

SIMPLE STOCHASTIC TEMPORAL CONSTRAINT NETWORKS

Vadim Kirillov
Agricultural Information Systems Division
Institute of Agricultural Economics
Kharkov, Ukraine
kvp@svet.kharkov.ua

Vasant Honavar¹
Artificial Intelligence Research Group
Department of Computer Science
Iowa State University
Ames, Iowa 50011
honavar@cs.iastate.edu

ABSTRACT

Many artificial intelligence tasks (e.g., planning, situation assessment, scheduling) require reasoning about events in time. Temporal constraint networks offer an elegant and often computationally efficient framework for formulation and solution of such temporal reasoning tasks. Temporal data and knowledge available in some domains is necessarily imprecise - e.g., as a result of measurement errors associated with sensors. This paper introduces stochastic temporal constraint networks thereby extending constraint-based approaches to temporal reasoning with precise temporal knowledge to handle stochastic imprecision. The paper proposes an algorithm for inference of implicit stochastic temporal constraints from a given set of explicit constraints. It also introduces a stochastic version of the temporal constraint network consistency verification problem and describes techniques for solving it under certain simplifying assumptions.

1. INTRODUCTION

Many problems in artificial intelligence (AI) involve temporal inference - i.e., reasoning about events that occur at (or during) different time instants (or intervals). Examples of such problem domains include, among others, planning, scheduling, natural language processing, complex situation assessment, and commonsense reasoning. The events may be instantaneous (represented by points along the time line) or they may span a certain duration (represented by intervals along the time line). The events can be in different temporal relations with respect to each other. These relations instantiate a set of constraints that must be satisfied by the events in question. A central task in temporal reasoning involves inference of relations between a given set of events. A related task is that of inferring the occurrence of events that satisfy the given relations between known events. Also of interest is the detection of inconsistency among a given set of temporal

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relations between events. This inference is carried out in light of the available domain knowledge and might be of interest to the reasoning agent.

The past several years has seen much progress in temporal reasoning starting with the pioneering work of Allen [1983] on reasoning with time intervals using topological (or qualitative relations). Other notable early research on temporal inference includes Villain and Kautz's point algebra [1986], Dean and McDermot's time map [1987], and linear inequalities used by Malik and Binford [1983] and Valdes-Perez [1986]. More recently, several researchers sought a unified framework for temporal inference using the formalism and tools of constraint satisfaction [Montanari, 1974; Tsang, 1993]. In particular, Van Beek [1992a, 1992b] and Ladkin and Reinefeld [1992] provided effective solutions to some temporal reasoning tasks involving topological, i.e. qualitative, temporal relations introduced earlier by Allen [1983]. Maddux [1993] contributed new results on the complexity of some temporal reasoning tasks. Dechter, Meiri, and Pearl [1991] developed constraint-based techniques for reasoning with metric (i.e. quantitative) temporal relations. Kautz and Ladkin [1991] and Meiri [1991] proposed two different frameworks to handle both topological as well as metric temporal relations. More recently, Schwalb and Dechter [1993] examined a more general structure of temporal constraints, i.e. disjunctive constraints. These frameworks have contributed an impressive array of theoretical results as well as practical tools for dealing with temporal reasoning tasks. But they do not adequately address imprecision of temporal knowledge and or data which is typical of many practical application domains.

Imprecision of time measurements may stem from natural or artificial clutter which affects sensors. Metric temporal relations may contain estimates of temporal parameters distorted by random errors. This type of imprecision may be described using a probabilistic model and is therefore called stochastic imprecision. As an illustration of temporal reasoning under stochastic imprecision, consider a modified version of the narrative which was used by Dechter, Meiri, and Pearl [1991]. We have introduced stochastic imprecision into a simplified version of that narrative (with the disjunctive constraints eliminated).

***Example 1.1.** John goes to work by car and his trip to work takes an average of 35 minutes, with the standard deviation being 4 minutes. Fred goes to work by bus and his trip takes an average of 45 minutes, with the standard deviation being 8 minutes). Today, John left home at about 7:15 am (± 3 minutes), and Fred arrived at work exactly at 8:05. We also know that John arrived at work about 25 minutes (± 6 minutes) after Fred left home.*

Given all necessary additional information about statistical properties of the random variables used in this narrative, one might wish to answer queries such as: "To what extent are the assertions in the narrative consistent?", "What are the possible times at which Fred could have left home for work with a probability 0.997 ?", and so on.

With the exception of recent attempts by Kirillov [1994] and Goodwin, Neufeld, and Trudel [1994], current literature on temporal inference has remarkably little to offer in terms of approaches to dealing with stochastic sources of imprecision in various temporal reasoning tasks. Given the ubiquitous presence of stochastic sources of uncertainty in

many practical applications of interest, this represents a significant gap in the current repertoire of techniques for temporal reasoning.

The work of Goodwin, Neufeld, and Trudel [1994] deals with a rather restricted temporal inference task (namely, that of propagating the truth values assigned to subintervals to the truth value of the interval covering the subintervals when the knowledge of subintervals may be imprecise in a probabilistic sense) motivated by a particular meteorological application. Unfortunately, it does not address the general task of temporal inference under stochastic imprecision.

The work presented in Kirillov [1994] was concerned primarily with heuristic solutions to the situation assessment task under stochastic uncertainty with emphasis on military surveillance applications. This work offers a framework for the representation of stochastically imprecise time points and intervals, as well as for reasoning with such events using imprecise metric time relations. But it does not support (among other things), the use topological time relations. Neither does it take advantage of the general framework of constraint-based approach to temporal inference. Thus there is clearly a need for extending this approach to reasoning under stochastic uncertainty and integrating it with the constraint-satisfaction based approaches to various temporal reasoning tasks.

