ELASTIC WAVE SCATTERING BY AN INTERFACE CRACK IN LAYERED MATERIALS

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INTRODUCTION

Interfaces play an important role in structural performance of composite materials, which are widely used in many industrial applications. Composite materials are usually made in layered structure, where two adjacent materials are bonded together along their common faces. Therefore, the inspection technique for determining the quality of the bonding interface is of great interest. The elastic wave scattering method for characterization is often used for this purpose[1].

In this study, we focus our attention on the problem of the elastic wave scattering by an interface crack in layered structure. In References [2] and [3], it has been shown that the interface crack between two solids can be characterized by the amplitude spectra of scattered waves. The spectrum analysis has been proved to be a good method for crack size estimation in high frequency range($2k_La > \pi$, $2a$ is crack length). However for a small crack ($k_La < 1$), the spectrum analysis method would become difficult in crack size detection, because of the limitation of the actual transducer bandwidth. In this case, the characteristics of the scattering amplitude in low frequency range would be of more interest. The aim of this paper is to make clear the behavior of the scattering amplitude in low frequency range. In the process the complete multi-scattering in the upper solid layer is considered and the present formulation can be applied to the scattering problem of the layer with any thickness. It is shown here that the amplitude spectra of scattered waves by the interface cracks of different size are different in low frequency range as well as at the resonance point of the layer.

PROBLEM STATEMENT

The layered structure of interest is illustrated in Figure 1. Two solids D and $\bar{D}$ of different elastic properties are bonded together along common faces and immersed in water D_0. The interface B_0 between water and solid D is located at

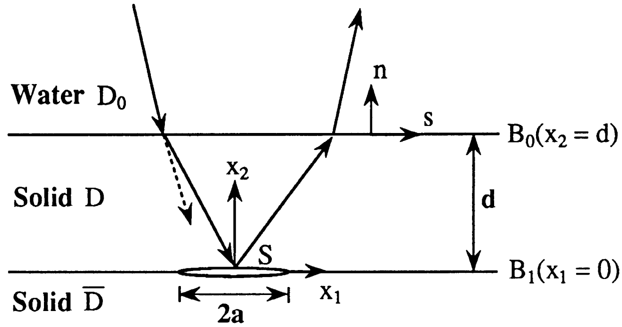

Figure 1. Layered structure with an interface crack S between two solids D and $\bar{D}$.

$x_2 = d$ and the interface B_1 between two solids D and $\bar{D}$ is at $x_2 = 0$. It is assumed that two solids are bonded perfectly at interface B_1 except the debonded part S, which is regarded as an interface crack. When the bonding layer is thin enough, both interface B_1 and debonded area S can be described by spring model as was done in Ref.[2]. The boundary conditions at the interface $B_1(x_2 = 0)$ between two solids are then expressed by

$$\mathbf{t} = -\bar{\mathbf{t}} \ , \qquad \mathbf{t} = \mathbf{K} \cdot (\bar{\mathbf{u}} - \mathbf{u}) \tag{1a, b}$$

where $\mathbf{u}$ and $\mathbf{t}$ are the displacement and traction vectors on the interface B_1 in the side of solid D. The bar denotes the quantities in the side of solid $\bar{D}$. The matrix $\mathbf{K}$ is the spring stiffness constants given by

$$\mathbf{K} = \begin{bmatrix} K_t & 0 \\ 0 & K_n \end{bmatrix} \tag{2}$$

where K_t and K_n are the transverse and normal stiffness of the interface B_1. For perfect bonding, $K_t, K_n \to \infty$ and for complete debonding or traction free crack, $K_t, K_n = 0$.

