SCATTERING OF OBLIQUELY INCIDENT RAYLEIGH WAVES BY A SURFACE-BREAKING CRACK

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ABSTRACT

Recent results on reflection, transmission and scattering of obliquely incident Rayleigh surface waves by an infinitely long surface-breaking crack are reviewed. Sets of crack-opening displacements for infinitely long cracks with various depths are used to construct approximate crack-opening fields for the scattering of a Rayleigh wave by a surface-breaking crack of large length-to-depth ratio. The scattered surface-wave fields for the finite-length crack are subsequently obtained by the use of a representation integral over an appropriate Green's function and the approximate crack-opening displacements. Polar diagrams are presented for the amplitude of the scattered surface-wave field.

INTRODUCTION

Scattering of both surface waves and body waves by a sub-surface crack has been analyzed in Refs.[1] and [2]. Scattering by a surface-breaking crack, under normal incidence, has been treated by Mendelsohn et al.[3]. More recently some interesting resonance effects have been found for the scattering by a near-surface parallel crack (see Ref.[4]). In addition, various approximate methods to compute scattered fields for surface-breaking and sub-surface cracks have been discussed in Refs.[5]-[8]. Scattering of horizontally polarized waves by sub-surface and surface-breaking cracks of arbitrary orientation has been analyzed by Mal [9] and Datta [10], respectively.

In this paper we first review some results of Ref.[11] for the reflection, transmission and scattering of a plane Rayleigh surface wave by an infinitely long surface-breaking crack, when the wave is incident under arbitrary angle. As in Ref.[3] it is convenient to decompose the formulation into two problems. The first problem deals with the physically symmetric fields relative to the plane of the crack, and the second problem deals with the antisymmetric fields. The symmetric problem is governed by a single singular integral equation for the opening-mode dislocation density. The antisymmetric problem is, however, governed by a system of two coupled singular integral equations for the two sliding-mode dislocation densities. In Ref.[11] the singular integral equations have been solved numerically. Once the dislocation densities are known, the fields of reflected and transmitted surface waves can be computed by the use of representation integrals.

The scattering problem is then reconsidered for the case of a surface-breaking crack of finite dimensions. The results obtained previously for an infinite crack are used to derive an approximation to the field scattered by a finite crack. Polar plots show the angular variations of the scattered-field amplitude at large distances from the crack.

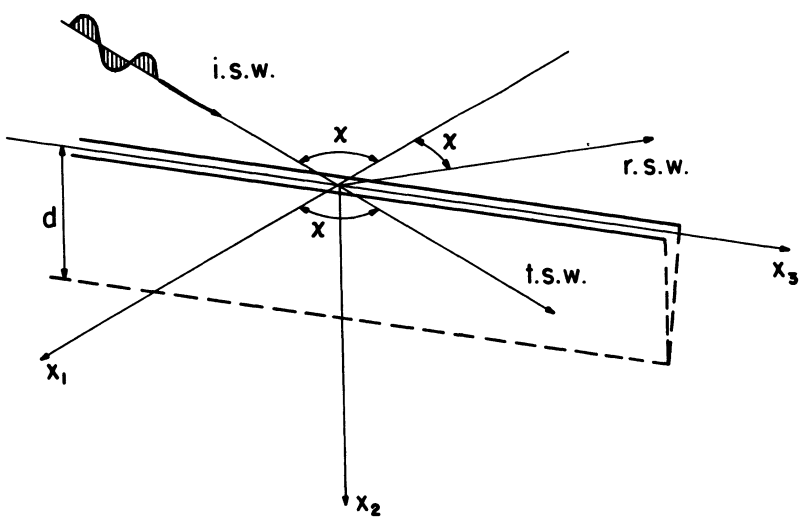

Fig. 1: Reflection and transmission of a Rayleigh surface wave (incident surface wave) incident on an infinite crack of depth d.

INFINITELY LONG CRACK

We consider a homogeneous, isotropic, linearly elastic half-space which contains a surface-breaking crack of depth d, normal to the free surface, as shown in Fig. 1. The crack lies in the plane $x_1 = 0$ and extends to infinity in the $\pm x_3$ directions. Let μ and ν denote the shear modulus and Poisson's ratio. The slownesses of surface, longitudinal and transverse waves will be referred to as s_R, s_L and s_T, respectively. It is then convenient to define the following parameters:

$$\eta_R = s_T/s_R, \quad \varepsilon = s_L/s_T = \sqrt{\frac{1-2\nu}{2(1-\nu)}}, \quad \alpha_L = \sqrt{1-(\varepsilon\eta_R)^2}, \alpha_T = \sqrt{1-\eta_R^2} \quad . \tag{1}$$

The incident Rayleigh surface wave travels over the free surface $x_2 = 0$. The angle of incidence χ is measured from the normal. Let u_o and ω denote an amplitude factor and the frequency of the incident wave.

