DEVELOPMENT AND COMPARISON OF BEAM MODELS FOR TWO-MEDIA ULTRASONIC INSPECTION

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INTRODUCTION

This paper reports on an effort to model the radiation pattern of a submerged ultrasonic transducer exciting a beam which is incident on a liquid-solid interface. The important aspects of this process are the diffraction of the beam as it propagates in the liquid and solid media, focussing of the beam due to a lens at the transducer face and/or the curvature of the interface, and aberrations induced by refraction at the interface.

Ray tracing techniques have commonly been used to describe focussing and aberrations introduced in ultrasonic beams due to refraction and reflection at surfaces [1]. These methods generally ignore beam spreading due to diffraction. If aberrations are neglected, simple formulae are available to predict the effects of diffraction on the axial fields of unfocussed piston sources [2] and the full fields of Gaussian sources [3] after refraction through a planar or cylindrically curved liquid-solid interface. Presented here are two models which treat both diffraction and aberration effects as an ultrasonic beam passes through a planar or cylindrical interface. The approximate Gauss-Hermite model is presented as a working computational tool. The accuracy of its predictions are evaluated by comparison to those of the more exact and computationally intensive Green's function model.

GAUSSIAN-HERMITE MODEL

The Gauss-Hermite (G-H) beam model is based on an expansion of the radiation field in a complete set of orthonormal Gauss-Hermite functions. The use of this type of solution for transducer radiation fields was proposed by Cook and Arnoult [4]. Thompson and Lopes [5] combined this description of the diffraction of a propagating sound beam with a ray tracing model for the refraction of the fields at an interface to produce a hybrid model which accounts for both diffraction and aberrations.

In the fluid, the G-H formalism expresses the velocity potential, ϕ, as the sum of eigenfunctions

$$\phi(x,y,z) = \sum_{mn} C_{mn}\psi_m(x,z)\psi_n(y,z)e^{j(\omega t-kz)} \qquad (1)$$

where the C_{mn} are constant complex coefficients and the beam is directed along the z-axis. The eigenfunctions, ψ, are composed of transversely varying Gaussian exponential and Hermite polynomial factors, along with axially varying amplitude and phase terms (see Ref. [5] for a complete description). These eigenfunctions satisfy a reduced wave equation in which a term of the order $\partial^2\psi/\partial z^2$ has been dropped [4]. This approximation is equivalent to the Fresnel approximation and should be good for the well collimated beams often used in ultrasonic NDE.

The coefficients, C_{mn}, can be determined by employing orthogonality relationships [6], provided that the potential ϕ is known on some plane, such as the plane containing the transducer face,

$$C_{mn} = e^{-j(\omega t - kz_o)} \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \psi_m^*(x,z_o)\psi_n^*(y,z_o)\phi(x,y,z_o). \tag{2}$$

This integral may be evaluated exactly for a Gaussian distribution in the initial plane, but must be numerically integrated for a piston or other more complicated source. One benefit of this method is that, in principle, any type of source may be treated with equal ease.

Ray Tracing Through interface

Figure 1 illustrates the geometry of the procedure for treating refraction at the liquid-solid interface. Suppose a cylindrically curved liquid-solid interface (bold arc) is illuminated by a transducer whose central ray is given by the central solid line. An incident field in the fluid may be defined, via the transducer radiation pattern, on a surface just before the interface (dashed arc), assuming no influence by the solid. A transmitted plane is defined perpendicular to the refracted central ray of the beam. One would predict a set of virtual fields on this transmitted plane which, were the plane fully embedded in the solid, would produce the actual radiation field in the solid. A ray tracing analysis is used to relate the complex amplitudes of the incident and transmitted virtual fields. The fields on the transmitted plane can be used to generate a new set of coefficients for the G-H functions in order to describe the propagation of the beam into the solid.

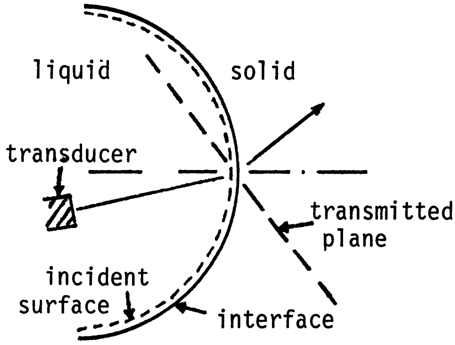

Fig. 1. Geometry of interface transmission computation.

