RAYLEIGH AND LOVE WAVES IN CLADDED ANISOTROPIC MEDIUM

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INTRODUCTION

Early work on dispersion of Rayleigh and Love waves was by Love [1], who gave the first comprehensive treatment of the case of an elastic solid half-space covered by a single solid layer.

Stoneley [2] investigated the effect of the ocean on the transmission of Rayleigh waves. Many other authors considered the dispersion of surface waves in isotropic layered medium. Ewing et al., [3] gave a review of the works done by many authors, such as, Haskell [4], Jeffreys [5], Kanai [6], Tolstoy and Usdin [7], and Newlands [8]. In all the publications mentioned above the media were considered isotropic. Farnell and Adler [9] discussed guided wave propagation in thin layers. Their discussion included a detailed description of Rayleigh and Love waves in isotropic layered medium, and a general description of Rayleigh and Love waves in anisotropic layered medium. As numerical examples, they discussed in detail the case of cubic crystals when the Love-Rayleigh mode separation was valid, and only one coupled case, when the direction of propagation was rotated 5^0 from the crystal axis. Earlier Crampin [10] and Crampin and Taylor [11] considered surface waves in layered anisotropic media. Of particular relevance to the present work is that of Crampin and Taylor [11], who discussed several cases of anisotropic half-spaces. However, they did not consider the case of an isotropic layer overlying a transversely isotropic half-space when the axis of symmetry is parallel to the interface.

In this paper we investigate the effect of anisotropy of the transversely isotropic substrate on the dispersive wave propagation in the isotropic layer. This problem is of interest in electronics and in space structures (Farnell and Adler, [9]; Pelka, [12]). The two substrates considered here, beta-quartz and graphite fiber reinforced epoxy, are transversely isotropic. It is assumed that in both cases the axis of symmetry is parallel to the layer. Numerical results are presented for two cases: a gold layer on beta-quartz and an aluminum cladding on graphite-epoxy fiber reinforced medium.

PROBLEM FORMULATION

We consider here a layered half space as shown in Fig. 1. This is composed of an isotropic layer of thickness H bonded to a half space of a transversely isotropic material with the x-axis being the unique axis.

If $\vec{U} = (u,v,w)$ is the displacement vector referred to the cartesian coordinate system O(x,y,z), then the equations of motion are, in the absence of body forces,

$$\mathrm{div}\,\underline{\underline{\sigma}} = \rho\ddot{\vec{U}} \tag{1}$$

where $\underline{\underline{\sigma}}$ is the stress tensor and

$$(\dot{\cdot}) \equiv \frac{\partial()}{\partial t} \cdot$$

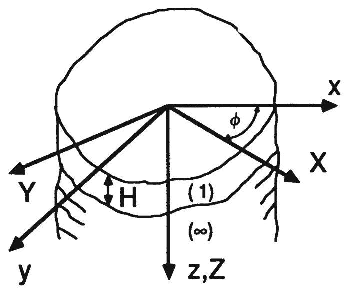


Figure 1. Geometry of the problem. Layered half-space.

Following Buchwald [13] the displacement components satisfying the equations of motion (1).

$$u = \frac{\partial\Theta}{\partial x} \tag{2}$$

$$v = \frac{\partial\Phi}{\partial y} + \frac{\partial\Psi}{\partial z} \tag{3}$$

$$w = \frac{\partial\Phi}{\partial z} - \frac{\partial\Psi}{\partial x} \tag{4}$$

where Θ, Φ, and Ψ can be chosen as, assuming propagating waves in the plane of $x,y,$.

$$\Theta = g_1(z)\ e^{i(kx+\xi y-\omega t)} \tag{5}$$

$$\Phi = g_2(z)\ e^{i(kx+\xi y-\omega t)} \tag{6}$$

$$\Psi = g_3(z)\ e^{i(kx+\xi y-\omega t)}. \tag{7}$$

By substituting the above potentials in (2-4), and in turn in (1), it can be shown that the solutions have the following forms.

