Area angle can monitor cascading outages with synchrophasors

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***Abstract*—We monitor the severity of multiple line outages inside an area of the power system according to the limitations on a bulk power transfer through the area when the outages occur. The monitoring combines together synchrophasor measurements around the border of the area to form an angle across the area that can be tracked in real time. This is an approach based on physical principles to extract actionable information by suitably combining synchrophasor measurements. We show the capabilities of the method on a model of the WECC system on an area with approximately 500 lines.**

I. Introduction

With increasing and variable demands placed on the power transmission system, areas of the power system are often stressed as bulk power is transferred through the area. Each line in the area has a power flow limit that is a thermal limit or arises as a proxy for other kinds of limits. Under contingencies such as line outages, these individual line limits become more binding on the bulk transfer of power through the area, and in severe cases, the bulk power flow through the area will have to be restricted. It is important to be able to quickly determine the severity of the outages so that the appropriate remedial actions can be taken. Especially in the case of multiple outages, a quick response could prevent further cascading and a blackout. Many observed cascading blackouts start with a few outages occurring more slowly, which gives a possibility of quick action to forestall the subsequent, faster cascading processes that lead to a widespread blackout.

This paper demonstrates how to combine synchrophasor measurements around the border of an area to quickly monitor the severity of multiple outages inside the area. Alternatively, after some delay for state estimation calculations, one can also monitor outages via SCADA and state estimation. However, state estimation is less reliable for multiple outages. If the state estimation fails, our approach using synchrophasors changes from a faster alternative to the only indicator.

More generally, synchrophasor measurements provide fast monitoring of bus voltages over a wide area. As more synchrophasors are deployed, one of the challenges is summarizing and understanding the new data. One advantageous approach is to use physical principles to combine together synchrophasor measurements into quantities that are more meaningful and actionable. In this paper we combine voltage angles around the border of an area of the power system into a bulk angle across the area. The concept of the voltage angle across a power system area is new and is described in detail in [1], [2], including how it derives from circuit theory principles. The area angle concept is a generalization of the angle across a cutset area concept developed and proposed for stress monitoring in [3], [4], [5]. Throughout this paper we use a DC load flow model with voltage phasor angles and real power flows.

We note that synchrophasor measurements around the border of an area can be advantageous for other applications such as combining AC voltage measurements in a transmission corridor to monitor voltage collapse [6] or locating line outages in the area [7]. More generally, the border measurements can be used to effectively decouple the area from the rest of the interconnection [8]. These methods that apply to power areas will be particularly useful when utilities or ISOs in large interconnections restrict their attention to network models and phasor measurements for only their own area.

II. Stress monitoring using area angle

Fig. 1 illustrates a simple example of three equal, parallel lines connecting two buses a and b. We compare monitoring the angle difference between the buses with monitoring the power transferring between them in the case of a double outage.

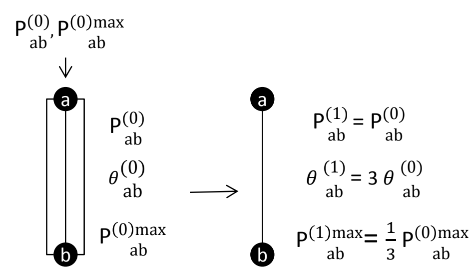

Fig. 1. Simple example of three parallel lines with double outage

We assume lossless lines and we observe how the angle difference between buses and the power entering into bus a varies from the base case to the double outage case. The superscripts 0 and 1 stand for the base case and double outage case respectively. In both the base case and the double outage case, the power P_{ab} entering into bus a remains the same, while the angle difference $\theta_{\rm ab}$ between the buses increases and triples in the case of a double outage. This increment in the angle as the double outages occur is a good indicator of increased stress in the system.

To quantify stress caused by line outages, we consider the maximum power that could enter the system. As the outages become more severe, they cause the other lines to reach their thermal limits and so the capacity of the system to transfer more power reduces. For instance, as the double line outages occur in Fig. 1, the maximum power that could enter bus a decreases. We can see that as the stress increases, or in other

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words, the maximum power that could enter bus a decreases, the power entering the area remains constant, but the area angle gets larger and indicates the increased system stress.

