MODELING AND PARAMETER ESTIMATION OF ULTRASONIC BACKSCATTERED ECHOES

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INTRODUCTION

Ultrasonic backscattered echoes represent not only the impulse response of the ultrasonic transducer, but also contain information pertaining to the inhomogeneity of the propagation path, effect of frequency dependent absorption and scattering, dispersion effect, and geometric shape, size and orientation of reflectors. Therefore, a well-defined modeling of the backscattered echoes leading to the estimation of arrival time, echo skewness, center frequency, and bandwidth is highly desirable for the nondestructive evaluation of materials [1]. In this paper, we model the backscattered echoes, assuming that all parameters describing the shape of the echo are unknown. Then, iterative parameter estimation techniques such as the Gauss-Newton method [2] or Simplex-Search method [3] have been applied to estimate echo parameters. These algorithms have been evaluated in terms of rate of convergence, sub-optimal estimation due to local minima, and presence of noise.

MODELING OF ULTRASONIC ECHOES

In general, as shown in Figure 1, the received ultrasonic signal is composed of two terms:

$$r(t) = h(t) + \upsilon(t) \qquad (1)$$

where h(t) represents superimposed back-scattered echoes corresponding to the geometry and property of materials, and $\upsilon(t)$ is measurement noise.

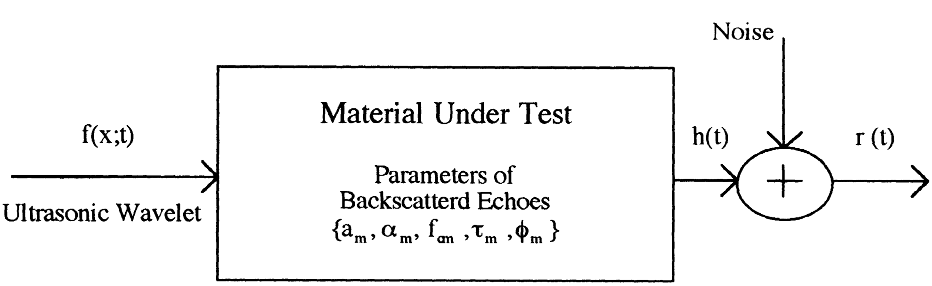

Figure 1. System Description of Backscattered Echoes

Backscattered echoes can be modeled as the sum of M echoes,

$$h(t) = \sum_{m=1}^{M} a_m f(\bar{x}_m;t) \tag{2}$$

where $\bar{x}_m$ is a vector of parameters given by

$$\bar{x}_m = \left[\alpha_m \quad f_{cm} \quad \tau_m \quad \phi_m\right] \tag{3}$$

and $f(\bar{x}_m;t)$ is the echo waveform modeled as

$$f(\bar{x}_m;t) = e^{-\alpha_m (t-\tau_m)^2} \cos\{2\pi f_{cm} (t-\tau_m) + \phi_m\} \tag{4}$$

The parameters of the backscattered echoes are :

a_m : amplitude
α_m : bandwidth factor
τ_m : arrival time
f_{cm}: center frequency
ϕ_m : phase

The goal of this investigation is to estimate the above parameters in the presence of noise for the nondestructive evaluation of materials.

The signal-to-noise ratio (SNR) for a single echo (M =1) is defined as the ratio of signal power to noise power,

$$\mathrm{SNR} = 10\log \frac{\frac{a^2}{2}\sqrt{\frac{\pi}{2\alpha}}\{1 + e^{-\frac{2\pi^2 f_c^2}{\alpha}}\}}{\sigma_v^2} \tag{5}$$

Where noise is assumed to be Guassian with a variance of σ_v^2.

Vector Presentation of Signal

Consider the discrete values of t given by

$$\bar{t} = [0 \quad \Delta\tau \quad 2\Delta\tau \quad . \quad . \quad N\Delta\tau]^T \tag{6}$$

where $\Delta\tau$ is the sampling period and $T = N\Delta\tau$ is the signal duration. Then, the backscattered echoes can be written in a vector form:

$$\bar{r} = \bar{m} + \bar{\upsilon} \tag{7}$$

where

$$\bar{r} = [r(0) \quad r(\Delta\tau) \quad r(2\Delta\tau) \quad . \quad . \quad r(N\Delta\tau)]^T \tag{8}$$

