THE AUTOMATION OF THE BORN INVERSION FOR ULTRASONIC FLAW SIZING

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ABSTRACT

The Born approximation has been widely employed as a basis for determining flaw sizes using individual pulse-echo waveforms together with the assumption of an ellipsoidal flaw geometry. A major difficulty in implementing such algorithms has been the determination of the time delay corresponding to the flaw centroid. However, both the time delay calculation and the flaw size determination itself can be performed in an optimal fashion using statistical estimation techniques with an appropriate error model. We will discuss the application of these techniques to an automated flaw-sizing algorithm requiring a minimum of operator input, and will compare the results obtained by this method with those obtained by previous operator-intensive methods.

INTRODUCTION

A simple inversion algorithm for determining the projected radii of ellipsoidal flaws from individual pulse-echo waveforms has been discussed by Rose et al (1980). This algorithm is based on the Born approximation, and is therefore strictly applicable only to the weakly scattering case. In practice, however, it has been found to give reliable results for strong scatterers (e.g. voids) also, due to its sensitivity to the front-face echo, and hence the algorithm is gaining wide popularity as an NDE tool. The procedure involves two main steps, namely: (1) the determination of the centroid time delay (corresponding to the time at which an echo would have been received from the flaw centroid), and (2) the inversion itself, which yields the characteristic function, from which the projected radius (distance from centroid to front-face tangent plane along an axis corresponding to the incident direction) may be inferred.

An optimal procedure for estimating the centroid time delay has been proposed by Richardson and Elsley (1980). This technique is based on the variation of phase with frequency, at low frequencies, and incorporates a statistical model of the noise. In this paper we employ an extended version of the same technique. In addition, we discuss the application of statistical estimation to part (2) of the procedure, i.e. the estimation of the flaw size itself. The advantages of this technique over the more traditional procedure of calculating the characteristic function are firstly that it avoids the noise-vulnerable deconvolution step, and secondly (and most important) it can, in principle, yield an error estimate for the flaw size.

The goal of this work was to produce a robust automated algorithm for flaw sizing, with a minimum of operator input.

CENTROID TIME-DELAY CALCULATION

It can be shown that the L-L scattering amplitude spectrum for a general scatterer with limited spatial extent in all directions can be expressed as a polynomial with respect to the angular frequency ω (Richardson 1981), i.e.

$$A(\omega) = \sum_{n\ \mathrm{even}} A_n \omega^n + i \sum_{n\ \mathrm{odd}} A_n \omega^n$$

Furthermore, for a scatterer with inversion symmetry, and whose centroid corresponds to the origin of the coordinate system, we have (Richardson 1981):

$$A_3 = 0$$

If the centroid is displaced from the origin by an amount corresponding to round-trip travel time τ, then we can write

$$A(\omega) = A^{(0)}(\omega)\ \exp(i\omega\tau)$$

where superscript 0 has been used to indicate that the quantity is relative to the preferred origin.

Expressing in terms of a polynomial, we have

$$A(\omega) = A_2^{(0)}\omega^2 + iA_2^{(0)}\tau\omega^3 + (A_4^{(0)} - A_2^{(0)}\frac{\tau^2}{2})\omega^4 + \ldots$$

Thus in principle one could determine τ from a measured spectrum by expressing the scattering amplitude as a polynomial with respect to ω, and taking the ratio of the first two terms. In practice, a more efficient procedure is to truncate the series after a few terms, and restrict the frequency range of the measurement equations. This was the approach taken by Richardson and Elsley (1980), who truncated the series at the ω^3 term. There is, however, some advantage in extending the series to the ω^4 term, since A_4 is related to the 2nd moment of the spatial distribution of the scatterer in the Born approximation, and hence gives a useful preliminary indication of the flaw size. It is easy to show that for an ellipsoid of projected radius a, that:

$$A_4^{(0)}/A_2^{(0)} = -(2a/c)^2/10 \tag{1}$$

where c is the speed of sound in the host medium.

An additional advantage of including the ω^4 term is that the τ determination is then based on measurements over a wider frequency range, and hence has made more efficient use of the data.

Our measurement model is then of the form:

$$f(\omega) = p(\omega)(A_2\omega^2 + i\,A_3\omega^3 + A_4\omega^4) + \nu(\omega)$$

where $f(\omega)$ and $p(\omega)$ represent the measured waveform and the transducer response respectively, and $\nu(\omega)$ represents the noise term. The covariance function of ν, representing its expectation value over the statistical ensemble of possible measurements, can be expressed in terms of an assumed noise model, as:

