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Risk assessment and optimal scheduling of serial projects

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Abstract

The valuation and planning of complex projects are becoming increasingly challenging with rising market uncertainty and the deregulation of many industries, which have also raised the need for efficient risk management. We take the perspective of a private firm interested in sequential capacity expansion of a project and develop a framework for measuring the downside risk of the serial project and optimising the sequence of the stages. Under general distributional assumptions for the duration of each stage, we present an accurate representation of the project's net present value (NPV) distribution based on a Pearson curve fit, leading to closed-form expressions for the associated risk measures. We then assess the impact of duration variability on the value at risk and demonstrate its role in stochastic project scheduling. We also account for the trade-off between maximising the expected NPV and minimising the risk exposure, and obtain the optimal schedule for risk-averse decision-makers. It becomes obvious that both the duration variability of each stage and the decision-makers' risk preferences can significantly affect the optimal sequence of the stages and that high duration variability is not always undesirable, even for risk-averse decision-makers.

Keywords Decision analysis · Modularity · NPV distribution · Project scheduling · Risk analysis

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1 Introduction

The deregulation of many industries poses a formidable challenge to private firms managing multi-stage projects, since the associated uncertainties over both future revenue streams and completion time of different stages (technological uncertainty) complicate the assessment of risk. This, in turn, affects critical managerial decisions, such as project scheduling. Examples of such projects include the Elizabeth line, London's new railway, that was originally designed to deliver a series of stages between 2017 and 2019, yet was not fully operational until May 2023 with an additional cost of £3 billion over the original budget (Tucker 2017; Keay 2022). In addition, the High Speed 2 and the Heathrow expansion can be treated as serial projects, where each stage has an uncertain duration, cost and benefit (Edgington 2020; Thijssen 2021). More specifically, the former has a full network of 330 miles and will be executed in two phases. While its first phase (140 miles) is under construction and due for completion between 2029 and 2033, the second phase is split into three sub-phases with a target completion date between 2040 and 2045. Similarly, the Heathrow expansion, which aims to increase capacity from about 80 to 142 million passengers per annum, will be delivered in four stages with Stage 1 to be completed by 2026 and all expansions by 2050 (https://www.heathrow.com/company/about-heathrow/expansion/documents). Other examples include the development and capacity expansion of renewable energy projects, such as the Walney Extension and the Hornsea offshore wind farm (Vaughan 2019). The latter is planned to have a total capacity of up to 6 gigawatts and its construction has been split into four phases to be consecutively executed due to limited budget and workforce (https://orsted.co.uk/energy-solutions/offshore-wind/our-wind-farms?gad=1 &gclid=EAIaIQobChMIwrWkxLex_wIVDsXtCh1XsA vEAAYASAAEgLAc_D_BwE).

While the traditional literature on project scheduling assumes discrete cash flows (Brucker et al. 1999; Herroelen and Leus 2005; Demeulemeester and Herroelen 2006), this is not suitable in the case of large infrastructure projects where revenues accrue continuously (Pogue 2004; Almond and Remer 1979; Tanchoco et al. 1981; Remer and Nieto 1995). Additionally, as this literature focuses on maximising the net present value (NPV) or minimising the makespan of a project under risk neutrality, the implications of attitudes towards risk remain an important open research direction. To address these disconnects, we develop a continuous-time framework in which we derive the probability distribution and risk measures of the NPV of a serial project as well as the optimal sequence of stages under economic and technological uncertainty. The former is modelled via a continuous-time stochastic process, while the latter by a generic probability distribution. We contribute in three ways. First, we derive an accurate approximation for the probability distribution of the NPV of a multi-stage capacity expansion using a Pearson curve fit, from which we can obtain the closed-form expression for the value at risk (VaR) and the conditional VaR (CVaR) of the project. Second, we investigate the trade-off between expected NPV maximisation and risk minimisation, thereby deriving the solution to the optimal scheduling problem for risk-averse decision-makers. Third, we present the implications of economic and technological uncertainty on project scheduling and present managerial insights.