is the received signal vector,

$$\bar{\upsilon} = [\upsilon(0) \quad \upsilon(\Delta\tau) \quad \upsilon(2\Delta\tau) \quad . \quad . \quad \upsilon(N\Delta\tau)]^T \tag{9}$$

is the noise vector, and

$$\bar{m} = \begin{bmatrix} f(\bar{x}_1;0) & f(\bar{x}_2;0) & . . & f(\bar{x}_M;0) \\ f(\bar{x}_1;\Delta\tau) & f(\bar{x}_2;\Delta\tau) & . . & f(\bar{x}_M;\Delta\tau) \\ . & . & . . & . \\ . & . & . . & . \\ f(\bar{x}_1;N\Delta\tau) & f(\bar{x}_2;N\Delta\tau) & . . & f(\bar{x}_M;N\Delta\tau) \end{bmatrix} \times \begin{bmatrix} a_1 \\ a_2 \\ . \\ . \\ a_M \end{bmatrix} \tag{10}$$

is the proposed model vector. Rewriting the above equation in a more compact form by grouping the elements of each column vector,

$$\bar{m} = \left[\bar{f}(\bar{x}_1) \quad \bar{f}(\bar{x}_2) \quad . \; . \quad \bar{f}(\bar{x}_M)\right] \times \bar{a} \tag{11}$$

or

$$\bar{m} = A \times \bar{a} \tag{12}$$

where A represents the backscattered model matrix. The model matrix A is formed such that each column of A represents an echo vector of length N, and the corresponding a_i is the amplitude of that particular echo.

Parameter Estimation Criteria

Once we construct the non-linear model matrix A, then we search for an estimate of the amplitude vector, $\tilde{\bar{a}}$, by minimizing $\left|\bar{r} - \left(A \times \tilde{\bar{a}}\right)\right|$ in the least square sense. Then, with the estimate $\tilde{\bar{a}}$, we reconstruct the model as

$$\tilde{\bar{m}} = A \times \tilde{\bar{a}}, \tag{13}$$

and define the error function as

$$\bar{e} = \bar{r} - \tilde{\bar{m}}, \tag{14}$$

If we rewrite $\bar{m}$ in terms of echo amplitudes and echo waveforms,

$$\bar{m} = a_1\bar{f}(\bar{x}_1) + a_2\bar{f}(\bar{x}_2) + \ldots\ldots + a_M\bar{f}(\bar{x}_M) \tag{15}$$

By grouping the parameter vectors into a matrix $X = \left[\bar{x}_1 \quad \bar{x}_2 \quad . \; . \quad \bar{x}_M\right]$, we can write $\bar{e} = \bar{e}(X)$. Now, the problem becomes the minimization of the error function with respect to the variable matrix X. Since the error function is a vector of length N, the criteria for minimizing a vector could be minimizing its norm. Let's define the objective function as the norm of the error function as, $\varphi(X) = |\bar{e}(X)|$. Then, the general problem description can be stated as

$$\underset{X \in \Re^n}{\text{minimize}} \quad \varphi(X), \tag{16}$$

A problem of this type is called an *unconstrained optimization problem*, since there is no constraint provided on X. In the next section, we will present methods for minimizing the objective function with respect to parameter matrix X.

Optimization Techniques

The optimization routines, after each iteration, update the parameter matrix X such that the objective function $\varphi(X)$ converges to a minimum value. The global minimum of the objective function provides us the optimum solution for the parameter matrix X*. However, there are local minimums as well. A local minimum, of course, does not give the optimal solution. Depending on the initial guess and the applied optimization technique, there is always a possibility for the objective function to remain in one of the local minimums. A good guess may avoid the local minimums. The Simplex Search parameter estimation method, with a reasonable initial guess of parameters, is less sensitive to the local minimums although it slowly converges to the global minimum. On the other hand, least-square methods like Gauss-Newton, are sensitive to local minimums although they converge faster than the Simplex Method. The performance of these two methods of optimization are investigated in this study.

PARAMETER ESTIMATION RESULTS

Applying these methods, we have been able to detect and estimate all parameters of a noise-free echo with a 100% accuracy. (Fig. 2-b,c). However, in the presence of noise (Fig 2-d), the optimum parameter vector is not the same as the original one, because the noise changes the shape of the echo. The parameter estimation of an echo corrupted by noise with 6.12 dB SNR is robust (see Fig 2-e,f). This estimation improved the SNR from 6.12 to 26.34 dB.