Actually, the scalar G-H solution is not rigorous in the solid media. However, for longitudinal waves, it should be a good approximation to the behavior of scalar elastic displacement potential. For transverse waves the problem is more severe. But, nonetheless, for many cases in which one component of the vector displacement potential is dominant, the scalar solution may be an accurate representation. For example, a well collimated beam passing at a moderate angle through a planar or gently curved interface should be a valid geometry for modeling shear waves.

Numerical Results - Axicon Transducer

Previously, the G-H model has been applied to beams generated by planar, spherically focussed and cylindrically focussed piston and Gaussian transducers. In order to demonstrate the applicability of the G-H model to general types of transducers, some results are presented here for a conically focussed, or axicon, transducer. This type of transducer has received some attention due to an apparent extended depth of focus [7-9]. The geometry of the axicon is shown in Fig. 2. The probe is characterized by a radius, "a", a cone angle, "α", and a frequency, "f".

Figures 3a and b show the axial profile of an axicon transducer radiating into water at 2.5 MHz. For this case, $\alpha = 5.7°$ and a = 2.0 cm. The G-H prediction shown in Fig. 3a was obtained with a 65 x 65 term expansion, and is compared with the calculations of Dietz [7], shown in Fig. 3b. In Fig. 3b, the dashed line corresponds to a numerical integration of the Rayleigh diffraction integral and the solid line represents an approximation using the Method of Stationary Phase (MSP). The G-H model agrees well with the numerical result of Dietz, except in the very near field where convergence is sensitive to the number of terms taken in the expansion. The G-H expansion is also seen to be superior to the MSP approximation. Figures 3c,d,e show the full fields of an axicon with f = 2.25 MHz, $\alpha = 4.3°$, and a = .635 cm. In Fig. 3c, the transducer is radiating into water. In Figs. 3d,e, the fields are shown after passage through a plane water-steel interface for 45° refracted shear waves and 70° refracted longitudinal waves, respectively. Note that for the 45° T wave case, the beam retains a good focal region, whereas for the 70° L case the beam shape has been degraded severely by aberrations.

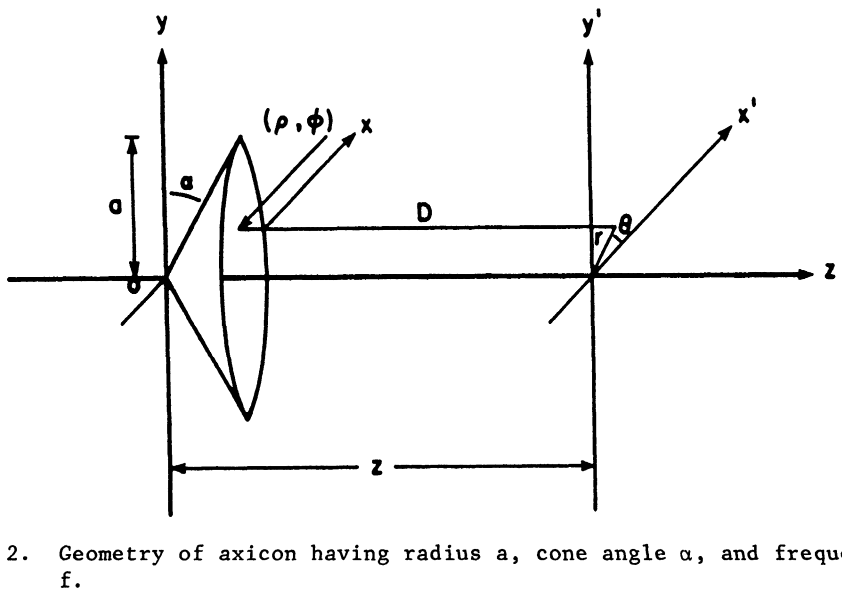

Fig. 2. Geometry of axicon having radius a, cone angle α, and frequency f.

