REFLECTION OF ELASTIC WAVES BY AN ARRAY OF INTERFACE CRACKS

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INTRODUCTION

An imperfect diffusion bond between two dissimilar materials may be modeled by an interface with an array of interface cracks. The analytical treatment of the reflection and transmission of elastic waves by an array of interface cracks provides the prerequisite for nondestructive characterization of the diffusion bond by ultrasonic techniques. This paper is to develop approximate solutions for the reflection and transmission coefficients from an array of interface cracks.

It is assumed in this paper that the interface cracks are uniformly distributed along the interface. The distribution is characterized by the crack distribution density. By the use of a differential self-consistent scheme in conjunction with the backscattering signal strength formula [1], the multiple scattering problem from a distribution of interface cracks is reduced to finding the crack opening displacement of a single interface crack. Once this single scatterer problem is solved (numerically), the effective reflection and transmission coefficients are obtained by solving a simple differential equation. Finally, applications of the results to nondestructive evaluation of diffusion bonds are discussed.

Reflection and transmission of elastic waves by an array of cracks in a homogeneous solid have been studied extensively, *e.g.*, see [2]-[4].

PROBLEM STATEMENT

Consider an isotropic, linearly elastic bimaterial of infinite extent. A Cartesian coordinate system (x_1, x_2, x_3) is chosen such that the interface is given by $x_2 = 0$. For convenience, we call the material in the upper half-space material 1, the one in the lower half space material 2. Let λ_i, μ_i be the Lamé constants and ρ_i be the mass density, where the subscript $i = 1, 2$ corresponds to the materials with which these constants are associated. It is assumed that the interface contains an array of cracks of average length $2a$ as shown in Fig. 1. The density of crack distribution can be represented by

$$c = aN/L \quad , \tag{1}$$

where N is the number of cracks within distance $2L$.

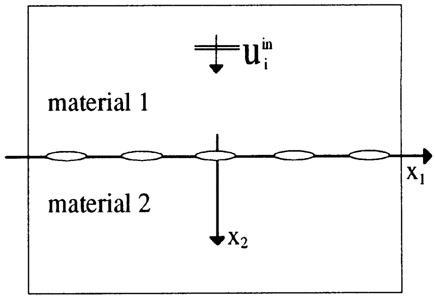

Fig. 1 An array of interface cracks.

Next, let a plane, longitudinal, time-harmonic wave travel in the direction of the positive x_2 axis from $x_2 = -\infty$. Assume the wave has an amplitude factor u_0 and frequency ω. If the steady state term $\exp(-i\omega t)$, which is common to all field variables, is omitted, the displacement field generated by this plane wave in material 1 can be written as

$$u_i^{in} = u_0 \delta_{i2} \exp(ik_L^{(1)} x_2) \ , \tag{2}$$

where δ_{ij} is the Kronecker delta and $k_L^{(1)}$ is the longitudinal wavenumber in material 1.

Because of the interaction between the incident wave and the interface cracks, the wave field near the interface is very complicated. However, for reasonably low frequencies, the field far from the interface will be dominated by longitudinal plane waves [2]. Therefore, the total displacement field may be written as

$$u_i = \begin{cases} u_i^{in} + R(c) u_0 \exp(-ik_L^{(1)} x_2) \ , & x_2 < 0 \\ T(c) u_0 \exp(ik_L^{(2)}) \ , & x_2 > 0 \end{cases} \ , \tag{3}$$

where R and T are termed the effective reflection and transmission coefficients, respectively. Their dependence on the interface cracks distribution is explicitly indicated in (3) by denoting them as functions of c, although R and T also depend on frequencies.

The total displacement field given by (3) implies that the cracked interface may be treated effectively as a perfect interface with an effective reflection coefficient that depends on the crack distribution. The objective of this paper is to find R and T in terms of c and the incident frequency.

BACKSCATTERING AMPLITUDE

Backscattering from the cracked interface contains two components. One is from the cracks, the other is from the interface. In this section, the signal strength formula derived by Auld [1] will be used to obtain the backscattering from a perfect interface and from a single crack on the interface, respectively.