The boundary conditions of the interface $B_0(x_2 = d)$ between water D_0 and solid D are given by

$$\mathbf{n} \cdot \mathbf{u} = (\rho_F \omega^2)^{-1} \mathbf{n} \cdot \nabla p \ , \qquad \mathbf{n} \cdot \mathbf{t} = -p \ , \qquad \mathbf{s} \cdot \mathbf{t} = 0 \tag{3a, b, c}$$

where p is the wave pressure in water; ρ_F is mass density of water; and ω is angular frequency of incident wave. Vectors $\mathbf{n}$ and $\mathbf{s}$ are normal and tangential unit vectors to the interface B_0.

FREE-FIELD SOLUTIONS

Free-field distribution could be calculated by considering the interface between two solids with no debonded part[4]. The distribution of free waves in three regions are described in Figure 2. For the oblique incidence from water, there is one reflected wave in water, four multi-reflected partial waves in solid D, and two transmitted waves in solid $\bar{D}$. All these free waves are expressed by

$$p^{free} = p_0 e^{ik_F \mathbf{p}^{-F} \cdot \mathbf{x}} + p_0 R^{FF} e^{ik_F \mathbf{p}^{+F} \cdot \mathbf{x}} \qquad in \ \ D_0 \tag{4a}$$

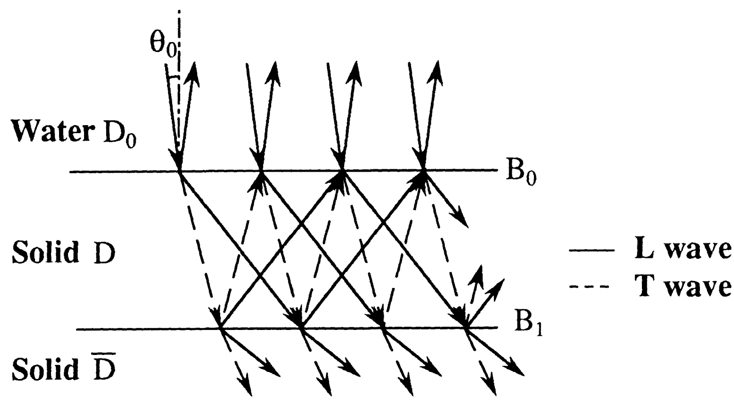

Figure 2. Free-field distributions: reflected and transmitted waves.

$$u_i^{free} = p_0 \sum_{\beta=L,T} R^{-F\beta} d_i^{-\beta} e^{ik_\beta \mathbf{p}^{-\beta}\cdot\mathbf{x}} + p_0 \sum_{\beta=L,T} R^{+F\beta} d_i^{+\beta} e^{ik_\beta \mathbf{p}^{+\beta}\cdot\mathbf{x}} \quad in \;\; D \qquad (4b)$$

$$\bar{u}_i^{free} = p_0 \sum_{\beta=\bar{L},\bar{T}} R^{-F\beta} d_i^{-\beta} e^{ik_\beta \mathbf{p}^{-\beta}\cdot\mathbf{x}} \quad in \;\; \bar{D} \qquad (4c)$$

where p_0 is the amplitude of incident pressure; R^{FF} is the reflection coefficient in water and the symbol F denotes the quantities in water; $R^{\pm F\beta}$ ($\beta = L, T, \bar{L}, \bar{T}$) are the coefficients of transmitted waves in two solids; and $k_\beta = \omega / c_\beta$ is the wave number ($\beta = F, L, T$, or $\bar{L}, \bar{T}$, c_β: phase velocity). Vectors $\mathbf{p}^{\pm\beta}$ and $\mathbf{d}^{\pm\beta}$ are propagation and polarization vectors.

The reflection coefficient R^{FF} and transmission coefficients $R^{\pm F\beta}$ can be determined by substituting Equations (4a,b) into boundary conditions of interface B_0 given in Equations (3a,b,c), and Equations (4b,c) into boundary conditions of interface B_1 in Equations (1a,b). Seven equations are for seven unknown coefficients, so the free-field can be obtained.

