CHARACTERIZATION OF SURFACE WAVE SCATTERING BY SURFACE BREAKING CRACKS

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ABSTRACT

Various analytical methods based on the real reciprocity relation are applied to the problem of Rayleigh wave scattering by a surface crack. In one formulation, the reflection coefficient observed at the transducer terminals is expressed in terms of an integral over the crack surface of the product of the perturbed and unperturbed fields. This integral is then converted to a volume integral and the Born Approximation is applied. In the other formulation a "Kirchoff" type approach is used such that the effect of the crack is expressed as an equivalent body force distribution. That force distribution is then approximated and normal mode techniques are used to find the scattered field amplitude. The two methods are compared with each other and with the results obtained from geometrical diffraction theory. An experimental procedure is also proposed for the inverse problem.

INTRODUCTION

New experimental results have been obtained on the surface scattering of Rayleigh waves in the medium and short wavelength regimes during the past year, especially by Khuri-Yakub and co-workers at Stanford, by Tittman at Rockwell, and Quentin and co-workers at the University of Paris. The theoretical calculations we present here apply in the same regime and show agreement as far as the angular scattering from the surface crack is concerned. In the frequency response, the theory does not predict the resonances observed experimentally, but agrees well with the general behavior of the scattering.

We basically used two approaches to the problem. In the first one we used Auld's reciprocity relation[1] in volume integral form and applied the Born Approximation. In the second one, we replaced the crack by approximately equivalent body sources and found the scattered field using normal mode theory. We called this approach a "Kirchoff" method due to the fact that we calculate the scattering from a set of secondary sources that do not satisfy the boundary conditions everywhere.

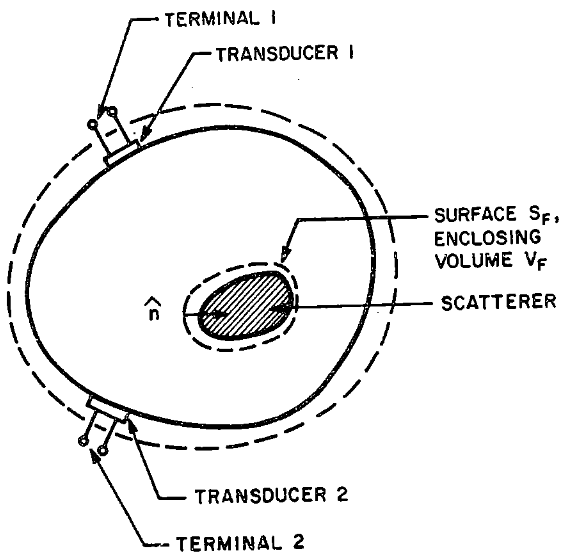

Fig. 1. General scattering geometry

Born Approximation — Consider the general scattering geometry shown in Fig. 1 . Starting from the piezoelectric reciprocity relation in a source free medium

$$\nabla \cdot \left\{ \underline{V_1} \cdot \underline{\underline{T_2'}} - \underline{V_2'} \cdot \underline{\underline{T_1}} + \underline{E_1} \times \underline{H_2'} - \underline{E_2'} \times \underline{H_1} \right\} = 0 \qquad (1)$$

It can be shown that[1]

$$\delta\Gamma_{21} = \frac{1}{4P} \int_{S_F} \left\{ \underline{V_1} \cdot \underline{\underline{T_2'}} - \underline{V_2'} \cdot \underline{\underline{T_1}} \right\} \cdot \hat{n}\, dS \qquad (2)$$

where the field quantities with subscript 1 are those excited by incident power P at terminal 1 in the absence of the crack and those with subscript 2 are excited with an incident power P at terminal 2 in the presence of the crack. Here, $\delta\Gamma_{21}$ is the change in the transmission coefficient from terminal 1 to terminal 2 due to the presence of the crack. We use primes with subscript 2 to emphasize the presence of the crack.

