EXPERIMENTAL OBSERVATION OF THE SLOW SQUIRTING MODE IN SOLID/FLUID/SOLID TRILAYERS

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INTRODUCTION

The problem of guided waves in solid/fluid/solid trilayers has been investigated in great detail both theoretically and experimentally. Lloyd and Redwood were the first to investigate guided wave propagation in a layered plate composed of two solids with perfect contact, slip, or a fluid layer at their interface [1]. Rokhlin et al. studied the elastic interface wave guided by a thin film between two solids and utilized it in predicting the strength of adhesive bonds [2,3]. Couchman et al. [4] and Guyott and Cawley [5] reported on the phenomenon of resonance splitting in the vicinity of classical Lamb modes in symmetrical trilayers. Laperre and Thys investigated elastic wave dispersion in both symmetric and asymmetric trilayers composed of two solid plates separated by a fluid layer [6]. A number of papers have appeared on the use of trilayers as antireflection coating [7-9].

The main goal of this work is to experimentally observe the slow squirting mode of a solid/fluid/solid trilayer. A mode with similar characteristics has been predicted by Franklin in the case of a fluid layer between an elastic plate and a solid substrate [10]. The well-known lowest order asymmetric mode in an immersed plate [11], that exhibits similar dispersion behavior but different frequency dependence, has been experimentally observed by Desmet et al. [12].

LOW-FREQUENCY ASYMPTOTIC BEHAVIOR OF THE SQUIRTING MODE

Figure 1 shows a symmetric trilayer composed of a thin fluid film between two identical, isotropic and homogeneous plates with vacuum on each side. The two plates have the same thickness of $2h$ and are composed of the same material, characterized by its density ρ_s, its shear wave velocity c_T, and its longitudinal wave velocity c_L. The density, sound velocity, and thickness of the fluid layer are ρ_f, c_f and d_f, respectively. The z-

direction is chosen along the thickness of the plates which are infinite in the perpendicular x and y directions.

The dispersion equations of the Global Symmetric Modes (GSM) and Global Antisymmetric Modes (GAM) that exist in such a structure are given by Eqs. (4) and (5), respectively, in Ref. 13. It must be emphasized that this symmetry or antisymmetry is global with respect to the center of the structure. The squirting mode we are focusing on is a member of the GSM family as its displacement profile is symmetric with respect to the center of the thin fluid film. Following the mathematical method of Achenbach [14] Coulouvrat et al. derived a low-frequency asymptotic expression (Eq. 8 of Ref. 13) for the phase velocity of what they call the F_0^+ mode (F for fluid, $+$ for symmetric, and 0 for lowest-order) from the general solution of Global Symmetric Modes. For convenient comparison to our following results we include their equation here

$$\frac{k_x}{k_T} \approx \left[\frac{3\rho_f h}{4(1-\frac{c_T^2}{c_L^2})\rho_s d_f} \right]^{\frac{1}{6}} (k_T h)^{-\frac{2}{3}}, \tag{1}$$

where $k_x = \omega / c_x$ is the wave number in the x-direction, c_x is the phase velocity of the guided mode, and $k_T = \omega / c_T$ is the wave number of the shear wave mode in the plate. The appearance of the plate's density in Eq. (1) is just an artifact caused by the normalization process based on the shear wave number in the solid, $k_T = \omega / c_T = \omega \sqrt{\rho_s / \mu}$, where μ is the shear modulus of the solid. Substituting this expression for k_T along with the ratio between the shear and longitudinal velocities in the solid expressed in terms of Poisson's ratio ν, we arrive at the following expression for the phase velocity

