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Non-linear Inverse Analysis of Transportation Structures Using Neuro-adaptive Networks with Hybrid Learning Algorithm

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ABSTRACT

The load-bearing capacity of pavement structures is a fundamental structural performance metric of transportation infrastructure networks in the context of safe and efficient movement of people and goods from one place to another. Non-destructive test (NDT) methods are typically employed to routinely evaluate the structural condition of pavement structures, their lifespan and the appropriate maintenance activities to be carried out. This involves computing the Young's modulus of each layer of the pavement structure through inverse analysis of acquired NDT data. Over the past two decades, soft computing techniques such as Artificial Neural Networks (ANNs), Genetic Algorithms (GAs), and Fuzzy Logic Approach (FLA) have been applied in numerous civil engineering fields for pattern recognition, function approximation, etc. This paper proposes the use of an Adaptive-Network-based Fuzzy Inference System (ANFIS) combined with Finite Element Modeling (FEM) for inverse analysis of multi-layered flexible pavement structures subjected to dynamic loading. Using the proposed approach, it will be possible for pavement engineers to characterize the non-linear, stress-dependent modulus of the pavement layers based on the NDT data in real time, identify the pavement defects, and better determine the appropriate rehabilitation strategy.

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INTRODUCTION

The United States has 4 million miles of roadways that have been constructed, rehabilitated, and maintained over the previous century, and they represent a huge national investment that has provided a safe and comfortable means of transportation for both private and commercial vehicles. Since pavement structures wear down and deteriorate under heavy axle loadings and environmental influences, they need to be maintained and rehabilitated on a regular basis. This requires a very significant commitment of resources on the part of nation's highway agencies at the State, Federal and local levels. For instance, total highway expenditure by all units of government in 2000 was $126.7 billion, a 203 percent increase compared to 1980 (average annual increase of 10 percent) (NCHRP 2004). The sheer magnitude of annual expenditures on highway maintenance justifies the application of best available test procedures and technologies to optimize the use of highway funds.

Various NDT methods have been developed to routinely assess the existing pavement structural condition and subsequently identify the necessary corrective actions. Among them, the Falling Weight Deflectometer (FWD) is the most commonly used NDT device for evaluating the structural state of pavements. A picture of FWD equipment used by the Iowa Department of Transportation (DOT) is shown in Figure 1. The FWD can either be mounted in a vehicle or on a trailer and is equipped with a weight and several velocity transducer sensors. To perform a test, the vehicle is stopped and the loading plate (weight) is positioned over the desired location. The sensors are then lowered to the pavement surface and the weight is dropped. The advantage of an impact load response measuring device over a steady state deflection measuring device is that it is quicker, the impact load can be easily varied, and it more accurately simulates the transient loading of moving traffic. Sensors located at specific radial distances monitor the deflection history. The deflections measured at radial distances away from the load form the deflection basin. In order to calculate the pavement structural capacity accurately, the deflection basins should be measured and analyzed accurately. Although there are numerous methods for evaluating the structural capacity of pavements from deflection basin data, there is no standard or universally accepted procedure that presently exists (PCS/Law Engineering 1993).

Figure 1. Falling Weight Deflectometer (FWD) Equipment Used for Non-Destructive Testing of Pavements

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Inverse or back analysis is used to determine the Young's modulus of pavement layers based on measured deflection data. In this process, more commonly referred to as backcalculation, a numerical optimization method is employed so that the measured deflection basin agrees with the deflections given by a numerical model. The optimization process is an iterative method which modifies the elastic modulus of the pavement layers until a better adjustment is produced. Moreover, the optimization process can be carried out by employing an algorithm of parameter identification like non-linear least-squares algorithm, research in a database, or soft computing methods such as Artificial Neural Networks (ANNs), Genetic Algorithms (GAs), etc. Especially, in recent years, ANNs have been shown to be capable of predicting the pavement layer moduli using the FWD field deflection measurements (Meier and Rix, 1995, Ceylan et al. 2007).