$$C(\omega) = C_0 + |p(\omega)|^2(C_4\omega^4 + C_{10}\omega^{10}) \tag{2}$$

where C_0 and C_4 represent the coefficients of the electrical noise and grain noise terms respectively, and can be determined from a measured spectrum in the absence of a flaw. C_{10} represents the coefficient of the model error term, resulting from the fact that we have truncated the polynomial expansion for the amplitude at the ω^4 term, and hence the errors in amplitude will be $O(\omega^5)$. The unknowns are therefore A_2, A_3, A_4, and C_{10}. In order to determine the maximum likelihood values of these quantities (corresponding to the most probable values provided the a-priori statistical distribution is flat) we have to maximize

$$\ln P(f|A_2, A_3, A_4, C_{10}) =$$

$$-0.5\left[\sum_\omega \frac{|p(\omega)|^2}{C(\omega)} \left|A_2\omega^2 + i\,A_3\omega^3 + A_4\omega^4 - \frac{f(\omega)}{p(\omega)}\right|^2 + \sum_\omega \ln C(\omega)\right]$$

We can perform the maximization on A_2, A_3, and A_4 analytically for an assumed value of C_{10}, by minimizing

$$S = \sum_\omega \frac{|p(\omega)|^2}{C(\omega)} \left| A_2\omega^2 + i\,A_3\omega^3 + A_4\omega^4 - \frac{f(\omega)}{p(\omega)}\right|^2$$

and obtain

$$\hat{A}_2 = \frac{a_8 b_2 - a_6 b_4}{a_4\,a_8 - a_6^2}$$

$$\hat{A}_3 = \frac{b_3'}{a_6}$$

$$\hat{A}_4 \quad \frac{a_4 b_4 - a_6 b_2}{a_4 a_8 - a_6^2}$$

where

$$a_n = \sum_\omega \frac{|p(\omega)|^2\,\omega^n}{C(\omega)}$$

$$b_n = R\left(\sum_\omega \frac{p^*(\omega) f(\omega)\omega^n}{C(\omega)}\right)$$

$$b'_n = I(\sum_\omega \frac{p^*(\omega)f(\omega)\omega^n}{C(\omega)}) \quad .$$

The uncertainties in these estimates are given by:

$$\sigma(A_2) = \sqrt{\frac{S_1 \ a_8}{(N - 2)(a_4 a_8 - a_6^2)}} \quad ,$$

$$\sigma(A_3) = \sqrt{\frac{S_2}{(N - 1)a_6}} \quad ,$$

$$\sigma(A_4) = \sqrt{\frac{S_1 \ a_4}{(N - 2)(a_4 a_8 - a_6^2)}} \quad ,$$

where

$$S_1 = \sum_\omega \frac{|p(\omega)|^2}{C(\omega)} (A_2\omega^2 + A_4\omega^4 - R[\frac{f(\omega)}{p(\omega)}])^2 \quad ,$$

$$S_2 = \sum_\omega \frac{|p(\omega)|^2}{C(\omega)} (A_3\omega^3 - I[\frac{f(\omega)}{p(\omega)}])^2 \quad .$$

and N is the number of frequency-domain measurements.

The maximization with respect to C_{10} must be performed numerically. The procedure is to obtain the best estimates of A_2, A_3, A_4 for a series of assumed values of C_{10} and select the set for which $P(f|A_2, A_3, A_4, C_{10})$ is maximum. The best estimate of τ is then obtained from

$$\hat{\tau} = \hat{A}_3/\hat{A}_2$$

From (2), we see that the value of C_{10} determines the frequency range in which a sharp increase in $C(\omega)$ occurs, due to the effect of the ω^{10} term. Since the weighting factor in the solution for A_2, A_3, and A_4 is proportional to $1/C(\omega)$, the value of C_{10} thus determines the effective cutoff frequency of the solution, above which the truncated polynomial expression is no longer a valid representation of the measured spectrum. As will be discussed later, this fact is of advantage in the automatic selection of a transducer frequency. We may conveniently define the cutoff frequency f_c as the frequency at which the model error is equal to the sum of the electrical noise plus grain noise.

FLAW SIZE DETERMINATION

Having obtained the arrival time corresponding to the flaw centroid the next step in determining the flaw size is to estimate the arrival time corresponding to the front-face echo. To accomplish this, we first shift the time origin of the waveform by τ, so as to coincide with the flaw centroid, and then obtain the maximum likelihood estimate of the projected radius 'a' based on the Born approximation, and assuming an ellipsoidal flaw.

The impulse response function in pulse-echo mode, for an ellipsoid whose centroid is at the origin, is given by

$$R(\alpha, T, t) = \alpha\{1(T, t) - T[\delta(t-T) + \delta(t+T)]\}$$

where $T = 2a/c$, $1(T,t)$ is a unit rectangular function of halfwidth T, and α is dependent on the material parameters. Using the expressions given by Gubernatis et al (1977) it is easily shown to be proportional to the fractional deviation in acoustic impedance, $\delta(\rho c)/\rho c$.

