

EFFICIENT RECONSTRUCTION OF ELASTIC STIFFNESSES IN ISOTROPIC AND ANISOTROPIC PLATES

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INTRODUCTION

Because composite materials are often used in safety critical applications, such as in aerospace, it is desirable to have a quick and reliable confirmation of their material properties. The aim of this paper is to introduce the idea of inferring elastic and/or viscoelastic material properties from plate wave measurements with a minimum amount of data. Material characterization of anisotropic materials has been a focus point of research for many years. A good review of this literature can be found in a recent review article [1]. Rokhlin and collaborators [2-4] have also studied this problem extensively, using propagation characteristics and the transmission coefficient to reconstruct elastic properties. The advantage of reconstructing the stiffnesses from the zeros of the transmission coefficient lies in the fact that in contrast to the zeros of the reflection coefficient the transmission coefficient zeros are independent of the fluid properties. A drawback of this method is that the plate must be accessible from both sides.

Safaeinili, *et al* [5] have determined the isotropic and anisotropic viscoelastic stiffnesses of several low-density plates in air. A synthetic aperture method based on position and angle scanning was employed to reconstruct property information. These workers also found that the imaginary parts of the stiffnesses play an important role in some aspects of the ultrasonic reflection and transmission behavior. The same conclusion was reached independently by Deschamps and Hosten [6], who studied fiber-reinforced plastics and graphically demonstrated the effect of losses on the scattering coefficients in composites.

Because of the inherently periodicity in the secular equations for guided waves in a plate, much of the measured data often acquired in such test is highly redundant.

That is, it contains no new information on the elastic properties of the solid material guiding the wave. Therefore, in the best case its use in elastic property extraction only slows convergence of the optimization algorithm to a final result. In the worst case the redundant data may contain a weak frequency dependence owing to velocity dispersion caused by the inhomogeneous nature of the composite [7] or characteristics of the matrix. Our aim in this paper is to simplify elastic property extraction in engineered materials by reducing or eliminating the reliance on redundant plate wave data. We demonstrate an efficient, but accurate, preliminary method to extract elastic stiffnesses from plate wave reflection spectra for the limited cases of isotropic and transversely anisotropic materials, which ignores most of the measured data that is simply not needed.

PROBLEM STATEMENT AND METHOD

The transducer voltage calculation, including material property interactions and finite beam effects of both transmitter and receiver, has been given previously as [8]

$$V(x_i, f) = \frac{1k_f}{4\pi^2} \int_0^{2\pi} d\phi \int_0^{\pi/2-100} d\theta R(\theta, f) D_T(\theta, \phi) D_R(\theta, \phi) \sin(\theta). \quad (1)$$

The reflection coefficient is denoted by $R(\theta, f)$, and the transducer directivity functions D_T and D_R can be found in [8]. Previous analysis [8] has shown that the product of the two piston beam directivity functions can be accurately replaced by the product of Gaussian beam directivity functions. The replacement is an excellent approximation and permits the analytical asymptotic reduction of the integrals Eq. (1).

The measurement procedure is essentially the same one used by Chimenti and Nayfeh [9] in earlier studies of guided wave propagation and consists of the transducers deployed as indicated schematically in Fig. 1. The incident angle is α , and the transducer and receiver are adjusted at the opposite angles. The angle between the axis labeled ρ and the z -axis is θ_0 . Essential for the design of the experiment is the beam shift parameter x_i . In earlier investigations Lobkis, *et al.* [8] have shown that if we select x_i so that it is equal to the transducer radius a , the minima of the voltage will be coincident with the zeros of the reflection coefficient. Our experimental arrangement exploits this important finding to simplify the accurate extraction of elastic stiffnesses. This is the first of the data simplifications we apply. With this procedure we can circumvent the evaluation of the integral expression in Eq. (1) and return confidently to the much easier and quicker task of calculating the plane-wave reflection coefficient.

The experimental data are acquired by exciting the transducer with a 100 μ s chirp signal whose bandwidth spans 2 to 10 MHz. The data are recorded as reflection spectra. For comparison with the calculations the raw data are normalized with the diffraction corrected transducer frequency response. A more complete description of the technique used here can be found in [8].

