

AN EFFICIENT APPROXIMATE MODEL FOR ELASTIC WAVE SCATTERING IN PLATES

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INTRODUCTION

The objective of this work is to develop a fast method for modeling time-domain ultrasonic wave scattering in plates. Due to the possible excitation of multiple plate modes and dispersive behavior of these waves, the scattered signal can be very complicated. The model that is presented in the following, can be valuable tool in real-time inspection or development of new inspection techniques for the aircraft safety inspection. Inspection of the structural integrity of an aircraft involves, in part, the inspection of rivots for existence of cracks. Rivots can be checked using eddy-current probes or regular contact ultrasonic transducers. However, for inspection of rivots that are hidden by a top layer (e.g. lap-joints), a better technique may be to excite a guided ultrasonic wave which can travel in the plate under the joint and interrogates the hidden rivot. The scattered signal can be picked up by the same transducer in the pulse-echo mode.

This problem can be solved exactly by utilizing brute force algorithms such as finite element or finite difference techniques. However, these algorithms require large amount of computer time and are impractical for real time inspection applications. Since speed in modeling is an important criteria, in this work we will only focus on an approximate model for both transducer and the scattering process. An example of a common set-up is shown in 1. Our approach is based on the assumption that the transducer is in the far-field of the signal scattered from the hole/crack. Also, it is assumed that the transducer has finite temporal and spatial bandwidth. These are not limiting assumptions since in practice they are always true.

In the following, we present a brief review of plate wave theory. We will model the real transducer excitation by the plane wave superposition. In other words, we solve the plate problem with a plane wave incident on one side and then reconstruct the finite transducer case by superposing the plane wave solutions. Next, we take advantage of the reciprocity theorem and calculate the voltage in the receiving transducer due to backscattered or transmitted signal. Finally, we compare the modeled signal with the experimental signals for both transmission and reflection case.

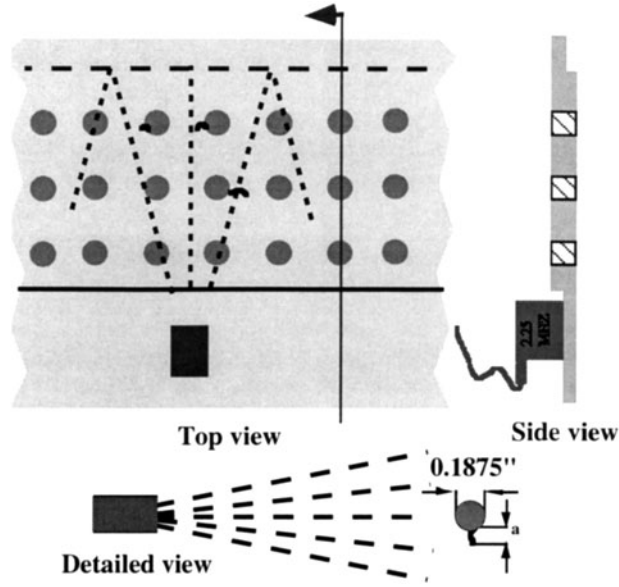


Figure 1: Inspection of rivot-holes for the existence of cracks using oblique incidence contact transducers.

PROBLEM STATEMENT

The focus of this paper is to solve for the scattering field due to reflection and/or transmission from the rivot-holes with or without cracks in thin plates. To solve this problem, we first present solutions to the wave propagation in plates. This solution will form a basis for the work presented in this paper. The propagation of ultrasonic wave in elastic plates is a classical problem and can be found in many textbooks (for example see [1]). The standard method for solving this problem is to express the solution in terms of plane waves which are characteristic modes of an infinite medium made of the plate material. So the governing equations for the waves propagating in plates can be written as

$$\tau_{ij,j} + \rho\omega^2 u_i = 0 \quad (1)$$

$$\tau_{ij} = \lambda\delta_{ij}u_{k,k} + \mu(u_{i,j} + u_{j,i}) \quad (2)$$

where τ_{ij} is the stress component in the solid, u_i is the displacement component, ρ is the density, λ and μ are the Lamé constants, and ω is the temporal frequency. In addition to the above equations, a set of boundary conditions is also prescribed for both sides of the plate. Displacement component u_i can be written in terms of scalar potential function ϕ and a vector potential function ψ . In this paper, we are only concerned with a two-dimensional case where there is no displacement or variation in the \hat{x}_2 direction. Hence, the solution to the plate problem can be written in the form