The objective of this paper is to investigate the feasibility of using Adaptive-Network-based Fuzzy Inference System (ANFIS) for inverse analysis of multi-layered flexible pavement structures based on FWD data. In this approach, a Finite Element (FE) model is employed to envisage the response of the pavement to FWD load with known characteristics of pavement materials. The FE model captures the non-linear, stress-dependent behavior of geo-materials used in the underlying unbound pavement layers resulting in realistic materials characterization and modeling of responses.

Fwd testing and inverse analysis of pavement systems

The FWD equipment measures pavement surface deflections from an applied dynamic load that simulates a moving wheel (FAA 2004). There are many advantages to using FWD tests in lieu of, or to supplement, traditional destructive tests for pavement structural evaluation. Without FWD testing, structural data must be obtained from numerous cores, borings, and excavation pits on existing highway/airport pavements. This process can be very disruptive to highway/airport operations. FWD tests are economical to perform and data can be collected at up to 250 locations per day. FWD devices have earned a major role in pavement management. The Strategic Highway Research Program (SHRP) adopted the FWD device as a key piece of equipment for assessing structural capacity of long-term pavement performance (LTPP) test sections. Under the LTPP program, FWD testing is used at all general pavement studies (GPS) and specific pavement studies (SPS) test sites.

During FWD testing, typically, a 9,000-lb load is applied to the pavement surface by the intermediary of a circular plate (with a diameter of 12 in.) and the generated duration of the half-sine pulse is typically 30 ms. It corresponds to the loading time produced by a truck moving at 40 to 50 mph. The resulting pavement surface deflections are measured with six geophones at offsets of 0 (D0), 12 in. (D12), 24 in. (D24), 36 in. (D36), 48 in. (D48), and 60 in. (D60) intervals from the center of the load. The pavement properties are then backcalculated from the observed dynamic response of the pavement surface to an impulse load (the falling weight) through inverse analysis. Backcalculation of pavement layer properties is a very useful pavement design tool to evaluate the structural condition of in-service pavements and to characterize the layer properties as inputs into available numerical or analytical programs.

For flexible pavements considered in this study, several pavement layer moduli backcalculation programs have been proposed in the literature such as the AREA method (Hoffman et al. 1982), ELMOD, MODULUS, WESDEF (Van Cauwelaert 1989), MODCOMP (Irwin and Szenbenyi 1991, Irwin 1994), etc. Researchers have also

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developed ANN-based approach to backcalculation after the pioneering application by Meier and Rix (1995). In this paper, an Adaptive-Network-based Fuzzy Inference System (ANFIS) based approach is presented for the backcalculation of non-linear stiffness properties of multi-layered flexible pavement structures modeled as 2-D axisymmetric FE structures.

Since Jang (1993) proposed the ANFIS, its applications are numerous in various fields including engineering, management, health, biology and even social sciences. Jang et al. (1997) pointed out the following major areas for ANFIS applications: automatic control, pattern recognition, robotics, nonlinear regression, nonlinear system identification and adaptive signal processing. This paper proposes the application of ANFIS for adaptive backcalculation of pavement layer properties through nonlinear input-output mapping.

Figure 2. Schematic of Multi-layered Flexible Pavement Structure

ADAPTIVE-NETWORK-BASED FUZZY INFERENCE SYSTEM (ANFIS) METHODOLOGY

One of the most important and promising research fields in recent years has been *Nature-Inspired Heuristics*, an area utilizing some analogies with natural or social systems for deriving non-deterministic heuristic methods to obtain better results in combinatorial optimization problems (Colorni et al. 1996). Fuzzy logic approach (FLA) is one such heuristic method (Zadeh 1965).

In contrast to classical set theory, where membership of the elements are assessed in binary terms (an element either belongs to or does not belong to the set), fuzzy sets are sets whose elements have degrees of membership. The fuzzy set theory permits the gradual assessment of the membership of elements in a set with the aid of a membership function valued in the real unit interval [0, 1].