ALUMINUM PLATE

For the isotropic case the reconstruction consists of first extracting V_l from data acquired at an incident angle α above the critical longitudinal angle. We use only the information in the first measured voltage minimum. For aluminum the critical longitudinal angle lies at $\theta_{\text{long}} = 13.65^\circ$, so the transducers are adjusted to $\alpha = 20^\circ$. This first minimum, which corresponds to the A_1 mode, is found at a frequency of 1.84 MHz. With our assumption of zero and minimum coincidence, we can fit V_l by iterative solution of the Lamb characteristic equation in the vicinity of the A_1 zero, or alternatively by direct solution of the Lamb equation for V_t , holding V_l constant and substituting the measured zero for the wavevector. The iterative solution of the Lamb wave equation is done by employing a direct search method, in our case the Hooke and Jeeves [12] pattern search. The execution time for the reconstruction was 0.1 s, and convergence was reached after 9 iterations.

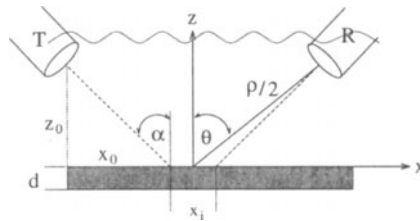


Figure 1. Schematic diagram of the experimental setup, where T represents the transmitter and R the receiver.

To determine the longitudinal velocity V_l the transducers are adjusted to an angle below the critical longitudinal angle. Now, both partial longitudinal and partial transverse waves propagate in the plate, the modes occur closer together, and voltage minima are therefore no longer clearly defined. The longitudinal velocity is therefore determined by employing a nonlinear least-square minimization of the difference between the data and the voltage as calculated from 1 in a small frequency range ($\Delta f = \pm 0.3\text{MHz}$) about 2 MHz, while varying V_l with V_t held constant. Convergence was reached after 13 iterations with an execution time for the minimization of V_l of 2.3 sec, much less than reconstructing both constants by minimizing a least square algorithm over the entire frequency range. A consistency check for the value of V_t , given the inferred V_l , is then made.

Using the procedure described above on synthesized data computed with Eq. (1), the constants for an aluminum plate are reconstructed and yield exactly the input constants ($V_l=6.37\text{ km/s}$, $V_t=3.11$) to within 0.1%. The position of the first minimum is sufficient to reconstruct the transverse velocity. Also, the longitudinal velocity can

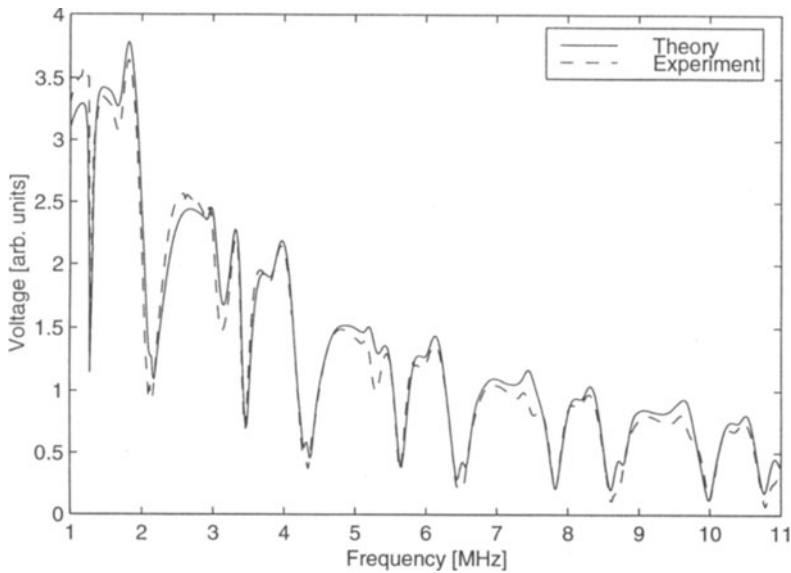


Figure 2. Experimental and predicted reflection spectrum using reconstructed material properties for an aluminum plate with $\alpha = 10^\circ$.

be accurately reconstructed with our procedure. A comparison of the experimental reflection spectrum with a calculated one using the reconstructed constants is shown for an incident angle of 10° in Fig. 2. The experiment has been normalized with the transducer frequency response. The theoretical prediction (dashed curve) is in good agreement with the data (solid curve). Also the position of the minima align very well, as expected.