We assume a measurement model of the form

$$f(t) = R(\alpha, T, t) * p(t) + \nu(t)$$

$$= \alpha\left(\int_{t-T}^{t+T} p(t')dt' - T[p(t-T) + p(t+T)]\right) + \nu(t)$$

Where f(t) and p(t) represent the measured waveform and the transducer response in the time domain respectively, and $\nu(t)$ represents the noise term. The covariance of ν could be obtained using the same noise model as for the time-delay calculation, i.e., equation (2), transformed to the time domain. For this problem, however, one can, to a good approximation regard $\nu(t)$ as white noise whose expectation value is σ_f.

The maximum likelihood values of α and T are obtained by minimizing

$$S = \sum_t [f(t) - R(\alpha, T, t) * p(t)]^2$$

with respect to these two quantities. The minimization on α can be performed analytically, whereby

$$\hat{\alpha} = \frac{\sum_t f(t)\, m(t,T)}{\sum_t m(t, T)^2}$$

where

$$m(t,T) = \Delta t \sum_{t-T}^{t+T} \xi\, p(t') - T[p(t-T) + p(t+T)]$$

and

$$\xi = \begin{cases} 1/2 & t' = T-t \text{ or } T+t \\ 1 & \text{otherwise} \end{cases}$$

The minimization with respect to T (in order to obtain a) must be performed numerically.

We can determine the uncertainty in our resulting estimate of a by approximating the conditional variance cov(a,α|f) by A^{-1}, where A is the curvature tensor of the chi-squared function $\phi = S/\sigma_f^2$, given by (Richardson and Evans 1980):

$$A = \frac{1}{2} \left(\frac{\partial}{\partial x} \frac{\partial}{\partial x}^T \phi\right)_{x=\hat{x}}$$

where x is the vector whose components are a and α.

The standard deviation of a, i.e. σ_a, is then given by:

$$\sigma_a^2 = (A^{-1})_{11}$$

The above assumes that the noise term $\nu(t)$ is gaussian. This condition would be violated if the assumed impulse response function were not a good approximation to reality, e.g. in the case of a non-ellipsoidal scatterer. The required condition in practice is that the reduced chi-square χ_ν^2 is of the order of unity, where in this case:

$$\chi_\nu^2 = \frac{\sum_t (f - R(\hat{\alpha}, \hat{a})*p)^2}{(N - 3)\ \sigma_f^2}$$

where N is the number of time-domain measurements.

σ_a represents the uncertainty in a, assuming that the origin of the coordinates corresponds to the flaw centroid. Since there is an additional source of uncertainty in the location of the centroid, this must be included in the overall uncertainty in a. Since the centroid time delay calculation and the flaw size determination are sensitive to different frequency regimes of the measurements, they may, to a reasonable approximation, be considered independent quantities, and the overall uncertainty in a determined by

$$\sigma_{total}^2 = \sigma_a^2 + c^2\ \sigma_\tau^2$$

The above procedure for obtaining the flaw radius has the advantage of avoiding the noise-vulnerable deconvolution step. In addition, one can take advantage of the fact that for a strong scatterer the early part of the waveform is better described by the Born approximation than the latter part, by only including the early part of the waveform in the measurement equations.

PRACTICAL IMPLEMENTATION OF THE ALGORITHM

The above considerations have formed the basis of an automated algorithm for flaw-sizing from individual pulse-echo waveforms, in which the only operator inputs are the time windows for the reference (transducer response) and flaw waveforms. An optional additional input is a time window for the noise calculation, in order to estimate C_0 and C_4 for the centroid calculation. It was found however, that the exact values are not critical, and that setting $C_4 = 0$ and $C_0 = 10^{-4}\ |f(\omega)|^2_{max}$ usually gives satisfactory results. By far the most critical noise parameter is C_{10}, which is determined by the semi-adaptive procedure discussed earlier. The calculation of the flaw radius 'a' and its associated uncertainty are then automatic. In addition, the algorithm can determine whether a higher or lower frequency transducer should be used in further measurements. It accomplishes this by first checking to see if the required accuracy has been achieved in the calculation of A_2, A_3, and A_4 (according to some arbitrary criterion, e.g. 5-sigma). If not, the cutoff frequency f_c for the centroid calculation, is compared to the frequency range of the transducer response. If f_c falls outside the usable frequency range of the transducer, an appropriate recommendation is made as to the optimum frequency range of future measurements.

An important additional consideration in the practical implementation of the algorithm is that flaw waveforms are frequently contaminated by "front-face ringing," resulting from the response of the front-face of the host medium to the ringing of the transducer. This contributes non-stationary noise to the measurements, violating the statistical assumptions of the algorithm. This ringing signal can, however, be removed quite simply by acquiring an additional waveform with the transducer laterally displaced from its original position (so as not to illuminate the flaw), and subtracting this waveform (representing the ringing signal) from the original waveform (flaw + ringing). Care must be exercised, however, in properly registering the two waveforms before substracting. The registration can be based on some feature which is prominent in both waveforms. This approach would, however, require an additional input by the operator, in the form of the time window defining the feature.