Against this background, this paper develops a mathematically well-founded generalization of constraint-based approaches to temporal inference to deal with practical applications which involve temporal reasoning under stochastic imprecision. It builds on foundational work on temporal reasoning and constraint satisfaction by various authors to deal with stochastic imprecision. (Non-stochastic imprecision (e.g., fuzziness of temporal relations) is the subject of a forthcoming paper [Kirillov and Honavar, 1996]). In particular, we consider a constraint-based approach to temporal reasoning with *simple* (i.e., non-disjunctive) stochastic constraints. Extension of this model to deal with disjunctive constraints is currently under investigation.

The paper is organized as follows: An overview of Metric Temporal Constraint Networks is provided in Section 2. A simple stochastic generalization of metrical temporal constraint framework is outlined in Section 3. Section 4 describes inference of implicit stochastic temporal constraints and section 5 further elaborates on stochastic temporal inference techniques under the assumption that stochastic uncertainty can be modeled using Gaussian probability distributions. Section 6 defines a measure of consistency of stochastic temporal constraint networks and provides an algorithm for consistency checking. Section 7 concludes with a summary of the paper and an outline of some future research directions.

2. OVERVIEW OF METRIC TEMPORAL CONSTRAINT NETWORKS

This section briefly summarizes the mathematical formulation of the *temporal constraint satisfaction problem* (TCSP) due to Dechter, Meiri, and Pearl [1991] to lay the groundwork for the discussion of *stochastic temporal constraint networks* that follows. A

temporal constraint network is a *directed graph* whose nodes correspond to a set of (typically real-valued) temporal variables, X_1, \dots, X_n , having a continuous domain, and a set of unary and binary constraints. Generally, each unary constraint T_j on some temporal variable X_j is represented by a set of closed intervals. That is,

$$T_j = \{I_1, \dots, I_{k_j}\} = \{[a_1, b_1], \dots, [a_{k_j}, b_{k_j}]\}. \quad (1)$$

Thus, a *unary constraint* T_j , restricts the domain of a variable time point X_j to the given set of intervals, and represents the disjunction

$$(a_1 \leq X_j \leq b_1) \vee \dots \vee (a_{k_j} \leq X_j \leq b_{k_j}). \quad (2)$$

A binary constraint T_{ij} restricts the permissible values for the difference in values that can be simultaneously assumed by X_j and X_i ; thus it stands for the disjunction

$$(a_1 \leq X_j - X_i \leq b_1) \vee \dots \vee (a_{k_{ij}} \leq X_j - X_i \leq b_{k_{ij}}). \quad (3)$$

Thus, a unary temporal constraint can be naturally interpreted as specifying a time datum. For instance, if $k_j = 1$, then (2) simply means an assignment of an interval estimate to a time point variable X_j . A binary constraint describes a relationship between two temporal variables within a given scenario. Dechter, Meiri, & Pearl (1991) further simplified the mathematical specification of a TCSP by introducing a fictitious temporal variable X_0 , which is typically assigned a value of 0 to represent the "*beginning of the world*". This makes it possible to replace unary constraints of the form specified in (2) with an equivalent binary constraint:

$$(a_1 \leq X_i - X_0 \leq b_1) \vee \dots \vee (a_{k_i} \leq X_i - X_0 \leq b_{k_i}) \quad (4)$$

The above specification of temporal constraints maps quite naturally to a simple constraint network in the form of a directed constraint graph, in which nodes represent temporal variables and edges represent the known binary constraints. Thus, an edge $i \rightarrow j$ represents a given (known) binary constraint T_{ij} .

An n -tuple $X = (x_1, \dots, x_n)$ is called a *solution* to a given TCSP if the assignment $\{X_1 = x_1, \dots, X_n = x_n\}$ satisfies all the constraints of the TCSP. A constraint network is said to be *consistent* if at least one solution exists. A value v is said to be a *feasible value* for a variable X_i if there exists a solution in which $X_i = v$. The set of all feasible values of a variable is called the *minimal domain* of the variable.

Major reasoning tasks with metric temporal constraint networks include

1. checking a given network for consistency;
2. inferring (not explicitly prespecified) constraints between any pair of variables, X_i and X_j ; and
3. finding the solutions for a given TCSP.

Clearly, the latter two tasks are meaningful only when the constraint network is consistent. The task 3 can be reduced to solving several instances of task 2 (with different pairs of variables). The minimal domain for a variable X_i ($i \neq 0$) can be obtained by solving task 2

for the pair $\langle X_0, X_i \rangle$. A solution to a TCSP can be constructed by selecting values from minimal domains of different variables and instantiating the corresponding uninstantiated variables. Because the assignment of some value to a variable X_i requires specifying the constraint for the variable pair $\langle X_0, X_i \rangle$, in the worst case, a search for all TCSP solutions entails performing the task 2 for all n pairs $\langle X_0, X_1 \rangle, \dots, \langle X_0, X_n \rangle$.

The mathematical framework outlined above supports reasoning tasks with precise time points using crisp metric relations (i.e., distances) between them. If reasoning with time intervals is required, each interval is treated as an ordered pair of time points, and this ordering is represented as an additional constraint. This facilitates the use of essentially the same type of constraint network for reasoning about metric relations involving both time points as well as time intervals.