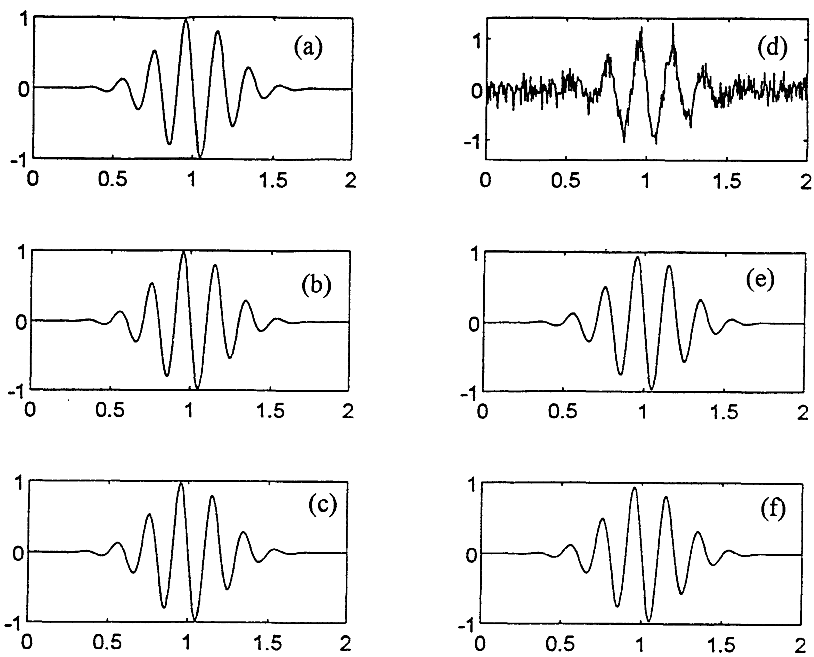

Figure 2. a) A noise-free echo with the parameter vector $\bar{x}_1 = [10 \quad 5 \quad 1 \quad 0.5\pi]$; b) Detection of the echo by Gauss-Newton method after 17 iterations with an initial guess of $\bar{x}_1^{\,o} = [15 \quad 8 \quad 0.8 \quad 0]$ where the estimated parameter vector is the same as $\bar{x}_1$; c) Detection of the echo in (a) by Simplex-Search method after 461 iterations using the same initial guess; d) The echo in (a) corrupted with additive white Gaussian noise (SNR=6.12 dB); e) Detection of the echo in (d) by the Gauss-Newton method after 16 iterations where the optimum parameter vector is $\bar{x}_1^{\,*} = [9.60 \quad 4.96 \quad 1.01 \quad 0.5\pi]$ and the SNR is improved to 26.35 dB; f) Detection of the echo in (d) by the Simplex-Search method after 401 iterations where the optimum parameter vector is found to be the same as in (e).

The estimation of two distinct echoes (see Fig 3-a) greatly increases the number of iterations for the Simplex method due to local minimums. For the Gauss Newton method the number of iterations does not change significantly but the accuracy of estimation becomes dependent on the initial guess. Note that in Fig. 3-c, a better initial guess is used, compared to the one for the Simplex Search shown in Fig. 3-b Among the initial guesses of parameters, the one for the arrival time is crucial because frequent local minima exist due to the oscillatory nature of echoes. A better initial guess for the arrival time not only guarantees the global minimum but also reduces the number of iterations.