Fuzzy inference systems (FIS) are powerful tools for the simulation of nonlinear behaviors utilizing fuzzy logic and linguistic fuzzy rules. In the literature, there are several inference techniques developed for fuzzy rule-based systems, such as Mamdani (Mamdani and Assilian, 1975) and Sugeno (Takagi and Sugeno, 1985). In the Mamdani fuzzy inference methodology, inputs and outputs are represented by fuzzy relational equations in canonical rule-based form. In Sugeno FIS, output of the fuzzy rule is characterized by a crisp function and it was developed to generate fuzzy rules from a given input-output data set. Neuro-fuzzy systems are multi-layer feed forward adaptive

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networks that realize the basic elements and functions of traditional fuzzy logic systems (Oh et al. 2002). Since it has been shown that fuzzy logic systems are universal approximators, neuro-fuzzy control systems, which are isomorphic to traditional fuzzy logic control systems in terms of their functions, are also universal approximators. ANFIS is an extension of the Sugeno fuzzy model.

The Sugeno model allows the fuzzy systems to learn the parameters using adaptive backpropagation learning algorithm. In general, ANFIS is much more complicated than FIS. A FIS can be considered to be a parameterized non-linear map or a crisp function in a consequence called *f*, namely:

$$f(x) = \frac{\sum_{l=1}^{m} y^l \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i) \right)}{\sum_{l=1}^{m} \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i) \right)} \tag{1}$$

where y^l is a part of output if Mamdani reasoning is applied or a constant if Sugeno reasoning is applied (Jang et al. 1997). The membership function $\mu_{A_i^l}(x_i)$ corresponds to the input $x = [x_1, ..., x_n]$ of the rule *l* and *m* is the number of fuzzy rules. For the *i*th input predictor variable, x_i is the real data (for example, the measured FWD deflection) in one point from the set of observed values. The output values, *f(x)* are the estimated values (for example, the back-calculated pavement layer modulus) of simulation function within the range of input set (Abolpour et al. 2007). The center of gravity method is used for defuzzification. This can be further written as

$$f(x) = \sum_{l=1}^{m} w_l b_l(x) \tag{2}$$

Where w_i = y^1 and

$$b_l(x) = \frac{\prod_{i=1}^{n} \mu_{A_i^l}(x_i)}{\sum_{l=1}^{m} \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i) \right)} \tag{3}$$

If F_S is a set of continuous estimated value functions on domain *D*, then *f* can approximate F_S to any desired accuracy. Let F_S be a bounded function on [*a*, *b*] and *D* = $\{x^1,..,x^h\}$, a set of points in [*a, b*]. Then there exists the least squares polynomial of degree $\leq r$ between F_S and Q^h, which minimizes the following expression:

$$\sum_{j=1}^{h} \left| F_S(x^j) - Q(x^j) \right|^2 \tag{4}$$

Overall polynomial's degree is equal to or less than *r*. Where Q^h is real data of output values over *h*th point of input set (for each input predictor variable *i* = 1, 2,…, *n* and for each point of real world data *j* = 1, 2,…, *h*).

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In the Mamdani type of fuzzy system, the real data of the output values can be classified into classes such that the length of each class is equal to [*a, b*]. But in the Sugeno type, the length of [*a, b*] is only determined over input data set (*D*), and *f* can be approximately equal to F_S; hence, F_S is the output values of simulation model. In the interest of space, the derivation of equations for development and evaluation of rule base are not presented in this paper, but can be found in Jang et al. (1997). 'Learning' process in ANFIS methodology, namely adaptation of membership functions to emulate the training data, is commonly performed by two techniques: backpropagation and hybrid learning algorithms. The hybrid optimization method is a combination of Least Squares Error (LSE) and backpropagation descent method. In hybrid learning algorithm, consequent parameters are identified in forward computation by LSE algorithm, and premise parameters are adjusted in backward computation using backpropagation algorithm.