$$g_1(z) = F_1\Omega_1^+ + G_1\Omega_2^+ \quad (8)$$

$$g_2(z) = F_2\Omega_1^+ + G_2\Omega_2^+ \quad (9)$$

$$g_3(z) = F_3\Omega_3^- \quad (10)$$

Here

$$\Omega_1^+ = A_{11}\, e^{is_1 z} + A_{12}\, e^{is_1(H-z)} \quad (11)$$

$$\Omega_1^- = A_{11}\, e^{is_1 z} - A_{12}\, e^{is_1(H-z)} \quad (12)$$

$$\Omega_2^+ = A_{21}\, e^{is_2 z} + A_{22}\, e^{is_2(H-z)} \quad (13)$$

$$\Omega_2^- = A_{21}\, e^{is_2 z} - A_{22}\, e^{is_2(H-z)} \quad (14)$$

$$\Omega_3^- = A_{31}\, e^{irz} - A_{32}\, e^{ir(H-z)} \quad (15)$$

with

$$r = \sqrt{\frac{k_2^2 - k^2 - \epsilon\xi^2}{\epsilon}} \quad (16)$$

$$s_{1,2}^2 = -\frac{\left[\gamma k^2 - k_2^2\,(1+\beta)\right]}{2\beta} \pm \frac{\sqrt{\left[\gamma k^2 - k_2^2\,(1+\beta)\right]^2 - 4\beta\left[k^2 - k_2^2\right]\left[\alpha k^2 - k_2^2\right]}}{2\beta} - \xi^2 \quad (17)$$

In the above the non-dimensional material constants have been defined as

$$\alpha = \frac{c_{11}}{c_{55}}\,,\quad \beta = \frac{c_{22}}{c_{55}}\,,\quad \delta = 1 + \frac{c_{12}}{c_{55}},\ \epsilon = \frac{c_{44}}{c_{55}}\,,\ \gamma = 1 + \alpha\beta - \delta^2$$

Also, we have defined

$$k_2^2 = \frac{\rho\omega^2}{c_{55}}\,.$$

The constants appearing in equations (8-10) may be taken as

$$F_1 = G_1 = F_3 = 1 \tag{18}$$

$$F_2 = \frac{k_2^2 - \alpha k^2 - \xi^2 - s_1^2}{\delta\left[\xi^2 + s_1^2\right]} \tag{19}$$

$$G_2 = \frac{k_2^2 - \alpha k^2 - \xi^2 - s_2^2}{\delta\left[\xi^2 + s_2^2\right]} \quad . \tag{20}$$

Using (5-10) in (2-4) and the stress-strain relations, the displacement and traction components can be written in matrix form as follows.

$$\{u\}_m = [E]_m \, [D]_m \, \{v\}_m \tag{21}$$

Here m is the layer identifier, (1 or ∞).

$$\{u\}_m = [u, v, w, \sigma_{yz}, \sigma_{zz}, \sigma_{xz}]_m^T \tag{22}$$

$$\{v\}_m = [A_{11}, A_{21}, A_{31}, A_{12}, A_{22}, A_{32}]_m^T \tag{23}$$

and $[D]_m$ is a (6 X 6) diagonal matrix

$$\mathrm{diag}[D]_m = [e^{is_1 z}, e^{is_2 z}, e^{irz}, e^{is_1(H-z)}, e^{is_2(H-z)}, e^{ir(H-z)}]_m . \tag{24}$$

$[E]_m$ is a 6 X 6 matrix whose elements depend on the material and wave parameters. This can be found in [14]. The vectors $\{u\}_m$, $\{v\}_m$ and matrices $[E]_m$, $[D]_m$ in equation (21) are partitioned to improve the efficiency and accuracy of the numerical computations. We write

$$\{u\}_m = \begin{Bmatrix} u_D \\ \cdots \\ u_S \end{Bmatrix}_m \qquad \{v\}_m = \begin{Bmatrix} v^- \\ \cdots \\ v^+ \end{Bmatrix}_m \tag{25}$$

$$[E]_m = \begin{bmatrix} Q^m_{11} & Q^m_{21} \\ \cdots & \cdots \\ Q^m_{12} & Q^m_{22} \end{bmatrix}, \quad [D]_m = \begin{bmatrix} \left[D^m_{11}\right]_z & 0 \\ \cdots & \cdots \\ 0 & \left[D^m_{22}\right]_z \end{bmatrix} \tag{26}$$

where the subscript z stands for the z level at which the [D] matrices are evaluated and the subscripts D, and S denote displacements and tractions, respectively.

Using the solution given by equation (21) and applying the appropriate boundary conditions of continuity of tractions and displacements at the interface between the layer and the half space, the traction-free boundary condition at the upper surface, and the radiation condition as $z \to \infty$, one arrives at the following dispersion equation.

$$\det[B - AC^{-1} D] = 0 \tag{27}$$

Where

$$[A] = Q^{\infty}_{11} \left[D^{\infty}_{11}\right]_0 \tag{28}$$

$$[B] = Q^{1}_{11} \left[D^{1}_{11}\right]_H \left\{Q^{1}_{21} \left[D^{1}_{11}\right]_0\right\}^{-1} Q^{1}_{22} \left[D^{1}_{22}\right]_0 - Q^{1}_{12} \left[D^{1}_{22}\right]_H \tag{29}$$

$$[C] = Q^{\infty}_{21} \left[D^{\infty}_{11}\right]_0 \tag{30}$$

$$[D] = Q^{1}_{21} \left[D^{1}_{11}\right]_H \left\{Q^{1}_{21} \left[D^{1}_{11}\right]_0\right\}^{-1} Q^{1}_{22} \left[D^{1}_{22}\right]_0 - Q^{1}_{22} \left[D^{1}_{22}\right]_H \tag{31}$$

Given the material properties of the media, equation (27) can be solved to obtain the dispersion curves showing the dependence of the phase velocity on frequency. This is discussed in the following section.