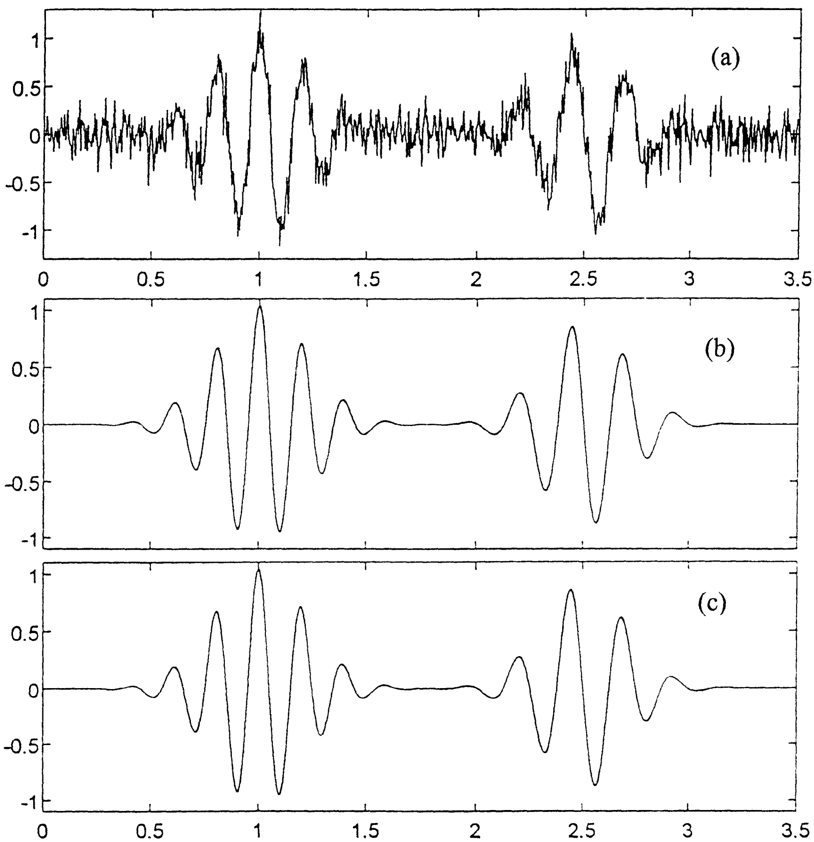

Figure 3. a) Two echoes with 6.05 dB SNR, and parameter vectors $\bar{x}_1 = [10 \;\; 5 \;\; 1 \;\; 0]$, $a_1 = 1.0$ and $\bar{x}_2 = [12 \;\; 4 \;\; 2.5 \;\; 0.5\pi]$, $a_2 = 0.9$; b) Detection of the echoes by the Simplex Search method after 3055 iterations with an initial guesses of $\bar{x}_1^{o} = [6 \;\; 4 \;\; 0.9 \;\; 0]$, and $\bar{x}_2^{o} = [7 \;\; 3 \;\; 2.4 \;\; 0]$ where the optimum parameter vectors are $\bar{x}_1^{*} = [10.58 \;\; 5.03 \;\; 1 \;\; 0]$, $a_1^{*} = 1.04$ and $\bar{x}_2^{*} = [12.49 \;\; 4.04 \;\; 2.50 \;\; 0.55\pi]$, $a_2^{*} = 090$, and the SNR is improved to 24.50 dB; c) Detection of the echoes by the Gauss-Newton method after 24 iterations, using a better initial guess for $\bar{x}_1^{o} = [6 \;\; 4 \;\; 0.95 \;\; 0]$, where the same optimum parameter vectors are found.

For the case of two <u>interfering</u> echoes (see Fig 4-a), the problem becomes more complicated since the routine needs to differentiate between the two echoes. This results in a greater number of iterations for the Simplex Search algorithm (Fig 4-b), and requires a better initial guess for the Gauss Newton method.