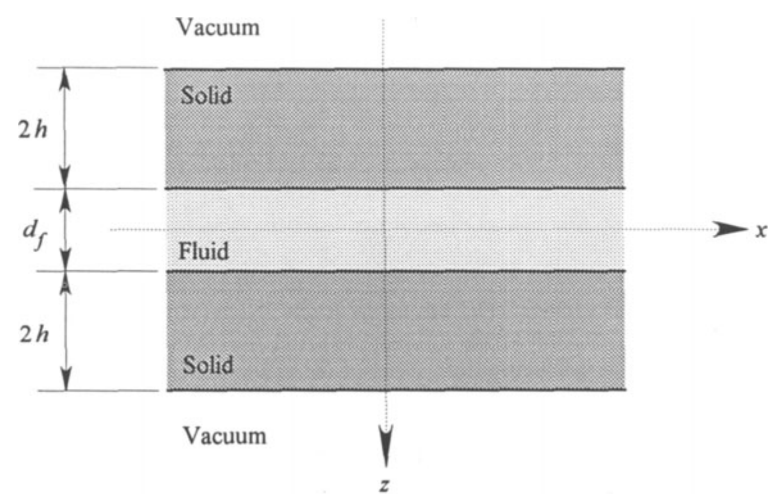

Figure 1. Geometry of the solid/fluid/solid trilayer and the coordinate system.

$$c_x \approx \left[\frac{E h^3 d_f \omega^4}{3(1-\nu^2)\rho_f}\right]^{\frac{1}{6}}, \tag{2}$$

where E denotes Young's modulus. Equation (2) is equivalent to Eq. (1). However, it clearly shows that this mode depends only on the stiffness of the plates and the density of the liquid film between them. One surprising aspect of Eq. (2) is that the phase velocity is not simply a function of the fluid mass since the fluid density is not multiplied but rather divided by the thickness of the fluid layer. Later we will demonstrate that this intriguing feature is due to the squirting effect of the thin gap between the plates, that accelerates the fluid more and more as the gap narrows.

PHYSICAL MODEL FOR THE SQUIRTING MODE

The squirting mode of a thin fluid layer bounded by two thin solid plates corresponds to longitudinal vibrations of the fluid caused by the symmetric transverse vibrations of the plates. This implies that as the two plates undergo small symmetric transverse vibrations in opposite directions they force the fluid in the very thin gap between them to experience a much larger displacement in the longitudinal direction, hence the name "squirting" mode. Based on this simple physical model, we present the following derivation that leads to Eq. (2) starting from the basic principles of strength of materials and fluid mechanics.

Let us consider again the solid/fluid/solid trilayer shown in Figure 1. We will start from the well-known differential equation governing the bending deformation of a plate:

$$\frac{2h^3 E}{3(1-\nu^2)} \frac{\partial^4 w}{\partial x^4} = -p, \tag{3}$$

where w is the transverse displacement in the z-direction and p denotes the fluid pressure. The balance of momentum equation for the fluid can be written as follows

$$\rho_f \frac{\partial^2 u}{\partial t^2} = -\frac{\partial p}{\partial x}, \tag{4}$$

where u denotes the longitudinal displacement of the fluid. Exploiting the continuity equation for incompressible fluid we can write

$$w = \frac{d_f}{2} \frac{\partial u}{\partial x}. \tag{5}$$

Combining Eqs. (3), (4) and (5) yields the following wave equation

$$\frac{E h^3 d_f}{3(1-\nu^2)\rho_f} \frac{\partial^6 w}{\partial x^6} = \frac{\partial^2 w}{\partial t^2}. \tag{6}$$

For harmonic vibrations of $w = A\, e[i(k_x x - \omega t)]$, we can write the dispersion equation as

$$\frac{E h^3 d_f}{3(1-\nu^2)\rho_f}\, k_x^6 = \omega^2 . \tag{7}$$

which can be rearranged to a form identical to Eq. (2) by substituting $k_x = \omega / c_x$.

Figure 2 shows the phase velocity of the squirting mode plotted as a function of frequency for the case of two 5-mm-thick aluminum plates bordering a 1-mm-thick water layer. The shear and longitudinal wave velocities in aluminum were taken as $c_T = 3100$ m/s and $c_L = 6380$ m/s, respectively, while the density of aluminum was taken as $\rho_s = 2800$ kg/m^3. The sound velocity in water was taken as $c_f = 1470$ m/s, and its density as $\rho_f = 1000$ kg/m^3. The solid line represents the dispersion curve obtained by numerically solving the exact dispersion equation (Eq. 4 in Ref. 13).The dotted line represents the low-frequency asymptote given by Eq. (1). At very high frequencies the squirting mode approaches the Stoneley-Scholte velocity.