Dechter, Meiri, & Pearl (1991), have shown that the search for a TCSP solution does not need any backtracking, if for every constraint, the interval set of equation (1) consists of a single time interval. This case corresponds to non-disjunctive constraints (also referred to as *simple* constraints).

This paper proposes a stochastic generalization of simple TCSP.

3. SIMPLE STOCHASTIC TEMPORAL CONSTRAINT NETWORKS

There are at least three possible ways to generalize a TCSP to incorporate stochastic imprecision in some form:

1. Assume that for some i, j , the parameters a_{ij}, b_{ij} are stochastically imprecise, and hence, the constraints defined by equation 3 hold with some probability.
2. Assume that time variables X_1, \dots, X_n are random, and express the known constraints on them in the form of a-priori specified probability distributions.
3. A combination of the above two approaches.

To keep things simple, the discussion in this paper is limited to the second of the three approaches enumerated above. After Dechter, Meiri and Pearl (1991), we formulate the problem using a fictitious time variable X_0 which allows us to replace unary constraints on a variable X_i ($i \neq 0$) with an equivalent binary constraint for the variable pair $\langle X_0, X_i \rangle$. Also, following the notational convention that is common in probability theory literature, we reserve upper-case letters for random variables and lower-case ones for their values.

A *simple binary stochastic constraint* stochastically constrains the permissible values for the distance

$$\Delta_{ij} = X_j - X_i \quad (5)$$

by asserting that the probability density function of Δ_{ij} is $f_{ij}(\delta)$; $1 \leq i, j \leq n$.

We further assume that

$$f_{ij}(y) = f_{ji}(-y) \quad (6)$$

That is, the density function is an *even* function.

Further, to simplify analysis using standard techniques of probability theory (e.g. Von Mises, 1964), we also assume that the probability density functions satisfy the assumptions under which the maximum likelihood approach can be used.

A *network of binary stochastic constraints* consists of a set of random variables X_1, \dots, X_n , and a set of binary stochastic constraints, $f_{ij}(\delta)$, imposed on the distances Δ_{ij} between some of these variables.

As in the non-stochastic case, such a network can be represented by a directed constraint graph whose nodes correspond to variables and edges represent explicit constraints. Thus, an edge $j \rightarrow i$ indicates that an explicit constraint T_{ij} for the random distance

$\Delta_{ij} = X_j - X_i$ is specified; it is labeled by the respective probability density function.

Assumption (6) means we need only to consider either $j \rightarrow i$, or $i \rightarrow j$, but not both for a given pair of nodes $\langle X_i, X_j \rangle$. To keep things simple, we will further assume that any two random distances, Δ_{ij} and Δ_{kl} corresponding to two different edges in the constraint graph are *stochastically independent*. In a number of applications of practical interest, it is often reasonable to require that the distributions are Gaussian. In this case, each binary stochastic constraint is characterized completely by the mean m_{ij} and standard deviation σ_{ij} . In the example that follows, we assume that $\forall i, j: (1 \leq i, j \leq n)$, the distribution of random distance Δ_{ij} is Gaussian with mean m_{ij} and standard deviation σ_{ij} (respectively).

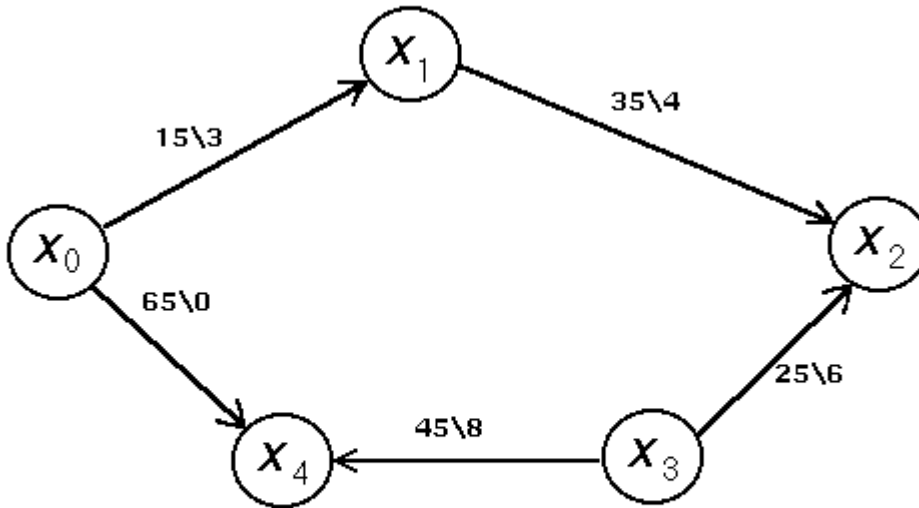


Fig.1. A constraint graph representing Example 1.1.

The stochastic temporal constraint graph for Example 1.1 (see Section 1 above) is shown in Fig. 1. Here $[X_1, X_2]$ and $[X_3, X_4]$ (respectively) are time intervals, during which John and Fred travel to work.. The beginning of the world is $X_0=7:00$ AM, which makes it convenient to express time intervals in minutes. Since this example assumes that any given probability density function, $f_{ij}(\delta)$ can be uniquely characterized by its mean m_{ij} , and the standard deviation σ_{ij} , the edges of the constraint graph in Fig. 1 are labeled with these two parameters (m_{ij}, σ_{ij}) .

The sections that follow discuss the inference of derived constraints (i.e., those not specified explicitly a-priori) and network consistency checking in such a stochastic temporal constraint network .