Backscattering Signal Strength Formulas

For a two transducer system, Auld [1] has derived a steady-state reciprocal relation which can be applied to flaw detection and characterization. Transducer I with power P produces the incident field. Transducer II is the receiver. The ratio of received electrical signal strength over incident signal strength is denoted by Γ. Auld's formula gives the change of Γ due to scattering by an imperfection:

$$\delta\Gamma = [(E_{\mathrm{II}})_{\mathrm{flaw}} - (E_{\mathrm{II}})_{\mathrm{no\,flaw}}]/(E_{\mathrm{I}})_{\mathrm{flaw}} , \tag{4}$$

where E_{I} and E_{II} are the strengths of the electrical signals in transducer I and II, respectively. For backscattering, (4) is simplified to

$$\delta\Gamma = -\frac{i\omega}{4P}\int_S (\sigma_{kj}^{(2)} u_k^{(1)} - \sigma_{kj}^{(1)} u_k^{(2)}) n_j dS \ , \tag{5}$$

where S is an arbitrary surface which surrounds the scatterer and n_j is the normal of the surface defined positive inward. The quantities $\sigma_{kj}^{(1)}$ and $u_k^{(1)}$ are the stress and displacement fields induced by the exciting transducer I with power P in the absence of the scatterer, while $\sigma_{kj}^{(2)}$ and $u_k^{(2)}$ are the stresse and displacement fields in the presence of the scatterer. One may also call $\sigma_{kj}^{(1)}$ and $u_k^{(1)}$ the incident fields and call $\sigma_{kj}^{(2)}$ and $u_k^{(2)}$ the total fields.

When the scatterer is a traction-free crack, (5) can be further simplified to

$$\delta\Gamma = \frac{i\omega}{4P}\int_{A^+} \sigma_{kj}^{(1)} \Delta u_k n_j dS \ , \tag{6}$$

where Δu_k is the crack opening displacement defined by

$$\Delta u_k = u_k(A^+) - u_k(A^-) \tag{7}$$

and $A^{\pm}$ indicate the illuminated and un-illuminated crack faces, respectively.

In what follows, (5) and (6) will be used to calculate the backscattering from the interface and from an interface crack.

Backscattering from the Interface

In this section, we consider a perfect interface with reflection coefficient R. For the incident wave given in (2), the relevant displacement and stresses on the interface are:

$$\text{Incident fields:} \quad u_2^{(1)} = u_2^{in} = u_0 \ , \quad \sigma_{22}^{(1)} = \sigma_{22}^{in} = ik_L^{(1)} u_0 (\lambda_1 + 2\mu_1) \tag{8}$$

$$\text{Total fields:} \quad u_2^{(2)} = u_2 = u_0(1+R) \ , \ \sigma_{22}^{(2)} = \sigma_{22} = ik_L^{(1)} u_0 (\lambda_1 + 2\mu_1)(1-R) \ . \tag{9}$$

If the entire lower half-space is considered as a scatterer, the backscattering from the lower half-space can be calculated through (5). For practical purposes, let us assume that the incident beam has a bounded cross-section. Let (-L,L) denote the insonified region on the interface by the incident beam. Then, (5) becomes

$$\delta\Gamma_1 = -\frac{i\omega}{4P}\int_{-L}^{L}(\sigma_{22}u_2^{in} - \sigma_{22}^{in}u_2)dx_1 \quad . \tag{10}$$

Making use of (8) - (9) in (10) yields

$$\delta\Gamma_1 = u_0^2\left(\frac{-i\omega}{4P}\right)\left[4i(\lambda_1 + 2\mu_1)k_L^{(1)}RL\right] \ , \tag{11}$$

where, again, L is the half-length of the insonified region by the incident beam on the interface.

Backscattering from an Interface Crack

Consider a single crack on the interface. When the incident wave u_i^{in} is given by (2), the total wave field can be decomposed into three components. For example, the stress field in material 1 ($x_2 < 0$) can be written as

$$\sigma_{ij} = \sigma_{ij}^{in} + \sigma_{ij}^{r} + \sigma_{ij}^{s} \quad , \tag{12}$$

where σ_{ij}^{in} is the stresses induced by the incident wave, σ_{ij}^{r} is the reflected wave from the interface in the absence of the crack, and σ_{ij}^{s} is the scattered field from the crack in the absence of the interface.

To obtain the backscattering from the crack only, we choose the incident field in (6) to be

$$\sigma_{ij}^{(1)} = \sigma_{ij}^{in} + \sigma_{ij}^{r} \quad . \tag{13}$$

The pertinent component on the crack surface is thus given by

$$\sigma_{22}^{(1)} = ik_L^{(1)}u_0(\lambda_1 + 2\mu_1)(1 - R) \quad . \tag{14}$$

When the incident wave is given by (13), the principle of linear superposition allows us to write the corresponding crack opening displacement Δu_i as

$$\Delta u_i = u_0(1 - R)\Delta v_i \quad , \tag{15}$$

where Δv_i is the crack opening displacement of an interface crack due to the incident displacement wave of unit amplitude.

Substitution of (14) and (15) into (6) yields the backscattering from an interface crack

$$\delta\Gamma_2 = u_0^2\left(-\frac{i\omega}{4P}\right)(\lambda_1 + 2\mu_1)[ik_L^{(1)}a(1 - R)^2 V] \quad , \tag{16}$$

where

$$V = \int_{-1}^{1}\Delta v_2(x)dx \tag{17}$$

is the crack opening area due to an incident displacement wave of unit amplitude. From the definition, it is clear that V is independent of the crack length a. However, V is a function of frequency.