Take the acoustic field equations

$$\nabla \cdot \underline{\underline{T_2'}} = i\omega\rho_2 \underline{V_2'} \qquad (3)$$

$$\nabla_s \underline{V_2'} = i\omega \underline{\underline{s_2}} : \underline{\underline{T_2'}} \qquad (4)$$

$$\nabla \cdot \underline{\underline{T_1}} = i\omega\rho_1 \underline{V_1} \qquad (5)$$

$$\nabla_s \underline{V_1} = i\omega \underline{\underline{s_1}} : \underline{\underline{T_1}} \qquad (6)$$

Multiplying (3), (4), (5), (6) by

$$-\underline{V_1},\ \underline{\underline{T_1}},\ \underline{V_2'},\ -\underline{\underline{T_2'}}$$

respectively, adding them up, using the tensor identity

$$\nabla \cdot (\underline{V} \cdot \underline{\underline{T}}) = \underline{V} \cdot \nabla \cdot \underline{\underline{T}} + \underline{\underline{T}} : \nabla_s \underline{V}$$

and the constitutive relations

$$\underline{\underline{T_2}} = \underline{\underline{c_2}} : \underline{\underline{S_2}} \;, \quad \underline{\underline{T_1}} = \underline{\underline{c_1}} : \underline{\underline{S_1}}$$

one gets

$$\nabla \cdot (\underline{V_2'} \cdot \underline{\underline{T_1}} - \underline{V_1} \cdot \underline{\underline{T_2'}})$$
$$= i\omega(\rho_1 - \rho_2)\underline{V_2} \cdot \underline{V_1} + i\; \underline{\underline{S_2}}:(\underline{\underline{c_1}} - \underline{\underline{c_2}}): \underline{\underline{S_1}} \qquad (7)$$

Taking the volume integral of both sides, applying the divergence theorem and comparing with Eq. (2), we obtain finally

$$\delta\Gamma_{21} = \frac{i\omega}{4P}\int_{V_F}\left\{(\rho_1 - \rho_2)\underline{V_2'} \cdot \underline{V_1} + \underline{\underline{S_2'}}:(\underline{\underline{c_1}} - \underline{\underline{c_2}}):\underline{\underline{S_1}}\right\} dV \qquad (8)$$

For an air filled crack, this result reduces to

$$\delta\Gamma_{21} = \frac{i\omega}{4P}\int_{V_F}\left\{\rho\; \underline{V_2'} \cdot \underline{V_1} + \underline{\underline{S_2'}}:\underline{\underline{c}}:\underline{\underline{S_1}}\right\} dV \qquad (9)$$

where ρ is the density and $\underline{\underline{c}}$ is the stiffness matrix of the surrounding medium.

Now we make the key assumption that $\underline{V_2'}$ and $\underline{\underline{S_2'}}$ are the fields excited with an incident power P at terminal 2 in the absence of the crack (Born Approximation). In other words, we assume that the disturbance of the fields in the void region is negligible. Therefore, from here on, we will drop the primes in the field quantities with subscript 2 to indicate that they are unperturbed fields; i.e.,

$$\delta\Gamma_{21} = \frac{i\omega}{P}\int_{V_F}(\rho\; \underline{V_2} \cdot \underline{V_1} + \underline{\underline{S_2}}:\underline{\underline{c}}:\underline{\underline{S_1}})\; dV \qquad (10)$$

This is a very drastic assumption, but has been found to produce useful results for volume voids and cracks.[2] Throughout the calculations, we will assume that

i) The crack is in the far field of the transducer, therefore, for constant power input and constant efficiency of the transducer with frequency, the fields vary proportionally to ω. There is an $\omega^{1/2}$ factor coming from the transducer itself[3] and another $\omega^{1/2}$ factor due to diffraction.

ii) The crack is small enough that, in the vicinity of the crack, the plane wave approximation to the fields is possible in the frequency range of interest.