NEAR-FIELD SOLUTIONS

The total wave field in each region is the combination of free wave field and scattered field and expressed by

$$p = p^{free} + p^{sc} \quad in \;\; D_0 \qquad (5a)$$

$$u_i = u_i^{free} + u_i^{sc} \quad in \;\; D \qquad (5b)$$

$$\bar{u}_i = \bar{u}_i^{free} + \bar{u}_i^{sc} \quad in \;\; \bar{D} \qquad (5c)$$

where p^{sc}, u_i^{sc} and $\bar{u}_i^{sc}$ are scattered waves in water D_0, solid D and solid $\bar{D}$, respectively. The boundary integral equations for scattered waves in three regions are given by[2,5]

$$\frac{1}{2}p^{sc}(\mathbf{x}) = -\int_{B_0} P(\mathbf{x},\mathbf{y})\{\mathbf{n}(\mathbf{y})\cdot\nabla_y p^{sc}(\mathbf{y})\}ds_y + p.v.\int_{B_0}\{\mathbf{n}(\mathbf{y})\cdot\nabla_y P(\mathbf{x},\mathbf{y})\}p^{sc}(\mathbf{y})ds_y \quad \mathbf{x}\in B_0 \tag{6a}$$

$$\frac{1}{2}u_i^{sc}(\mathbf{x}) = \int_{B_0+B_1+S} U_{ij}(\mathbf{x},\mathbf{y})t_j^{sc}(\mathbf{y})ds_y - p.v.\int_{B_0+B_1+S} T_{ij}(\mathbf{x},\mathbf{y})u_j^{sc}(\mathbf{y})ds_y \quad \mathbf{x}\in B_0\cup B_1\cup S \tag{6b}$$

$$\frac{1}{2}\bar{u}_i^{sc}(\mathbf{x}) = \int_{B_1+S} \bar{U}_{ij}(\mathbf{x},\mathbf{y})\bar{t}_j^{sc}(\mathbf{y})ds_y - p.v.\int_{B_1+S} \bar{T}_{ij}(\mathbf{x},\mathbf{y})\bar{u}_j^{sc}(\mathbf{y})ds_y \quad \mathbf{x}\in B_1\cup S \tag{6c}$$

where P and U_{ij} are fundamental solutions for the liquid and the solid full spaces, respectively.

Substitution of Equations (5a,b,c) into Equations (6a,b,c) and the use of boundary conditions in Equations (1a,b) and (3a,b) lead to the boundary integral equations for the total wave field. Through the discretization of boundary integral equations and using boundary constraint condition in Equation (3c), we can get the system of coupled equations with unknown quantity of p, $\mathbf{u}$ and $\bar{\mathbf{u}}$. The system can be solved for the given free-field. The crack opening displacement $[u_j]$ on the interface crack S can be obtained as

$$[u_j] = u_j - \bar{u}_j\ . \tag{7}$$

FAR-FIELD SOLUTION

The scattered wave pressure in water caused by the crack opening displacement $[u_j]$ on S can be expressed by

$$p^{sc}(\mathbf{x}) = \int_S G_j(\mathbf{x},\mathbf{y})\{u_j(\mathbf{y}) - \bar{u}_j(\mathbf{y})\}ds_y \tag{8}$$

where G_j is the Green's function which represents the scattered pressure at point $\mathbf{x}$ in water caused by a unit discontinuity of displacement component in x_j-direction at a point $\mathbf{y}$ on interface crack S. The boundary conditions to determine the Green's function can be written as

$$G_{ij}^*(\mathbf{x},\mathbf{y}) - \bar{G}_{ij}^*(\mathbf{x},\mathbf{y}) = \delta(x_1-y_1)\delta_{ij}\ ,\ \ T_{ij}^*(\mathbf{x},\mathbf{y}) - \bar{T}_{ij}^*(\mathbf{x},\mathbf{y}) = 0 \ \ on\ x_2=0 \tag{9a, b}$$

$$n_iG_{ij}^* = (\rho_F\omega^2)^{-1}n_iG_{j,i}\ ,\ \ n_iT_{ij}^* = -G_j\ ,\ \ s_iT_{ij}^* = 0\ \ on\ x_2 = d \tag{9c, d, e}$$

