EVALUATION OF DISPERSION MODES FOR LAYERED STRUCTURES USING FOCUSED ACOUSTIC WAVES

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INTRODUCTION

The evolution of Lamb wave dispersion has a great significance for the characterization of layered materials such as composite plates, bonded joints, and coating materials. The method traditionally used to evaluate locally the dispersion Lamb modes refers to the use of two wide-band transducer in "pitch-catch" disposition. By varying the emitter-receiver at the opposite incident-reflection angle θ, and scanning the emission tone-burst frequency f, one distinguishes the minima in the reflection coefficient *R(f)* of the test sample at each θ, the dispersion relation of group velocity v_g versus frequency f can be established by $v_g = v_0 / \sin\theta$ where v_0 is the wave velocity in coupling liquid [1-3]. However, there are some inconveniences for this method. Firstly, it needs an ultrasonic goniometer to adjust two probes at a varying oblique incident angle, which is not practical for NDE utilization. Secondly, with finite-sized transducers where the incident beam is not really a plane wave, the angular and spectra resolution will be limited, and in addition there will be a dead angle for small incident angle. Thirdly, there exists the so called "non-specular reflection" occurring at Lamb critical angles and it causes perturbations for reflection wave detection [1,2]. To overcome these disadvantages, we employ a focused acoustic beam of large angular aperture at normal incidence to evaluate the reflection coefficient $R(\theta, f)$ for layered structures. This method demands only one probe and the problem of the non-specular reflection can be avoided as the wave reflection at the different angle is all captured at the same time. The principle is based on registration of Acoustic Material Signature or *V(z)* curve of the sample. An inversion algorithm of Fourier transform permits us to reconstruct $R(\theta)$ if the *V(z)* is measured in amplitude and in phase. By scanning *f* for each *V(z)* measurement, we obtain the reflection coefficient function $R(\theta, f)$. The Lamb wave modes is then be evaluated from the minima appearing in the magnitude of $R(\theta, f)$ data.

THEORY

Determination of Lamb Modes from the Reflection Coefficient Function for a Layered Structure

When a plane wave reaches a layered plate immersed in a fluid at a certain incident angle θ, the incident wave can be decomposed into two components - one normal to the plate and

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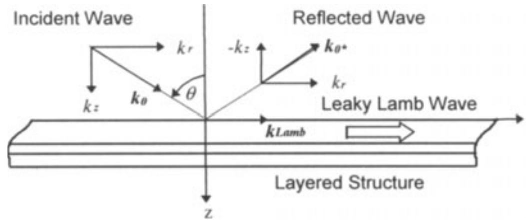

Fig. 1. Lamb wave generated by plane wave in a layered structure.

another parallel to the plate (see Fig. 1). If the wave vector of the later wave k_r coincides with the Lamb wave vector k_{Lamb} of the plate, the Lamb wave will be generated and propagate along the plate. The generation of Lamb wave will give a loss in the reflection wave and appear minima in the reflection coefficient when the incident angle or the frequency varies.

The similarity between the Lamb wave dispersion and the minima evolution of reflection coefficient function of a single layer has been studied by Chimenti and Rokhlin [4]. They demonstrated that when the density of the coupling fluid is much less than that of the layer, the zeros of the reflection coefficient function converge to the poles which are the solutions of dispersion equation for Lamb waves. In ultrasonic NDE, it is often the case that one uses water as coupling fluid and testing materials having a mass density greater than that of water. For a multi-layered structure, this condition can be generalized if the impedance ratios between the coupling fluid and the top and bottom layers of the structure are small. To give an example, we show in Fig. 2 the dispersion curves of Lamb waves and in Fig. 3 the evolution of the reflection coefficient function $R(\theta, f)$ in its magnitude for a three-layer bonded structure.