NUMERICAL RESULTS AND DISCUSSION

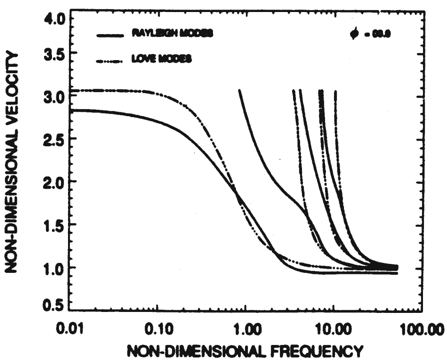
In this paper, two cases are studied. A beta-quartz transversely isotropic substrate coated with a gold layer and an aluminum cladded graphite-epoxy composite whose material properties are listed in table 1.

Table 1. Material Properties.
(All the stiffnesses are in the Units 10^9 N/m^2).

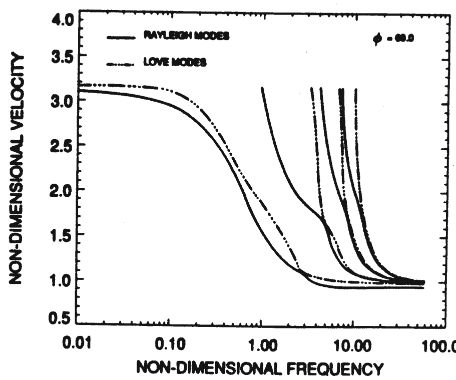
Material	ρ(g/cm^3)	c_{11}	c_{22}	c_{12}	c_{44}	c_{55}
Gold	19.3	203.0	203.0	147.0	28.0	28.0
Beta-quartz	2.65	110.4	116.6	32.8	49.95	36.1
Aluminum	2.7	109.0	109.0	56.0	26.5	26.5
Graphite-epoxy	1.2	160.7	13.92	6.44	3.5	7.07

For a gold layer on the beta-quartz, three angles of propagation are considered. The corresponding Love and Rayleigh-mode dispersion curves are plotted in Figs. 2(a,b,c). The non-dimensional velocity is c/β^1 and KH c/β^1 is the non-dimensional frequency. The superscript on the shear velocity is the layer identifier. For gold β^1 = 1.2045 Km/s. Figure 2(a) shows the dispersion curves for the first eight modes when the direction of propagation is along the symmetry axis ($\phi = 0^0$). The Love-modes start at the horizontally polarized shear wave velocity of the substrate $\beta^{\infty}{}_{55}$ = 3.6909K m/s and approach asymptotically the shear velocity of the layer β^1 as (KH $\to \infty$). The two subscripts on the shear velocity are the subscripts of the stiffness constant that relates the direction of propagation to the direction of polarization.

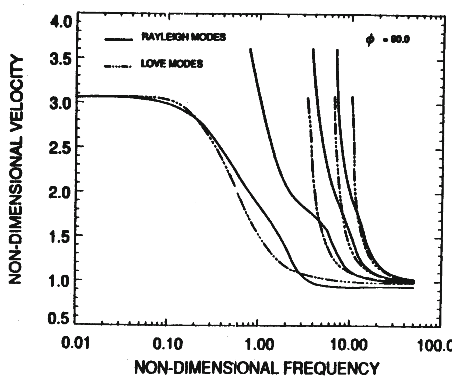
The first Rayleigh-mode starts at the velocity of the Rayleigh wave of the substrate in the direction of symmetry, c^{∞}_R = 3.4320 Km/s as (KH $\to$ 0) and approaches the free-surface Rayleigh wave velocity of the layer, $c^1{}_R$ = 1.1274 Km/s.



a) Direction of propagation along the symmetry axis.



b) Direction of propagation rotated 69⁰ from symmetry axis.



c) Direction of propagation normal to symmetry axis.