where G_j is the pressure and G_{ij}^* is the displacement component in x_i-direction due to the unit displacement source in x_j-direction. The traction kernel $T_{ij}^* = T_{ik}^n(\partial_x)G_{kj}^*$ and the differentiation is $(\cdot)_{,i} = \partial(\cdot)/\partial x_i$. The point $\mathbf{y} = (y_1, 0)$ is the location of the displacement source at interface B_1 (See, Figure 3.). Fourier transform pair is now defined as

$$\tilde{\phi}(k_1, y_2) = \int_{-\infty}^{\infty} \phi(y_1, y_2) e^{-ik_1 y_1} dy_1 \ , \quad \phi(y_1, y_2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{\phi}(k_1, y_2) e^{ik_1 y_1} dk_1 \ . \qquad (10a, b)$$

Then Fourier transforms of boundary conditions in Equations (9a $\sim$ e) become

$$\tilde{G}^*_{ij} - \tilde{\bar{G}}^*_{ij} = \delta_{ij} e^{-ik_1 x_1}, \quad \tilde{T}^*_{ij} - \tilde{\bar{T}}^*_{ij} = 0 \quad on\ x_2 = 0 \qquad (11a, b)$$

$$n_i \tilde{G}^*_{ij} = (\rho_F \omega^2)^{-1} n_i \tilde{G}_{j,i} \ , \quad n_i \tilde{T}^*_{ij} = -\tilde{G}_j \ , \quad s_i \tilde{T}^*_{ij} = 0 \quad on\ x_2 = d \ . \qquad (11c, d, e)$$

The scattered waves in three regions are shown in Figure 4. There are four scattered partial waves in solid D, one in water and two in solid $\bar{D}$. All the scattered waves can be expressed by

$$\tilde{G}_j = A_j^{+F} e^{-ik_F \tilde{\mathbf{p}}^{+F} \cdot \mathbf{x}} \qquad in\ D_0 \qquad (12a)$$

$$\tilde{G}^*_{ij} = \sum_{\beta = L,T} A_j^{+\beta} \tilde{d}_i^{+\beta} e^{-ik_\beta \tilde{\mathbf{p}}^{+\beta} \cdot \mathbf{x}} + \sum_{\beta = L,T} A_j^{-\beta} \tilde{d}_i^{-\beta} e^{-ik_\beta \tilde{\mathbf{p}}^{-\beta} \cdot \mathbf{x}} \qquad in\ D \qquad (12b)$$

$$\tilde{\bar{G}}^*_{ij} = \sum_{\beta = \bar{L},\bar{T}} A_j^{-\beta} \tilde{d}_i^{-\beta} e^{-ik_\beta \tilde{\mathbf{p}}^{-\beta} \cdot \mathbf{x}} \qquad in\ \bar{D} \qquad (12c)$$

where j denotes the component of displacement source which causes the scattering. A_j^α is the amplitude of scattered α-wave($\alpha = F, L, T$ or $\bar{L}, \bar{T}$). $\tilde{\mathbf{p}}^{\pm\beta}$ and $\tilde{\mathbf{d}}^{\pm\beta}$ are unit directional vectors of propagation and polarization of scattered waves. The component $\tilde{p}_1^{\pm\beta}$ should meet the Snell's law with the component of wave vector k_1. So, we have $k_1 = k_F p_1^{+F} = k_\beta p_1^{\pm\beta}$. Substituting Equations (12a,b,c) into Equations (11a $\sim$ e), we can obtain all coefficients A_j^α. The Green's function for the scattered wave in water caused by the point source of the opening displacement at the interface crack S can be obtained by taking the inverse Fourier transform of Equation (12a) as

$$G_j(\mathbf{x}, \mathbf{y}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{G}_j e^{ik_1 y_1} dk_1 = \frac{k_F}{2\pi} \int_{-\infty}^{\infty} A_j^{+F} e^{-ik_F \tilde{\mathbf{p}}^{+F} (\mathbf{x} - \mathbf{e}_1 y_1)} d\zeta \qquad (13)$$

where $\zeta = p_1^{+F}$.