$$\phi(x_1, x_2) = (A^{Lc} \cos k_2^p x_3 + A^{Ls} \sin k_3^p x_3) e^{i(k_1 x_1 - \omega t)} \quad (3)$$

$$\psi_2(x_1, x_2) = (A^{Tc} \cos k_3^t x_3 + A^{Ts} \sin k_3^t x_3) e^{i(k_1 x_1 - \omega t)} \quad (4)$$

where k_1 is the wavenumber and $k_2^\alpha = \sqrt{k_\alpha^2 - k_1^2}$ ($\alpha = p, s$ for pressure and shear). By prescribing a boundary condition, we can solve for unknown coefficients $A^{L\alpha}$ and $A^{T\alpha}$ where $\alpha = c, s$. For the case under study, the two sides of the plate have traction-free boundary condition except for the contact transducer footprint where a traction is prescribed. Clearly,

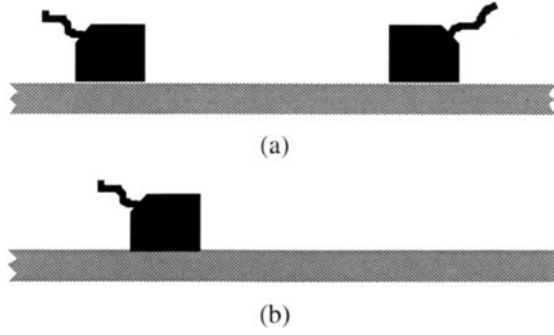


Figure 2: Contact transducers on a thin plate: a) the transmission set-up (pitch-catch), and b) reflection set-up (pulse-echo).

four independent equations are needed to solve for the four unknown coefficients. The traction boundary conditions for the two surfaces of the plate at $x_3 = +h, -h$ (assuming a plate with thickness $2h$) provides the needed four equations. As shown in Figure 2, contact transducer is placed on the top surface of the plate. The boundary conditions for this problem can be approximated with prescription of a stress field on the area of the contact transducer footprint. The stress field due to this transducer can be expressed in terms of plane waves weighted with an appropriate angular spectrum.

$$\tau_{33}^{inc}(x_1, h) = \int \hat{\tau}(k_1) e^{ik_1 x_1} dk_1 \quad (5)$$

$$= \int e^{-(k_1 - k_c)^2} e^{ik_1 x_1} dk_1 \quad (6)$$

where k_c is the center spatial frequency of the contact transducer. All stresses are zero for the surface at $x_2 = -h$. The problem can be solved by expanding the solution in terms of the plane waves solution. For a plane wave incidence case, the coefficients can be calculated by solving the linear system of equations $TA = B$. T is a known matrix that depends on the plate thickness and material property, A is a vector containing the unknown wave amplitudes, and B is a vector representing the forcing function.

The transmission problem consists of modeling the pitch-catch signal traveling in a plate, as shown in Figure 2a. By using Auld's reciprocity theorem [2] and assuming receiver and the transmitter having the same response, the signal of the receiver can be written as

$$S(t) = \int \int \tau_{33}^{receiver}(x_1) u_3(x_1) dx_1 d\omega \quad (7)$$

$$= \int \int \tau^{inc}(x_r - x_1) u_3(x_1) dx_1 d\omega \quad (8)$$

τ and u can be represented by Fourier integrals (angular spectrum). Consequently, the voltage can be written as

$$S(t) = \int \int [\int \hat{\tau}(k_1) e^{ik_1(x_r - x_1)} dk_1] [\int \hat{u}(k_1) e^{ik_1' x_1} dk_1'] dx_1 d\omega. \quad (9)$$

By switching the order of integration and simplifying

$$S(t) = \int \int \hat{\tau}(k_1) \hat{u}(k_1) e^{ik_1 x_r} dk_1 d\omega. \quad (10)$$

Since $\hat{\tau}(k_1)$ is small for $real\{k_1\}$ much greater or smaller than k_c , above integral can be calculated on a finite interval.