In Fig. 2, we have: (a) the dispersion curves for the three-layer plate immersed in water and (b) the dispersion curves for the same plate free of charge (in vacuum). In order to compare them with the reflection coefficients, these dispersion curves are not plotted as common used type as group velocity v_g versus frequency *f*, but an equivalent incident angle θ instead of group velocity for the vertical axis. This angle corresponds to the radiation angle of leaky energy in surrounding fluid (water) when Lamb wave propagates (Fig. 2 (a)). The relationship between v_g and θ is $v_g = v_0 / \sin\theta$. As the Lamb waves are leaky, v_g is complex and the propagation velocity takes its

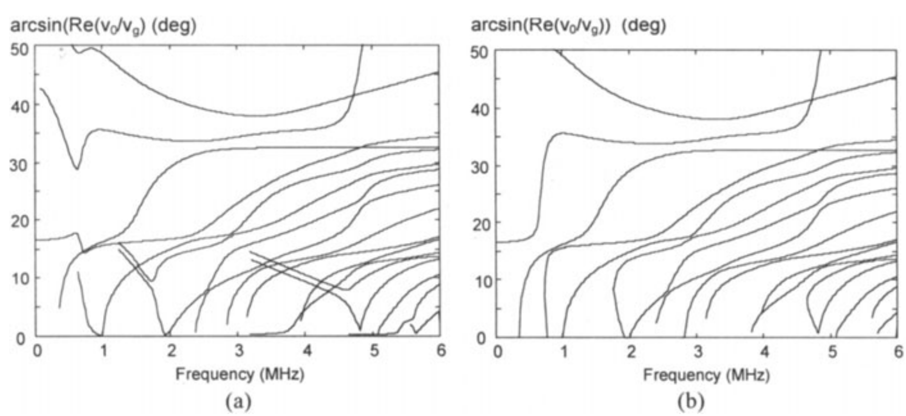

Fig. 2. Dispersion curves of Lamb waves for a bonded joint of Aluminum (1.6mm)-Epoxy(0.2mm)-Aluminum(0.6mm) (a) immersed in water (leaky Lamb mode) (b) free of charge (Lamb mode).

real part. So, we have $\theta = \arcsin\left(\mathrm{Re}\left(v_0/v_g\right)\right)$. When the plate is free of charge (Fig. 2 (b)), there is no coupling fluid and v_0/v_g means the inverse of normalized Lamb wave velocity. In Fig. 3, the complex reflection coefficient function $R(\theta, f)$ is calculated for the same structure and is given in its magnitude with an image representation as used by Chimenti [5]. The value of the reflection coefficient is represented by the gray-scale from zero to unity. As we talked above that the minima in the reflection coefficient function identify the conditions for Lamb wave generation, the black traces in the image correspond to the dispersion relation of the Lamb waves.

It must be pointed out that the dispersion behavior for the layered plate with charge and without charge is not the same as we compare the Fig. 2 (a) with (b). The minima evolution of the reflection coefficient function obtained for plate immersed in water is only comparable to the Lamb mode dispersion for the plate without charging but not for water coupled one.

V(z) Method

The *V(z)* curve or its terminological name the Acoustic Materials Signature was first introduced to understand the mechanism of image contrast in scanning acoustic microscope. It was then developed to characterize quantitatively the elastic properties of the testing materials. The *V(z)* curve is obtained by measuring the amplitude *V* of the signal received by the acoustic lens at normal incidence of a testing sample when the relative distance *z* between the lens and the sample is translated. For a volume sample, the most explored information from a *V(z)* measurement is the Rayleigh surface wave velocity of the material. The *V(z)* curve is explained as the interference response between the reflection wave and the excited Rayleigh surface wave when the testing sample is insonified by a focused beam having an aperture angle which covers the Rayleigh critical