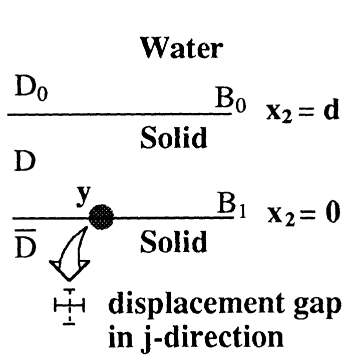

Figure 3. Displacement source on the interface B_1.

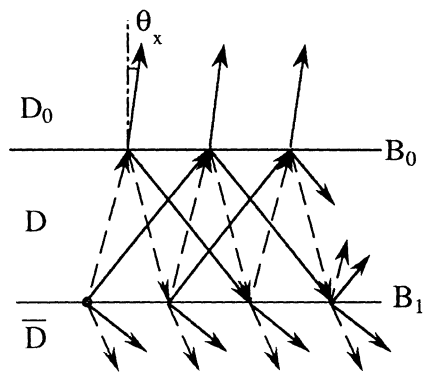

Figure 4. Waves in layered structure.

In the far-field of $k_F|\mathbf{x}| \gg 1$ and $|\mathbf{x}| \gg |\mathbf{y}|(\mathbf{y} \in S)$, by using the steepest descent method[6], the Green's function G_j can be expressed approximately as

$$G_j(\mathbf{x},\mathbf{y}) \approx \sqrt{\frac{1}{2\pi k_F|\mathbf{x}|}} e^{i(k_F|\mathbf{x}|-\frac{\pi}{4})} k_F \tilde{\nu}^F A_j^{+F} e^{ik_F\tilde{p}_1^{+F} y_1}|_{\zeta=\sin\theta_x} \tag{14}$$

where θ_x =arcsin$(-x_1/|\mathbf{x}|)$ is the observation angle. Substitution of Equation (14) into Equation (8) leads to the far-field expression of the scattered field

$$\begin{aligned} p^{sc}(\mathbf{x}) &\approx \sqrt{\frac{1}{2\pi k_F|\mathbf{x}|}} e^{i(k_F|\mathbf{x}|-\frac{\pi}{4})} \int_S k_F \tilde{\nu}^F A_j^{+F} e^{ik_F\tilde{p}_1^{+F} y_1}|_{\zeta=\sin\theta_x}\{u_j(\mathbf{y}) - \bar{u}_j(\mathbf{y})\} ds_y \\ &\equiv \sqrt{\frac{1}{2\pi k_F|\mathbf{x}|}} e^{i(k_F|\mathbf{x}|-\frac{\pi}{4})} \times \Omega_F(k_F,\theta_x) \end{aligned} \tag{15}$$

where Ω_F represents the amplitude of scattered pressure at far-field in water excluding the factor of distance attenuation and is defined by

$$\Omega_F(k_F,\theta_x) = \int_S k_F \tilde{\nu}^F A_j^{+F} e^{ik_F\tilde{p}_1^{+F} y_1}|_{\zeta=\sin\theta_x}\{u_j(\mathbf{y}) - \bar{u}_j(\mathbf{y})\} ds_y\ . \tag{16}$$

NUMERICAL RESULTS

Two kinds of materials, steel and aluminum, are considered here for numerical analysis. The length of interface crack is 2a. The thickness of solid layer D is d. The spring constants of interface between two solids are set to $aK_t/\mu = aK_n/\mu = 10000$ for perfect interface B_1 and $K_t = K_n = 0$ for interface crack S. The interfaces B_0 and B_1 are truncated in a finite length of 10a for near-field calculation. The material constants for calculation are as follows

water:	$c_F = 1480m/s$		$\rho_F = 1000kg/m^3$
steel:	$c_L = 5900m/s$	$c_T = 3200m/s$	$\rho = 7800kg/m^3$
aluminum:	$c_L = 6420m/s$	$c_T = 3040m/s$	$\rho = 2700kg/m^3$