To generalize this simple example to the real system, we consider a connected area of the power system with border buses M. The border buses M comprise the a border buses near the generation and the b border buses near the loads. The area is mostly transferring power between the generation and load. The power entering into the area (similar to the power entering bus a in the simple example) is the sum of the powers entering into the area along the tie lines connected to the border buses a. We apply the new concept of area angle to get the angle difference across the area from the a buses to the b buses (similar to angle difference of the buses a and b in the simple example). As described in detail in [1], the area angle θ_{area} is a weighted combination of the angles around the area:

$$\theta_{\text{area}} = \frac{\sigma_a B_{eq} \theta_m}{b_{\text{area}}} = w\theta_m. \tag{1}$$

Here σ_a is the vector of the size of the number of border buses M which has the entry 1 corresponding to each a border bus and entry 0 in the rest. B_{eq} is the susceptance matrix of the Kron reduction of the area to the border buses (this Kron reduction is electrically equivalent to the original area from the perspective of the border buses). θ_m is the vector of the angles of the border buses M. b_{area} is the susceptance of the area which can be calculated as

$$b_{\text{area}} = \sigma_a B_{eq} \sigma_a^T. \tag{2}$$

As we can see, the area susceptance and area angle are inversely related. We discuss this relationship in detail in [10].

w in (1) is a row vector of weights that depend only on the area topology and the susceptances of lines in the area. The a buses have positive weights and the b buses have negative weights. Thus in (1) to get the area angle across the area from the a buses to the b buses, the weighted combination of the angles in the b border buses is subtracted from the weighted combination of the angles in the a border buses.

It is important to choose the area so that the area angle is meaningful and useful for power systems operation. In this paper, we choose an area of the transmission system between major generation and major load to try to describe with the area angle the stress resulting from the transfer of power through the area and how the stress varies with line outages inside the area.

III. Stress Monitoring Using Area Angle

A. Problem set up

Our goal is to monitor a single quantity for the area that captures the severity of multiple outages inside the area. Ideally, the monitored quantity changes from its base case value if a line outages, and the amount of change should indicate the severity of the outage. Our results will show that while the real power entering the area P_a^{into} remains constant after the outages, the area angle θ_{area} increases as the outages becomes more severe and tracks the severity of the outages inside the area. Thus the area angle θ_{area} is a better indicator of area stress than the real power entering the area P_a^{into}.

We evaluate the severity of the outage inside the area with the maximum power that could enter the area P_a^{intomax}. We increase the power entering and leaving the area by assuming a particular pattern of load and generation injection at the border buses that increases the power transferred through the area. This power transferred through the area is increased until the first line in the area reaches its power flow limit. Each line in the area has a limit on its real power flow that corresponds to the line thermal limit or is a proxy for other system limits. As the generation and load increase, there is increased stress on the transmission system, and lines may approach or reach their limits, especially under contingency conditions in which another line outages. We calculate the area angle and the maximum power that could enter the area that satisfy all line limits after each outage. It is of interest to find out how much monitoring θ_{area} gives an indication of the outage severity as evaluated by P_a^{intomax}. Note that since P_a^{intomax} involves a hypothetical increase of the power entering the area from the current situation, it cannot be monitored directly.

The objective is to show how area angle can track the severity of the outages inside the area. However, there are some outages inside the area which for the area angle can not track the severity well. Plotting and ordering the relationship between the maximum power that could enter the area and the corresponding area angle can reveal and identify these outages that are outliers. In particular, we plot area angle and the maximum power transfer for all single outages, order them based on the maximum power transfer, find these outliers, and handle these exceptional cases separately. Note that this screening for the outliers can be done using the single outages only. After removing these outlier lines from the list of all lines inside the area, we can track the outages of remaining lines, plot the result and observe the relationship between the area angle and the corresponding maximum power transfer.

B. Formulation

We need to calculate area angle, area susceptance and the maximum power entering the area after outages. We use formulas (1) and (2), after outages to determine area angle and area susceptance. Furthermore, after finding the extra power injection in border buses after outages that stresses the area until the first limit is reached, we can calculate the maximum power could enter the area without violating any line limits. This section explains this calculation in detail.

For a general area that has paths around the area that are parallel to the power flow through the area, an outage inside the area will cause some change in the power into the area tie lines. But if there are no such parallel paths around the area, the power in the tie lines does not change, and the power entering into the area will remain constant. In our results we use an area that has high impedance parallel paths so that the the power entering into the area will remain approximately constant.