We will consider time-harmonic waves. Since there is translational invariance with respect to the x_3-axis, the term $\exp[-i\omega(t-s_R x_3 \sin\chi)]$ is common to all field variables. In the sequel this term is omitted. The displacements generated by the incident wave can then be written as:

$$u_1(x_1,x_2)=u_o\cos\chi \; V_1(x_1,x_2), \quad u_2(x_1,x_2)=iu_o P(\eta_R) \; V_2(x_1,x_2) \; ,$$

$$u_3(x_1,x_2)=u_o\sin\chi \; V_3(x_1,x_2) \; , \tag{2a,b,c}$$

where

$$V_j(x_1,x_2) = [X_j e^{-\omega s_R \alpha_L x_2} + (1-X_j)e^{-\omega s_R \alpha_T x_2}]\exp(i\omega s_R x_1 \cos\chi), \quad (j = 1,2,3) \tag{3}$$

$$X_1 = X_3 = 2/\eta_R^2 \; , \quad X_2 = 1 - X_1 \; , \tag{4}$$

$$P(\eta_R) = (2\alpha_L\alpha_T + \eta_R^2 - 2)/(\alpha_T\eta_R^2) \; . \tag{5}$$

In the plane of the crack ($x_1 = 0$), the stresses associated with the displacements (2) take the form:

$$\sigma_{1i}(0,x_2) = \mu\, u_o\, \sigma_i(x_2)\ , \quad (i = 1,2,3)\ , \tag{6}$$

where the functions σ_i are readily obtained by the use of Eqs.(2) and Hooke's law.

The total field in the half-space can be analyzed as the superposition of the incident field in an uncracked half-space and the scattered field in the cracked half-space. The scattered field, which can be thought of as being generated by prescribed surface tractions on the faces of the crack, is conveniently decomposed into physically symmetric and antisymmetric fields relative to the plane $x_1 = 0$. The prescribed surface tractions on the crack faces are equal in magnitude, but opposite in sign, to the tractions of Eq.(6). The symmetric and antisymmetric fields can be determined from the solutions of two independent mixed-boundary-value problems for the quarter-space defined by $x_1 \geq 0$, $x_2 \geq 0$ and $-\infty < x_3 < \infty$. The symmetric problem is defined by the boundary conditions:

$$\sigma_{2i} = 0\ , \quad x_2 = 0\ , \tag{7a}$$

$$\sigma_{11} = -\mu\, u_o \sigma_1(x_2)\ , \quad 0 < x_2 < d, \quad x_1 = 0\ , \tag{7b}$$

$$\sigma_{12}, \sigma_{13} = 0\ , \quad x_1 = 0\ , \tag{7c}$$

$$u_1 = 0\ , \quad d < x_2 < \infty\ , \quad x_1 = 0\ . \tag{7d}$$

The antisymmetric problem is defined by:

$$\sigma_{2i} = 0\ , \quad x_2 = 0\ , \tag{8a}$$

$$\sigma_{12} = -\mu\, u_o \sigma_2(x_2)\ , \quad 0 < x_2 < d\ , \quad x_1 = 0\ , \tag{8b}$$

$$\sigma_{13} = -\mu\, u_o \sigma_3(x_2)\ , \quad 0 < x_2 < d\ , \quad x_1 = 0\ , \tag{8c}$$

$$\sigma_{11} = 0\ , \quad x_1 = 0\ , \tag{8d}$$

$$u_2, u_3 = 0\ , \quad d < x_2 < \infty\ , \quad x_1 = 0\ . \tag{8e}$$

In both Eqs.(7) and (8), the subscript i runs from 1 to 3.

It is shown in Ref.[11] that the formulation of the problem can be reduced to two systems of singular integral equations of the first kind. The unknown functions in these equations are the symmetric-mode (I) and antisymmetric-mode (II,III) dislocation densities across the crack faces. The symmetric-mode dislocation density is governed by a single integral equation, while the two antisymmetric-mode dislocation densities are governed by a pair of coupled integral equations. The singular integral equations are stated in Ref.[11].

The integral equations can be solved numerically by using the methods discussed in Refs.[13] and [14]. It suffices to prescribe the values of three dimensionless parameters. They are: Poisson's ratio ν, the angle of incidence χ, and the frequency of excitation $\bar{\omega} = d/\lambda_R$, where $\lambda_R = 2\pi/\omega s_R$ is the wavelength of surface waves in the solid. All other dimensionless parameters can be written in terms of these three basic parameters.

