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ABSTRACT

The reflection coefficient of a Rayleigh wave at a surface crack is measured in both the long wavelength and the short wavelength limits. From the long wavelength measurement, the maximum value of the stress intensity factor is evaluated. Using glass samples, we have formed surface cracks, measured their reflection coefficient, and predicted failure stress with an error of less than 5% from the measured fracture stress. In short wavelength limit, the reflection coefficient is measured versus frequency. Length and depth resonances of the crack are observed and used to estimate the crack geometry with an accuracy of 10-20% for EDM notches in steel.

INTRODUCTION

It is important to characterize surface cracks, so as to be able to determine their size, and effect on the breaking stress of a structure. In this work, we describe two different techniques for characterizing surface cracks. The reflection coefficient of a Rayleigh wave at the crack is measured versus frequency in both the long wavelength and the short wavelength limits. From the long wavelength measurement, the maximum value of the stress intensity factor is calculated, and the fracture stress due to the presence of the surface crack is estimated. For larger cracks, the short wavelength measurement of the reflection coefficient versus frequency is employed. Length and depth resonances are detected by measuring maxima and minima in the reflection coefficient and used to estimate the length and depth of the crack. The fracture stress is then estimated from the knowledge of the exact dimensions of the crack. It is important to note both the long wavelength and short wavelength measurements are done at one angle of incidence only.

STRESS INTENSITY FACTOR MEASUREMENT OF SURFACE CRACKS

In order to understand the principles of the measurement, consider first a sample with a flat elliptical shaped crack present in the interior. When a far field stress is applied with a component σ_{zz} normal to the face of the crack, the local stress σ in the plane of the crack will be of the form

$$\sigma = \frac{K_I(s)}{\sqrt{2\pi r}} = \frac{k_I(s)\sigma_{zz}}{\sqrt{2\pi r}} \quad (1)$$

where r is the distance along the normal from the crack edge and $K_I(s)$ is known as the stress intensity factor at a distance s along the crack edge. The crack will grow, and hence a brittle sample will break at the point where the maximum value of $K_I(s)$ exceeds the fracture toughness K_{Ic} of the material.

It was shown by Budiansky and Rice¹ that by carrying out three measurements of the reflection coefficient of an acoustic wave with a wavelength much larger than the dimensions of the crack, the maximum value of the normalized stress intensity factor (SIF), $k_I(s)$ defined in Eq. (1) could be

determined. They showed that the measured normalized maximum SIF should vary by less than 10% for an elliptic crack where the ratio of major to minor axis varies from 20 to 1 for cracks of the same area.

Our aim has been to check this theory experimentally. For this purpose we decided to work with surface cracks of a roughly semi-elliptical shape, for they are easier to produce. We assumed that the same type of theory would hold for surface cracks as for cracks in the bulk. We decided to carry out the experiments with a brittle material, glass, so that the critical maximum SIF could be checked by a simple fracture test. Finally, we modified the theory to make it appropriate for use with acoustic surface waves, and to simplify the experimental measurement technique as much as possible. This was done by supposing that the crack location and alignment could be found by a high frequency measurement or in our case by locating it by eye. Then, we showed a single measurement of the reflection coefficient of a surface acoustic wave incident normal to the surface of the crack is needed to evaluate the maximum value of the normalized SIF.

The reflection coefficients of a surface acoustic wave is given by the relation²

$$S_{11} = \frac{i\omega}{4} \int_{S_{\text{crack}}} \sigma_{zz} \Delta u_z dS \quad (2)$$

for the crack configuration in Fig. 1, where S_{crack} is the crack area, σ_{zz} is the stress at the crack, before introduction of the crack, associated with an incident surface wave of unit power, and Δu_z is the displacement of crack surface. The parameters Δu_z and σ_{zz} are proportional to the equivalent quantities for an applied static stress when the depth of the crack is much smaller than a wavelength. It has been shown by Budiansky and Rice that for elliptic cracks, the displacement jump integral is also given by the relation^{1,2}

$$\int \Delta u_z dS \approx \frac{1-\nu^2}{3E^2} \sigma_{zz} \frac{\pi^3}{8} k_{I\text{max}}^2 \quad (3)$$