First, to check the present method for layered structure, we let both solids be the same material, steel. Thus, the problem is reduced to the scattering by a crack in a homogeneous solid near liquid-solid interface which has been investigated in Ref.[5]. The results of scattered amplitude in normal direction to the interface for a normal incidence ($\theta_0 = 0$) as a function of ak_F comparing with the results in Ref.[5] are presented in Figure 5. Here, d is the thickness of solid layer D (in Ref.[5], it is the depth of the crack from the interface). It can be seen that they agree well. Figure 6 shows the scattering amplitudes versus d/a as a function of ak_F by the interface crack in the structure of water/steel/aluminum. For the same crack size a, when the distance d between crack and interface B_0 increases, the peak value of the scattering amplitude will decrease and peak position will shift to high frequency. In other words, for a certain thickness d, the larger the crack size, the large the peak of scattering amplitude and the lower frequency point the peak appears at.

Figure 7(a) shows the same results as those in Figure 6 but for different a/d and as a function of dk_L (k_L is the wave number of longitudinal wave in steel). We can see that the peak of scattering amplitude is larger and shifts to lower frequency for the crack of larger size.

All the results in Figures 5 ~ 7(a) discussed above are in the case of low frequency range ($ak_L \approx ak_F/4 < 1$). In more high frequency range, one can image that the resonance phenomenon due to the multi-reflections in solid layer D will occur. The example for the structure of water/steel/aluminum is shown in Figure 7(b). Here, we can see that (1)the resonances appear at the positions near $dk_L = \pi$, (2)the resonance peaks are much larger than those in low frequency range, (3)for the crack of larger size, the resonance peak of scattering amplitude is also larger.

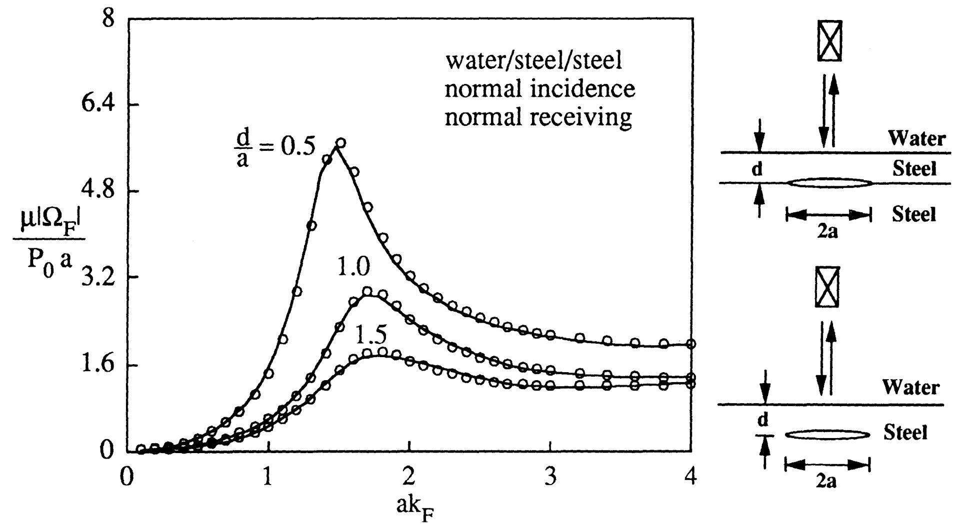

Figure 5. Scattering amplitude versus d/a as a function of ak_F in the structure of water/steel/steel (Solid line: present results, Circle: results in Ref.[5]).