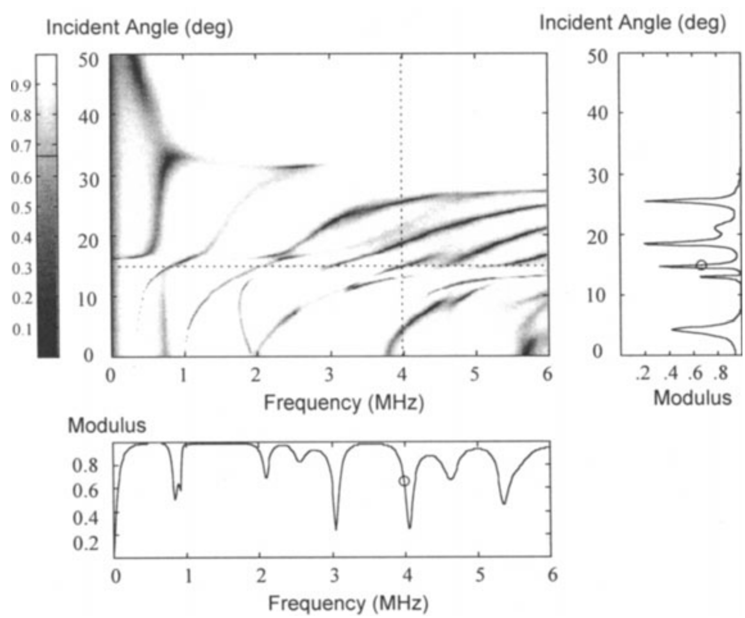

Fig. 3. An image representation of the reflection coefficient function $R(\theta, f)$ in its magnitude with two profiles cut at incident angle θ = 15 degree and frequency f = 4 MHz. It is obtained for the same structure as used for dispersion calculation in Fig. 2.

angle. If the sample is a layered material, there are a number of Lamb waves excited instead of a single Rayleigh wave and the *V(z)* curve is difficult to be interpreted directly from its amplitude measurement. But if we register the complex quantity of the *V(z)*, which means the amplitude and the phase measurement of the received signal, we can obtain in the *V(z)* response the information containing the reflectance function $R(\theta)$ of the sample at a certain working frequency whatever it is a volume material or a layered one.

Inversion Algorithm to Obtain the Reflection Coefficient *R(θ)* from *V(z)* Curve

An inversion algorithm between *V(z)* and $R(\theta)$ has been mentioned earlier in optics and is further demonstrated in a work of Hildebrand and Liang [6,7] in acoustics for the reflection coefficient function measurement at liquid-solid interface of substrate materials. We here generalize the method for the layered structure problem where the reflection coefficient function $R(\theta, f)$ is not only an incident angle dependent function but also a frequency dependent one.

Referring to Fig. 4, consider only the case of a circular acoustic probe (acoustic lens or shell transducer). Suppose that $U_f^+(k_r)$ is the angular spectrum of the emission field in the focal plane. This spectrum becomes $U_f^+(k_r)\exp(-j\sqrt{k_0^2 - k_r^2}z)$ at the surface of the layered sample where k_0 is the wave number in the coupling fluid. When it is reflected by the sample, the retro-propagating field returned to the focal plane will be:

$$U_f^-(k_r) = U_f^+(k_r)R(k_r)\exp(-j2\sqrt{k_0^2 - k_r^2}z) \tag{1}$$

where $R(k_r)$ is the reflection coefficient function of the sample and $k_r = k_0 \sin\theta$. If $U_0^-(k_r)$ is the voltage signal received by the transducer when a plane wave of unity amplitude is re-emitted from the focal plane toward the lens, the whole received signal of the probe is the integral for all components in the spectrum of the reflected field:

$$V = \int_0^{+\infty} 2\pi k_r U_0^-(k_r) U_f^r(k_r)\, dk_r \tag{2}$$

This formula is the so called Acoustic Material Signature as the vertical distance *z* between the probe and the sample varies:

$$V(z) = \int_0^{+\infty} 2\pi k_r U_0^-(k_r) U_f^+(k_r) R(k_r)\exp(-j2\sqrt{k_0^2 - k_r^2}z)\, dk_r \tag{3}$$

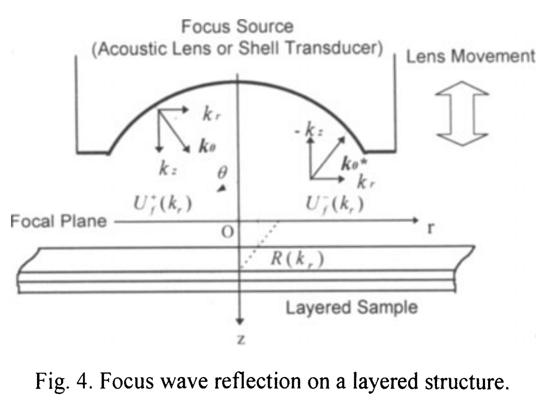

Fig. 4. Focus wave reflection on a layered structure.