where E is Young's modulus. By using the scattering theory, it can then be shown that the following expression for $k_{I\text{max}}$ of a surface crack can be obtained in terms of the reflection coefficient

of a surface wave,

$$k_{I\max} = \left[\frac{3}{2\pi^5} \frac{V_E^2 \lambda_R^3 Z}{V_S^2 \omega f_z} \frac{|S_{11}|}{[1 - (V_S/V_L)^2]^2} \right]^{1/6} \quad (4)$$

where λ_R is the Rayleigh wavelength, V_S , V_E , and V_L the shear, extensional, and longitudinal wave velocities, respectively, Z the distance of the transducer from the crack, and f_z a normalized quantity ~ 0.5 tabulated by Auld⁷ for Rayleigh waves.^{3,4} We note that $k_{I\max}$ is proportional to $|S_{11}|^{1/6}$. This is because k_I is proportional to the square root of the crack size while S_{11} is proportional to the cube of the crack size. Hence a large experimental error in measuring $|S_{11}|$ results in a much smaller error in $k_{I\max}$.

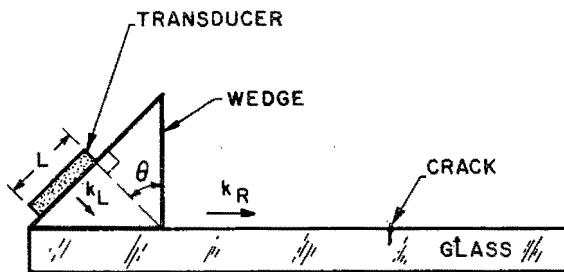


Fig. 1. Schematic diagram of surface crack and test set up.

Pyrex discs 3 mm thick by 7.6 cm in diameter in the annealed condition were prepared for fracture toughness testing by introducing small, semi-elliptical surface cracks in the center of each disc. These pre-cracks were made in a controlled way by applying a bending moment of approximately 2 Newton-meters to the disc, and on the side in tension, pressing a Knoop microhardness indenter quickly into the specimen with a force of 50-100 grams. This technique produces roughly half penny shaped cracks with radii between 100 and 500 micrometers (μm) depending on the combinations of bending moment and force on the Knoop indenter. Such small cracks were required to limit the crack size to less than the acoustic wavelength ~ 9 mm. Additionally, these half penny shaped cracks may be coaxed to an extended semi-elliptical shape by applying additional bending moment to the disc after the initial crack is started.

The strength of each disc in biaxial flexure was determined by the method of Wachtman, et al.^{5,6} This involves supporting the specimen on three equally spaced balls concentric with the load, which are applied to the center of the specimen by

a flat small diameter, flat end piston.

The discs were loaded to fracture at a loading rate of approximately 60 MPa per second using an MTS System 810 servohydraulic testing machine operated in stroke control mode. The loading rate was chosen to minimize possible slow crack growth effects.⁷ To further insure against environmental influences (humidity), the specimens were pumped down in a vacuum for 1 hour prior to strength testing, and subjected to a stream of dry nitrogen gas for 3 minutes prior to and during the flexure test.

Once the specimens were fractured, the geometry of the precracks was studied by examining the fracture surface with a metallograph using reflected light and at a magnification of 50 - 100 X. The aspect ratio (a/c) of the semi-elliptical surface precracks could then be easily measured using an eyepiece with a properly graduated reticle.

The stress intensity factor for the precrack is evaluated by the method of Shaw and Kobayashi.⁸ This analysis was chosen because it not only takes into account the proximity of the back surface to the precrack, but also allows for the presence of linearly varying stress field along the glass width. The expression for the maximum stress intensity is

$$K_I = \frac{M_B \sigma_B \sqrt{\pi a}}{E(k)} \quad (5)$$

where $E(k)$ is an elliptic integral of the second kind, M_B is a magnification factor which takes into account the aspect ratio and the proximity of the crack depth to the neutral axis, σ_B is the maximum bending stress at the surface of the specimen, and a is the length of the semi-minor axis (depth) of the precrack.