We have investigated two types of geometries. The first one is the single transducer system Fig. 2) where only one transducer moved around a circular path, is used for both transmission and reception. In Eq. (10) this can be simulated by changing the subscript 2 to 1 everywhere. The other is the double transducer system (Figure 3) where the receiving transducer stays fixed in the direction normal to the crack and the transmitting transducer is moved around a circular path. By angular scattering, we mean the variation of transmission (or reflection) coefficient (normalized with respect to the value for $\theta = 0$) with the angle θ defined in Figs. 2 and 3. By frequency response, we mean the variation of the (arbitrarily normalized) reflection coefficient for normal incidence ($\theta = 0$) with frequency. The arbitrary normalization is due to the fact that we cannot evaluate P in Eq. (10) without the knowledge of the transducer used in an actual experiment.

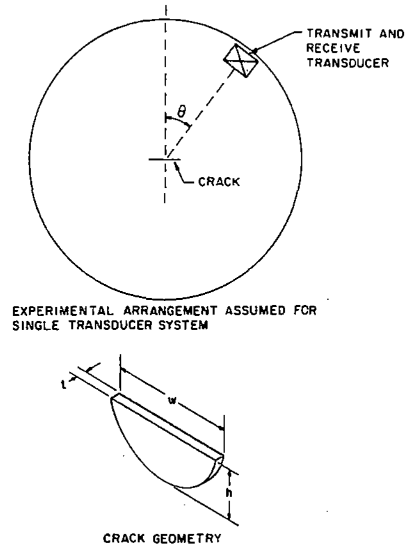

Fig. 2. Single transducer system and crack geometry

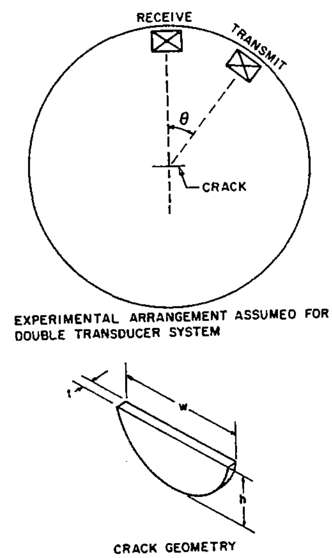

Fig. 3. Double transducer system and crack geometry

Throughout the analysis, we assume that the crack is elliptic in cross section and uniform in thickness.

"Kirchoff" Approach — Consider an electromagnetic problem in which a set of sources are radiating in the presence of a conductive obstacle (Fig. 4a). Using the induction theorem,[4] one can replace the incident field by equivalent surface currents in the presence of the obstacle. The problem is still as hard as the original problem, but for some geometries, one can use image theory to simplify the problem. Using the same idea we can say that the effect of the incident ultrasonic field on a void can be represented by a body force layer with the void present (Fig. 4b). In what we call "Kirchoff" approach, we neglect the effect of "images" to be considered and replace the crack with a body force layer which cancels the effect of normal traction of the incident field (Fig. 5).

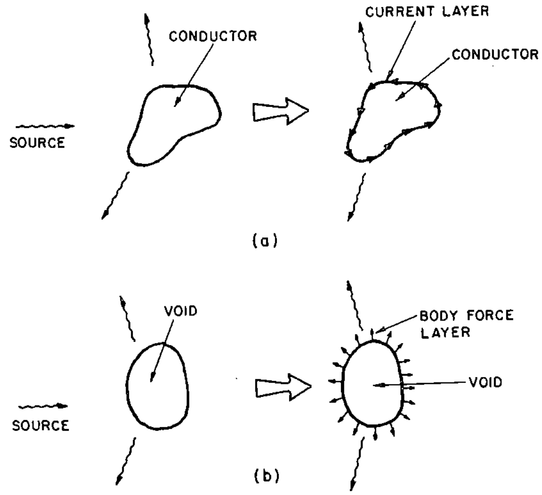

Fig. 4. Induction theorem (a) in electromagnetics, (b) in ultrasonics.