We notice in (3) that $U_0^-(k_r)$ and $U_f^+(k_r)$ depend only on the characteristics of the used probe, they can be considered respectively as the emission and response function of the probe. By modifying integral variable as $\sqrt{k_0^2 - k_r^2} = \pi p$, the relation (3) becomes:

$$V(z) = \int_0^{+\infty} 2\pi^3 p U_0^-(p) U_f^+(p) R(p) \exp(-j2\pi pz) dp \qquad (4)$$

This relation shows that the *V(z)* response is the Fourier integral of the reflectance function *R(p)* weighted by a function $2\pi^3 p U_0^-(p) U_f^+(p)$ depending only on the characteristics of the probe. The weighting function can be considered as the multiplication of the direct and inverse pupil function of the acoustic lens and it can be obtained by

$$2\pi^3 p U_0^-(p) U_f^+(p) = F^{-1}\{V_g(z)\} \qquad (5)$$

where F^{-1} is the inverse Fourier transform operation and $V_g(z)$, called the probe's geometrical response, is a $V(z)$ response corresponding to that obtained from an ideal reflection having $R(p) \equiv 1$ for any incident angle and frequency. So we can get the *R(p)* under normalized form:

$$R(p) = \frac{F^{-1}\{V(z)\}}{F^{-1}\{V_g(z)\}} \qquad (6)$$

Knowing that $k_r = \sqrt{k_0^2 - \pi^2 p^2}$ and $k_r = k_0 \sin\theta$, we obtain the reflection coefficient $R(\theta)$ reconstructed at values:

$$\theta = \arccos(\frac{\pi p}{k_0}) \qquad (7)$$

<u>Reconstruction *R(θ,f)* from *V(z,f)* Measurement</u>

The inverse algorithm established above permits us to obtain the reflectance function $R(\theta)$ from the *V(z)* measurement at a fixed frequency. To get the 2-dimentional function $R(\theta, f)$, we need to vary the frequency f between a desired range. In principle, we have two methods for frequency scanning. One is the use of a tone-burst as the emission signal of the acoustic lens. By stepping f, we invert the *V(z)* curves for each frequency to get $R(\theta, f)$. Another is working the lens with pulsed excitation. By doing the frequency spectrum *V(f)* of the received signal at each step z, we get $R(\theta, f)$ by inverting *V(z,f)*. The latter method is much easier and faster for the experimental procedure but has poor signal-to-noise ratio. Because in the latter case, the emission energy is distributed to all the frequency containing in the spectrum and in the former case it is emitted only at a fixed working frequency. For this reason, we adopt the first method in our following *V(z,f)* measurement.

EXPERIMENTS

<u>Complex *V(z,f)* Measurement</u>

Fig. 5 (a) shows the schematic of the experimental set-up developed for complex *V(z,f)* signal acquisition. For the mechanical part, we have two independent orientation stages to align the probe and the sample at their relative normal position. A micro meter stepping motor is employed to perform the z translation of the acoustic probe. For the electronics part, we used a programmable function synthesizer delivering a synchronous tone burst at a desired frequency to excite the probe,