To find the maximum power that could enter the area after outages, we need to calculate at first how much more power can be injected in the border buses and then add this injection to the base case power entering to the area. We use the power transfer distribution factor of the lines with respect to injections in border buses and the real power limits of lines to calculate the extra injection in the border buses. We increase the power entering the area with a specific pattern of injection in the border buses until the first line violates its maximum power flow limit. The pattern of injection is proportional to the base case power entering each border buses along the tie lines connected to that bus. This has the effect of increasing the area stress in the same pattern as the base case stress.

We use the following notation:

$P_{\mathrm{line}k}$	power flow through line k
P_{line}	vector of power flows through lines
$\Delta P_{\mathrm{inj}}^{rs}$	amount of extra power injected positive in bus r and negative in bus s to stress the system
θ	vector of voltage angles at buses
θ_{line}	voltage angle in each line
B	susceptance matrix
Λ	diagonal matrix of line susceptances
A	bus line incidence matrix
ρ_k^{rs}	power transfer distribution factor for line k with respect to injections in buses r and s

We describe the variables above in different conditions using the following notation:

X	generic variable
$X^{(i)}$	X evaluated for contingency number i. The base case is contingency number 0.
$X^{k\max}$	X evaluated at the maximum stressed case obtained by applying stress until line k reaches its maximum power flow rating.
$X^{(i)\max}$	X evaluated for the maximum stressed case obtained under contingency number i
X^{limit}	operating limit established for X

Contingency i can be single or cascading outages.

To calculate area angle after contingency i we use (1) as follows:

$$\theta_{\mathrm{area}}^{(i)} = \frac{\sigma_a B_{eq} \theta_m^{(i)}}{b_{\mathrm{area}}} = w\theta_m^{(i)}. \tag{3}$$

Note that, as discussed further in [10], (3) uses the weights w calculated from the susceptance matrix and area susceptance evaluated *before* the outage of line i, but it uses the border buses angles $\theta_m^{(i)}$ measured *after* contingency i. The susceptance matrix and an updated topology of the area before the outage are generally available to a control center [11].

As mentioned, we need to calculate the maximum power that could enter the area and for that we need to calculate the extra injection in border buses. It is convenient to first consider just border buses r and s and calculate the extra injection in buses r and s after contingency i. Injection in buses r and s means we add this injection in border bus r on the generation side and subtract the injection from border bus s on the load side. To find out this extra injection in border buses r and s, we need to find the margin of power flow and the generation shift factor of each line k in the area with respect to the injection in border buses r and s. To find out the margin of power flow in line k we do the following steps.

The voltage angles across the lines are

$$\theta_{\mathrm{line}}^{(i)} = A^T \theta^{(i)}, \tag{4}$$

and the power flows in lines are

$$P_{\mathrm{line}}^{(i)} = \Lambda^{(i)} \theta_{\mathrm{line}}^{(i)}\ , \tag{5}$$

where $\Lambda^{(i)}$ is the diagonal matrix of the susceptances after contingency i.. The margin of power flow of line k until its limit is reached is:

$$\Delta P_{\mathrm{line}k}^{(i)} = P_{\mathrm{line}k}^{\mathrm{limit}} - P_{\mathrm{line}k}^{(i)}\ , \tag{6}$$

where $P_{\mathrm{line}k}^{\mathrm{limit}}$ is the power flow limit of line k.

To find the generation shift factor, suppose that contingency i happens and that line k joins bus u to bus v. Then the power transfer distribution factor for line k is the amount of the increase in the power flow in line k due to a unit injection of power in bus r and a unit decrement of power in bus s:

$$\rho_k^{rs(i)} = b_k(e_u^T - e_v^T)(B^{(i)})^{-1})(e_r - e_s) \tag{7}$$

Here e_r denotes a vector with 1 at entry r and all other entries zero. Now, the maximum amount of injection in bus r and decrement from s until line k reaches its line limit is

$$\Delta P^{rs(i)k\max} = \frac{\Delta P_{\mathrm{line}k}^{(i)}}{\rho_k^{rs(i)}}. \tag{8}$$

Then the maximum possible or extra injection at the border buses r and s which satisfies all the line limits is the minimum amount of the maximum extra injections for all the lines:

$$\Delta P_{\mathrm{inj}}^{rs(i)} = \mathrm{Min}\{\Delta \mathrm{P}^{\mathrm{rs(i)1max}}, \Delta \mathrm{P}^{\mathrm{rs(i)2max}}, \ldots, \Delta \mathrm{P}^{\mathrm{rs(i)nmax}}\},$$

where n is the total number of lines inside the area.