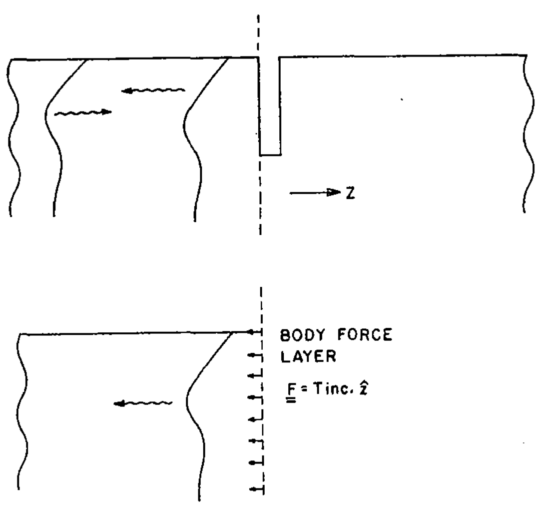

Fig. 5. "Kirchoff" approach to the scattering problem.

For the general geometry shown in Fig. 6 , defining the Rayleigh wave field modes, as

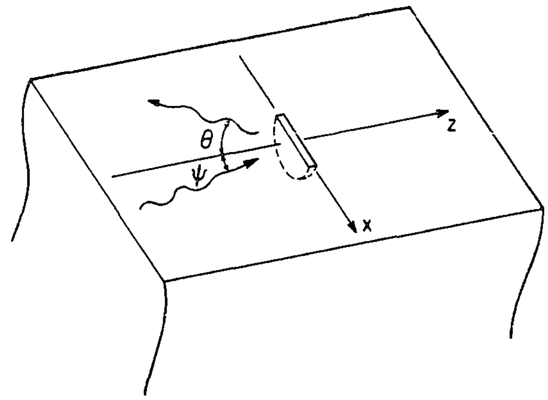

Fig. 6. Coordinate system used in defining Rayleigh wave modes.

$$\underline{V}_{\mu} = \underline{V}_{\mu}(x,y)e^{-i\mu z} \qquad (11)$$

$$\underline{\underline{T}}_{\mu} = \underline{\underline{T}}_{\mu}(x,y)e^{-i\mu z} \qquad (12)$$

where

$$\mu = \beta \cos \psi \qquad (13)$$

and using mode expansion techniques,[3] it can be shown that

$$a(\theta) \propto \frac{1}{\lambda}\int_{S_F} (V \cdot F)\, dxdy \qquad (14)$$

where

F is the body force distribution over the crack surface in Figure 5,
V is the modal field propagating in the θ direction,
θ is the angle between the propagation vector of the scattered wave component and normal vector to the crack,
a(θ) is the amplitude of the scattered field in the θ direction.

Numerical Methods — The integrals in Eqs. (10) and (14) were first converted to a single integral along the depth of the crack. That integral was then evaluated using 16 point Gauss' Integration Formula.[5] The calculations were carried out and plotted using an HP 9820 calculator. The results were also checked for accuracy with those calculated using the IBM 370 computer in double precision mode.

Comparison of Methods — For deep cracks ($h > \lambda$) where the width is in the order of several wavelengths, one can form a model based on geometrical and optical diffraction theory[6,7] and can deduce that the backscatter pattern for a single transducer system goes as $\sin x/x$ where