UNIDIRECTIONAL GRAPHITE-EPOXY PLATE

We demonstrate that the viscoelastic properties of an unidirectional graphite-epoxy plate can also be reconstructed with this method. Now instead of two independent constants we must contend with five. Before designing a strategy for the reconstruction, it is always a good idea to perform an elastic property sensitivity analysis [10]. The sensitivity analysis will tell us which parts of the dispersion behavior are strongly or weakly dependent on which elastic constants. Since the most easily interpreted measurements are done along and normal to the fiber direction, only these data will be used in the reconstruction procedure. In general, we follow the same idea for the reconstruction of the elastic constants as in the isotropic case. We attempt to use the minima positions of the voltage for the extraction above the critical longitudinal angle and use a least square minimization with a subset of the data below the critical angle.

Sensitivity curves are shown in Fig. 3, where a dispersion plot of the RC zeros along the fibers, accompanied by a series of lines demarking the sensitivity zones.

The arrows indicate the regions where certain constants are most sensitive. From

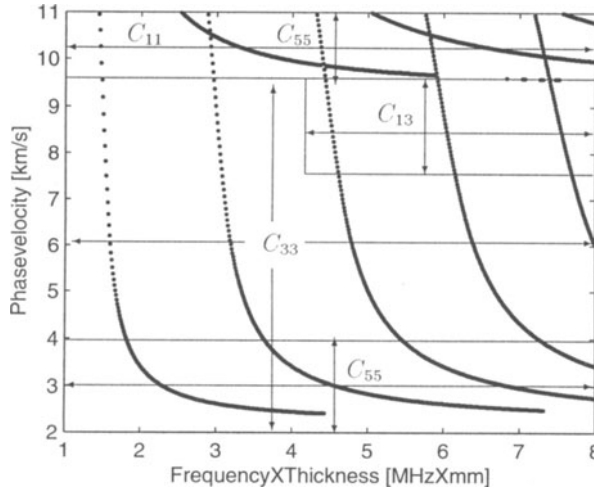


Figure 3. Reflection coefficient zero spectrum for a unidirectional graphite-epoxy plate along the fibers.

the figure we can see that we get a reasonable estimation of C_{13} only in the frequency-thickness range above 4 MHz·mm. The constant C_{55} is sensitive at lower and higher velocities, whereas C_{33} is sensitive at almost all angles except the lower ones. Because of its unique dependence C_{11} is determined a different way from that described here. That procedure, to assess the inplane longitudinal constant, is described elsewhere in these proceedings [11].

Using the sensitivity analysis shown above, the reconstruction procedure proceeds as follows, where ϕ is the angle formed by the plane of incidence and the fibers and α is the incident angle. The reconstruction algorithm is divided into several parts. First, C_{55} and C_{33} are simultaneously determined by minimizing the object function $\mathcal{A}(f_{30}; \theta = 30^\circ)^2 + \mathcal{A}(f_{20}; \theta = 20^\circ)^2$, where f_{20} and f_{30} are the frequencies of the first voltage minima at incident angles of 20° and 30° , respectively. We seed the algorithm with values of the constants that are 40% different from the nominal values. Thereafter, C_{13} can be extracted from data at $\alpha = 10^\circ$. This completes the elastic stiffnesses that can be inferred in the fiber direction. As a last step, we change to $\phi = 90^\circ$, normal to the fibers, to extract the slow shear stiffness C_{44} from data at $\alpha = 20^\circ$. The same extraction routine — using the Hookes and Jeeves pattern search — is used for this task. The total execution time is 8.2 sec on a Silicon Graphics workstation. Here the the strategy really pays off, since the minimization of a least-square expression all three control variables would take considerably longer.