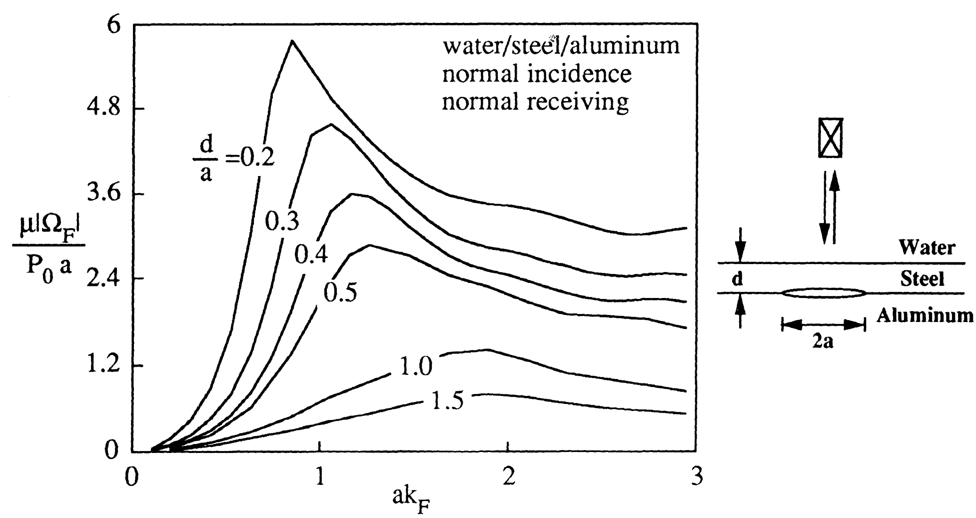

Figure 6. Scattering amplitude versus d/a as a function of ak_F in the structure of water/steel/aluminum.

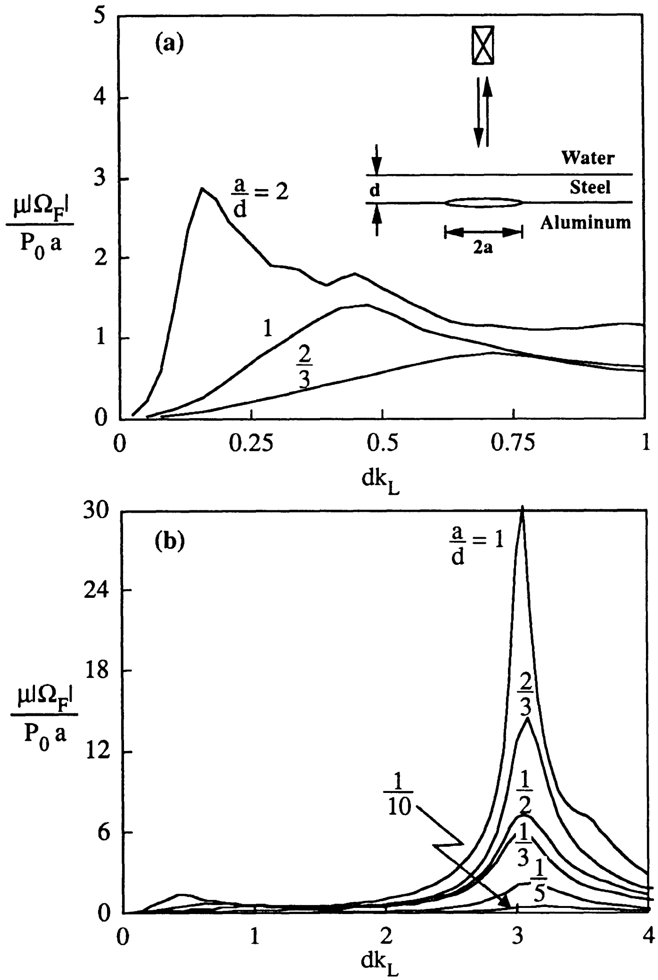

Figure 7. Scattering amplitude versus a/d as a function of dk_L in the structure of water/steel/aluminum ((a)in low frequency range of $dk_L < 1$, (b)in more high frequency range).

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