$$x = 2\pi \frac{W}{\lambda} \sin \theta$$

and the transmission pattern for double transducer goes as $\sin x/x$ where

$$x = \pi \frac{W}{\lambda} \sin \theta \quad .$$

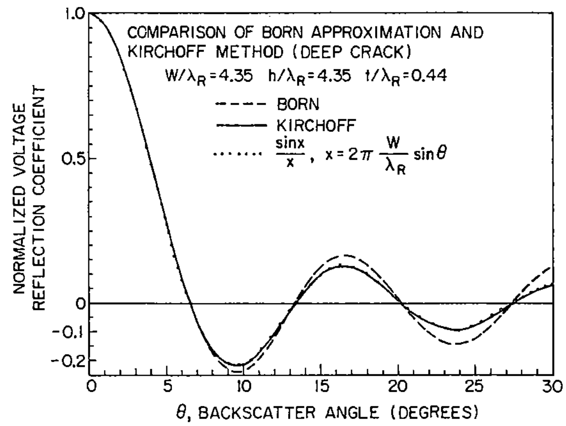

Fig. 7. Comparison of Born, Kirchoff and the Geometrical Diffraction methods for a deep crack.

In Fig. 7 we compare our results for Born, Kirchoff, and the Geometrical Diffraction methods. The crack is fairly deep and single transducer geometry is used. All three methods agree reasonably well. From the position of the nulls, one can determine the width of a deep crack. In Fig. 8 the same comparison is made for a shallow crack. We observe that the nulls of the patterns shift for a shallow crack. Note that the amplitude level for Kirchoff approximation is almost the same as that for $\sin x/x$ but the amplitude for Born is always greater. Experimentally, the reflection coefficient amplitude is very much larger than $\sin x/x$. For this reason we consider that the Born Approximation gives more realistic results than Kirchoff.

In Figs. 9 and 10 we show the shift of null for another crack with two different thicknesses. As is easily seen, the thickness change does not cause a null shift. In Fig. 11 we plot the effect of thickness change more explicitly. The only effect is the increase in relative amplitudes.

For the double transducer system, we observe the same behavior except that the position of the first null occurs at the position of the second null for the single transducer system. In Fig. 12 we show the null shift for the double transducer case.

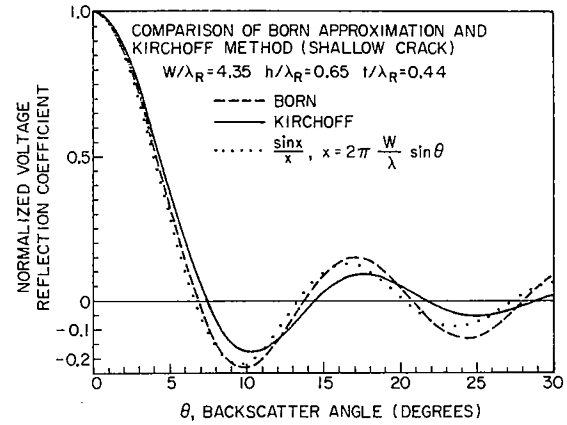

Fig. 8. Comparison of Born and Kirchoff methods for a shallow crack.

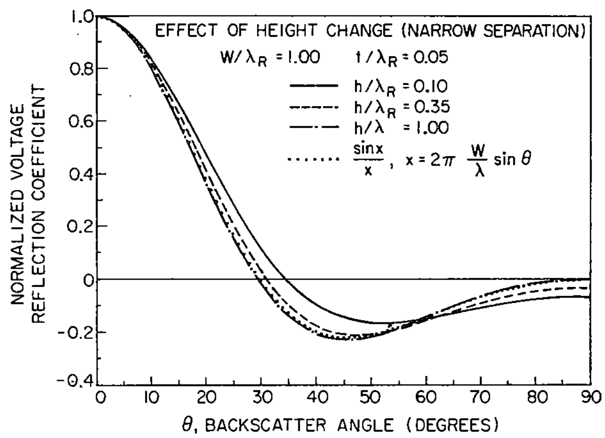

Fig. 9. Effect of height change for a thin crack.

In Figs. 13 and 14 we show the frequency response (both curves normalized with the same value estimated from experimental results) at normal incidence for some EDM notches[8] tested by Khuri-Yakub and co-workers[9] at Stanford. Although we do not obtain the resonances observed experimentally, the theoretical calculations predict reasonably well the background variation of scattering with frequency.