The crack-opening displacements are readily obtained by integration once the dislocation densities are known. Curves for the displacements in the x_1-direction are shown in Fig. 2. Four angles of incidence (χ = 0°, 45°, 75° and 89.5°) and two frequencies (corresponding to d/λ_R = 0.2 and 0.9) have been chosen.

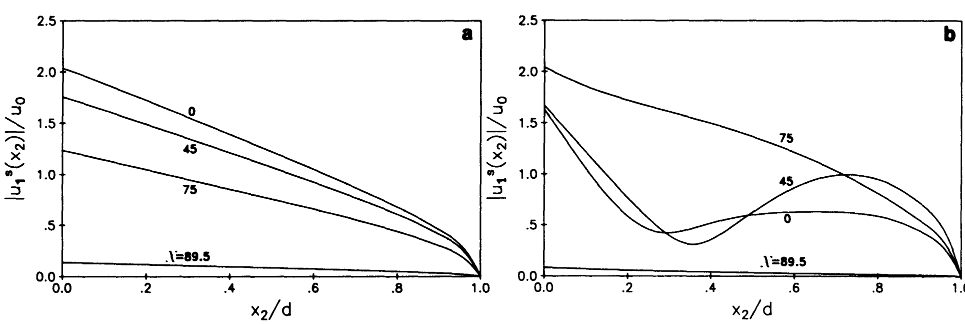

Fig. 2: Crack-opening displacements for various angles of incidence χ, for d/λ_R = 0.2 (a) and d/λ_R = 0.9 (b); ν = 0.3.

REFLECTION AND TRANSMISSION COEFFICIENTS

The radiated wave motion at a large distance from the plane of the infinitely long crack can be investigated by writing an asymptotic expansion for the displacement field as $|x_1| \to \infty$. It is found that the leading terms in the asymptotic expansions are non-decaying, while the next terms decay as rapidly as $1/|x_1|^{\frac{1}{2}}$, as $|x_1| \to \infty$. The non-decaying terms follow from the presence of pole singularities in the representation integrals. The details are given in Ref.[11].

The far-field displacements have the same form as those of the incident Rayleigh wave (see Eq.(2)). When x_1 is positive, the symmetric and antisymmetric displacements are for Rayleigh waves propagating under an angle χ with the positive x_1 axis (see Fig. 1), with amplitudes u_o^S and u_o^A respectively. When x_1 is negative, they are for Rayleigh waves propagating under an angle χ with the negative x_1 axis, with amplitudes u_o^S and $-u_o^A$.

We define the reflected Rayleigh surface wave as the sum of the two waves propagating in the negative x_1-direction. For $x_1 > 0$, the transmitted Rayleigh surface wave is defined as the sum of three waves: the incident wave and the two waves propagating in the positive x_1-direction. The amplitudes of the reflected and transmitted surface waves are $u_o^S - u_o^A$ and $u_o + u_o^S + u_o^A$, respectively. The reflection coefficient A_r and the transmission coefficient A_t are defined by

$$A_r = (u_o^S - u_o^A)/u_o, \quad A_t = (u_o + u_o^S + u_o^A)/u_o \ . \tag{9}$$

In general, the displacements u_o^S and u_o^A are complex numbers. Hence, the displacements of the reflected and transmitted surface waves differ from those of the incident surface wave both in phase and modulus. The fractions of time-averaged incident energy flux carried by the reflected and transmitted surface waves are equal to $|A_r|^2$ and $|A_t|^2$, respectively. The quantity

$$\langle P_{rad} \rangle = 1 - |A_r|^2 - |A_t|^2 \tag{10}$$

is then the fraction of time-averaged incident energy flux radiated into the solid by body waves.

In Ref.[11] curves have been presented for the reflection and transmission coefficients both versus the angle of incidence χ for fixed frequency and versus d/λ_R (λ_R = wavelength of surface waves) for fixed angle of incidence. For angles of incidence larger than a critical angle $\chi_c = \arcsin(\eta_R)$ the nature of the scattering process changes. For $\nu = 0.3$ we have $\chi_c \cong 68°$. When $\chi > \chi_c$, the apparent speed of the incident wave along the faces of the crack is less than the speed of transverse waves. Consequently, only non-propagating body-wave modes are excited over the crack faces (see Freund [15]). All energy transported to the crack by the incident wave must be carried away by the reflected and transmitted surface waves. Hence, it follows from Eq.(10) that

$$< P_{rad} > = 0 \ , \text{ for } \chi > \chi_c . \tag{11}$$

This result has indeed been verified numerically for two different frequencies. In fact, Eq.(11) provides us with a criterion for checking the validity of the numerical calculations in the range $\chi > \chi_c$. For all values of χ tested in this range it is found that $< P_{rad} >$ is less than 10^{-13} in absolute value. A figure for $< P_{rad} >$ versus the angle of incidence, for two values of d/λ_R, is presented in Ref.[11].