The reconstructed complex stiffnesses are

$$\begin{aligned}
 C_{11} &= 143.0 & C_{13} &= 7.6 & C_{33} &= 16.0 \text{ GPa} \\
 C_{44} &= 4.2 & C_{55} &= 8.3 & & \\
 C_{11}^* &= 1.68 & C_{13}^* &= 0.3 & C_{33}^* &= 0.3 \text{ GPa} \\
 C_{44}^* &= 0.08 & C_{55}^* &= 0.25 & &
 \end{aligned}
 \tag{2}$$

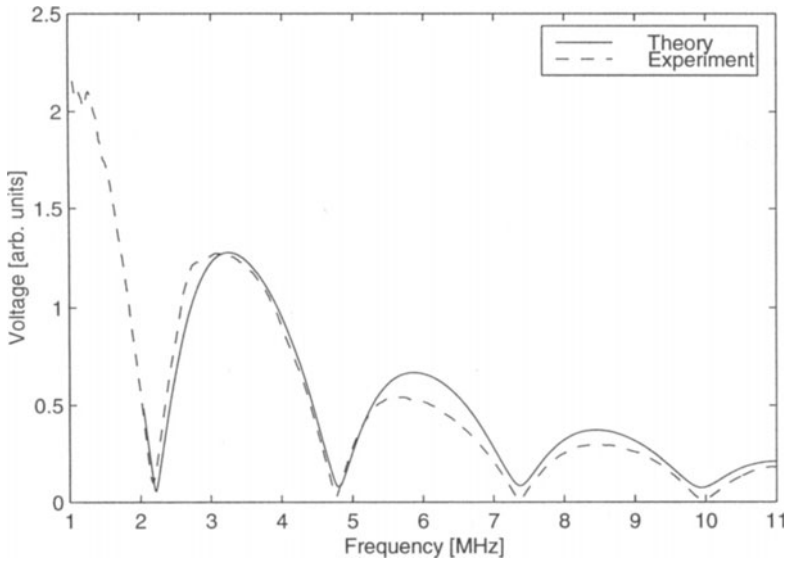


Figure 4. Experimental and predicted reflection spectrum using reconstructed properties for a unidirectional graphite-epoxy plate with $\alpha = 30^\circ$ and $\phi = 0^\circ$.

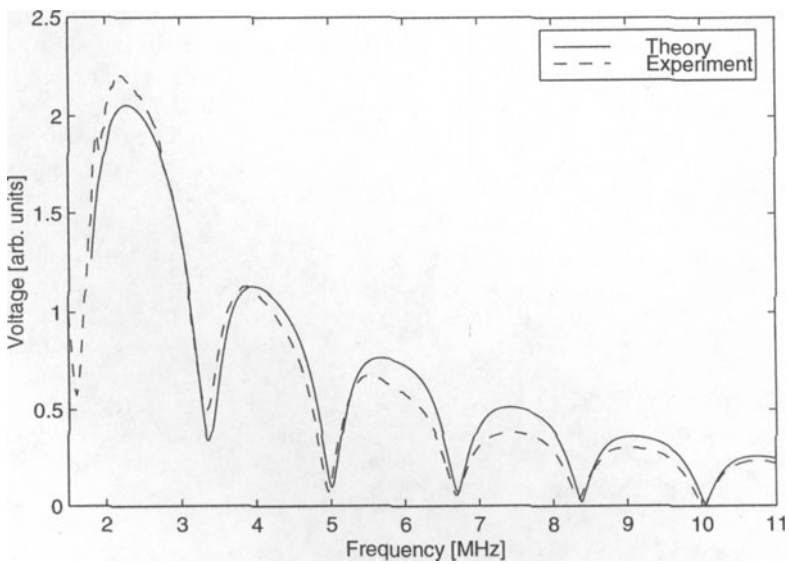


Figure 5. Experimental and predicted reflection spectrum using reconstructed properties for a unidirectional graphite-epoxy plate with $\alpha = 10^\circ$ and $\phi = 0^\circ$.