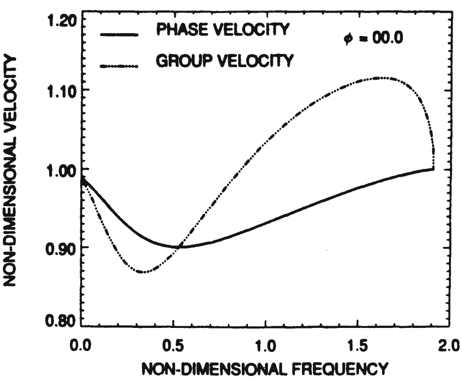
Figure 2(a,b,c). Phase velocity dispersion of the eight first modes of gold on beta-quartz.

The higher Rayleigh-modes start at the vertically polarized shear wave velocity of the substrate, β^{∞}_{55} and approach the shear velocity of the layer β^1 for high wave numbers.

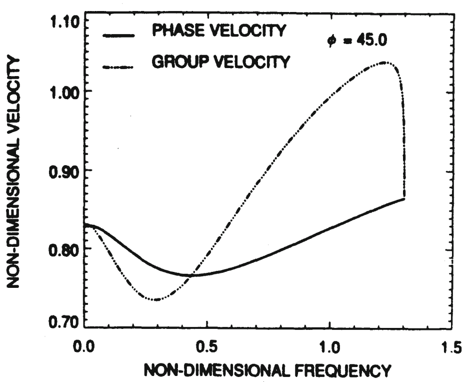
In Fig. 2(c) the dispersion curves for direction of propagation normal to the symmetry axis (ϕ = 90^0) are shown. In this case and for propagation at 0^0 (Fig. 2(a)) the Love-modes and the Rayleigh-modes separate, which is a very useful check for our three-dimensional formulation. The Love-modes start at the horizontally polarized shear wave velocity β^{∞}_{55} of the substrate and approach the shear wave velocity β^1 of the layer. The first Rayleigh mode starts at the velocity of the Rayleigh wave of the substrate in the direction perpendicular to the symmetry axis, and is determined to be equal to β^{∞}_{55}. As (KH $\rightarrow \infty$), it approaches c^1_R of Gold. The higher modes start at the vertically polarized shear wave velocity β^{∞}_{44} = 4.3415 Km/s and as (KH $\rightarrow \infty$), they approach β^1 asymptotically.

The dispersion curves for a 69⁰ angle of propagation are shown in Fig. 2(b). This specific angle was chosen, because the Free-surface Rayleigh wave velocity of the beta-quartz is maximum. The fundamental mode tends to the surface wave velocity in the half-space, $c_R^{\infty} = 3.740$ Km/s, at low frequency and approaches $c^1{}_R$ at high frequency. The higher modes start at the lowest quasi-transverse velocity in the half-space, $\beta^{T_1} = 3.7804$ Km/s. It is noticeable that the modes do not cross each other as in the previous cases, but they pinch as observed by Crampin and Taylor [11].

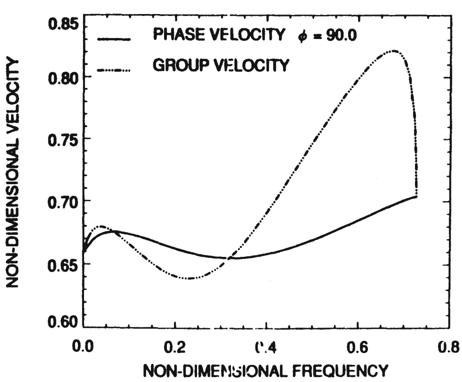
For the aluminum cladded graphite-epoxy half-space, results for three different directions of propagation are shown in Figs. 3(a,b,c). For this combination of material properties, the layer is found to stiffen the substrate as mentioned by Farnell and Adler [9], when they discussed the isotropic case. The common feature of these dispersion curves is that there is only one Rayleigh-like mode that propagates in a limited range of KH. They all start at the free-surface Rayleigh velocity and ends at the lowest shear velocity of the substrate, which happens to be the quasi-transverse wave velocity for the graphite-epoxy. The slopes of the dispersion curves are negative for ($\phi = 0^0$) as (KH $\rightarrow$ 0) and increase as the direction of propagation makes larger angles with the symmetry axis.



a) Direction of propagation along the fiber axis.



b) Direction of propagation rotated 45⁰ from the fiber axis.



c) Direction of propagation normal to fiber axis.

Figure 3. Phase and group velocities for a layer of aluminum on graphite-epoxy.

CONCLUSION

The effect of anisotropy of the substrate on the surface wave propagation in the isotropic layer has been studied in this paper. Two cases were considered: one relevant for electronic applications and the other for aerospace structures. It is shown that the relative material property differences in the two cases give rise to quite different behaviors of the surface waves. Implications of these differences to the scattering by defects (interface or surface cracks) will be reported in a subsequent paper.

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