Next, we present an approximate solution to the transducer response to the reflected signal from the edge of a semi-infinite plate. Unlike the case for the infinite plate, there is no closed-form solution for this case. Since most of the energy is reflected back from the edge of the plate, it is reasonable to assume that the reflected signal is similar to that reflected from an infinite half-space. By assuming this, we are ignoring the corner scattering and interaction between top and bottom quarter spaces. Using Auld's reciprocity theorem, the voltage due to the reflected signal can be written as

$$S(t) = \int \int \tau_{ij}^{inc} u_j n_i dx_i d\omega \quad (11)$$

where n_i is the unit vector normal to the surface and τ^{inc} can be calculated using only the incident coefficients and total displacement u is given by

$$U = U^{inc} + U^{ref:L} + U^{ref:T} \quad (12)$$

where U^{inc} is the incident displacement, $U^{ref:L}$ is the reflected displacement due to the L-incidence and $U^{ref:T}$ is the displacement due to the T-incidence. The reflected signal from a hole or a crack can be approximated by the reflected signal from the back-edge of the plate with an amplitude correction factor. The factor by which the amplitude is decreased can be calculated independently.

The integrals in Equations (10) and (11) can be evaluated numerically. The numerical evaluation of these integrals can be carried out in two different stages. First, the integral over the wave-number k_1 is calculated for a fixed ω . Then, the second integral can be evaluated using a fast Fourier transform (FFT) algorithm.

All the experiments, were carried out using a contact transducer as shown in Figure 2. The contact transducer was placed on the plate coupled with a thin layer of fluid. Acquiring signal in this fashion can be a difficult task due to signal instability. It was observed that minor movements of the transducer could change the shape of the signal. This could be due to the sensitivity of the measurement to the couplant thickness that changes with changing pressure inserted by hand on the transducer.

A pair of Stavely 2.25 MHz contact Rayleigh wave (30 degree angle) transducers are used for the experiment. The transducer's frequency response can be accurately approximated with a Gaussian centered at 2 MHz and with a 1/e value at 2.0 ± 0.3 MHz. The spatial spectrum is also modeled by a Gaussian with a 1/e value at $k_c \pm 2.5$ 1/mm. The transducer is excited with a broad-band pulse and the signal is received by the same (pulse-echo mode) (Figure 2b) or a similar (pitch-catch mode) (Figure 2a) transducer. The time signals are digitized using a Tektronix digital scope. The plate is made of aluminum with a thickness of 1.55 mm. The hole is circular with a diameter of 5 mm and has smooth boundary. To simulate cracks, a 2 mm notch was cut along the radius, i.e. at a 90 degree angle to the perimeter of the hole. In this paper, all crack measurements were done assuming a crack normal to the wave-front. In all cases, the experiment is done by manually positioning the transducer.

RESULTS

In this section, analytical results are compared with the experimental signals. In the transmission experiment, the transducers are placed 52 mm apart on an aluminum plate with a thickness of 1.5 mm. First, we consider the transmission case where no hole or flaw is present as shown in Figure 2a. In this case, pure plate modes are present and the analytical result is exact. As shown in Figure 3, there is a good agreement between the model and experiment. The signal is composed of two distinct parts which are contributions of first symmetric and antisymmetric modes (S_0 and A_0). There is an additional hump in the tail

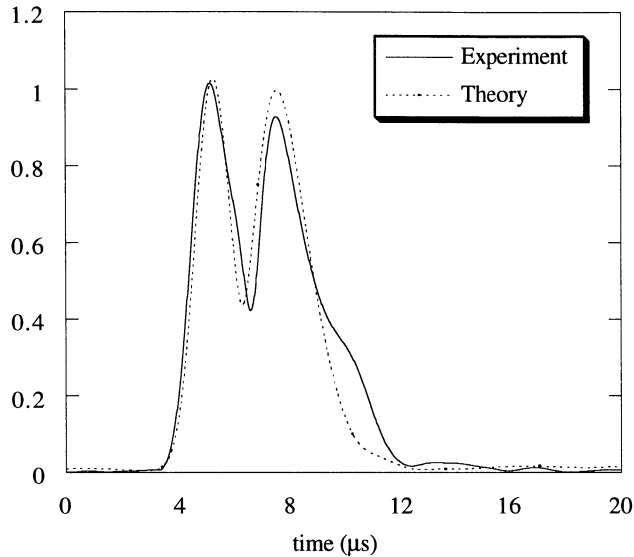


Figure 3: Experimental and theoretical transmission signals on a thin plate, signal shown for a case with $fd = 3$.

of the experimental signal that is not predicted by the model. This component may be due to an internal echo in the contact receiver. Figure 4 and 5 show the transmission signals in the presence of the hole with and without a crack, respectively. The model signal is obtained by multiplying the transmitted signal shown in Figure 3 by an attenuating factor that is determined from the geometry of the shadowing object using Kirchoff approximation. Note that model and experimental signals are very similar except for a difference in amplitude. Differences in amplitude are in the order of uncertainties that is expected in an experiment using a contact transducer. These uncertainties are due to contact force and couplant variations.