Our findings suggest that both the duration variability and the decision-makers' risk preferences can affect the optimal sequence of stages of a serial project significantly, and that their effect depends also on the expansion cost. More specifically, using a benchmark example (i.e. each stage with equal capacity size, cost and expected duration), we demonstrate that duration variability is undesirable if capacity expansions are costly, in which case stages with lower duration variability must be executed first. However, contrary to the intuition that an increase in uncertainty entails greater downside risk, we find that a project with higher duration variability is not always associated with higher risk exposure, especially when the cost of each stage is relatively low.

We proceed by first discussing some related work in Sect. 2. In Sect. 3, we introduce our model, the benchmark case of a single-stage capacity expansion, and the extension to a multi-stage project. In Sect. 4.1, we analyse the impact of duration variability on the project's expected NPV and risk exposure using a benchmark example, while Sect. 4.2 provides a general model for obtaining the optimal sequence of stages under risk aversion. Section 5 presents numerical results and managerial insights into the stochastic project scheduling problem and Sect. 6 concludes the paper offering directions for further research.

2 Related work

Until the 1980s, the need to allow for uncertainty in decision-making was not particularly pronounced, as many industries were subject to state regulation. However, the deregulation of many industries in the 1980s exposed firms to various types of uncertainty. This, in turn, raised the importance of extending traditional capital budgeting techniques, such as the NPV rule, to account for uncertainty and risk assessment (Wiesemann and Kuhn 2015). In the real options literature, this application potential has been exploited in decision-making under uncertainty by analysing the interaction between uncertainty in cash flows and managerial flexibility (McDonald and Siegel 1986; Dixit and Pindyck 1994; Trigeorgis 1996). A strand of this literature focuses on the sequential nature of investment decisions and the value creation of modularity (Gollier et al. 2005; Gamba and Fusari 2009; Baldwin et al. 2000; Kort et al. 2010; Chronopoulos et al. 2017). However, the underlying methodology, which is based on dynamic programming, is not particularly suitable to address critical aspects of serial projects, e.g. scheduling, that require robust optimisation techniques.

Herroelen and Leus (2005) tackle the stochastic project scheduling problem framing it as a multi-stage decision process wherein project activities' uncertainty is accounted for to prevent schedule disruptions. By allowing for uncertainty in projects' makespans, costs and revenues, a generic reformulation of the stochastic NPV maximisation problem is proposed by Sobel et al. (2009) and an algorithm is presented for identifying an optimal adaptive policy for project scheduling. More recent examples in the same line of work, where various scheduling policies and algorithms are developed and tested in a stochastic environment, include Wiesemann et al. (2010), Liang et al. (2019), Leyman and Vanhoucke (2017), Zheng et al. (2017), Ding and Zhu (2015). Although NPV-oriented models for stochastic project scheduling focus on the financial aspect of optimisation problems and provide significant flexibility in sequential decision processes, they tend to assume a risk-neutral decision-maker (Wiesemann and Kuhn 2015; Gutjahr 2015). However, for extreme values of duration and cash flow distributions, decision-making that is based only on the expected NPV and ignores attitudes towards risk may not be particularly accurate (Blau et al. 2000; Browning 2014; Chao et al. 2014; Rezaei et al. 2020).