ANFIS BASED APPROACH TO PAVEMENT STRUCTURAL EVALUATION

Recently, researchers have attempted to employ FIS and ANFIS methodologies to model the pavement deflection behavior under dynamic loading (Saltan et al. 2007) and backcalculate the mechanical properties of flexible pavements (Goktepe et al. 2004), respectively. These research studies have shown FLA to be a promising approach for rapid pavement structural evaluation, especially in handling the uncertainty and noise associated with field data. In this study, the feasibility of ANFIS methodology for backcalculating non-linear pavement layer moduli from NDT data is further explored.

As a first step towards employing ANFIS methodology in learning the inverse mapping between known input (pavement layer thickness, moduli, and Poisson's ratio) and output patterns in a supervised manner, synthetic training and testing databases were generated using a 2-D axisymmetric pavement finite-element software (Raad and Figueroa, 1980). The Asphalt Concrete (AC) surface layer was characterized as a linear elastic material. Stress-dependent elastic models along with Mohr-Coulomb failure criteria were applied for the base and subgrade layers. The *stress-hardening* K-θ model was used for the base layer:

$$E_R = \frac{\sigma_D}{\varepsilon_R} = K\theta^n \qquad (5)$$

where E_R is resilient modulus (psi), θ is bulk stress (psi) and K and n are statistical parameters.

The fine-grained low-strength subgrade was modeled using the *bi-linear* model for characterizing the resilient modulus:

$$\begin{aligned} E_R &= E_{Ri} + K_1.(\sigma_d - \sigma_{di}) \quad for\, \sigma_d < \sigma_{di} \\ E_R &= E_{Ri} + K_2.(\sigma_d - \sigma_{di}) \quad for\, \sigma_d > \sigma_{di} \end{aligned} \qquad (6)$$

where E_R is resilient modulus (psi), σ_d is applied deviator stress (psi), and K_1 and K_2 are statistically determined coefficients from laboratory tests. The bi-linear model is a commonly used resilient modulus model for subgrade soils. The value of the resilient

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modulus at the breakpoint in the bi-linear model, E_{Ri}, can be used to classify fine-grained soils as being soft, medium or stiff. The effect of 9,000 lb FWD impact loading on the flexible pavement structure was simulated in the FE software over typical ranges of AC surface and base layer thicknesses and moduli ranges (Table 1).

In developing the ANFIS-based backcalculation approach, input parameters were partitioned using grid partitioning technique and Gaussian membership functions. Input variables were fuzzified by dividing them into 3 partitions. First order Sugeno FIS with linear output function was selected as the inference system. ANFIS structure was completed by the selection of hybrid learning algorithm. In the rulebase, fuzzy variables were connected with T-norm (fuzzy AND) operators and rules were associated using max-min decomposition technique. Goktepe et al. (2004) used 9 input variables and 1250 training patterns which resulted in an extremely large rule-base and long computing hours. Therefore, they concluded that ANFIS methodology and fuzzy partitioning are not appropriate for a multivariate nonlinear approximation problem comprising 9 input variables. In the same study, Goktepe et al. (2004) employed ANFIS in a scenario involving considerable amount of uncertainty or having incomplete deflection data and found the ANFIS approach to be successful.

Table 1. Ranges of Pavement Layer Properties

Pavement Layer	**Thickness (inches)**	**Elastic Layer Modulus (psi)**
Asphalt Concrete	3 – 15	100,754 – 1,995,419
Base	4 – 22	K_b: 3,014 – 14,000 n_b: 0.2 – 0.6
Subgrade	∞	1,012 – 14,000