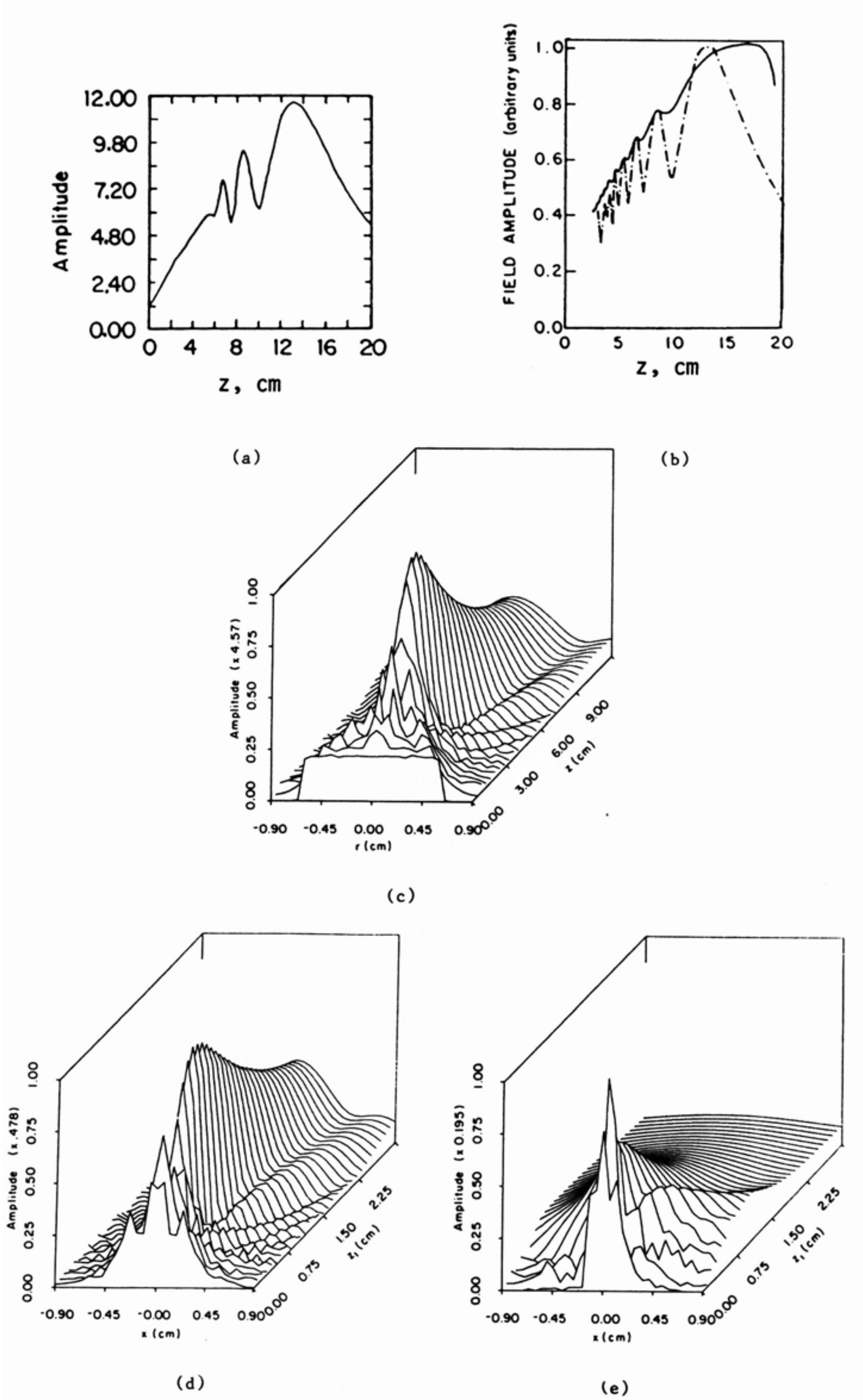

Fig. 3. Axicon Radiation: a) axial profile in water - G-H; b) axial profile in water - Dietz (dashed-numerical calculation, solid - MSP); c) full field in water; d) 45° T wave through water-steel interface (C_L= .596 cm/μs, C_T = .3235 cm/μs, standoff in water = 1.27 cm); e) 70° L wave through water-steel interface (standoff in water = 1.27 cm).