The ten samples tested gave a normally distributed set of fracture toughness measurement, and the mean value of fracture toughness, K_{IC} , at a confidence level of 90% is: $0.76 \pm 0.04 \text{ MPa m}^{1/2}$. This compares very well to Wiederhorn's experimental results of $0.75 \pm 0.010 \text{ MPa m}^{1/2}$ for 6 pyrex specimens and $0.778 \pm 0.011 \text{ MPa m}^{1/2}$ for 8 specimens.⁹

The experimental set-up for measuring the acoustic surface wave reflection coefficient $|S_{11}|$ is shown schematically in Fig. 1. A wide band, high efficiency wedge transducer¹⁰ is used to excite and detect the surface acoustic waves. The transducer has a center frequency of 3.4 MHz, a bandwidth of 50% and a one-way insertion loss of 9.2 dB. Calibration of the transducers was carried out by measuring the transmission between a pair of transducers, as described in reference 10. All measurements were taken in the far field of the transducer ($Z > w^2/\lambda_R$).

A set of acoustic measurements were taken with the front of the wedge a distance $Z = 2.3$ cm from the crack and with the acoustic surface wave normal to the crack surface. The measurements were taken in the bending jig, using a bending moment of approximately 2 Newtons-meters to ensure the crack was open.

A series of reflection measurements on 10 different cracks was made, and $k_{I\max}$ calculated from Eq. (4).

In Fig. 2 the acoustic prediction of σ_c is plotted against the mechanically measured value of σ_c for 10 specimens. A least squares fit of this data indicates a linear relationship between theoretical and experimental measurements with a slope of 0.85. We see that for larger, more irregular cracks, where the assumption of the theory that the diameter of the crack is less than 1/4 wavelength is not well satisfied, the error between theory and experiment increases. It is interesting to note that the effect of the ellipticity of some of the small surface precracks appears not to affect the accuracy of the acoustic prediction of k_{Imax} to any appreciable degree. This confirms the prediction of Budiansky and Rice¹ that effect, if any, should be less than 10%.

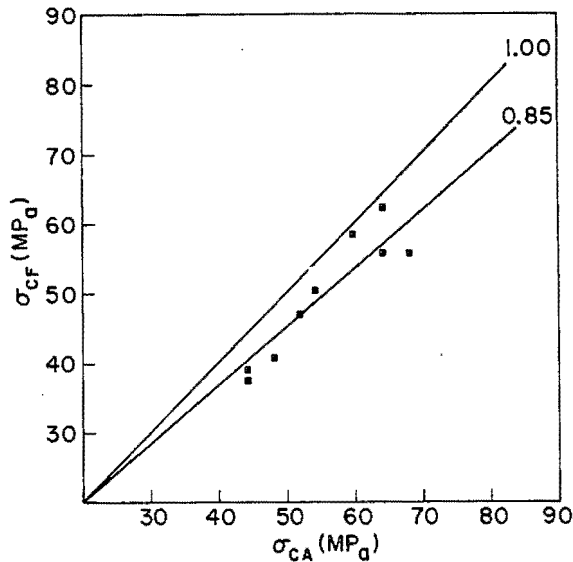


Fig. 2. Actual fracture stress versus predicted fracture stress.

LENGTH AND DEPTH MEASUREMENT OF SURFACE CRACKS

Several acoustic techniques for the purpose of determining crack size have been reported in the literature. For instance, Tittman et. al.¹¹ have measured the length of a crack accurately, by determining the scattering of a Rayleigh wave as a function of angle. As this function has a sinc θ dependence the length of the crack can be estimated from the distance between the nulls. The problems with this technique are that it assumes that space is available to carry out the angular scattering measurement, and gives no information about the depth of the crack. In another example, Lidington, et al.¹² discuss several techniques for determining the depth of a crack several wavelengths deep, but do not determine its length.