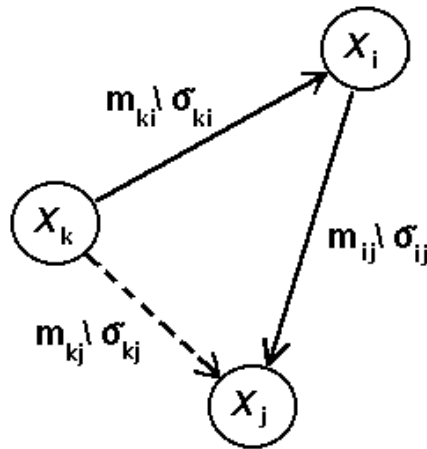


Fig.2. The simplest constraint graph with the implicit constraint shown as a dashed line

4. INFERENCE OF IMPLICIT STOCHASTIC TEMPORAL CONSTRAINTS

As in the case of non-stochastic temporal constraint networks which were outlined in section 2 above, one can distinguish between explicit (or a-priori explicitly specified) and implicit (or derived or inferred) stochastic constraints. This section discusses the inference of derived stochastic temporal constraints.

Implicit constraints result from the linear orderings that exist along the time line. This is illustrated in Fig. 2 for a simple network containing only two explicit constraints for pairs $\langle X_i, X_j \rangle$ and $\langle X_k, X_i \rangle$. The only inferable implicit constraint for $\langle X_k, X_j \rangle$ is represented by a dashed line which indicates the existence of the following relationship between the three random distances Δ_{kj} , Δ_{ki} , and Δ_{ij} :

$$\Delta_{kj} = \Delta_{ki} + \Delta_{ij} . \tag{7}$$

Thus, the implicit stochastic temporal constraint $k \rightarrow j$ corresponds to the random variable Δ_{kj} , whose probability distribution $f_{kj}(\delta)$ is given by:

$$f_{kj}(\delta) = f_{ij}(\delta) \otimes f_{ki}(\delta) \quad (8)$$

where $f_{ij}(\delta)$ and $f_{ki}(\delta)$ are the probability densities of the random variables Δ_{ki} and Δ_{ij} , respectively, and \otimes is the convolution operation defined by:

$$g(x) \otimes h(x) = \int_{-\infty}^{\infty} g(t)h(x-t)dt \quad (9)$$

Since only explicit temporal constraints (of the sort denoted by solid lines in the figures 1 and 2) are used in inference of derived constraints, it is easily shown that the total number κ of implicit constraints between a given pair of nodes, will be in the range given by:

$$0 \leq \kappa \leq 2^n - 1 \quad (10)$$

where n is the number of time variables. In the above formula, the lower bound on κ corresponds to the case when no path exists between the two given nodes in the constraint graph. The upper bound corresponds to the case of a complete graph of explicit temporal constraints, i.e. when an explicit constraint is provided for each pair of the time variables.

In the general case, the stochastic constraint network corresponds to a graph whose nodes are time variables and edges are the given binary stochastic constraints. The inference of a new stochastic constraint between any two nodes — say X_i and X_j (with the corresponding node indices i and j respectively) — from the given set of stochastic constraints can thus be reduced to finding a path between these two nodes in the (explicit) constraint graph. Let $i, j_{s_1}, \dots, j_{s_k}, j$ be the node indices of the nodes that lie along the s -th path Π_{ij}^s between the nodes X_i and X_j . This path yields the derived stochastic constraint $f_{ij}^s(\delta)$ (i.e., the probability density function of the random variable Δ_{ij}^s) between the nodes X_i and X_j where:

$$\Delta_{ij}^s = \Delta_{ij_{s_1}} + \Delta_{j_{s_1}j_{s_2}} + \dots + \Delta_{j_{s_k}j} \quad (11)$$

$$f_{ij}^s(\delta) = f_{ij_{s_1}}(\delta) \otimes f_{j_{s_1}j_{s_2}}(\delta) \otimes \dots \otimes f_{j_{s_k}j}(\delta). \quad (12)$$

Since more than one such path may connect two chosen nodes in a stochastic constraint graph, different constraints may be inferred from a given set of pairwise distances or binary stochastic constraints. This raises the issue of possible inconsistency in the set of explicit stochastic constraints (see section 5 below).

Given the mean values and variances of the explicit constraint probability distributions in (11), the mean value ${}^s m_{ij}$ and (assuming stochastic independence of the set of random variables that constitute the nodes of the explicit constraint graph), the variance ${}^s \sigma_{ij}^2$ for the inferred constraint $f_{ij}^s(\delta)$ (the probability density function of the random variable Δ_{ij}^s) are given by:

$${}^s m_{ij} = m_{ij_{s_1}} + m_{j_{s_1} j_{s_2}} + \dots + m_{j_{s_k} j}. \quad (13)$$

and

$${}^s \sigma_{ij}^2 = \sigma_{ij_{s_1}}^2 + \sigma_{j_{s_1} j_{s_2}}^2 + \dots + \sigma_{j_{s_k} j}^2. \quad (14)$$

Armed with this machinery for stochastic constraint-based reasoning, we can revisit Example 1.1, and attempt to answer the query, “When did John arrive at work?”. Bearing in mind that X_2 stands for “John arrived at work”, the query can be answered by inferring the implicit constraint for $\langle X_0, X_2 \rangle$, from the given explicit constraints imposed on $\langle X_0, X_1 \rangle$ and $\langle X_1, X_2 \rangle$. Calculation of the mean and standard deviation in accordance with (13), (14) results in the following result: “John arrived at 7:50 AM, with the standard deviation 5 minutes”. Note that although an answer of this sort would be quite satisfactory in many applications, a complete description of the inferred constraint requires the derivation of the probability distribution corresponding to the desired implicit constraint using formula (12).