VISCOUS ATTENUATION

Whenever an acoustic mode propagating in a fluid/solid structure produces large relative motion between the fluid and solid constituents, the viscosity of the fluid is expected to assume a crucial role. The primary effect of viscosity is usually significant, sometimes prohibitive, attenuation combined with a less significant effect on the velocity. In order to explore the effects of fluid viscosity on the propagation properties of the squirting

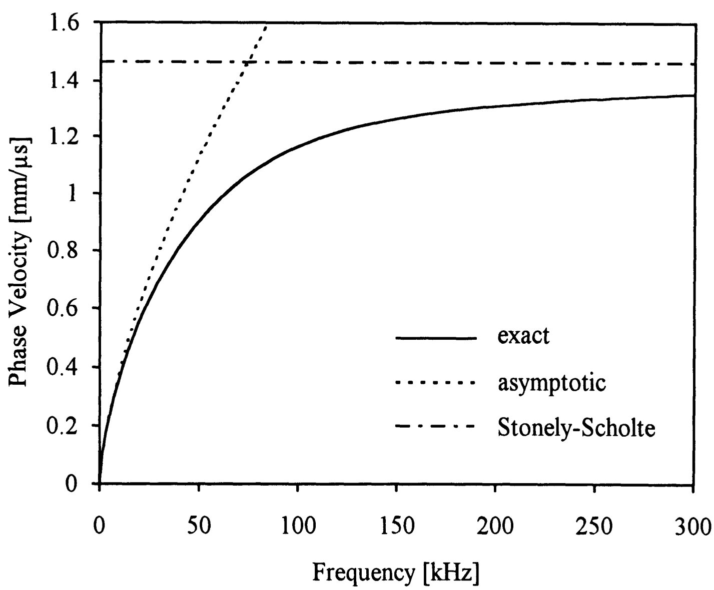

Figure 2. Phase velocity of the squirting mode plotted as function of frequency along with its low- and high-frequency asymptotes.

mode, we have to re-formulate the dispersion equation taking into account the nonvanishing shear stresses in the fluid. This analysis is detailed in Ref. 15. The results presented here are adapted from that reference and interested reader should consult it for specific details.

Figure 3 shows the attenuation of the squirting mode denoted by F_0^+ along with the next two higher-order symmetric modes (solid lines). The dimensions and material properties used in these calculations are the same as those used in generating Figure 2 except for the viscosity of the fluid which was taken as $\eta = 1.0$ kg / ms, one thousand times the viscosity of distilled water, in order to represent a viscous fluid. It is very clear from these curves that the squirting mode is much more attenuated by the viscosity of the fluid when compared to other modes. This is to be expected because of the very large relative fluid/solid displacement produced by the squirting mode. A simple explicit formula that approximates the attenuation coefficient of the slow squirting mode in the range $\delta \ll d_f < 2h \ll \lambda$, where δ is the viscous skin depth and λ is the wave length in fluid can be derived very easily. The final result can be written as:

$$\alpha \approx \frac{\delta}{6d_f}\frac{\omega}{c_x}. \qquad (8)$$

The prediction of this approximate model is also shown in Figure 3 as a dashed line. The most important conclusion we can draw from these attenuation calculations is that, in spite of its relative sensitivity to fluid viscosity, the attenuation of the squirting mode is quite acceptable over a wide frequency range of interest. It should be mentioned here that the

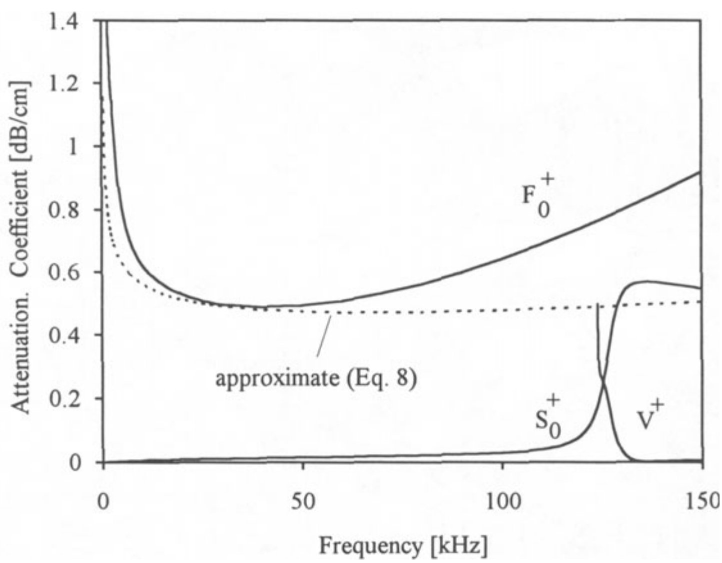

Figure 3. Attenuation coefficient versus frequency curves for the three lowest-order symmetric modes of the solid/fluid/solid trilayer (solid lines). The prediction of the approximate model for the squirting mode is also shown in dashed line.