RESULTS

We now present the results of some preliminary testing of the algorithm with two sets of data.

The first data set consisted of a series of measurements of a 400 micron spherical void in titanium, using a transducer of 5 MHz center frequency, with a number of different incident angles. The results are given in Table 1, together with the results obtained using the more tranditional operator-intensive method, shown for comparison. Although no "front-face ringing" subtraction was made, the estimated flaw sizes in all cases were quite satisfactory, and the error bars were reasonable.

The second data set consisted of measurements of three spherical flaws in titanium (radii 200, 400, and 600 microns respectively), which were each measured with three transducers (center frequencies 2.25, 5, and 10 MHz respectively). Each of the 9 flaw waveforms was accompanied by a "flawless" waveform to enable the front-face ringing to be subtracted. The results are

Table 1. Results of the Automated Born Inversion for a 400 μm Spherical Void, Viewed from Different Angles. Also Shown for Comparison are the Results Obtained by the Standard Operator-Intensive Method

DIRECTION OF INCIDENT WAVE		a(μm)	
θ(°)	ϕ(°)	MANUAL SKILLED OPERATOR	AUTOMATIC
0	0	388	412±10
30	30	389	390± 7
30	90	391	403± 7
30	150	393	394±10
30	210	388	450± 8
30	270	392	421± 4
30	330	379	408± 5
52	0	416	399± 8
52	60	380	396± 8
52	120	388	446± 7
52	180	388	417± 4
52	240	436	409± 7

presented in Table 2. In 3 of the 9 cases, no solution could be obtained, in 2 of these cases, a positive A_4 value was obtained (which from equation (1) is inconsistent with the Born approximation), and in the third, no significant minimum was obtained in the chi-squared function used in the flaw-size determination. Of the 6 cases for which a solution was obtained, the results were generally satisfactory, in that reasonable values were obtained for the flaw sizes even when the transducer frequency was far from optimum (i.e. ka considerably different from 1), the worst-case error being 50%. The optimum transducer for each of the 3 flaws is indicated in the table. The recommendation of desired transducer frequency (as made by the algorithm) is also indicated in the table, from which it can be seen that the algorithm made the correct decision in all 9 cases. It is interesting that the 3 cases for which no solution could be obtained coincided with those 3 measurements for which $ka \gg 1$. This underscores the necessity of having sufficient low frequency coverage in the measurements.

Table 2. Results of Automated Born Inversion for 3 Different Spherical Voids, Measured by 3 Different Transducers

	ACTUAL FLAW RADIUS (μm)	CENTER FREQUENCY OF TRANSDUCER (* indicates ka~1)	FROM AUTOMATED ALGORITHM: MAXIMUM LIKELIHOOD RADIUS (μm)	FROM AUTOMATED ALGORITHM: OPTIMUM TRANSDUCER	MANUALLY OBTAINED VALUES, FOR COMPARISION
A	200	2.25	317 ± 18	HIGHER	
B	200	5	283 ± 9	HIGHER	
C	200	10*	243 ± 9	OK	219
D	400	2.25	567 ± 7	HIGHER	
E	400	5*	444 ± 4	OK	263
F	400	10	POSITIVE A_4	LOWER	
G	600	2.25*	692 ± 5	OK	473
H	600	5	POSITIVE A_4	LOWER	
I	600	10	NO SIGNIFICANT MINIMUM	LOWER	

One problem which is apparent from Table 2, is that the errors in the flaw sizes were underestimated, as evidenced by the fact that multiple measurements of a single flaw were not consistent within the error bars. Preliminary indications are that this is due to the failure of the Born approximation to give a sufficiently accurate representation of the impulse response of a strong scatterer, leading to an invalid chi-squared value. A useful approach might therefore be to include a parametrized model-error term in the measurement model, and deal with it in a similar way to the C_{10} term in the centroid calculation. Further work is necessary to understand this problem.

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DISCUSSION

J.H. Rose (Ames Laboratory): I would like to ask about this C-10, when it tells you to move higher. Could you elucidate a little bit on that? I think that if you had perfect data, you would always extract C-10 if you were too low.

K.A. Marsh (Rockwell International Science Center): I did oversimplify that the maximum likelihood technique not only gives you the values of A2, A3, and A4, but it also gives you an estimate of the uncertainty. Now, you can set the criterion of the uncertainty that you want. For example, you might be satisfied with a 5 sigma measurement or better, or a 10 sigma measurement, so you put in an arbitrary criterion for that. If you have attained the required accuracy, the algorithm says nothing; if you have not, then it looks at the value of C-20 to determine whether or not a higher or lower frequency transducer would have improved the accuracy.