To measure the reflection signal, the transducer was placed 28 mm from the edge or the hole (see Figure 2b). Figure 6 shows good agreement between the model and experimental signals for the case of reflection from the back-edge of the plate. The simulated result of the reflection from the back-edge provides the basis for calculation of the reflected signal from the hole and crack. The signal from the hole or crack are obtained by multiplying the back-edge signal by a factor which is determined by the geometry of the reflector using Kirchoff approximation. Figures 7 and 8 show the results for the reflection from the hole and reflection from a hole with crack, respectively. The existence of the flaw is evident in Figure 8 as two extra peaks. The two extra peaks are additional time delayed reflections from the crack for both A_0 and S_0 modes. In cases where signal to noise ratio is high, the distance between transducer and the object can be increased to reduce interference between A_0 and S_0 signals.

The results indicate that the reflected signal is a better tool in detecting a crack emanating from the side of a rivet hole. This is because the effect of the crack on the reflected signal is evident as a separate and relatively strong component. While for the transmission case, only the amplitude of the signal is changed. Since an amplitude reduction can be a result of many different factors, it is not very useful. While time characteristic of the reflected signal changes significantly in the presence of the crack. The relative amplitude of the crack signal may also be used as an indicator of the length of the crack. Furthermore, the reflected signal should be present for each excited propagating mode in the plate.

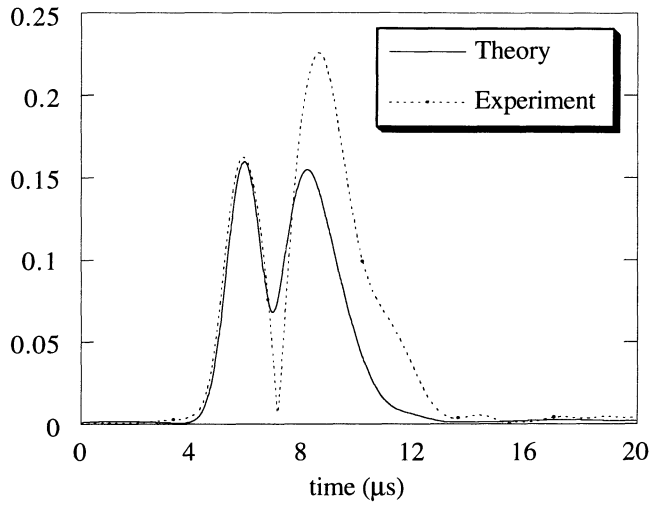


Figure 4: Experimental and theoretical transmission signals on a thin plate in the presence of a hole, signal shown for a case with $fd = 3$.

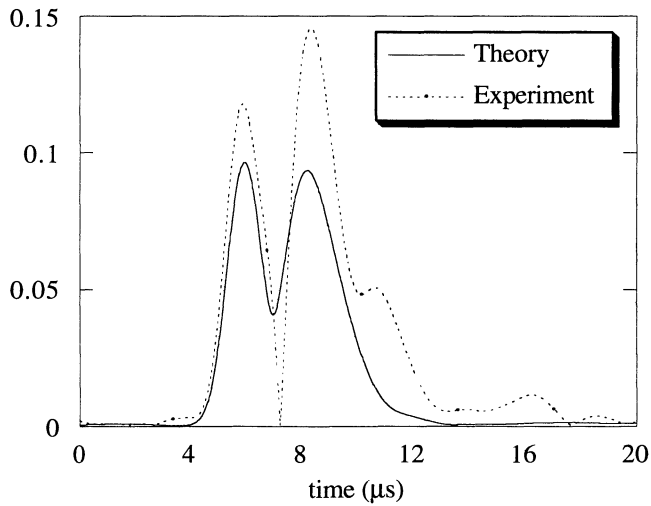


Figure 5: Experimental and theoretical transmission signals on a thin plate in the presence of a hole and a crack, signal shown for a case with $fd = 3$.