The variance of a project's revenues was often used to evaluate the risk of it (Markowitz 1968; Van Horne 1966), until the VaR was introduced as a more practical risk measure of the worst-case loss of an investment associated with a given probability. Nevertheless, despite its popularity, the VaR does not capture the shape of the tail of a loss distribution. To overcome the drawbacks of VaR while maintaining its advantages, Rockafellar and Uryasev (2000) introduced a coherent risk measure, known as the CVaR, aiming at quantifying the expected losses occurring beyond the VaR. Literature addressing the stochastic project selection and scheduling problem, considering decision-makers' risk preferences, includes Ke and Liu (2005), Beraldi et al. (2012), Huang and Zhao (2014), Wang and Ning (2018), who control the probability of occurrence of undesirable investment outcomes (e.g. a negative expected NPV or positive VaR) using a chance-constrained method. Moreover, the trade-off between risk minimisation and profit maximisation is commonly considered in mean-risk models (Colvin and Maravelias 2011; Chen et al. 2012; Bozorgi-Amiri et al. 2013; Alonso-Ayuso et al. 2014; Dupačová and Kozmík 2015; Huang et al. 2016; Zhao et al. 2018, 2019). For example, Alonso-Ayuso et al. (2014) consider a stochastic copper extraction planning problem under both risk neutrality and risk aversion, and their results clearly indicate the advantage of involving risk measures, such as the VaR and CVaR, of a project in the decision-making process.

The unknown distribution and variability (variance) of a project's makespan further emphasise the need for risk measures that facilitate efficient risk management. While different probability distributions for modelling project duration are examined within the stochastic resource-constrained project scheduling problem (RCPSP), the high- and low-variability settings of the duration distribution are often distinguished (Ashtiani et al. 2011; Ballestin and Leus 2009; Fang et al. 2015). The reason is that the optimal scheduling rule that minimises the expected makespan of a project often changes with respect to the variance of the duration variables. For example, Chen et al. (2018) evaluate the efficiency of 17 priority rules and show that the optimal one for the deterministic RCPSP does not perform best for the stochastic RCPSP. Their results confirm that the performance of the priority rules depends on project characteristics, e.g. the resource demand and duration variability of each activity. Therefore, different scheduling rules could be chosen according to the amount of information on duration distributions that a decision-maker has. Similarly, we investigate the impact of duration variability on the NPV distribution and, more importantly, show how the optimal schedule of a serial project can be obtained for decision-makers with different risk preferences.

A discrete and deterministic cash flow incurred at the start of each stage is not particularly relevant in the case of large infrastructure projects where revenues usually accrue continuously (Pogue 2004; Almond and Remer 1979; Tanchoco et al. 1981; Remer and Nieto 1995). For example, the annual average electricity price rise between 2004 and 2021 in UK is approximately 8% per year, from 4.16 pence per kilowatt hour (p/kWh) in 2004 to 15.08 p/kWh in 2019 (https://www.gov.uk/government/statistical-data-sets/gas-and-electricity-prices-in-the-non-domestic-sector). This was followed by a dramatic increase to 20.86 p/kWh in 2022 due to high market volatility, which clearly demonstrates that the revenue stream of a project fluctuates with time and that price uncertainty should also be taken into account. In addition, Cui et al. (2020) study the unbiased estimation by Monte Carlo simulation of the expected present value of a cumulative cash flow over an infinite horizon, dependent on an underlying stochastic process such as a geometric Brownian motion or a Cox–Ingersoll–Ross process. Therefore, in this paper, we develop a continuous-time framework for sequential capacity expansion under economic and technological uncertainty and derive the project's VaR and CVaR. Furthermore, we consider the trade-off between NPV maximisation and downside risk minimisation of a project due to alternative scheduling options[1]. Our results indicate that both the duration variability and the risk preferences can have a significant effect on the optimal sequencing of a multi-stage project, and that this depends on the expansion cost of each stage. Interestingly, we also find that higher duration variability does not necessarily imply higher risk exposure; differently from conventional intuition, it can be beneficial even for risk-averse decision-makers.

3 Risk assessment of serial project

3.1 The model

We take the perspective of a private firm that considers the capacity expansion of a project sequentially in discrete stages. While the construction process takes a random but finite amount of time, the project has an infinite lifetime,[2] accrues stochastic revenues and is subject to technological uncertainty, reflected in the random duration of each stage. Given a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, the σ-algebra $\mathcal{F}_t \subset \mathcal{F}$ reflects the information available at time $t \geq 0$. Without being a real restriction, we

[1] Note that while the value of waiting due to economic uncertainty is not the focus of this paper, we provide a real options framework in A to investigate its impact on the risk assessment and optimal scheduling of a two-stage project. A thorough study of the implications of discretion over timing is left for future work.

