ACOUSTOELASTIC WAVE VELOCITY IN METAL MATRIX COMPOSITE UNDER THERMAL LOADING

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INTRODUCTION

It is well known that microstresses are developed in a composite subjected to a temperature change due to the mismatch in thermal expansion between the fibers and the matrix. The stresses in the matrix can be large enough to cause the matrix to yield and deform plastically. The nonlinear thermal behavior is evidenced by experimentally observed thermal hysteresis in a metal matrix composite under thermal cycling [1]. Obviously, the thermal hysteresis plays an important role on the dimensional stability of the metal matrix composites, especially for graphite fiber reinforced composites.

Recently, experimental acoustoelastic data show that similar hysteresis loops can be observed in wave velocity versus temperature plots [2]. The analytical work given in [3] provides the correlation between the hysteresis of sonic velocity and the thermal strain hysteresis. Consequently, the sonic velocity measurement is likely to provide an efficient tool to characterize not only the residual stress but also the nonlinear thermal expansion behavior in a composite.

The goal of this task is to analyze the axial and transverse wave velocities in an unidirectional Graphite/Aluminum composite by utilizing a composite model of acoustoelasticity which is an extension of the simplified model reported in [3]. Parametric studies were performed to determine the sensitivity of the constituent properties to the composite acoustoelastic response under thermal cycling.

A COMPOSITE MODEL OF ACOUSTOELASTICITY

The theory of acoustoelasticity of a homogeneous medium, which relates the change in wave speeds to initial stresses, is based on superposition of a small dynamic disturbance on a prestressed continuum [4-6]. In applying the acoustoelastic theory to a composite material, each phase of the composite can be assumed to be governed by the equations of acoustoelasticity for homogeneous medium. As demonstrated in [3], it is convenient to use the simple form of equations of motion based on the first Piola-Kirchhoff stress tensor. Together with the constitutive equations for hyperelastic materials, which relates the second Piola-Kirchhoff stress to Lagrangian strain, it enables one to obtain the relation between the increment of the first Piola-Kirchhoff stress, t_{ij}, and the displacement gradient,

$u_{k,l}$, due to a small disturbance. For phase q in the composite,

$$t_{ij}^{(q)} = C_{ijkl}^{(q)} u_{k,l}^{(q)} \tag{1}$$

where $C_{ijkl}^{(q)}$ can be termed acoustoelastic "stiffness" of the phase q material. Corresponding to a prestessed state, $C_{ijkl}^{(q)}$ is expressed in terms of the second and third order elastic moduli of the phase material and the stress and elastic strain in the phase. Next, by considering the composite as a homogeneous material, the effective acoustoelastic "stiffness" of the composite, $\bar{C}_{ijkl}$, has the relation

$$\bar{t}_{ij} = \bar{C}_{ijkl} \bar{u}_{k,l}$$

where $\bar{t}_{ij}$ and $\bar{u}_{k,l}$ are the average stress increment and the average displacement gradient in the composite, respectively. Similar to the second order theories, the effective acoustoelastic stiffness of the composite, $\bar{C}_{ijkl}$, can be obtained from the acoustoelastic stiffnesses of the fiber and matrix phases using an analytical or numerical averaging scheme. Note that the acoustoelastic stiffness of a phase is a function of the prestressed states in that phase. In general, the stress and strain are not constant over each phase although they are usually independent along the fiber direction of a unidirectional composite. However, approximate results can be obtained using average stress and strain values obtained from the averaging scheme to provide a constant acoustoelastic stiffness over each phase.

Because of the unsymmetric nature of the stress tensor, t_{ij}, the results of the averaging scheme for the second order theory cannot be used directly in such acoustoelastic problems. Nevertheless, for the case of a unidirectional composite subjected to free thermal expansion, the prestresses and prestrains are purely extensional and satisfy the condition of axisymmetry. Therefore, instead of using the simple averaging method as demonstrated in [3], a more accurate averaging scheme, termed the composite cylinders assemblage [7], is adopted in the current study.