In our work, we measure the reflection of the crack versus frequency at normal incidence only. When a surface acoustic wave excites a crack, of the type shown in Fig. 3, its edges act as sources that excite surface waves which propagate along the crack surface. We postulate that when the crack is a multiple of a half wavelength long (the x direction) it acts as a resonator and tends to

absorb energy, whereas when it is $(2n+1)\lambda/4$ long, where λ is the wavelength, it exhibits very little displacement of its top edge and so there is a maximum in the back scattered reflected signal. Similarly, when the depth (y direction) of the crack is a multiple of a half wavelength long the top surface displacement of the crack tend to be very small, as it is at its tip, and so there is again a maximum in the reflection coefficient of a wave incident on the crack. Thus, even though it is difficult to calculate end effects, we might expect that we could estimate the crack length a and depth b from the formula

$$a = V_x / 2\Delta f_x \quad (6)$$

$$b = V_y / 2f_y \quad (7)$$

where V_x , V_y are the surface wave velocities in the x and y directions on the crack surface and Δf_x is the frequency separation between the reflection maxima, while f_y is the frequency for a reflection maximum when the depth is one half wavelength.

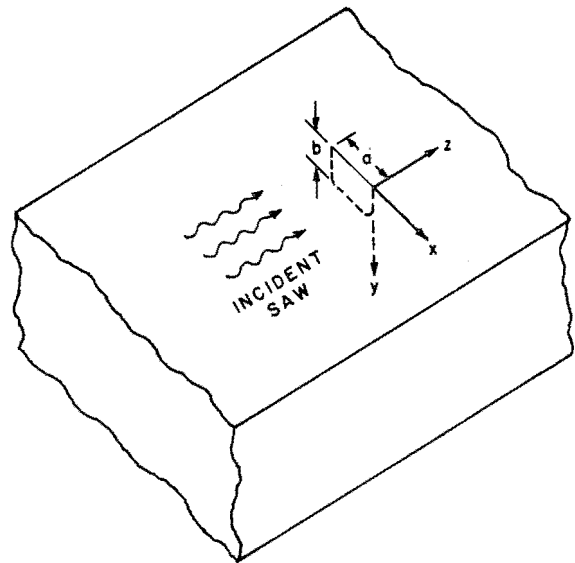


Fig. 3. Schematic diagram of crack configuration.

We carried out our experiments on 6 EDM notches of known length and depth in a steel sample.¹³ We use a highly efficient wide bandwidth wedge transducer to excite the SAWs. The transducer had a center frequency of 5 MHz and a 3 dB fractional bandwidth of 50%.¹⁰ The reflection coefficients of the notches versus frequency are shown in Figs. 4 and 5. The length resonances and a half wavelength depth resonances can be clearly seen. We use this data to estimate the lengths and depths of the notches. It will be noted that the depth resonances are typically stronger than the length resonances because the excitation of the depth resonance is uniform over the length of the crack. The results obtained are very similar in nature to those for volume wave backscattering from a penny shaped crack.¹⁴

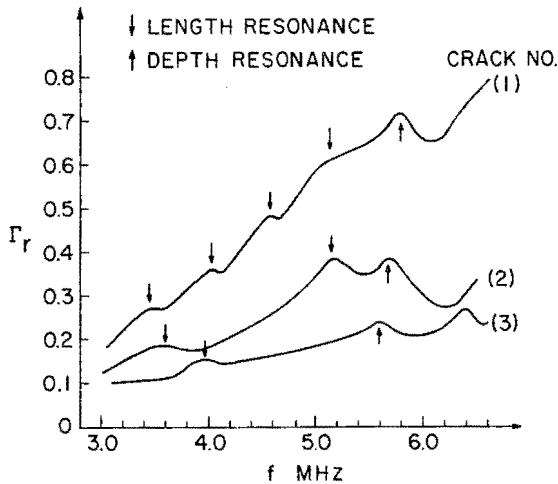


Fig. 4. Normalized reflection coefficient versus frequency of cracks 1, 2, 3.

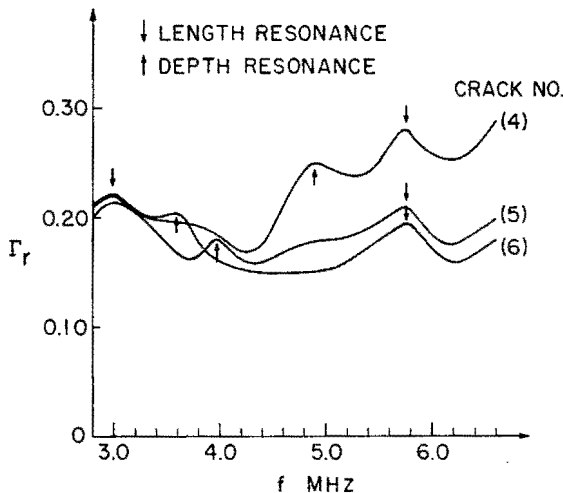


Fig. 5. Normalized reflection coefficient versus frequency of cracks 4, 5, 6.