GREEN'S FUNCTION MODEL

In order to assure the proper application of the Gauss-Hermite beam model, it is necessary to perform either an experimental or theoretical validation of the theory. Experimental data has been compared to the G-H model with favorable results for a variety of different cases [10]. However, there are limitations to this approach due to uncertainties regarding the degree to which the surface motion of actual probes corresponds to the "piston" assumption. Here, a more rigorous theoretical approach has been developed in order to provide a touchstone against which the G-H model may be evaluated.

Consider the semi-infinite volume, V, depicted in the left half of Fig. 4, which has a bounding surface, S. It is known that the time-harmonic displacement at a point in V can be represented as an integral over S of the form [11,12].

$$u_m(x) = -\int_S [\tau^G_{ijm}(x-X)u_i(X) - u^G_{im}(x-X)\tau_{ij}(X)]n_j dA(X) \tag{3}$$

where standard tensor index notation has been employed. In Eq. (3), u_i and τ_{ij} are the displacement and stress conditions at the surface S, n_j is the outward normal and u^G_{im} is the free-space Green's displacement tensor given by

$$u^G_{im} = (\rho\omega^2)^{-1}[-G_L(R)+G_T(R)]_{,im} + \mu^{-1}G_T(R)\delta_{im} \tag{4}$$

where

$$G_\beta(R) = (1/4\pi R)e^{ik_\beta R}, \qquad (\beta = L,T) \tag{5}$$

$$R = |x-X|. \tag{6}$$

The L and T subscripts denote properties of longitudinal and shear waves, respectively. The corresponding Green's stress tensor, τ^G_{ijm}, is obtained by substitution of Eq. (4) into Hooke's Law. From the physical point-of-view, U^G_{im} and τ^G_{ijm} may be thought of as the displacement and stress fields that would be radiated to the field point x by a localized body force f_m applied at the source point X, as sketched on the right hand side of Fig. 4.

If the surface S is considered to be a liquid-solid interface, then Eq. (13) provides a formal solution for the field in the solid provided that the boundary fields, U_i and τ_{ij}, on the solid surface due to a transducer beam illumination can be determined. The approach taken by the Green's function (GF) model then, is to determine these boundary fields by an appropriate approximation and evaluate the integral numerically.

The determination of the boundary fields has been based on the use of the G-H beam model, which is rigorously correct, within the Fresnel approximation, to describe the radiation pattern in the fluid. However, any appropriate transducer radiation model will work as well. First, the incident field is determined on the fluid side of the interface in the same manner as depicted in Fig. 1. Then, to calculate the actual fields at each point on the surface, the transducer beam is assumed to be locally a plane wave and the surface is assumed to be locally planar. Thus, everywhere on the surface the continuity of stress and displacement can be approximately introduced through the classical theory of plane wave incidence on a plane boundary between two media. The boundary fields are equal to those of the transmitted wave in this form of the Kirchhoff approximation. As the interval in the numerical integration procedure is reduced, the error in these approximations is decreased. One drawback to this method is that a small integration step is required to resolve the phase variations over the surface. The computation time is therefore quite lengthy.

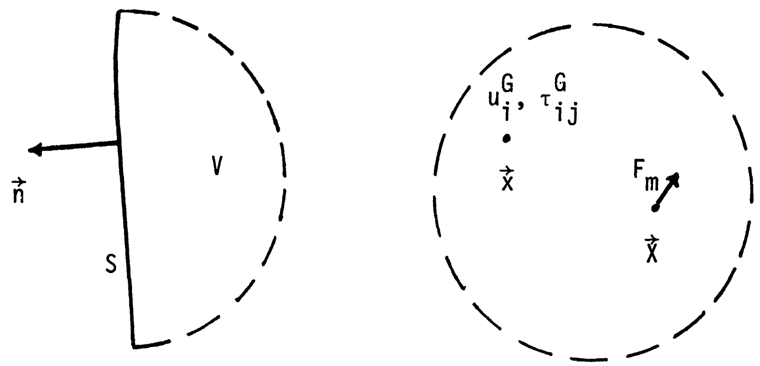

Fig. 4. Schematic geometry of Green's function formulation.