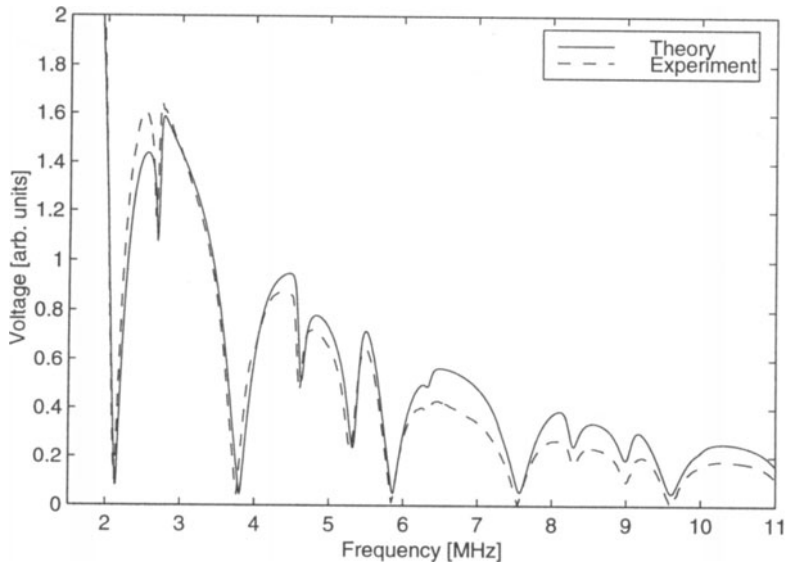


Figure 6. Experimental and predicted reflection spectrum for a graphite-epoxy plate with $\alpha = 10^\circ$ and $\phi = 90^\circ$.

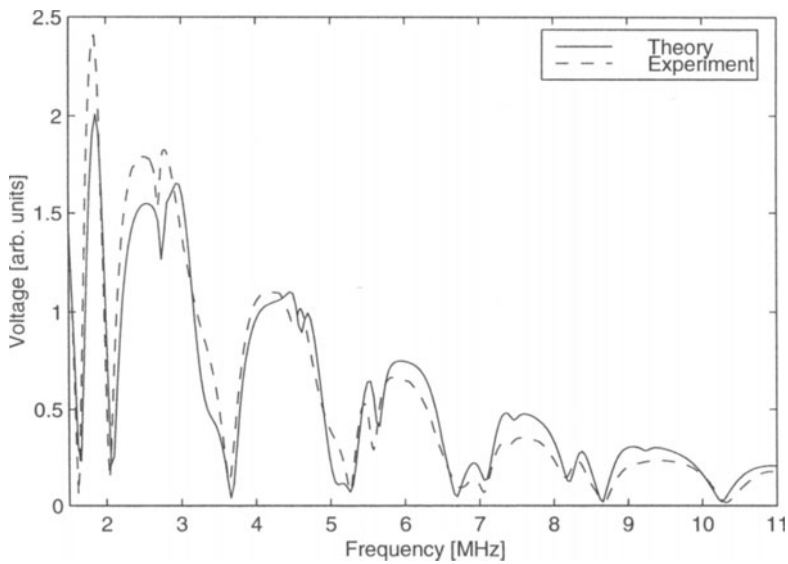


Figure 7. Experimental and predicted reflection spectrum for a graphite-epoxy plate with $\alpha = 10^\circ$ and $\phi = 60^\circ$.

The reconstructed constants of a uniaxial composite plate are determined by considering the data only along and normal to the fibers. Figures 4 through 7 show the experimentally obtained voltage (denoted by the dashed curve) compared to the calculation using inverted constants (solid curve). Good agreement of the experiment with the predicted curves can be seen in these graphs. Even curves for data acquired in non-principal directions show reasonably good agreement, as they must for consistent elastic constant extraction.

SUMMARY

The method we describe here gives a quick and reliable estimate of the viscoelastic material parameters of isotropic and transversely isotropic media. We have demonstrated that information in the first voltage minimum is sufficient to permit a reasonably accurate estimate of stiffness, determined at angles above the longitudinal critical angle. The remaining constants can be determined by employing a nonlinear least-square minimization procedure in conjunction with a detailed voltage model for the experimental signal.

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