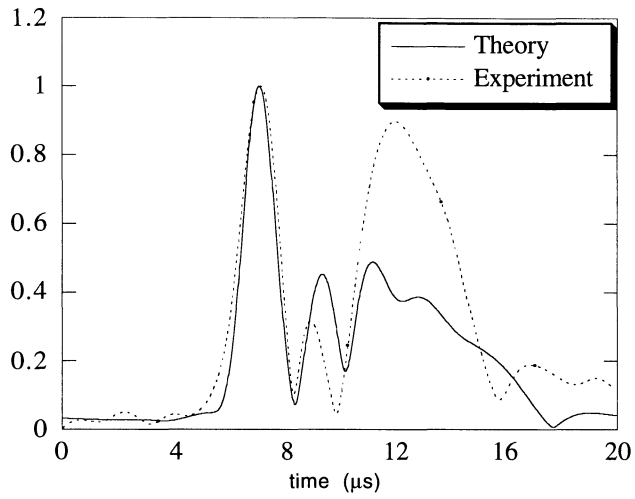


Figure 6: Experimental and theoretical reflected signals from the plate edge on a thin plate, signal shown for a case with $fd = 3$.

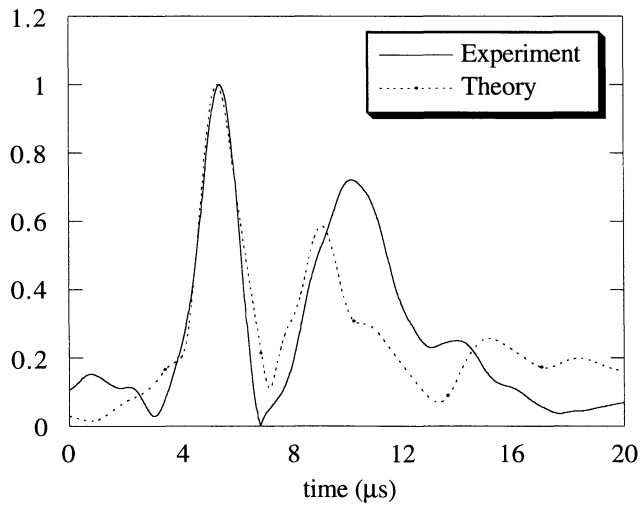


Figure 7: Experimental and theoretical reflected signals from a hole with no crack present on a thin plate, signal shown for a case with $fd = 3$.

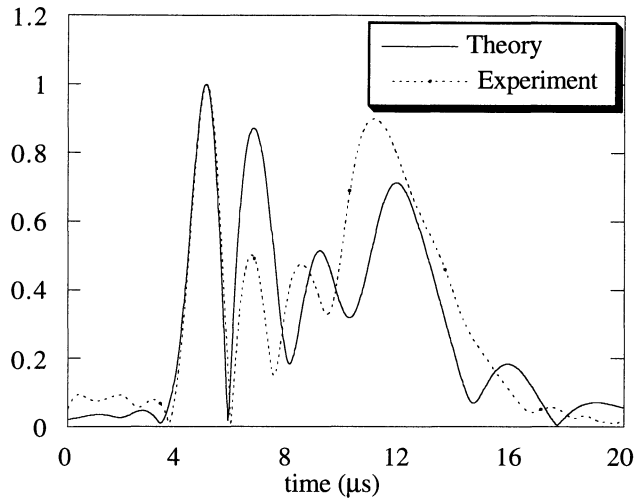


Figure 8: Experimental and theoretical reflected signals from a hole with a crack present on a thin plate, signal shown for a case with $fd = 3$.

Hence, it is possible to cross-correlate the reflected signals for different modes to achieve a better signal to noise ratio. This is specially true when distance between the transducer and the hole is large enough such that different modes separate in time. This is an important issue since for small separations the signals from different modes may interfere with each other. For cases where there is a small separation between the transducer and the rivot-hole, it is advantageous to use a transducer that can excite only one mode at a time, hence avoiding the interference problem.

SUMMARY

In this work, we have presented a fast approximate model for predicting transducer response to transmission and reflection signal from rivots and cracks emanating from them. This technique can model almost any type of commercial transducer response on plates. The comparison of the simulated results with the experimental result showed an very good agreement between the model and the experiment. In fact, due to instability of the measurement system, a more accurate model would not be advantageous. As the next step to further speed the numerical implementation, the integrals may be evaluated asymptotically. This task is currently under way and results will be presented in the future.

ACKNOWLEDGMENTS

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