NUMERICAL RESULTS

Results of the Green's function model are compared here with the predictions of the Gauss-Hermite model for an unfocussed piston illuminating a cylindrical interface. For this case, the transducer radius "a" is .635 cm, the frequency "f" is 5 MHz, the radius of curvature of the interface "R" is 7.62 cm, and the standoff in water "z_o" is for all cases 5.6 cm. The material properties used are those for water and fused silica: V_{water} = .15 cm/μs, V_L = .597 cm/μs, V_T = .376 cm/μs, and ρ_{solid}= 2.2 gm/cm^3. All results shown are for longitudinal waves.

Figure 5 shows the axial radiation fields for refracted angles of 0, 5, 15 and 30 degrees, where the refracted angle is that of the beam axis. The amplitude has been normalized to that at the transducer face, and has been divided by the transmission coefficient of the central ray. In Fig. 5a, the GF prediction is seen to have a slightly lower amplitude in the focal region than does the G-H profile. However, a case was run (dashed line) in which the Fresnel approximation was applied to the spherical wave functions of the GF code. The result agrees well with the G-H theory, suggesting that the difference between the G-H and GF models is due to the presence of the Fresnel approximation in the G-H model. The remaining plots demonstrate the good agreement of the G-H model with the GF predictions at higher angles.

In Fig. 6, two profiles are shown which are taken transverse to the central axis in a plane perpendicular to the cylindrical axis of the interface. The first is at normal incidence and the second is at a refracted angle of 30°. For both cases the profile is taken at a distance in the solid, z_1, of 2.5 cm. At normal incidence this corresponds to the geometrical focal point of the interface. The agreement is seen to be excellent at normal incidence and at 30° the G-H model approximates the structure of the profile very well.

SUMMARY

The Gauss-Hermite model for transducer radiation through an interface has been shown to be a versatile tool for modeling various types of transducers. Furthermore, when compared to a more rigorous model, it has proven to be remarkably accurate for the cases studied. The Green's function model will be very useful in further evaluation of the G-H and other models for more extreme cases and for shear wave cases.

ACKNOWLEDGEMENT

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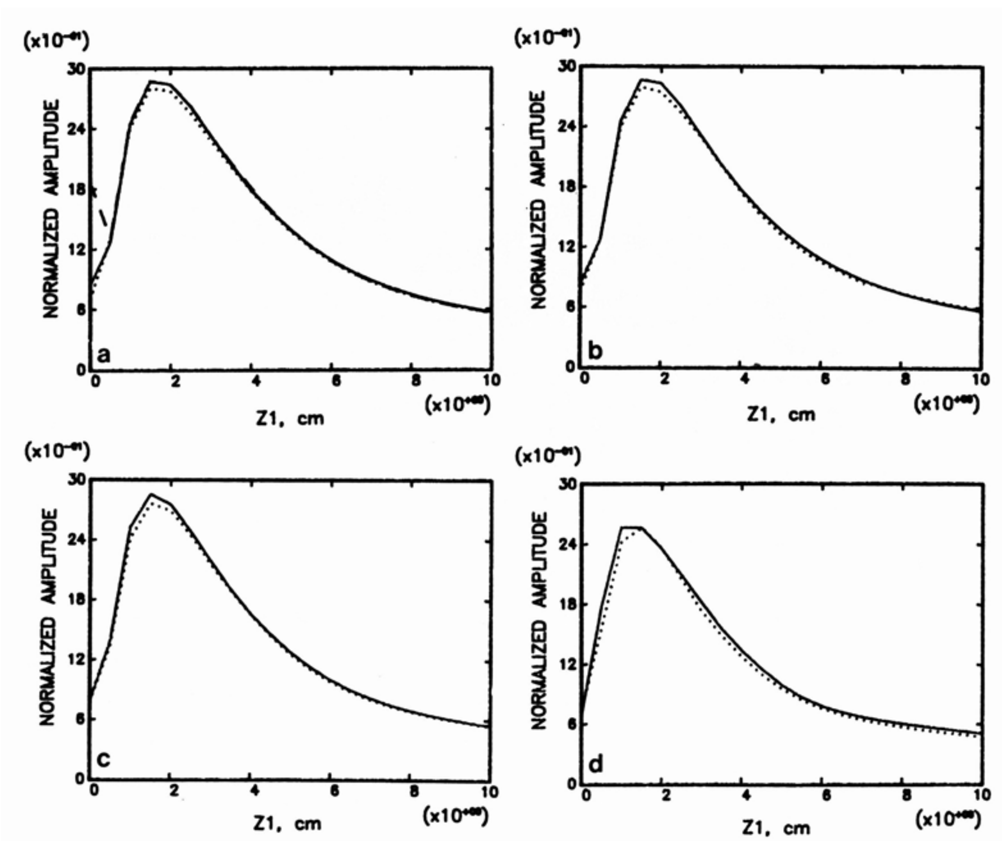