effect of fluid viscosity on the phase velocity of the modes propagating in the trilayer is negligible.

EXPERIMENTAL TECHNIQUE AND RESULTS

In the squirting mode of a solid/fluid/solid trilayer the solid plates contribute their stiffness while the fluid layer provides the necessary inertia. Consequently, this mode can be effectively generated and detected from either the solid or the fluid component of the trilayer system. We have taken advantage of this fact in designing an experimental setup to measure the phase velocity of the squirting mode. Figure 4 shows the schematic diagram of the experimental arrangement. A damped 200-kHz ultrasonic immersion transducer is used as a transmitter to shake the water filling the thin gap between the two aluminum plates. The transducer is excited by a 300 μs-long tone-burst signal generated by a function generator that is also used to trigger an oscilloscope which displays the rf signal that can be acquired by a computer for further processing. The vibrations generated in the trilayer structure are detected from the outside using a Polytec OFV 302 Helium-Neon laser-interferometer. An SRS 530 tracking filter was used to pre-filter the signal acquired by the interferometer before digitizing by a LeCroy 9410 oscilloscope.

The geometry of the aluminum/water/aluminum trilayer arrangement is shown in Figure 5. The gap between the two plates that can be filled with water is only 25 mm wide and its actual thickness as measured by ultrasonic transmission at 5 MHz was 0.8175 mm on the average, approximately 7 % larger than the thickness of the separator shims. The reduction in the width of the water layer was necessary to restrain water motion side ways thereby simulating the infiniteness in the theoretical model in the lateral direction. The whole arrangement was sealed using Silicone rubber.

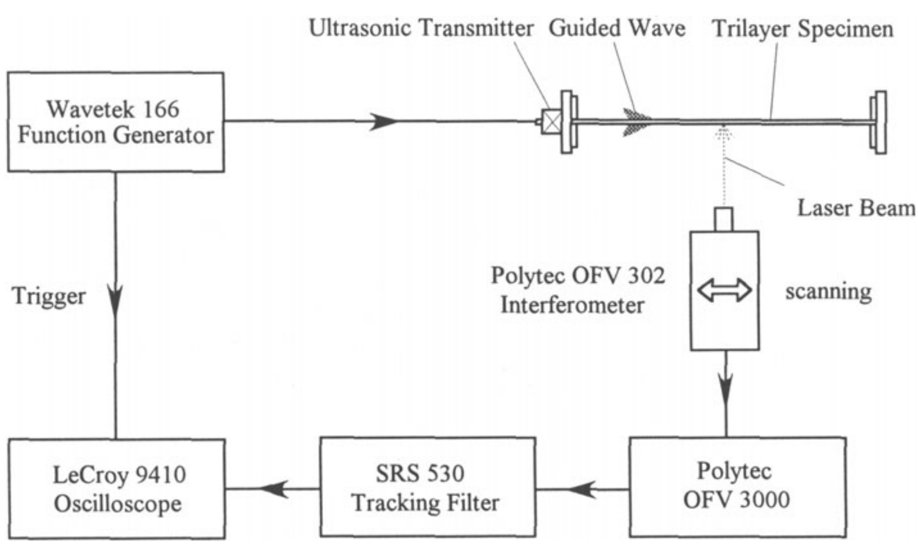

Figure 4. Schematic diagram of the experimental setup used in measuring the phase velocity of the squirting mode.