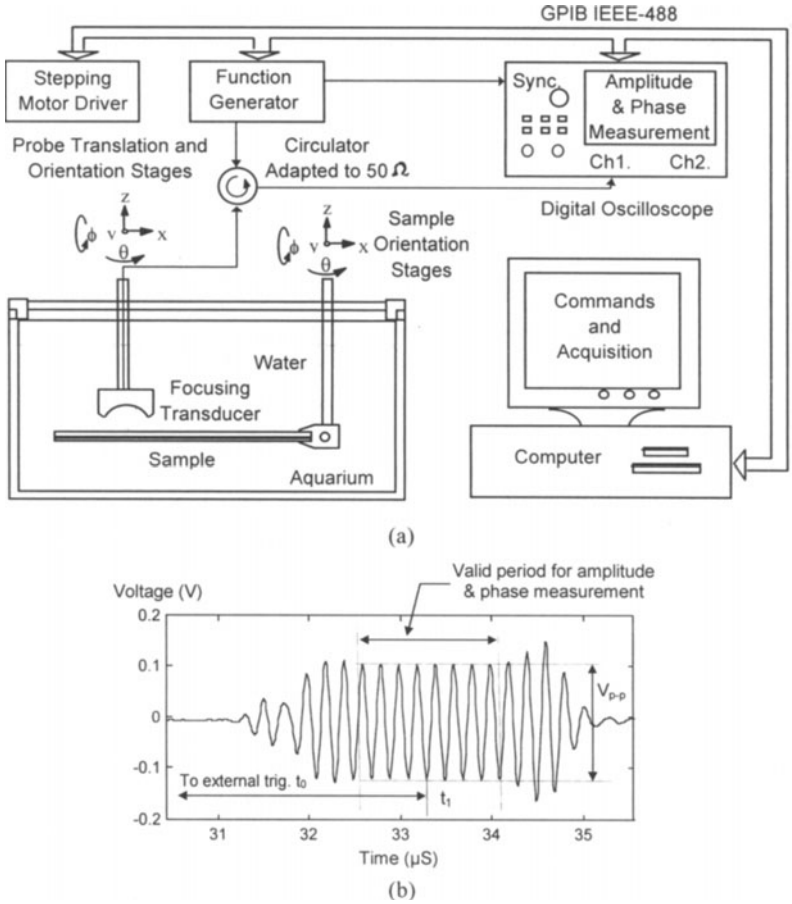

Fig. 5. Schematic of *V(z)* system (a) Experimental set-up; (b) Amplitude and phase determination in an interference *V(z)* echo.

a numerical oscilloscope to register the received signal and a PC to control the system via a GPIB bus.

Fig. 5 (b) gives an example of the interference *V(z)* signal registered by the oscilloscope and shows how to measure the complex quantity of the signal. In this tone-burst echo, we get its valid peak-to-peak amplitude V_{pp} between a stable period at the center of the echo. For phase information which means the phase delay between the emission burst and the received burst, we measure the time difference between t_1 and t_0 corresponding to respectively the time position of an arbitrary sine oscillation minimum (or maximum) in received burst and the time position of an arbitrary sine minimum (or maximum) in the emission burst. This is not a difficult task for a digital oscilloscope. The complex voltage value is then determined by

$$V_{complex} = V_{pp} \exp(-j2\pi f(t_1 - t_0)) \qquad (8)$$

A detailed description of the system and data acquisition procedure has been given in our previous work [8].

Lamb Modes Obtained for a Layered Plate

We studied an aluminum plate of 0.6mm thickness to valid the $V(z,f)$ inversion. Fig. 6 is the measured $V(z,f)$ data in its magnitude presented in a gray-scale image with two profiles of $|V(z,f)|$ cut respectively at f = 5 MHz and z = 0 mm. It is obtained by scanning f from 3.6 to 6 MHz with a step of 0.2 MHz and translating z from -12 to 5 mm with a step of 0.06 mm. The acoustic probe used is a spherical piezo-electrical shell transducer with a geometrical radius of 25 mm, an angular aperture covering between $\pm 45°$ and a central frequency of 5 MHz. For the probe response $V_g(z,f)$, we have registered a $V(z,f)$ obtained on an ideal reflector (a tense thin plastic membrane forming a plane surface between water and air) under exactly the same experimental conditions as to get $V(z,f)$ data for the layered sample.

By applying the algorithm above established to the complex $V(z,f)$ and $V_g(z,f)$ data, we have reconstructed the complex $R(\theta, f)$. Fig. 7 (b) is the $R(\theta, f)$ data given in its magnitude. For comparison, Fig. 7 (a) is the theoretical predicted $R(\theta, f)$ data obtained for the 0.6 mm thickness aluminum plate (the acoustical parameters of aluminum used for calculation are: longitudinal wave velocity vl = 6190 m/s, shear wave velocity vt = 3128 m/s and mass density ρ = 2740 kg/m^3). From the minima (black traces) corresponding to the dispersion-like curves of the Lamb modes in (a) and (b), we observe that for the experimentally obtained results (b), the two first Lamb modes at incident angles beyond 20 degree can be well distinguished and are comparable to the theoretical prediction. But for the low incident angle less than 20 degree, the mode minima are not clear. This default can be explained by the low signal-to-noise ratio of the emission energy of the probe at low incident angle. To solve this problem, we are proposing the method of signal processing to the acquired data and the results will be given in our next paper.