Then the maximum power $P_r^{\mathrm{into(i)max}}$ entering bus r corresponding to the maximum extra injection after contingency i, can be calculated as well:

$$P_r^{\mathrm{into(i)max}} = P_r^{\mathrm{into}} + \Delta P_{\mathrm{inj}}^{rs(i)} \tag{9}$$

Now the calculation given above for the extra injection at the buses r and s can be extended to the specific pattern of extra injections assumed at the border buses a and b by appropriately weighting the generation shift factors. We multiply the pattern ratios related to each pair of border buses to the value of generation shift factor for that pair and add them for all pairs of buses selected from a and b to get the final generation shift factor that relates increases in the injections in the given pattern to the change in power flow at each line k.

IV. Results

We use a 1553 bus model of WECC that was reduced from a larger model for cascading analysis [12]. We select the area shown in Figure 2 which covers roughly Washington and Oregon states. The 7 northern (and western) border buses are near the borders of Canada-Washington, Washington-Montana, Oregon-Idaho, and the 5 south border buses are near the Oregon-California border. There are approximately 400 buses and 515 lines inside this area. The transfer through the area of interest is north to south; that is, from the north border buses to the south border buses.

The area angle is

$$\begin{aligned}\theta_{\text{area}} = \; & 0.223\,\theta_1 + 0.006\,\theta_2 \\ & + 0.008\,\theta_3 + 0.01\,\theta_4 + 0.02\,\theta_5 + 0.18\,\theta_6 + 0.59\,\theta_7 \\ & - 0.39\,\theta_8 - 0.41\,\theta_9 - 0.004\,\theta_{10} - 0.03\,\theta_{11} - 0.18\,\theta_{12}\end{aligned}$$

In practice the measurements with very small weights could be omitted.

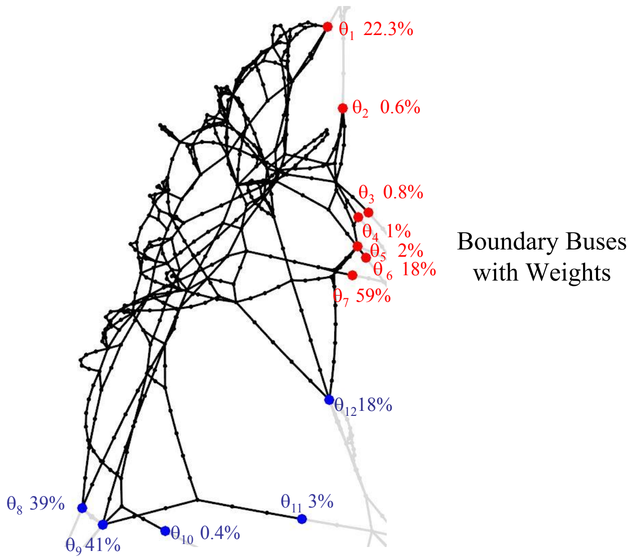

Fig. 2. Area of WECC system with area lines in black, north border buses in red and south border buses in blue. The border bus weights are shown as percentages. Layout detail is not geographic.

We first compute the area angle and the maximum power that could enter the area for all single line outages inside the area, and order the outages based on the decreasing value of the maximum power transfer, or, equivalently, in order of increasing stress. We observed that area angle increases as the maximum power transfer decreases in almost all cases, but there are some outliers to the general trend. We find these outlier lines, take care of them separately, and remove them from the list of lines considered. There are 53 outliers from 515 lines inside the area of which only 30 of them are really of concern, since we only need to detect alarm and emergency situations and do not need to perfectly track the severity by area angle. For all the remaining lines, we sample random combinations of double and triple outages. After ordering the results by severity in the same way as before, we observe in all of them that the area angle tracks the maximum power transfer well and can detect the severity of the multiple, and potentially cascading, outages inside the area. We will show some of these results later.

The main reason for the abnormal behavior of the outliers is that they change the local transfer of power, but not the bulk transfer of power through the area. These lines are typically near big generation and load inside the area so that their outage changes the local transfer of power. The area angle is approximately related to the susceptance of the area [10] and to the bulk transfer of power through the area, not the local power transfer. The other reason for these outliers is lack of coordination between the line limits and the susceptance of the area. Since lines limits affect the maximum power that could enter the area and the susceptance affects the area angle, the coordination between them affects the results. In the case of our test system, some of the lines have artificial line limits because the model is reduced from a larger grid model, and this could be one factor that reduces the coordination between the line limits. We take care of all the outlier line outages separately. Synchrophasor or SCADA signals would monitor the outages of these lines and their outages need to be mitigated separately with individual actions.