As already pointed out, in general, for a given pair of nodes in a constraint graph, there may be more than one path between them in the explicit constraint graph. This situation is illustrated in Figure 3, where two alternative ways of inferring the constraint for the distance, $\Delta_{02} = X_2 - X_0$, exist. Thus, it is possible in this case, to derive two different time references to the answer the query: “When did John arrive at work?”.

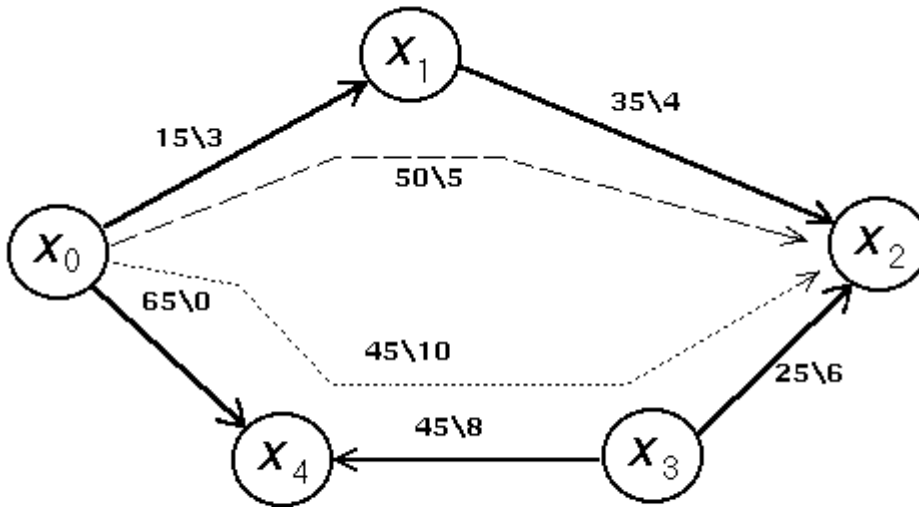


Fig.3. A constraint graph representing Example 1.1 with two alternative implicit constraints

It is natural then to raise the question as to how one may go about selecting the *best* result from the set of derived constraints between two nodes in the constraint graph. A simple rule of thumb suggests the selection of the derived implicit constraint that has the smallest

variance and ignore the rest. A potentially better alternative is suggested by the maximum likelihood approach in statistics (Von Mises, 1964).

Within a maximum likelihood framework, it is natural to treat possible mismatches between different estimates of a derived constraint obtained from using different paths in the constraint graph as a result of stochastic imprecision associated with the explicit constraints. Therefore, a natural interpretation of the different mean values of a derived constraint (calculated considering different paths) between a given pair of nodes X_i and X_j using equation (13), as random estimates of the unknown mean value, Δ , of the random distance, $X_i - X_j$. To emphasize the fact that equation (13) yields estimates of the actual mean value, in what follows, we will use Greek letters ${}^1\mu_{ij}, \dots, {}^N\mu_{ij}$ to denote these estimates instead of ${}^1m_{ij}, \dots, {}^Nm_{ij}$. Our objective then is to find the estimated value of Δ that would maximize some optimality criterion, $Q({}^1\mu_{ij}, \dots, {}^N\mu_{ij}, \Delta)$, for given values of ${}^1\mu_{ij}, \dots, {}^N\mu_{ij}$.

A natural criterion to optimize is the N -dimensional joint probability density function, $f({}^1\mu_{ij}, \dots, {}^N\mu_{ij}, \Delta)$. The estimation problem is thus reduced to a classic one-parameter maximization problem. Its solution is given by:

$$\frac{d}{d\Delta} f({}^1\mu_{ij}, \dots, {}^N\mu_{ij}, \Delta) = 0. \quad (15)$$

Before we can solve equation (15), we need to derive the distribution function, $f({}^1\mu_{ij}, \dots, {}^N\mu_{ij}, \Delta)$, based on what we know about the stochastic constraint network. Assuming stochastic independence of the explicit constraints, it is possible, in theory, to construct this function if we have the following knowledge about the constraint network:

1. the specific explicit constraints that determine each of the estimated means ${}^s\mu_{ij}$, and
2. the probability distributions associated with each such constraint.

Unless additional assumptions are made, the calculation of the desired distribution function becomes rather complicated. Therefore, in order to illustrate the general approach, we will limit our discussion in what follows to stochastic constraints that obey Gaussian statistics.

5. INFERENCE OF IMPLICIT CONSTRAINTS WITH GAUSSIAN STATISTICS

To illustrate the feasibility of the maximum likelihood approach to reasoning with stochastic temporal constraints, consider a particular case when all the time constraints are Gaussian i.e., each probability distribution function in equation (12) is of the following form:

$$f_{ij}(\delta) = \frac{1}{\sqrt{2\pi}\sigma_{ij}} \cdot \exp\left(-\frac{(\delta - m_{ij})^2}{2\sigma_{ij}^2}\right). \quad (16)$$

Since the sum of any number of Gaussian random variables yields a Gaussian random variable, the distribution on left-hand side of equation (12) is Gaussian (normal). Since an N -dimensional Gaussian distribution is fully specified by an $N \times N$ covariance matrix and an N -dimensional mean vector, the task of estimating $f({}^1\mu_{ij}, \dots, {}^N\mu_{ij}, \Delta)$ reduces to an estimation of the corresponding mean vector and covariance matrix.