Here we assume that X_3 is the fiber direction in a unidirectional composite or in a plate with bounding surfaces parallel to X_3. Utilizing the standard contracted notation, the axisymmetric prestressing condition under free thermal expansion provides that the prestresses and prestrains in the phase q are $\sigma_1^{(q)} = \sigma_2^{(q)}$ and $E_1^{(q)} = E_2^{(q)}$ with the shear stress and strain components being zero. Then, based on the results given in [3], equation (1) provides the relation of the normal stress increments, $t_i^{(q)}$, versus the normal strain increments, $\epsilon_j^{(q)}$, as

$$t_i^{(q)} = C_{ij}^{(q)} \epsilon_j^{(q)} \tag{2}$$

where $C_{11}^{(q)} = C_{22}^{(q)} = \sigma_2^{(q)} + k_{22}^{(q)}$, $C_{33}^{(q)} = \sigma_3^{(q)} + k_{33}^{(q)}$,

$C_{12}^{(q)} = C_{21}^{(q)} = k_{12}^{(q)}$, and $C_{23}^{(q)} = C_{32}^{(q)} = k_{23}^{(q)}$.

In a majority of fiber composites, the fibers can be considered to be transversely isotropic and the matrix can be treated as an isotropic material. For the transversely isotropic fiber, which is also assumed to be elastic, there are five independent second order moduli, b_i, and nine such third order moduli, d_i, [8]. The nonzero k_{ji} can be expressed as

$$k_{22}^{(q)} = 2\frac{\rho^i}{\rho^o} \lambda_2^4[(b_1+b_2)+(6d_1+4d_2+3d_3)E_2^{(q)}+(3d_1+d_2+d_4+d_9)E_3^{(q)}]$$

$$k_{33}^{(q)} = 2\frac{\rho^i}{\rho^o}\lambda_3^4[\sum_{i=1}^{5} b_i+2(3d_1+d_2+2d_4+d_5+d_6)E_2^{(q)}+3\sum_{i=1}^{9} d_iE_3^{(q)}]$$

$$k_{12}^{(q)} = 2\frac{\rho^i}{\rho^o}\lambda_2^4[b_1+(6d_1+2d_2)E_2^{(q)}+(3d_1+d_4)E_3^{(q)}]$$

$$k_{23}^{(q)} = 2\frac{\rho^i}{\rho^o}\lambda_2^2\lambda_3^2[(b_1+\frac{b_3}{2})+(6d_1+d_2+2d_4+d_9)E_2^{(q)}+(3d_1+d_2+2d_4+d_5+d_6)E_3^{(q)}]$$

Note that, for elastic fibers, $E_i^{(q)}$ are the differences between total and free thermal strains. The stretch ratios λ_i and the mass density ρ^i in the prestressed state can be written as

$$\lambda_i = 1 + E_i^{t(q)}; \quad i = 1,2,3, \quad \rho^i/\rho^o = 1 - \sum_{i=1}^{3} E_i^{t(q)}$$

where $E_i^{t(q)}$ denote total strains in prestressed state including both mechanical and thermal effects and ρ^o is the density in the stress free state.

For an isotropic matrix similar expressions hold with $(b_3,\ldots b_5)$ and $(d_4,\ldots d_9)$ equal to zero. Furthermore, we assume that the prestrains are not large in both phases. Additionally, it is assumed that when the prestressed state in the matrix is plastic, the third order effects enter through elastic mechanical prestrains only [6]. The five independent moduli of isotropic matrix can be expressed in terms of Lame (λ, μ) and Murnaghan (ℓ,m,n) constants

$$b_1 = \lambda/2, \quad b_2 = \mu, \quad d_1 = (2\ell-2m+n)/6, \quad d_2 = (m-n)/2, \quad d_3 = n/3$$

Note that the normal stress-strain relation of equation (2) satisfies the transversely isotropic condition for both the fiber and matrix phases. By neglecting the rigid body rotation of the imposed small disturbance, the transverse shear becomes symmetric and the transverse shear modulus satisfies the transversely isotropic condition, while the axial shear stresses remain unsymmetric. Therefore, the transversely isotropic condition is not completely satisfied. However, this provides no major concern in utilizing the composite cylinders assemblage model which requires that the phase stiffnesses are at most transversely isotropic since only normal wave speeds are of interest in the current study. In addition, the effect of the constituent axial shear properties on the composite normal properties is expected to be small (note that, in transversely isotropic case, the composite normal properties are independent of the constituent axial shear properties).