In making the estimates of length, it is important to use a reasonable approximation for the effective surface wave velocities along the crack surface in the x and y directions. As far as propagation in the x direction is concerned, we can regard the Rayleigh wave that is excited in the corner region as being a wave propagated along the corner region bounded by the crack surface and substrate surface. It has been shown by Lagasse et al.¹⁵ that such guided modes propagate at a velocity just slightly less than the Rayleigh velocity; so we have taken $V_x = V_R$ to calculate the length resonances and used a measured value of $V_R = 3.02 \cdot 10^5$ cm/sec. On the other hand for the depth resonances, we regard the wave as propagating in a surface waveguide of width a, for the edges of the crack are held rigidly. Thus, for the lowest order mode, we take the propagation constant k_y in the y direction to be given by the formula

$$k_y^2 = \frac{4\pi^2 f_y^2}{V_R^2} - \frac{\pi^2}{a^2} \quad (8)$$

for a half wavelength resonance $k_y b = \pi$ and as is given by Eq. (6). Hence we may write Eq. (7) in the form

$$b = (4f_y^2/V_R^2 - 1/a^2)^{-1/2} \quad (9a)$$

or

$$b = (V_R/2) [f_y^2 - (\Delta f_x^2)]^{-1/2} \quad (9b)$$

The estimated sizes from Eqs. (6) and (7) with $V_y = V_R$ and Eq. (9) are the estimated sizes compared to the actual sizes and the results are summarized in Table I. We have used Eq. (9b) for a corrected estimate of depth, and in brackets given results based on using the correct value of a in Eq. (9a). It will be seen that because the length resonance estimate is somewhat in error, the use of the correct value of a can give considerably better estimates of the depth of the crack. The maximum error will be seen to be of the order of 18% and the results are often considerably better than this.

It is worthwhile to note that such resonances should be obtained where the angle of incidence is not normal to the crack as the same phenomena takes place. Hence only one angle of incidence on a crack need be available for an accurate crack size estimation. Furthermore as the basic technique is one by which resonances are determined, we might expect to be able to generalize it to determine resonances of odd shaped cracks, and estimate their major dimensions. Working with B. A. Auld we are developing a theory to predict the nature of the scattering which we hope will provide further insight into the phenomenon.

ACKNOWLEDGEMENTS

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TABLE I. Estimate of Length and Depth

Crack #	Actual Length a (mm)	Frequency Shift Δf_x MHz	Estimated Length a - Eq.(6)	Error	Actual Depth b (mm)	Frequency f_y for Maximum Reflection	Estimated Depth Eq. (7) $v_x = v_R$	Corrected Estimate Depth Eq. (9)	Error in Estimated Depth %
1	2.34	0.56	2.69	+15	0.228	5.8	0.26	0.261 (0.261)	+14 (+14)
2	1.04	1.54	0.98	- 6	0.254	5.7	0.265	0.275 (0.274)	+8.3 (+8.3)
3	0.64	2.44	0.619	-3.3	0.254	5.6	0.269	0.296 (0.30)	+16.5 (+17)
4	0.66	2.8	0.53	-18	0.33	4.9	0.308	0.378 (0.348)	14.7 (5.8)
5	0.66	2.8	0.53	-18	0.347	4.0	0.378	0.529 (0.668)	15.7 (+1)
6	0.66	2.8	0.53	-18	0.587	3.6	0.419	0.668 (0.54)	+11 (+2)

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DISCUSSION

- M. Srinivasan (Carborundum): You discussed the results on glass. Do you expect imaging in ceramics to be more difficult? It can be very difficult with ceramics.
- B. T. Khuri-Yakub (Stanford): We believe that this work could be directed to ceramics and we are in the process of setting up experiments to try these with Tony Evans.