In general, V must be obtained numerically. In this paper, a system of two singular integral equations are derived for the dislocation density f_j

$$\beta f_j(x)+\frac{1}{\pi}\int_{-1}^{1}\frac{f_j(\xi)}{x-\xi}d\xi+\int_{-1}^{1}K_{jm}(x,\xi)f_m(\xi)d\xi=g_j(x) \ , \ j=1,2 \tag{18}$$

where the dislocation density is related to the crack opening displacement by

$$\Delta v_j(x)=\int_{-1}^{x}f_j(\xi)d\xi \ . \tag{19}$$

In (18), β is the second Dundur's bimaterial constant and K_{jm} is a regular kernel. The integral equation is solved by using the Jacobi polynomial technique [5]. Once the integral equation is solved for the dislocation density f_j, the crack opening displacement Δv_i can be obtained from (19). Consequently, the crack opening area V is obtained as a function of frequency from (17).

DIFFERENTIAL SELF-CONSISTENT SCHEME

Differential self-consistent scheme (DSCS) has been used extensively in the area of micromechanics of composite materials. In this section, the DSCS is used to derive a differential equation for the effective reflection coefficient $R(c)$ defined in (3).

The DSCS is based on the notion of incremental construction of the backscattering amplitude by adding one crack at a time to the interface. Suppose that at a given crack density c, the interface is treated as a perfect one with effective reflection coefficient $R(c)$. The backscattering amplitude from this effectively perfect interface can be obtained from (11). The fundamental assumption of DSCS is that when an additional crack is added to the interface, the change in backscattering due to this addition is the backscattering from a single interface crack. This procedure results in an initial value problem for the effective reflection coefficient R

To accomplish the DSCS procedure, let us consider three problems:

Problem 1: Assume the interface has a crack density $c = aN/(L - a)$.

For this problem, if the cracked interface is treated as a perfect interface with effective reflection coefficient $R(c)$, the backscattering from the effectively perfect interface can be calculated from (11)

$$\delta\Gamma_1(c)=u_0^2\left(\frac{-i\omega}{4P}\right)\left[4i(\lambda_1+2\mu_1)k_L^{(1)}R(c)L\right] \ . \tag{20}$$

Problem 2: Assume the interface has a crack density $c_1 = a(N+1)/L$.

Again, if the cracked interface is treated as a perfect interface with effective reflection coefficient $R(c_1)$, the backscattering from the effectively perfect interface also can be calculated from (11)

$$\delta\Gamma_1(c_1)=u_0^2\left(\frac{-i\omega}{4P}\right)\left[4i(\lambda_1+2\mu_1)k_L^{(1)}R(c_1)L\right] \ . \tag{21}$$

Problem 3: Assume a crack of length $2a$ is located on the interface having effective reflection coefficient $R(c)$, where $c = aN/(L - a)$.

It is conceivable that the total backscattering in this problem contains two components. One is from the crack, the other is from the effective interface. The component from the interface crack is given by (16)

$$\delta\Gamma_2(c) = u_0^2\left(-\frac{i\omega}{4P}\right)(\lambda_1 + 2\mu_1)\left\{ik_L^{(1)}a[1 - R(c)]^2 V\right\} \quad . \tag{22}$$

The component from the effective interface with effective reflection coefficient $R(c)$ is given by (20). Therefore, the total backscattering in Problem 3 is the sum of (20) and (22).

On the other hand, let us consider an interface having crack density *aN/L*, *i.e.*, there are *N* cracks in the region [-*L*, *L*]. If we want to add an additional crack of length 2*a* to this region, we must rearrange the existing *N* cracks so that a region of length 2*a* becomes available to accommodate the new crack. Therefore, after the addition, the actual region occupied by the previous *N* cracks is reduced to (2*L* - 2*a*). This means that the crack density outside the newly added crack is *aN*/(*L* - *a*) instead of the original *aN/L*. However, the actual crack density in [-*L*, *L*] becomes *a*(*N*+1)/*L* after the addition. This observation indicates that Problem 3 can be equivalently stated as the consequence of adding one more crack to an interface with crack density *aN/L*. Since both Problem 2 and Problem 3 have the same crack density *a*(*N*+1)/*L*, they should have the same backscattering amplitude.