In utilizing the composite cylinders assemblage, it is more convenient to obtain the composite compliance, $\bar{S}_{ij}$, from the fiber compliance and the matrix compliance. After obtaining $\bar{S}_{ij}$, the wave velocities of the in-plane extensional waves propagating in the X_2 and X_3 directions, v_2 and v_3, respectively, can be computed from the relations

$$v_2 = \sqrt{\frac{1}{\rho^i}\left(\frac{\bar{S}_{33}}{\bar{S}_{22}\,\bar{S}_{33} - \bar{S}_{23}^2}\right)}, \quad v_3 = \sqrt{\frac{1}{\rho^i}\left(\frac{\bar{S}_{22}}{\bar{S}_{22}\,\bar{S}_{33} - \bar{S}_{23}^2}\right)}$$

Since the acoustoelastic wave velocities are dependent on the stress and strain states in the constituents, it is necessary to compute the stresses and strains before the composite acoustoelastic theory can be applied. To accomplish this, the thermal elastoplastic phase average stress model developed in [9] was employed. Briefly, given the constituents properties, the composite properties are computed by employing the composite cylinders' assemblage model. Then, using the definitions of average stress and strain in the composite, the phase stress average model computes the average stress and strain increments in the fibers and matrix for a given load increment. The incremental composite load can be a temperature change or any of the six composite stress components. The computed matrix stresses are utilized in a Mises yield condition to determine the onset of yielding. Once the matrix has yielded, the matrix plastic strains are computed from the associated flow rule. A kinematic hardening rule is incorporated in the plasticity theory to account for the translation of the yield surface during plastic deformation. For simplicity, only linear work-hardening is considered here. During an elastoplastic load increment, the matrix elastic and plastic compliances are utilized to form an effective matrix compliance. Then, together with the fiber elastic compliance, a new composite compliance matrix can be computed for the next load increment. If the composite load increment causes the matrix to unload, the matrix is assumed to exhibit elastic properties and no plastic strains are computed.

RESULTS

For the purpose of evaluating the theory, we consider the experimental data on wave velocity changes in a unidirectional P55/Aluminum composite with 38% volume fraction of fiber due to thermal cycling [2]. Properties used for calculation are listed in Table 1 for graphite P55 and Aluminum 6061. In Table 1, the only available third order elastic modulus, d_7, for P55 is determined from [10]. The values of Murnaghan constants for 6061 alloy are taken from [11]. Also, shown in Table 1 are the yield strength σ_y and plastic modulus E^p for the bilinear stress-strain relation of 6061.

Table 1. Constituent Properties

Transversely Isotropic P55 Fiber

Young's Moduli: E_a = 55.0 Msi, E_t = 1.1 Msi

Shear Modulus: G_a = 2.16 Msi

Poisson's Ratios: ν_a = 0.41, ν_t = 0.45

Thermal Expan. Coeff.: $\alpha_a = -0.43 \times 10^{-6}$/°F, $\alpha_t = 7.57 \times 10^{-6}$/°F

Third Order Modulus: d_7 = 1185.0 Msi (ref. 10)

Mass Density: ρ = 0.071 lbs/in^3

Isotropic 6061 Aluminum Matrix

Young's Modulus: E = 10 Msi

Poisson's Ratio: ν = 0.33

Thermal Expansion Coefficient: $\alpha = 12.7 \times 10^{-6}$/F

Murnaghan Constants: ℓ = -6.82 Msi, m = -49.6 Msi, n = -36.0 Msi (ref. 11)

Mass Density: ρ = 0.098 lbs/in^3

Yield Strength: σ_y = 11 ksi; Plastic Modulus: E^p = 1.56 Msi

Temperature Dependent Properties:

dE/dT = -1.6 ksi/F, $d\alpha/dT = 3.3 \times 10^{-9}/F^2$, $d\sigma_y/dT$ = -5.13 psi/F

dE^p/dT = -1.6 ksi/F

The results of axial sonic velocities predicted by the vanishing fiber diameter model (V-F) employed in [3] and the composite cylinders assemblage model (CCA) together with experimental data are compared in Figure 1. A 38% fiber volume fraction and 6 ksi matrix residual stress were assumed. The temperature independent constituent properties in Table 1 were used except that the fiber axial Young's modulus was modified to 48.3 Msi. A matrix residual stress of 6 ksi in the fiber direction was also assumed. As a result of predicting different Young's moduli, the V-F provides a shift of the wave velocity curves from that of the CCA. The discrepancy is about 0.4% of the initial V-F wave velocity at 75°F However, this is about 10.6% of the amplitude of the sonic velocity hysteresis loop. The transverse wave velocities were also computed using CCA and V-F. The discrepancy is about 38% of the initial velocity of the V-F. This mainly is due to the very different values of transverse Young's modulus predicted by these two models. It is believed that the CCA should provide more accurate result on this aspect.