We select a random samples of double and triple outages from the remaining list of lines, compute the area angle and area susceptance after their outage and then plot them in order of increasing severity as shown in Figures 3 and 4.

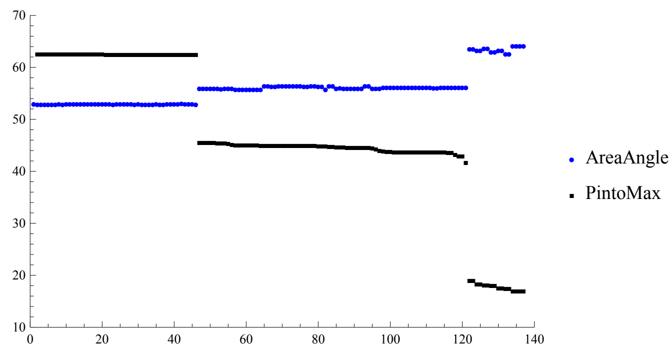

Fig. 3. Area angle $\theta^{(i)}_{\text{area}}$ in degrees, and maximum power into the area in per unit for a random sample of double outages in the area. Horizontal axis is outage number.

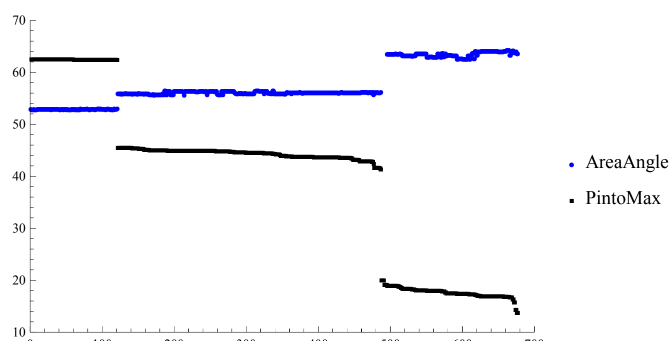

Fig. 4. Area angle $\theta^{(i)}_{\text{area}}$ in degrees, and maximum power into the area in per unit for a random sample of triple outages in the area. Horizontal axis is outage number.

As can be easily seen in the figures, the area angle tracks the severity of the outages. From left to right, as the maximum power that could enter the area decreases, the area angle increases and detects the severity of the double and triple outages. The plots also show that area angle can separate non-severe outages from the moderate or severe ones. There are three different levels of maximum power transfer that correspond to the safe, moderate and severe outages. There are also three levels of area angle corresponding to the three levels of severity. This suggests that thresholds can be set so that the area angle can distinguish safe outages from the moderate or severe ones and hence improve situational awareness. In real time, area angle can be calculated quickly from the weights and the angle data coming from synchrophasors and then it can be compared to its threshold to give an alarm in emergency situations. Moreover, the way we have formulated the outage severity indicates that the emergency action should reduce the bulk power transfer through the area.

To also show that area angle is related to the area susceptance, we did the same calculation for another random sample of triple outages and this time, we also calculate the area susceptance. The results in Figure 5 show that $\theta^{(i)}_{\rm area}$ is inversely related to the area susceptance $b^{(i)}_{\rm area}$, and as the outages becomes more severe from left to right, the susceptance of the area decreases and the area angle increases. Here also separation into different stress levels for all quantities can be easily seen.

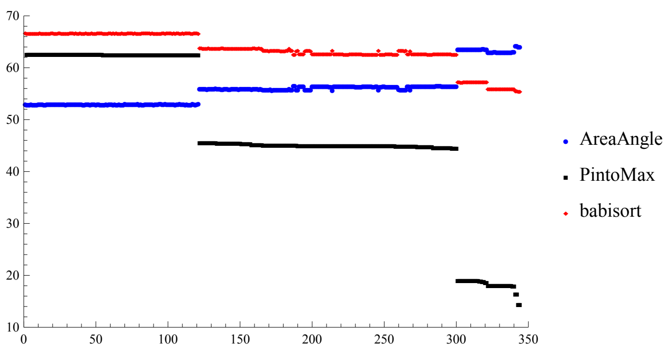

Fig. 5. Area angle $\theta^{(i)}_{\rm area}$ in degrees, area susceptance $b^{(i)}_{\rm area}$, and maximum power into the area in per unit for triple outages in the area. Horizontal axis is outage number.