To further simplify the analysis, we assume that different paths between any given two nodes of the constraint graph are disjoint, i.e. they have no common edges. This makes the variables, ${}^1\mu_{ij}, \dots, {}^N\mu_{ij}$, stochastically independent.

Armed with these simplifying assumptions, we derive the likelihood function, $f({}^1\mu_{ij}, \dots, {}^N\mu_{ij}, \Delta)$, for this particular case. We then solve equation (15) and illustrate it with an example.

To minimize notational clutter introduced by awkward indices, we omit below all references to the i -th and j -th nodes (since it is clear from context that the focus is on derived constraints between the variables \mathbf{X}_i and \mathbf{X}_j). We also move the upper-left indices to the convenient lower-right position. Also, we change the indexing of the elementary constraint variables so that Δ_{pk} stands for the k -th elementary constraint in the p -th path connecting the given pair of nodes in the constraint graph.

Thus we have a set of random variables, μ_1, \dots, μ_N , each being a sum of a few independent Gaussian variables:

$$\mu_p = \sum_{k=1}^{n_p} \Delta_{pk}, \quad p=1, \dots, N \quad (17)$$

The mean value of μ_p over all possible paths that result in a derived constraint of interest is the unknown parameter Δ that needs to be estimated.

The N -dimensional probability distribution of the set of random variables μ_1, \dots, μ_N is given by:

$$f(\mu_1, \dots, \mu_N, \Delta) = \frac{1}{(2\pi)^{n_p/2} |\mathbf{K}|^{1/2}} \cdot \exp\left(-\frac{1}{2}(\vec{\mu} - \vec{\Delta}) \mathbf{K}^{-1} (\vec{\mu} - \vec{\Delta})^T\right), \quad (18)$$

where $\vec{\mu} = (\mu_1, \dots, \mu_N)$ and $\vec{\Delta} = (\Delta, \dots, \Delta)$ are N -dimensional rows, and \mathbf{K} is the covariance matrix.

Given our assumptions, \mathbf{K} turns out to be a diagonal matrix. Furthermore, each diagonal element σ_k^2 of \mathbf{K} corresponds to a known variance (given by the sum of the variances of the stochastic constraints along the k -th path).

Differentiating both sides of equation (18) with respect to Δ , and equating it to zero (as in equation (15) above), yields a linear equation in Δ . Its solution $\hat{\Delta}$ is given by:

$$\hat{\Delta} = \frac{\sum_{p=1}^N \mu_p \cdot \sigma_p^{-2}}{\sum_{p=1}^N \sigma_p^{-2}}, \quad (19)$$

where σ_p^2 is the variance of the sum of stochastic constraints on the p -th path.

Because of the maximum likelihood estimate, $\hat{\Delta}$, being the weighed sum of Gaussian variables, has a normal distribution. Its mean is the unknown parameter Δ , and its variance is given by the formula:

$$\sigma_{\Delta}^2 = \frac{1}{\sum_{p=1}^N \sigma_p^{-2}}. \quad (20)$$

We now revisit Example 1.1, assuming that all the stochastic constraints used in it are Gaussian to illustrate the preceding analysis. We find that for the constraint imposed on $X_2 - X_0$,

$$\hat{\Delta} = \frac{50/5^2 + 45/10^2}{1/5^2 + 1/10^2} = 49.0, \quad (21)$$

and

$$\sigma_{\Delta}^2 = \frac{1}{1/5^2 + 1/10^2} = 20.0. \quad (22)$$

In other words, using all the information available implicitly in the constraint network, we find (using maximum likelihood estimates as discussed above) that John arrived at work at 7:49, where the standard deviation of the estimate is $\sqrt{20} \approx 4.47$ minutes. (Note that this estimate is more accurate than any of the other two estimates obtained by tracing individual paths from X_0 to X_2 in Section 4 above.)

Taking into account the fact that the estimated time is distributed normally, we can also conclude that, the probability John arrived at work at $7:49 \pm 13.42$ with a probability of 0.997.

6. CONSISTENCY OF STOCHASTIC CONSTRAINT NETWORKS

In the case of precise constraint networks, a set of constraints either holds or it does not. When a given set of constraints are not satisfiable, we say that the corresponding constraint network is inconsistent. In the stochastic networks, since constraints are stochastically imprecise, it makes it possible for a network to be consistent to various degrees. Therefore, we need suitable metrics that can quantify the degree to which a set of stochastic constraints are satisfied.

As shown above, for a given pair of nodes X_i and X_j in a stochastic constraint network, one or more implied constraints can be inferred, using different routes connecting these

nodes in the constraint graph. This is illustrated by Fig.3, where the two inferred constraints have different mean values that provide two different estimates of the time of John's arrival at work. A good measure of stochastic network consistency should help us decide if these differences are negligible.

Consider an arbitrarily chosen pair of nodes, $\langle X_i, X_j \rangle$, from a stochastic temporal constraint graph. (In what follows, we will use the same notations as in the previous section.)

All the inferable implicit constraints for this pair of nodes, i.e. random estimates, μ_1, \dots, μ_N , can be characterized by their respective distributions, $f_1(\mu), \dots, f_N(\mu)$, where N is the total number for different routes between the chosen pair of nodes. Also, in general, we should take into account the possibility that an explicit stochastic constraint could be imposed on this pair as well. We will denote the distribution associated with the latter by $f_0(\mu)$.