Note that the properties used in Figure 1 were so chosen that the sonic velocity curve predicted by the CCA could best match the experimental results. On the other hand, if a 55 Msi fiber Young's modulus is chosen, then the best fit of the experimental data can be achieved by using 33.2% fiber volume fraction and 7.5 ksi axial residual stress in the matrix. In both cases, the chosen material properties are not unreasonable due to the possible variations in properties which may occur during the manufacturing process. Therefore, parametric studies were performed to identify the key constituent properties which can most significantly influence the acoustoelastic response. This should provide valuable guidance for further acoustoelastic wave velocity measurement.

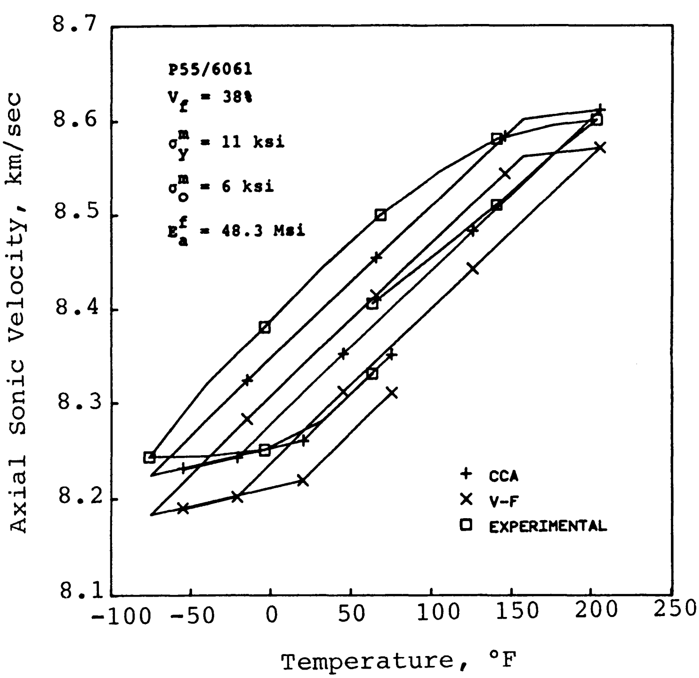

Fig. 1. Comparison of the Axial Sonic Velocities in a P55/6061 Composite.

To study the influence of a specific material property upon computed composite wave velocities, simulations were performed by varying a single property within a reasonable range while all other properties were kept unchanged. The results were compared with the baseline prediction of a Gr/6061 composite with 35% fiber volume fraction, 11 ksi matrix yield strength, 7.5 ksi matrix residual stress and the temperature independent properties shown in Table 1. All simulations were limited to one thermal cycle in which the temperature was changed from 75°F to -75°F, to 205°F and then to 75°F. The results of parametric studies demonstrated that the axial wave velocities were most strongly affected by the fiber axial modulus, the fiber volume fraction and the fiber third order elastic modulus; and much less affected by the matrix residual stress, matrix Young's modulus and temperature dependent matrix properties. The axial modulus and fiber volume fraction control the magnitude of the room temperature velocity but have little effect on the shape or size of the velocity hysteresis loop. The fiber third order elastic modulus controls the size and shape of the hysteresis loop and has a second order effect on the magnitude of the room temperature velocity, as demonstrated in Figure 2. It shows that, as the fiber third order modulus reduces to zero, a small hysteresis is obtained. Based upon the composite theory of acoustoelasticity, the change in composite sonic velocity is related to the change in stress state in each of the constituents through their individual third order elastic moduli. Thus, by eliminating the fiber third order modulus, i.e., $d_7 = 0$, the contribution of matrix third order moduli, Murnaghan constants, on the composite sonic velocity is demonstrated. This indicates that the sensitivity of the sonic velocity of the composite is primarily dependent on the third order elastic moduli of the fibers. In Figure 2, it is also interesting to note that an increase of d_7 reduces the initial sonic velocity at 75°F. This is because a tensile matrix residual stress results in a compressive fiber residual stress which coupled with the fiber third order modulus acts to reduce the composite sonic velocity.