All the results above are from the list of lines from which the outlier lines associated with local problems were all removed, but if one is only interested to only classify the outage severity into the three levels, this is possible by just removing 30 lines from the list of all lines. The advantage of this is that we need to take special account of fewer outlier lines compared to the other cases above, but we relax the exact tracking of the severity by the area angle. Figure 6 shows the area angle and the maximum power that could enter the area after triple combinations of outages chosen from such a list. It is evident that the outage severity classifications are preserved.

V. Conclusion

An area angle formed by combining together synchrophasor measurements around the border of the area can quickly track the severity of line outages inside the area. In particular, once outlier cases due to local effects inside the area are detected by analyzing the single outage cases and handled separately, we can quickly track the severity of multiple line outages with respect to limitations on the bulk power transfers through the area caused by individual lines reaching their power flow

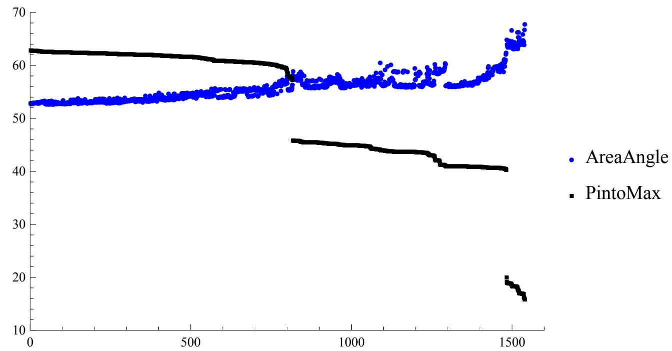

Fig. 6. Area angle $\theta^{(i)}_{\rm area}$ in degrees, and maximum power into the area in pu for a random sample of triple outages in the area with fewer special cases handled separately. Horizontal axis is outage number.

limits. This quick indication of outage severity could help forestall slowly developing cascading failures in the multiple outage case that is the most challenging case for complementary approaches based on state estimation. The separation of non-severe and severe outages also suggests setting thresholds of area angle corresponding to these severity levels to provide to operators improved situational awareness and recommended actions curtailing bulk power transfers when necessary.

VI. Acknowledgments

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References

[1] I. Dobson, Voltages across an area of a network, *IEEE Transactions on Power Systems*, vol. 27, no. 2, May 2012, pp. 993-1002.

[2] I. Dobson, New angles for monitoring areas, IREP Symposium, Bulk Power System Dynamics and Control - VIII, Buzios, Brazil, Aug. 2010.

[3] I. Dobson, M. Parashar, C. Carter, Combining phasor measurements to monitor cutset angles, Forty-third Hawaii International Conference on System Sciences, Kauai, Hawaii, January 2010.

[4] I. Dobson, M. Parashar, A cutset area concept for phasor monitoring, IEEE PES General Meeting, Minneapolis, MN USA, July 2010.

[5] G.J. Lopez, J.W. Gonzalez, R.A. Leon, H.M. Sanchez, I.A. Isaac, H.A. Cardona, Proposals based on cutset area and cutset angles and possibilities for PMU deployment, IEEE PES General Meeting, San Diego CA, July 2012.

[6] L. Ramirez, I. Dobson, Monitoring voltage collapse margin by measuring the area voltage across several transmission lines with synchrophasors, IEEE PES General Meeting, Washington DC, July 2014.

[7] H. Sehwail, I. Dobson, Locating line outages in a specific area of a power system with synchrophasors, North American Power Symposium (NAPS), Urbana-Champaign IL, Sept. 2012.

[8] H.Sehwail, I.Dobson, Applying synchrophasor computations to a specific area, *IEEE Trans. Power Systems*, vol 28, no 3, Aug 2013, pp. 3503-3504.

[9] A. Darvishi, I. Dobson, A. Oi, C. Nakazawa, Area angles monitor area stress by responding to line outages, NAPS North American Power Symposium, Manhattan KS USA, September 2013.

[10] A. Darvishi, I. Dobson, Synchrophasor monitoring of single line outages via area angle and susceptance, NAPS North American Power Symposium, Pullman WA USA, September 2014.

[11] J.E. Tate, T.J. Overbye, Line outage detection using phasor angle measurements, *IEEE Trans. Power Syst.*, vol 23, no 4, Nov 2008, pp. 1644-1652.

[12] M. Morgan, et al., Extreme Events Phase 2 final report, California Energy Commission, publication number CEC-MR-08-03, 2011