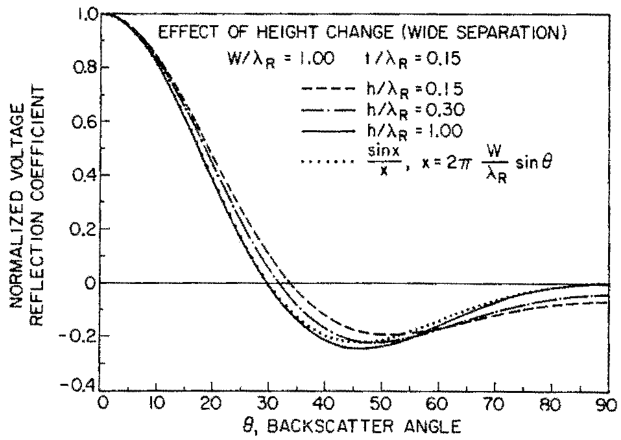

Fig. 10. Effect of height change for a thick crack (Born Approximation).

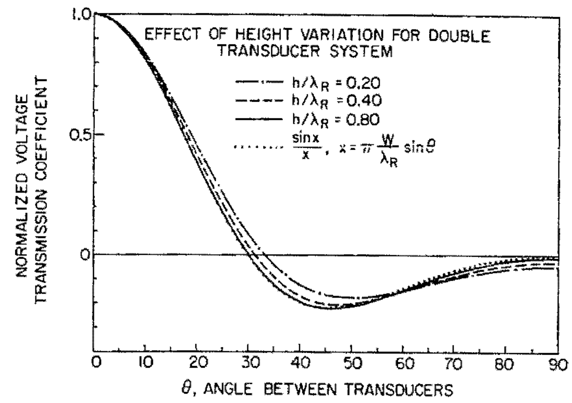

Fig. 12. Double transducer case, $w/\lambda_R = 2.00$.

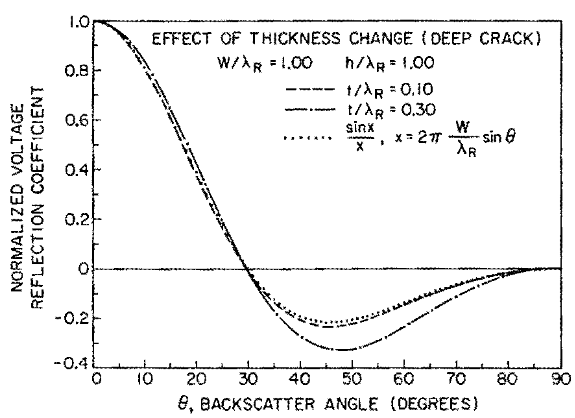

Fig. 11. Effect of thickness change (Born Approximation).

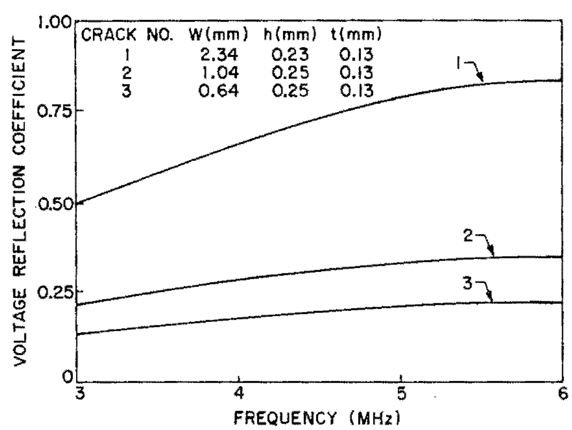

Fig. 13. Frequency response of cracks 1, 2, 3.[8,9]