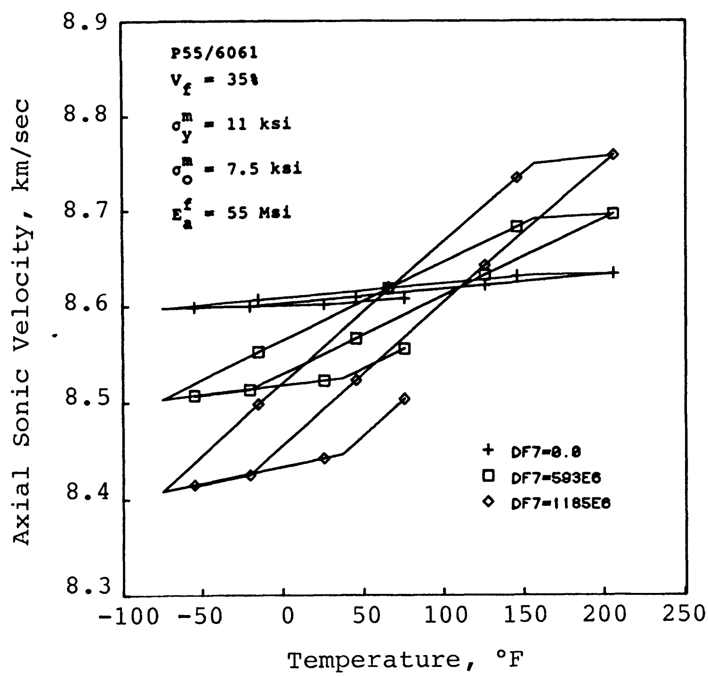

Fig. 2. Effect of Fiber Third Order Elastic Modulus, d_7, on the Axial Sonic Velocity Hysteresis in a P55/6061 Composite.

The transverse wave velocities were found to be most strongly affected by the matrix properties. Specifically, as shown in Figure 3, if the matrix modulus is allowed to be temperature dependent, the composite transverse wave velocity is predicted to increase with decreasing temperature. This effect is not predicted if the matrix is treated as if it has constant properties with temperature. This is because that when temperature dependence is considered, the matrix Young's modulus decreases with the increase of temperture and increases with decreasing temperature. This significantly shifts the slope of the velocity hysteresis loop to a negative value. Figure 3 also indicates that the temperature dependent matrix Young's modulus has a much stronger effect on the transverse sonic velocity than the third order elastic moduli. Due to the lack of experimental data, the effects of the temperature dependent fiber properties cannot yet be determined.

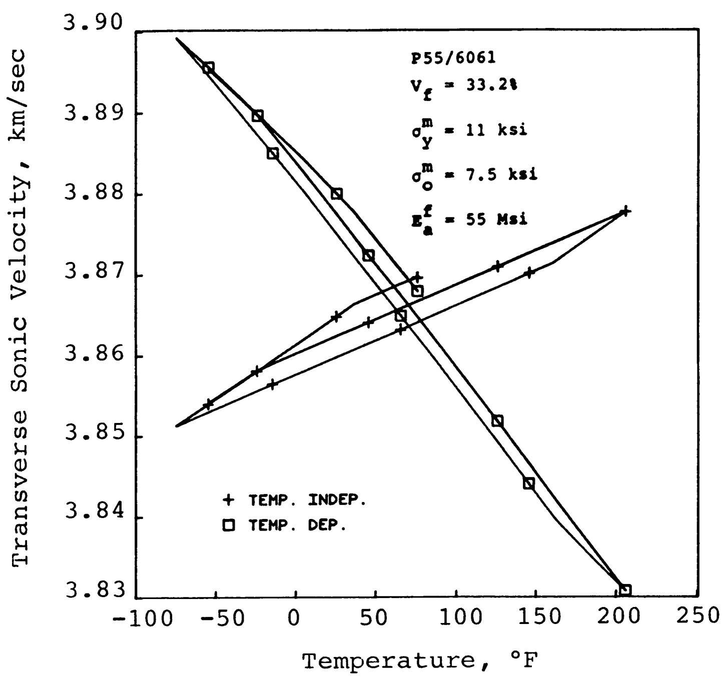

Fig. 3. Effect of Temperature Dependent Matrix Properties on Transverse Wave Velocities in P55/6061 Unidirectional Composites.

CONCLUSIONS

In the preceding sections, the composite model of acousto-elasticity has been shown to be capable of predicting the axial and transverse sonic velocity hysteresis loops that are caused by subjecting the unidirectional composite to thermal cycles. However, it is necessary to determine the material properties accurately so that the sonic velocity measurement can be utilized as a reliable tool to characterize the residual stress and the thermal strain in a composite.

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