Hence, we have $N+1$ different random estimates, $\mu_0, \mu_1, \dots, \mu_N$, for a non-random distance, Δ , between the time points, X_i and X_j . Their probability distributions, $f_0(\mu), f_1(\mu), \dots, f_N(\mu)$, all express different views on the single entity, the distance, Δ . It is natural to associate the stochastic consistency property of the constraint network with the difference of the first moments of these distributions, i.e. of their mean values. The larger the magnitude of this difference, the lower must be the consistency of the network under consideration.

It is also natural (as a first approximation) to require that in a perfectly consistent stochastic network, all the routes connecting a given pair of nodes should yield the same estimated mean distance, and the differences in the higher-order moments of the respective probability distributions be relatively small. This allows us to reduce the stochastic constraint network consistency checking problem to a set of $N(N+1)/2$ stochastic hypotheses testing problems, for each ordered pair of nodes in the constraint network.

For the given pair, $\langle X_i, X_j \rangle$, consider the simple hypothesis:

$$\mathbf{H}_0: \quad \forall(p, q) [\mathbf{M}(\mu_p) = \mathbf{M}(\mu_q)], \quad (22)$$

claiming that all the estimated time distances, $\mu_0, \mu_1, \dots, \mu_N$, along different routes have the same mean value (Here, $\mathbf{M}(x)$ denotes the mean of x). It should be tested against the composite alternative hypothesis:

$$\mathbf{H}_1: \quad \exists(p, q) [\mathbf{M}(\mu_p) \neq \mathbf{M}(\mu_q)], \quad (23)$$

which states that at least one of the estimates has a mean value that differs from the others.

Stochastic hypotheses testing problem have been treated in detail in the probability theory literature (Von Mises, 1964). Therefore, we limit our discussion below to an outline of the key results. The interested reader is referred to Von Mises (1964) or other advanced texts on probability theory for details.

An optimal decision making rule is based on the calculation of the so-called likelihood ratio, $L(\cdot)$, which is compared with a pre-assigned threshold, L_0 as follows:

$$L(\vec{\mu}) = \frac{f(\vec{\mu}, \Delta | \mathbf{H}_0)}{f(\vec{\mu}, \Delta | \mathbf{H}_1)} \underset{\mathbf{H}_1}{\overset{\mathbf{H}_0}{>}} L_0, \quad (24)$$

where $\vec{\mu} = \langle \mu_0, \mu_1, \dots, \mu_N \rangle$, and $f(\vec{\mu}, \Delta | \mathbf{H}_k)$ is an $(N+1)$ -dimensional conditional likelihood function for the hypothesis, \mathbf{H}_k . (The above rule simply states that hypothesis \mathbf{H}_0 is to be preferred over \mathbf{H}_1 if the value of the likelihood ratio is greater than the threshold and vice versa).

Use of this likelihood ratio in the determination of consistency of stochastic constraint networks is complicated by the lack of accurate knowledge of the parameter Δ . A couple of different ways of resolving this difficulty suggest themselves. We outline each of them in turn in what follows.

The first approach can be used when an explicit constraint for the pair of nodes under consideration, namely, $\langle X_i, X_j \rangle$, is given. In many applications (e.g., situation assessment tasks), it is natural to assume that the explicit constraint overrides any inferred constraints for the particular pair. In other words, the following relationship holds:

$$\Delta = \mu_0. \quad (25)$$

The second approach is useful when no explicit stochastic constraint is given for the pair of nodes under consideration, or when the application domain does not justify the use of a known explicit constraint to override the inferred constraints. To be specific, we will assume that μ_0 is unknown, and will use μ_1, \dots, μ_N (the mean values corresponding to the inferred constraint based on different paths through the constraint network). The missing parameter, Δ , is substituted with its estimated value, $\hat{\Delta}$, which is obtained by solving the maximum likelihood equation (15). Note that since the estimate is a function of μ_1, \dots, μ_N , this fact needs to be taken into account in the derivation of the likelihood functions. We shall refer below to the resulting functions as $f^*(\vec{\mu} | \mathbf{H}_k)$, $k=0,1$.

The decision rule in this case is given by:

$$\mathbf{L}^*(\vec{\mu}) = \frac{\mathbf{f}^*(\vec{\mu} | \mathbf{H}_0)}{\mathbf{f}^*(\vec{\mu} | \mathbf{H}_1)} \underset{\mathbf{H}_1}{\overset{\mathbf{H}_0}{>}} \mathbf{L}^*_0, \quad (26)$$

The likelihood ratio in this formula can be naturally interpreted as a useful measure of consistency of the stochastic constraint network with respect to a particular pair of nodes. The threshold value, L^*_0 , for a hypotheses testing problem of this sort, has to be chosen with respect to given probability α of the erroneous rejection of \mathbf{H}_0 . This choice is invariably domain-dependent.

Checking the entire network for consistency entails calculation of the likelihood ratio for each pair of nodes. We outline this process for the particular case of Gaussian stochastic constraints under the assumption that each of the multiple paths between a pair of nodes yields statistically independent estimates for the corresponding inferred constraint. Detailed derivation of the expression for the likelihood ratio for a similar case can be found in Kirillov (1994). We simply present the final results here. The numerator of the expression for likelihood ratio in equation (26) can be shown to be an N -dimensional normal distribution given by:

$$f^*(\vec{\mu}|\mathbf{H}_0) = \frac{1}{(2\pi)^{N/2} |\mathbf{C}|^{1/2}} \exp\left(-\frac{1}{2} \vec{\varepsilon} \mathbf{C}^{-1} \vec{\varepsilon}^T\right), \quad (27)$$

where $\vec{\varepsilon} = (\mu_1, \dots, \mu_N) - (\hat{\Delta}, \dots, \hat{\Delta})$, and $\hat{\Delta}$ is given by equation (19).