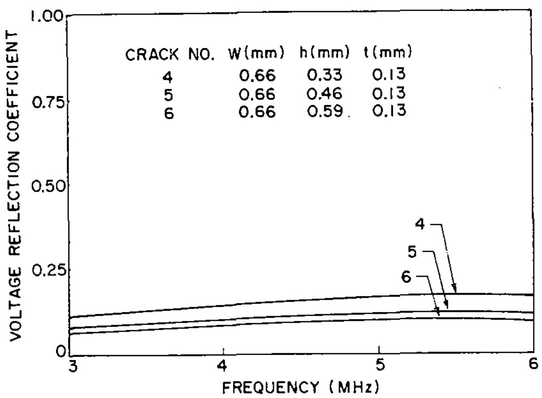

Fig. 14. Frequency response of cracks 4, 5, 6.[6,9]

Inverse Problem — The shift of the nulls with crack depth change suggests the following procedure for predicting the depth. Since at high frequencies the nulls are the same as those for sinx/x pattern, the first aim is to find a "high" frequency, f_h, where the crack is effectively deep. This can be achieved by measuring the position of a specified null (say the m-th null) at different frequencies and observing its convergence towards that of the sinx/x pattern. Once that high frequency is found, one can determine the width of the crack using the formula

$$\frac{w}{V_R} = \frac{m}{2 \sin \theta \cdot f_h} \tag{15}$$

where V_R is the Rayleigh wave velocity.

Then at a low frequency, f_L, one can compare the position of the null with the one obtained from

$$\sin \theta = \frac{m}{2(w/V_R)f_L} \tag{16}$$

and if a shift is observed, one can determine the height using a look-up table. If no shift is observed at the lowest frequency where a null is observed, the conclusion is that the depth is greater than ~3/4 λ at that frequency. The accuracy of the method depends on the choice of the frequency f_h. If it is chosen greater than necessary, the nulls occur with very close spacing in θ and the error in determining the width increases.

Conclusion — In this paper, we investigated the Rayleigh wave scattering from surface cracks using two approximate methods. Although we put emphasis on the Born Approximation in this paper, we are not sure which method gives better results on the shift of nulls with effective depth change. Owing to the fact that we did not have enough experimental data, we still are unable to favor one method over another.

The deficiency of the methods is that they do not predict the resonances observed experimentally in frequency response. We are now trying to obtain a variational expression for the form factor of the scattering pattern to be able to see the resonances in frequency response.

ACKNOWLEDGMENT

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DISCUSSION

Walter Kohn (University of California, San Diego): Could you please clarify one point for me? On the one hand you explain that you are referring to earlier remarks to the effect that, for the crack, the born approximation doesn't really give anything. On the other hand, you then show the results. Could you clarify this situation?

B.A. Auld (Stanford University): What I'm saying, Walter, when we're talking about born here, we're talking about EDM slots. They have a finite effect.

Walter Kohn: However, you seem to get good results even for a very small thickness. Is that because you just show relative values?

B.A. Auld: Yes. We are looking at the variation and we have not made a comparison with the actual amplitude of the return. We haven't made a comparison with the actual amplitude of the return because, in fact, I don't know that we have enough experimental data yet. We would like to look at that.

Walter Kohn: So, only when the thing becomes very thin, then the method fails.

B. A. Auld: Something terrible happens, yes, I'm sure.

Gordon Kino (Stanford University): I think there is a form of the born approximation which can be used for cracks which is fair. With an infinitesimally thin crack, you might turn to a surface integral rather than a volume integral and assume that the displacement on the surface is the unperturbed displacement. I believe you get results out of it that are okay.

B. A. Auld: Yes, I agree with you.

Gordon Kino: It comes down, in fact, to exactly what you have said. You then turn it back into a volume integral. Further, I would like to ask about variational principles. I think there have already been variational principles derived in this field by the reciprocity theorem which would work on this.

B. A. Auld: Yes, that is true and we do want to look at that. In fact, I think you did.