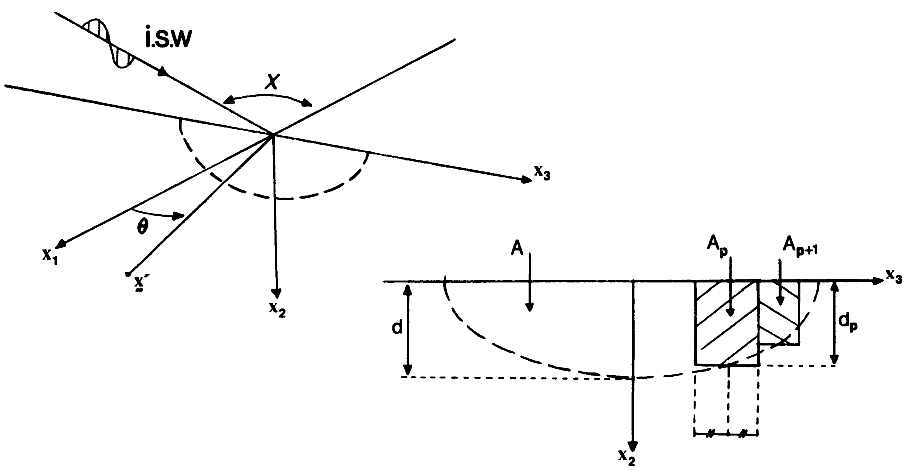

Fig. 3: Scattering of a Rayleigh surface wave (incident surface wave) incident on a crack of finite dimensions. Illustration of the method to evaluate the scattered field.

CRACK OF FINITE DIMENSIONS

Next we consider a crack of finite dimensions in the plane $x_1 = 0$, as shown in Fig. 3. The incident wave is the same as that of Eq.(2). In this section we investigate the radiated wave motion at an observation point $\underset{\sim}{x}'$ located on the free surface at a large distance from the origin in the direction θ. The total field in the half-space is written as the superposition of the incident field in an uncracked half-space and the scattered field in the cracked half-space. The scattered displacement-field at $\underset{\sim}{x}'$ can be written in the form

$$u_k(\underset{\sim}{x}') = \int_A \sigma^{GR}_{1j;k}(\underset{\sim}{x},\underset{\sim}{x}')\Delta u_j(\underset{\sim}{x})dA(\underset{\sim}{x}), \tag{12}$$

where

$$\Delta u_j = u_j^+ - u_j^- , \tag{13}$$

and $\sigma^{GR}_{ij;k}$ is the Rayleigh wave contribution to the Green's tensor $\sigma^{G}_{ij;k}$ (see Harris et al.[16]). The area of the crack is denoted by A. The plus superscript refers to the face that lies in the region $x_1 > 0$ and the minus superscript to the face in the region $x_1 < 0$.

The integral representation (12) is used to obtain an approximation to the scattered field. In fact, the exact crack-face displacements Δu_j, which are unknown, are replaced in Eq. (12) by the displacements obtained for <u>infinite</u> cracks of various depths. Figure 3 illustrates this approximation. The area A of the crack is broken up into n rectangular strips A_1, .., A_n. The p-strip has depth d_p. Then, instead of (12), we write:

$$u_k(\underset{\sim}{x}') \cong \sum_{p=1}^{n} \int_{A_p} \sigma^{GR}_{1j;k}(\underset{\sim}{x},\underset{\sim}{x}')\Delta u^p_j(\underset{\sim}{x})dA(\underset{\sim}{x}) , \tag{14}$$

where Δu^p_j is the displacement-discontinuity across the faces of an infinite crack of depth d_p. Using the results of Appendix A in Harris et al. [16], we find that:

$$\sigma^{GR}_{1j;k}(\underset{\sim}{x},\underset{\sim}{x}') = \frac{A_o}{(\omega s_R r)^{\frac{1}{2}}} U_k(\underset{\sim}{x}',\theta)\ \Sigma_j(\underset{\sim}{x}) , \tag{15}$$

where $U_k(\underset{\sim}{x}',\theta)$ are the displacements for a Rayleigh wave of amplitude unity, propagating at an angle θ from the x_1-axis (see Fig. 3 and Eq.2 and recall that the x_3'-dependence in Eq.(15) is of the form $\exp(i\omega s_R x_3' \sin\theta)$). The angle θ is defined by:

$$\cos\theta=(x_1'-x_1)/r,\ \sin\theta = (x_3'-x_3)/r,\ r = [(x_1'-x_1)^2+(x_3'-x_3)^2]^{\frac{1}{2}}. \quad (16)$$