The elements of the covariance matrix, \mathbf{C} , are given by:

$$c_{ur} = \delta_{ur} \cdot \sigma_u^2 - \frac{1}{\sum_{p=1}^N \sigma_p^{-2}}, \quad (28)$$

where $\delta_{ur} = 1$ if $u=r$, and 0 otherwise, and σ_u^2 is the variance of the estimated time distance along the u -th route in the constraint graph.

In this case, it is possible to further simplify the inequality in (26) by replacing both sides of the inequality by the corresponding logarithms. This yields:

$$\sum_{p=1}^N \sum_{q=1}^N d_{pq} \cdot (\mu_p - \hat{\Delta}) \cdot (\mu_q - \hat{\Delta}) \underset{\mathbf{H}_1}{\overset{\mathbf{H}_0}{>}} \lambda, \quad (29)$$

where d_{pq} is an element of \mathbf{C}^{-1} , and λ is the appropriately adjusted threshold.

As the size of the constraint network increases, typically, so does the number of different paths, N . It follows from equation (28) that as $N \rightarrow \infty$, the matrix \mathbf{C} becomes diagonal. This allows us to consider the asymptotically optimal version of the decision making rule given by:

$$\sum_{p=1}^N \frac{(\mu_p - \hat{\Delta})^2}{\sigma_p^2 + \sigma_{\Delta}^2} \underset{\mathbf{H}_1}{\overset{\mathbf{H}_0}{>}} \lambda. \quad (30)$$

This simply states that if the sum of the weighed squared differences between the mean values of time constraints found along different paths, μ_1, \dots, μ_N , and their average, $\hat{\Delta}$ (given by equation by (19)) exceeds the threshold, λ , the network meets the consistency test. This intuitively appealing rule is rather easy to implement for any given set of stochastic constraints if probability distributions have finite variances. However, it should be noted that if all the assumptions made in its derivation are not satisfied, it constitutes

only a sub-optimal rule for testing the consistency of stochastic temporal constraint networks.

7. SUMMARY AND DISCUSSION

We have developed a generalization of a constraint-based approach to temporal inference to deal with simple (i.e., non-disjunctive) stochastically imprecise temporal constraints. Stochastic imprecision of this sort is typical of environments where temporal data are captured by sensors or measurement devices that may be subject to random errors.

The framework developed in this paper provides some useful tools for situation assessment, validation of schedules and plans in such environments. This development took advantage of two simplifying assumptions concerning the stochastic independence of random variables.

First, we assumed that the constraints imposed on individual time distances, $\Delta_{ij} = X_j - X_i$, are all stochastically independent. If we drop this assumption, the only consequence would be that we would have to replace expressions (12) and (14) with somewhat more complicated formulae containing (given or estimated) joint probability distributions of the random variables, Δ_{ij} , for all pairs, $\langle X_i, X_j \rangle$ for which explicit constraints are given. If the individual distributions are Gaussian, this would give rise to a non-diagonal covariance matrix in (18) (since the individual distances are no longer assumed to be stochastically independent), and hence, somewhat more complicated expressions that would need to be used instead of (19) and (20).

Second, in Section 6, it was assumed that different paths that connect given two nodes of the constraint graph are disjoint. This guaranteed the stochastic independence of the inferred implicit constraints. This assumption was used in the derivation of formulae (28) and (30) only. If it does not hold, we would have to replace (28) with a more complicated formula, and the resulting test for stochastic network consistence would be less than optimal. Modulo these limitations, the framework for constraint-based temporal reasoning that is presented in this paper is quite general. However, a number of problems remain to be addressed. These include extensions of the proposed framework to handle disjunctive temporal constraints as well as interval constraints, and computational complexity considerations. We conclude with a brief sketch of an extension of the proposed approach to handle interval temporal constraints. Intervals can be treated as ordered pairs of time points and additional constraints can be imposed between the start and end of temporal intervals. Since these additional constraints are not metric but are topological, our framework needs to be modified to handle stochastic topological constraints. In particular, such a topological constraint between the start and end points of a temporal interval represented by the ordered pair of time points X_k and X_j can be formulated in terms of the probability that the size of the interval is greater than zero as follows:

$$Prob\{ X_k - X_j > 0 \} > \beta, \quad (31)$$

where β is a given confidence level.

Since the knowledge about the variables, X_k and X_j , are presented in the form of respective probability density functions, checking the constraint, (31), for consistency can be performed, as follows:

1. using (8), infer the stochastic constraint, $f_{kj}(\delta)$, for the distance, $\Delta_{kj} = X_k - X_j$, if this constraint is not given;
2. calculate the probability that the specified constraint holds, that is,

$$Prob\{X_k - X_j > 0\} = \int_0^{\infty} f_{kj}(\delta) \cdot d\delta ;$$

3. compare the calculated probability with the threshold value, β , and make the decision.

The test outlined above should be performed for each pair of variables, on which a topological constraint of the form given by equation (31) has been imposed. The results of these tests should be used in conjunction with the consistency tests for the metric stochastic constraints described in section 6 to determine the consistency of a network that includes both topological as well as metric constraints.

Applications of the framework for stochastic temporal inference to tasks such as schedule evaluation and planning are currently under investigation.

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