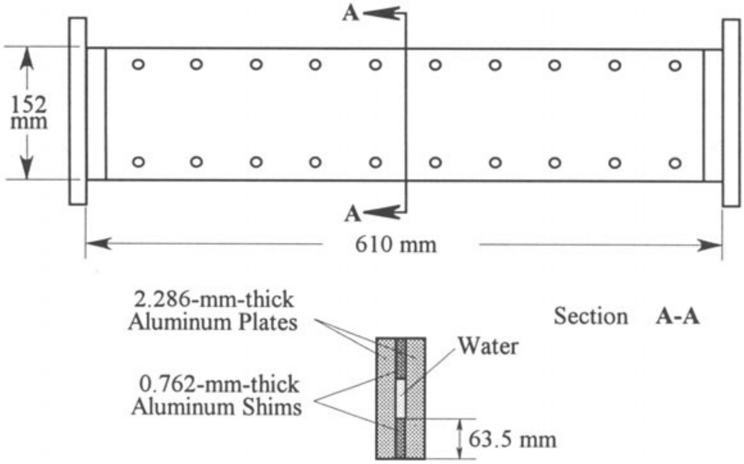

Figure 5. Geometry and dimensions of the aluminum/water/aluminum trilayer arrangement used in the dispersion experiment.

In order to accurately measure the phase velocity of the squirting mode as a function of frequency, individual measurements were performed by detecting the normal vibration of the plate at different points on the plate above the water layer by the interferometer at specific frequencies. A peak was identified in the detected rf signal and monitored in the time domain as the detection point was moved through a larger distance (50-100 mm) in numerous small (2-5 mm) steps in order to assure that the same peak was followed. The phase velocity of the squirting mode was then determined as the ratio of the total propagation distance to the accumulated propagation time. Figure 6 shows the results of the phase velocity measurements superimposed on the theoretically calculated dispersion curve of the squirting mode. The experimental error of our measurements, which is mainly caused by existing spurious interferences, is estimated at $\pm 7\%$. This uncertainty is indicated by error bars in Figure 6. Considering the numerous technical difficulties associated with the measurement, the agreement between the experimental and theoretical results is quite good.

CONCLUSIONS

The squirting mode in symmetric solid/fluid/solid trilayers is a special mode of wave propagation in which the two plates flex sideways in opposite phase thereby forcing the fluid in the thin gap between them to undergo large displacements in the longitudinal direction. This is a unique mode of wave propagation which can not be associated with any of the free Lamb modes in a single plate. A simple physical model that provides a better insight into the physics of this mode was presented. It was shown that the squirting mode is supported by both the solid (stiffness) and the fluid (inertia) and no analogous mode can exist if one of them is absent. It was found that the thickness of the fluid layer exerts a crucial effect on the attenuation of this mode while it does not significantly affect the higher-order modes. A dispersion experiment was designed to study the behavior of this mode. The experimentally measured phase velocity values turned out to be in good agreement with those predicted by theory. To the best of our knowledge, these measurements constitute the first conclusive experimental evidence of the existence of the slow squirting mode first predicted by Lloyd and Redwood in 1965.

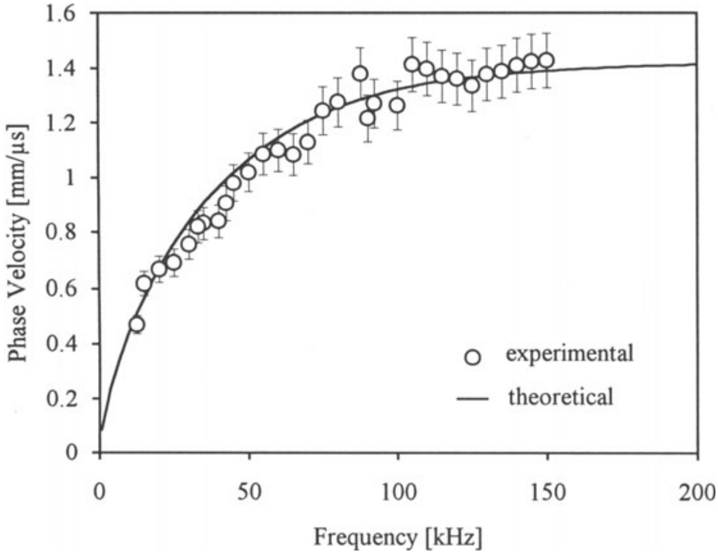

Figure 6. Experimentally measured phase velocity of the squirting mode in the aluminum/water/aluminum trilayer.

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