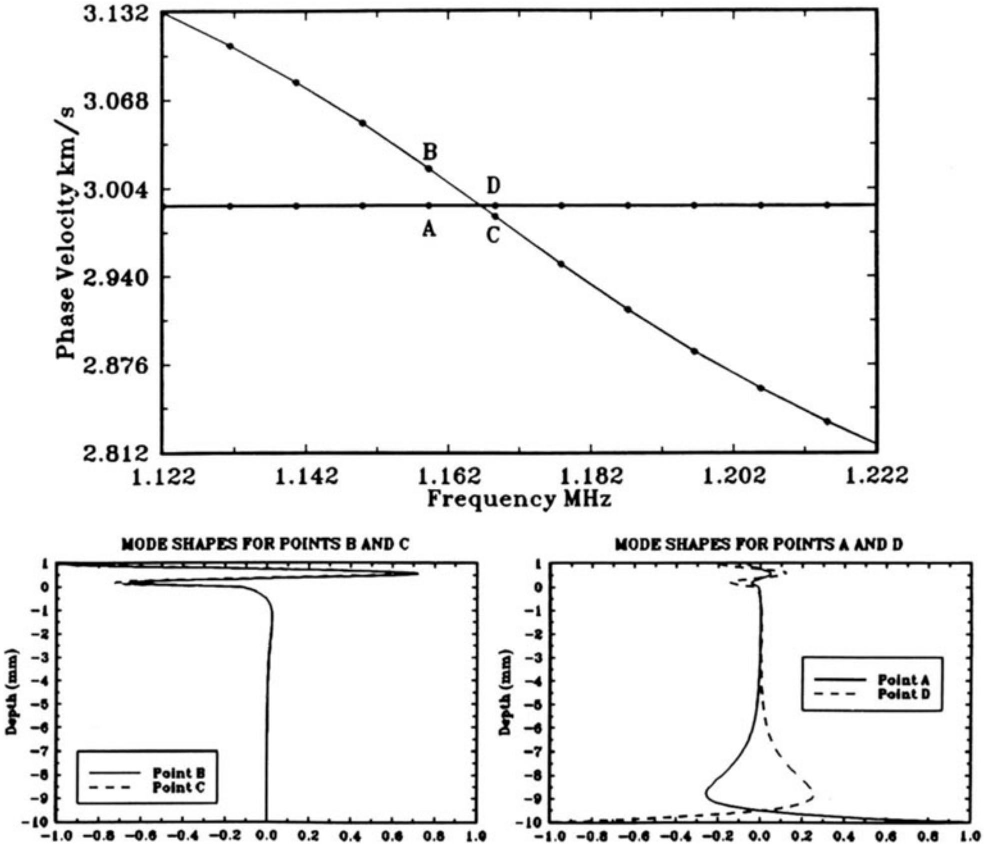

Figure 5 Diagram showing the intersection of two higher order modes and their x-displacement components on either side of the crossing for a 1mm coated steel plate.

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