Based on the reasoning above, the DSCS states that the backscattering amplitude from Problem 2 is the sum of those from Problem 1 and Problem 3, namely,

$$\delta\Gamma_1(c_1) = \delta\Gamma_1(c) + \delta\Gamma_0(c) \quad . \tag{23}$$

Substituting (20) - (22) into (23) yields

$$4R(c_1)L = 4R(c) + a[1 - R(c)]^2 V \quad , \tag{24}$$

or

$$\frac{R(c_1) - R(c)}{c_1 - c} = \frac{a[1 - R(c)]^2 V}{4L(c_1 - c)} \quad . \tag{25}$$

Since

$$\frac{a}{L(c_1 - c)} = \frac{1}{1 - c} \quad ,$$

it follows from (25) that

$$\frac{R(c_1) - R(c)}{c_1 - c} = \frac{[1 - R(c)]^2 V}{4(1 - c)} \quad . \tag{26}$$

In the limit $c_2 \to c$, (26) becomes

$$\frac{dR}{dc} = \frac{[1 - R(c)]^2 V}{4(1 - c)} \quad , \tag{27}$$

which is a first order differential equation for the reflection coefficient R as a function of the crack density c.

An initial condition is required to uniquely determine $R(c)$ from (27). Since c is the crack density, or the percentage of area that is cracked, it is obvious that $c = 0$ means no crack on the interface. In this case, the interface is truly a perfect one and the reflection coefficient R from a perfect interface is well know [6]

$$R(0) = R_0 = \frac{\rho_1/k_L^{(1)} - \rho_2/k_L^{(2)}}{\rho_1/k_L^{(1)} + \rho_2/k_L^{(2)}} \quad . \tag{28}$$

With (28), the differential equation (27) can be solved to yield a unique solution

$$R(c) = 1 - \frac{4(1-R_0)}{4 - V(1-R_0)\ln(1-c)} \quad . \tag{29}$$

This gives the reflection coefficient as a function of crack density c. The frequency dependence of R comes from V, the crack opening area as defined by (19).

To obtain the transmission coefficient $T(c)$, continuity conditions at the interface must be used. For a perfect interface, displacement and traction must be continuous at the interface. For cracked interfaces, displacement is no longer continuous. However, continuity of traction still holds, which, in this case, yields

$$T(c) = \frac{\rho_1 k_L^{(2)}}{\rho_2 k_L^{(1)}}[1 - R(c)] \quad . \tag{30}$$

Note that c may vary from 0 to 1. $c = 0$ means no crack and $c = 1$ means fully cracked interface, *i.e.*, separation. In the fully cracked case ($c = 1$), we expect $R(1) = 1$ and $T(1) = 0$. It is interesting to notice that the solutions given by (29) - (30) do satisfy these limiting conditions exactly, although they are approximate solutions.

As an example, the amplitude of R for a Cu/Al interface is plotted in Fig. 2 vs. non-dimensional wavenumber $k_T a$ for various values of crack density.

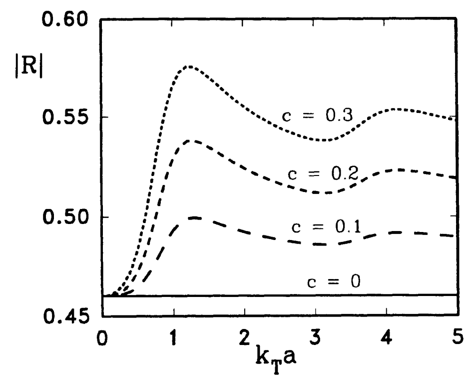

Fig. 2 Amplitude of the effective reflection coefficient for a Cu/Al interface vs. frequency.

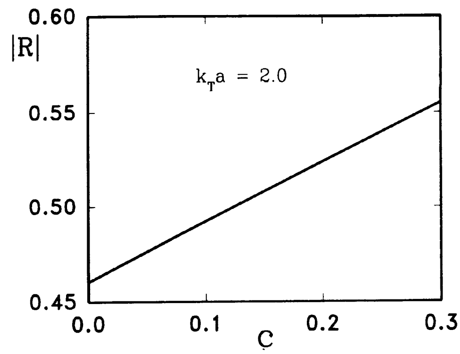

Fig. 3 Amplitude of the effective reflection coefficient vs. crack density.

APPLICATIONS TO NONDESTRUCTIVE EVALUATION OF INTERFACES

The solutions obtained in the previous section can be used to evaluate interface strength. For example, one of the important parameter in determine the remaining bond strength is the remaining bond area. When the debond are modeled by interface cracks, the remaining bond area is directly related to the crack density *c* defined in this paper.

For example, using Fig. 2, another plot can be generated for the Cu/Al interface, see Fig. 3. To evaluate the bond between Al and Cu, a pulse-echo test can be performed to measure the reflection coefficient *R*. Once R is known for a given frequency, the crack density (debond area) can be easily obtained from Fig. 3.

ACKNOWLEDGMENTS

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