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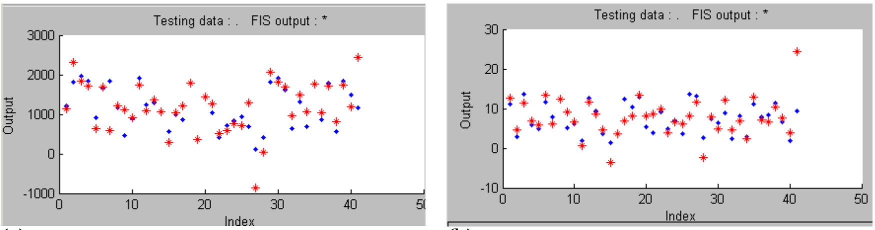
In this study, the first four FWD deflections (D0, D12, D24, and D36) along with the AC and base course thicknesses were used as inputs (a scenario involving incomplete datasets) and two separate ANFIS models were employed to predict AC modulus (E_{AC}) and non-linear subgrade modulus (E_{Ri}). This paper did not currently focus on backcalculating the base layer moduli due to the associated challenges identified in previous studies (Meier and Rix 1995). One hundred training patterns from the FE-based synthetic database were randomly selected as inputs for training in ANFIS and 40 testing vectors were independently selected from the synthetic database to check the prediction ability of developed ANFIS-based backcalculation model. The ANFIS methodology predictions for E_{AC} and E_{Ri} are shown in Figure 3. The red-colored markers represent the ANFIS predictions.

The average testing Root Mean Squared Error (RMSE) for E_{AC} and E_{Ri} predictions were reported to be 394 psi and 3.7 psi, respectively. These results are comparable with those obtained using Multi-Layer Preceptron (MLP) feed-forward ANN architecture (Ceylan et al. 2007) trained with a comprehensive database. The surface plots in Figure 4 illustrate the relationship between the input and output variables. Deflection, D36, is especially associated with having influence on subgrade modulus. The negative predictions for E_{AC} and E_{Ri} are not reasonable and highlight the need for refining the ANFIS based backcalculation model and rule-base further based on engineering experience.



(a) (b)

Figure 3. (a) ANFIS Predictions of AC modulus (E_{AC}); (b) ANFIS Predictions of Subgrade Modulus (E_{Ri})

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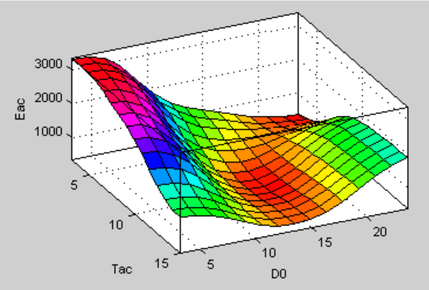
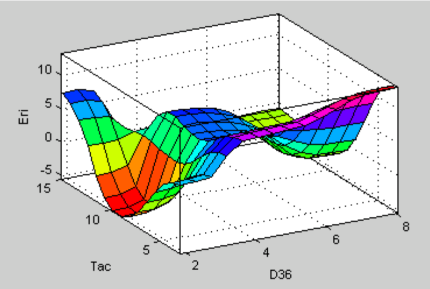


(a)



(b)

Figure 3. (a) ANFIS Surface Plots for AC modulus (E_{AC}); (b) ANFIS Surface Plots for Subgrade Modulus (E_{Ri})

SUMMARY AND CONCLUSIONS

Non-Destructive Test (NDT) equipment is used by highway engineers to routinely evaluate the structural condition of pavements by measuring their deflection response to impact loading and subsequently characterize the mechanical properties of pavement layers through inverse analysis (referred to as backcalculation). A new methodology based on Adaptive-Network-based Fuzzy Inference System (ANFIS) combined with Finite Element Modeling (FEM) is presented in this paper for backcalculating the non-linear pavement layer moduli in real-time based on measured pavement surface deflections. Since the input space partitioning and size of the rule-base are critical in ANFIS in terms of computational efficiency, this methodology is especially useful for solving problems with relatively smaller number of input variables and/or small to medium number of training dataset. It is shown that the developed ANFIS model inherits the fundamental capability of a fuzzy model to especially deal with nonrandom uncertainties associated with vagueness and imprecision associated with inverse analysis of transient pavement surface deflection measurements.

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