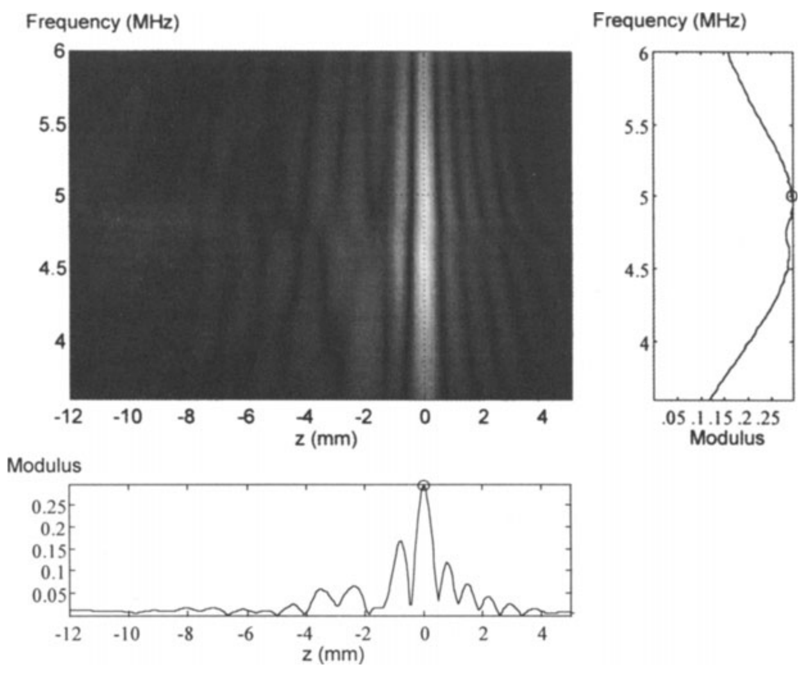

Fig. 6. A registration of $V(z,f)$ data obtained for a 0.6 mm thickness aluminum plate.

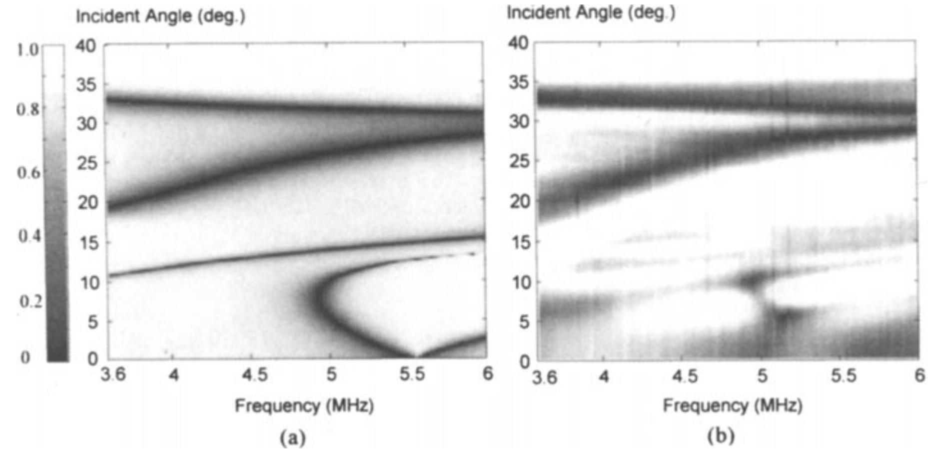

Fig. 7. Magnitude of the reflection coefficient function for the 0.6 mm thickness aluminum plate (a) Theoretically calculated (b) Reconstructed from *V(z,f)* measurement.

CONCLUSION

A focused wave method or *V(z)* method has been performed to evaluate the reflection coefficient function of layered structures. With the complex measurement of *V(z)* curve and applying an inverse algorithm based on Fourier transform, we can reconstruct the 2-dimentional reflectance function $R(\theta, f)$ permitting us to distinguish the Lamb modes from the minima evolution of the $|R(\theta, f)|$ image. The phase image can also be further explored. This method has a great simplicity for experimental procedure to determine locally and completely the Lamb modes and gives a efficient way to analyze many elastic properties of a layered structure such as anisotropy and interface adhesion.

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