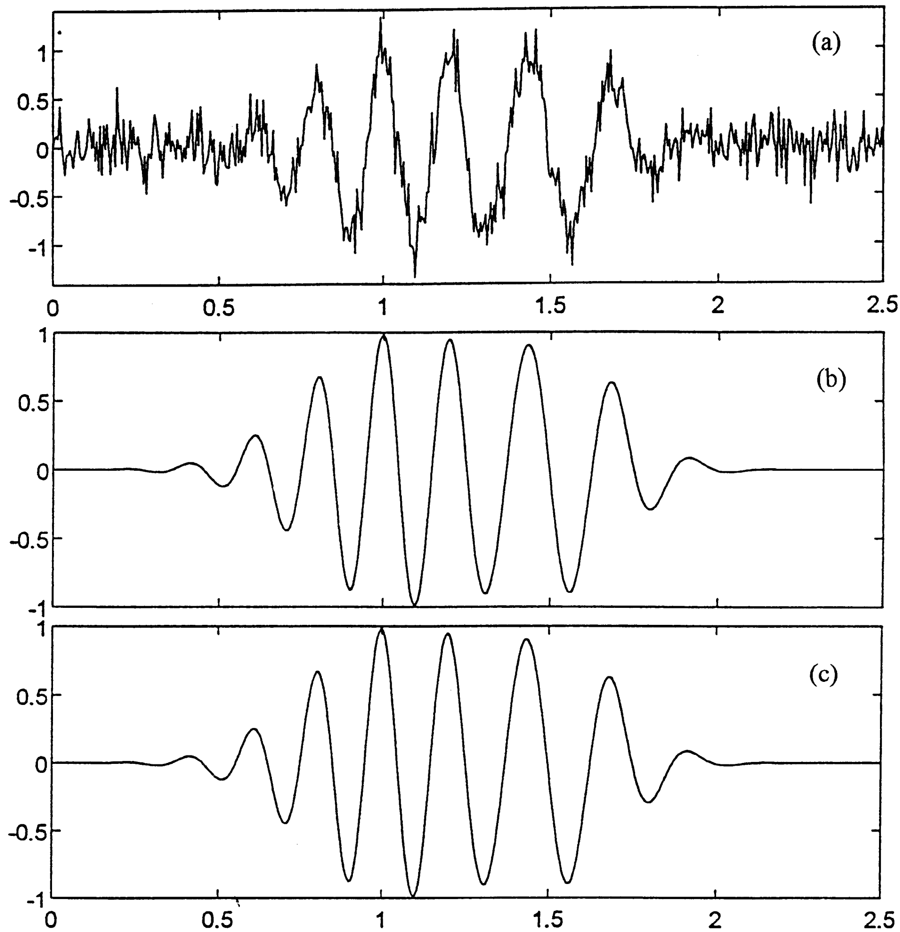

Figure 4. a) Two interfering echoes with 6.81dB SNR, and parameter vectors $\bar{x}_1 = [10 \quad 5 \quad 1 \quad 0]$, $a_1 = 1.0$ and $\bar{x}_2 = [12 \quad 4 \quad 1.5 \quad 0.5\pi]$, $a_2 = 0.9$; b) Detection of the echoes by the Simplex Search method after 3408 iterations with initial guesses of $\bar{x}_1^o = [6 \quad 4 \quad 1.1 \quad 0]$, and $\bar{x}_2^o = [7 \quad 3 \quad 1.4 \quad 0]$ where the optimum parameter vectors are $\bar{x}_1^* = [8.11 \quad 5.06 \quad 1.01 \quad 0.17\pi]$, $a_1^* = 0.98$ and $\bar{x}_2^* = [15.07 \quad 4.12 \quad 1.52 \quad 0.63\pi]$, $a_2^* = 0.97$, and the SNR is improved to 24.62 dB; c) Detection of the echoes by the Gauss-Newton method after 27 iterations, using better initial guesses for $\bar{x}_1^o = [6 \quad 4 \quad 1.05 \quad 0]$, and $\bar{x}_2^o = [7 \quad 3 \quad 1.45 \quad 0]$, where the same optimum parameter vectors are found.

If we increase the number of echoes in the data (Fig.5-a), the problem becomes even more complicated. It doubles the number of iterations and demands a good starting guess for the Gauss-Newton method (Fig. 5-c). Since the Simplex Search uses only function evaluations, the number of iterations greatly increases when the objective function has many variables (Fig. 5-b).