Also, we have

$$A_o = (\omega s_L \alpha_T/(\mu\varepsilon\eta_R^2 R'(1/\eta_R)))e^{i\pi/4}/(2\pi)^{\frac{1}{2}}\ , \quad (17)$$

$$\Sigma_j(\underset{\sim}{x})=\mu i\omega s_R\ T_j[Z_{j1}e^{-\omega s_R \alpha_L x_2}+ Z_{j2}e^{-\omega s_R \alpha_T x_2}]e^{-i\omega\Omega},\text{(no sum on j)},(18)$$

where

$$\Omega = s_R x_1 \cos\theta + s_R x_3 \sin\theta\ , \quad (19)$$

$$T_1 = 2/\eta_R^2,\ Z_{11} = \eta_R^2 + 2\alpha_L^2 - 2\sin^2\theta,\ Z_{12} = (\eta_R^2-2)\cos^2\theta\ , \quad (20a)$$

$$T_2 = -4i\alpha_L \cos\theta/\eta_R^2,\ Z_{21} = -Z_{22} = 1\ , \quad (20b)$$

$$T_3 = \sin(2\theta)\ ,\ Z_{31} = 1 -Z_{32} = 2/\eta_R^2\ , \quad (20c)$$

$$R(u) = (2u^2-1)^2- 4u^2(u^2-1)^{\frac{1}{2}}(u^2-\varepsilon^2)^{\frac{1}{2}}. \quad (21)$$

Next, recall the x_3-dependence of the crack-face displacements:

$$\Delta u_j^p(0,x_2,x_3) = \Delta u_j^p(0,x_2)\exp(i\omega s_R x_3 \sin\chi)\ . \quad (22)$$

Substitution of (15) and (22) into (14), together with the approximations

$$r \cong ((x_1')^2 + (x_3')^2)^{\frac{1}{2}},\ \tan\theta \cong x_3'/x_1'\ , \quad (23)$$

which are valid when the observation point is at a large distance from the crack, yields a result of the form

$$u_k(\underset{\sim}{x}') = E(\theta,\chi,d/\lambda_R)\left(\frac{d}{r}\right)^{\frac{1}{2}} U_k(\underset{\sim}{x}',\theta)\ , \quad (24)$$

where d is the depth of the crack at $x_3 = 0$ and E depends not only on the parameters θ,χ and d/λ_R, but also on the shape of the crack and Poisson's ratio.

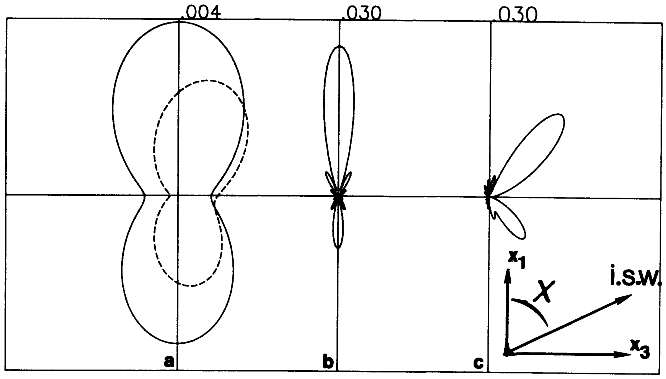

Fig. 4: Scattering coefficient for a semi-elliptical crack; (a) $k_R d = 1$, $\chi = 0°$(——) and $\chi = 45°$ (----); (b) $k_R d = 5$ and $\chi = 0°$; (c) $k_R d = 5$ and $\chi = 45°$; $\nu = 0.3$.

For a semi-elliptical crack defined by the equations

$$4x_2^2 + x_3^2 = 4d^2, \; x_2 \geq 0 \;, \tag{25}$$

and a choice of ten rectangular strips such that

$$|x_3|/2d = 0 \;, 1/3, 7/12, 47/60, 57/60, 1 \;, \tag{26}$$

the modulus of the scattering coefficient $E(\theta,\chi,d/\lambda_R)/(2u_o)$ is shown in Fig. 4. These curves are polar curves of the observation angle θ. Two angles of incidence ($\chi = 0°$ and $45°$) and two frequencies ($k_R d \equiv \omega s_R d = 1$ and 5) have been chosen. We observe diffraction lobes at the larger frequency.

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