Fig. 5. Axial profiles in solid (solid-G-H, dotted-GF, dashed-GF w/Fresnel approx.) a) θ_1=0°, b)θ_1=5°, c) θ_1=15°, d)θ_1= 30°.

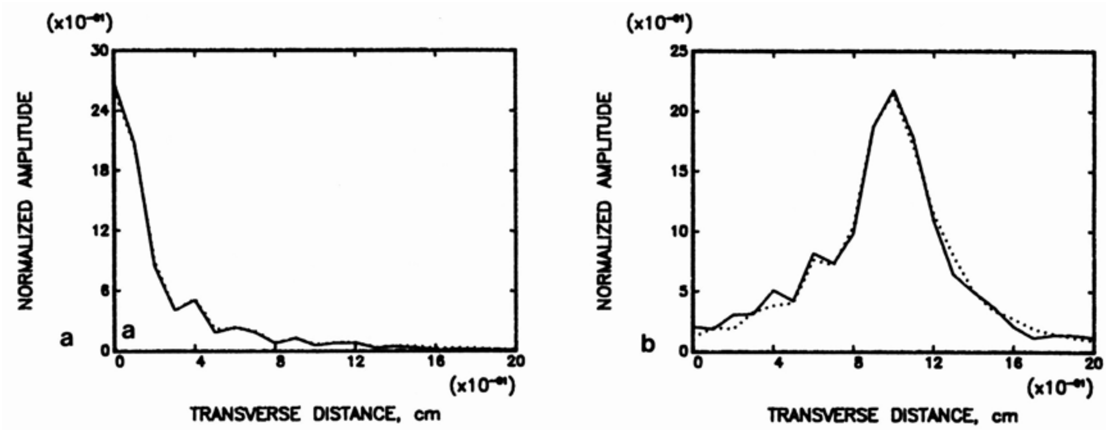

Fig. 6. Transverse profiles in solid (solid-GH, dotted-GF) a) $\theta_1=0°$, b) $\theta_1 = 30°$.

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DISCUSSION

From the Floor: Your Green's Functions were vector functions; is that right?

Mr. Newberry: Tensor functions.

From the Floor: How did you make the comparison to the scalar? What components did you pick? Did you pick the magnitude of the tensor?

Mr. Newberry: The result of the Green's Function model is a vector displacement in the solid, and the result of the Gauss-Hermite is a scalar displacement potential. We made an approximation of quasi-plane waves so we can deduce the displacement from the displacement potential.

From the Floor: So you calculated the displacement potential from your vector?

Mr. Newberry: We calculated a displacement from the displacement potential in the Gauss-Hermite model.

Mr. R. L. Ludwig, Colorado State University: Did you look at your beam model, where you tried to model a liquid-solid interface -- at the possibility of using a liquid as a first approximation to a couplant?

Mr. Newberry: I'm not sure I understand the question.

Mr. Ludwig: Well, what I had in mind was to use a fairly small layer of liquid so that you can actually use it as a couplant, as the first approximation for a couplant. Is this a possibility? Can you account for it?

Mr. Newberry: I think eventually we can. We (haven't reached) that problem where we just have a thin layer. We hope to extend this method to using wedges which would have a couplant in between. We haven't done that yet, no.

Mr. Ludwig: Thanks.

Mr. Fraser: Thank you.