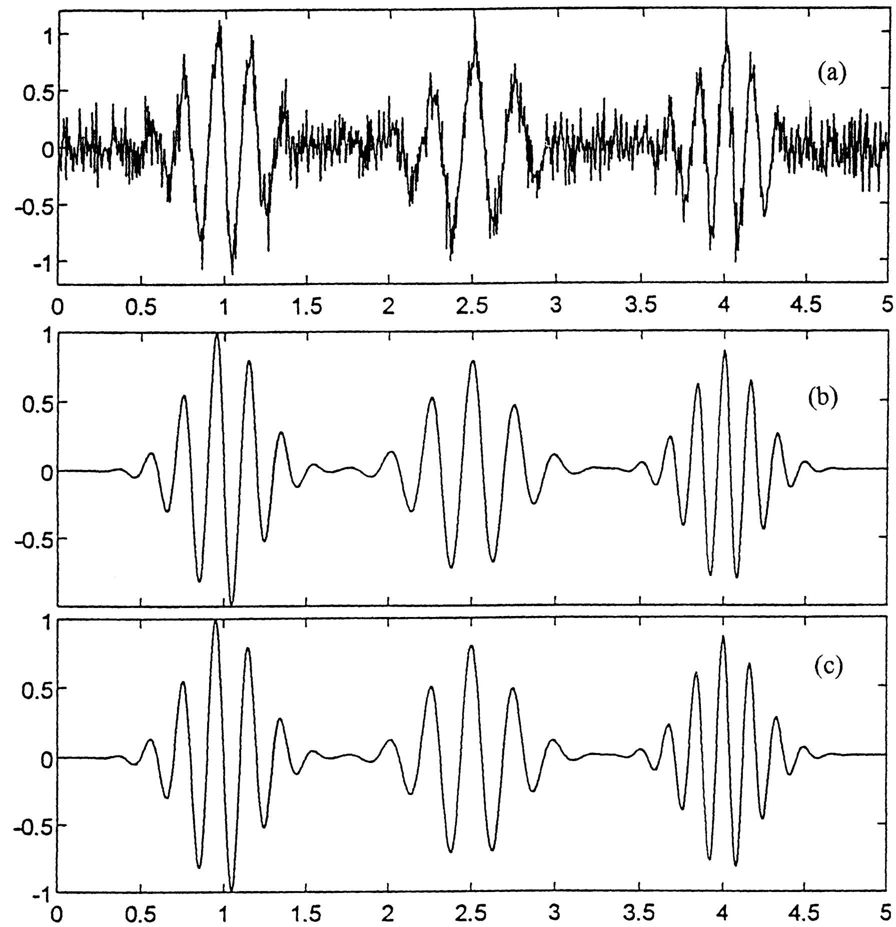

Figure 5. a) Three echoes with 6.26dB SNR, and parameter vectors $\overline{x}_1 = [10 \quad 5 \quad 1 \quad 0.5\pi]$, $a_1 = 1.0$, $\overline{x}_2 = [8 \quad 4 \quad 2.5 \quad 0]$, $a_2 = 0.8$, and $\overline{x}_3 = [12 \quad 6 \quad 4 \quad 0]$, $a_3 = 0.9$; b) Detection of the echoes by the Simplex Search method after 9680 iterations with initial guesses of $\overline{x}_1^{\,o} = [6 \quad 3 \quad 0.95 \quad 0]$, $\overline{x}_2^{\,o} = [5 \quad 2 \quad 2.3 \quad 0]$, and $\overline{x}_3^{\,o} = [9 \quad 4 \quad 3.8 \quad 0]$ where the optimum parameter vectors are $\overline{x}_1^{\,*} = [10.39 \quad 5 \quad 0.99 \quad 0.46\pi]$, $a_1^{\,*} = 1.02$, $\overline{x}_2^{\,*} = [7.6 \quad 4 \quad 2.49 \quad 0.12\pi]$, $a_2^{\,*} = 0.80$, and $\overline{x}_3^{\,*} = [11.27 \quad 6 \quad 4 \quad 0.07\pi]$, $a_3^{\,*} = 0.86$, and the SNR improved to 28.03dB; c) Detection of the echoes by the Gauss-Newton method after 36 iterations, using better initial guesses for $\overline{x}_2^{\,o} = [4 \quad 2 \quad 2.45 \quad 0]$, and $\overline{x}_3^{\,o} = [8 \quad 4 \quad 3.95 \quad 0]$ where the same optimum parameter vectors are found.

CONCLUSION

In this investigation, ultrasonic backscattered echoes are modeled as the superposition of several Gaussian echoes assuming that all parameters (amplitude, arrival time, center frequency, bandwidith and phase) describing the shape of the echo are unknown. Then, the Gauss-Newton and the Simplex-Search methods have been applied to estimate echo parameters. It has been observed that the Simplex-Search method performs well in estimating parameters of echoes with low signal-to-noise ratio, although the rate of convergence is slower by an order of magnitude when compared with the Gauss-Newton method. Estimation of echo center frequency, bandwidth and skewness is robust regardless of the values of the initial guess of these parameters. Care is required in the initial guess of the arrival time due to sub-optimal estimation, although this problem can be eliminated by performing the estimation with multiple random initial guesses and selecting the arrival time that offers the lowest estimation error. Overall, the results above indicate that the optimization routines are capable of detecting the ultrasonic backscattered echoes by their parameter vectors, while improving the SNR by about 20 dB.

REFERENCE

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