[2] The assumptions of infinite lifetime and perpetual revenue stream are commonly made in the real options literature because they not only support analytical tractability, but are also key features of different projects (Dixit and Pindyck 1994). For example, in the electricity sector, power generation facilities have an effective operation life of 30–50 years, while transmission facilities remain in service even longer. Hence, although the construction of a gas power plant or the installation of wind farm has a finite duration, its lifetime is significantly longer.

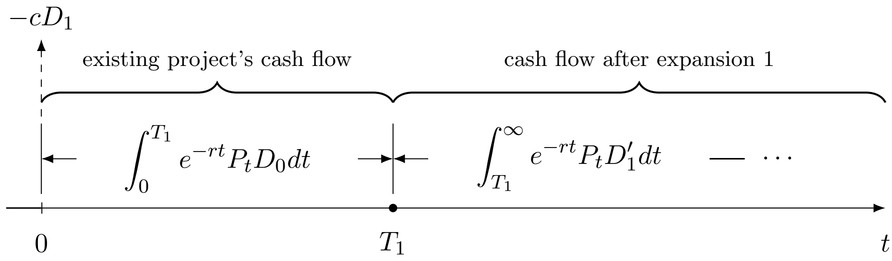

Fig. 1 Cash flow of single-stage capacity expansion taking place between time 0 and T_1

assume, in light of Dixit and Pindyck (1994) and Cui et al. (2020), that the price (per unit flow of output) process for the project follows a geometric Brownian motion

$$dP_t = \alpha P_t dt + \beta P_t dW_t, \quad P_0 \equiv P, \tag{1}$$

where P is the initial price, $\alpha > 0$ the growth rate, $\beta > 0$ the volatility, and W is a standard Brownian motion. Also, we denote by $r > \alpha$ the subjective discount rate defined exogenously.[3] Model (1) can be adapted to the users' preferences and the requirements of their respective application, as our build-up is general in terms of underlying model assumptions. Also, the particular choice, as said in Cui et al. (2020), is a viable candidate in project management.

The project comprises $n \in \mathbb{N}$ stages that are executed sequentially. For $j \in \{1, 2, \dots, n\}$, we denote by $D_j > 0$ the deterministic scale of each capacity expansion. Thus, $P_t D_j$ is the instantaneous revenue of the project in stage j. Following Huisman and Kort (2015), we assume a linear investment cost function, where a deterministic cost, $C_j = cD_j$, is incurred at the beginning of each capacity expansion and $c \geq 0$ represents the expansion cost per unit output. After the completion of stage j, the accumulated capacity is $D'_j = \sum_{k=0}^{j} D_k$, where $D_0 \geq 0$ is the initial capacity.

The duration of each stage is denoted by τ_j and has a general, continuous probability distribution with cumulative distribution function (cdf) $F_{\tau_j}(t)$ and probability density function (pdf) $f_{\tau_j}(t)$. Assuming that $\{\tau_j\}_{j=1}^{n}$ are independent (Steyn 2001; Chen et al. 2015), the completion time of stage j is given by $T_j = \sum_{k=1}^{j} \tau_k$. Finally, we denote by $V(\cdot)$ the resulting NPV of a project with associated cdf $G_V(v)$ and pdf $g_V(v)$.

[3] Risk neutrality is commonly assumed in corporate finance, however it also relies on market completeness. Hence, it may not be particularly relevant in our context of construction or capacity expansion in the absence of hedging instruments. For this, we use, instead, an exogenously defined (subjective) discount rate.

3.2 Single-stage capacity expansion

We begin with the basic case of a project that is subject to a single capacity expansion. As shown in Fig. 1, the expansion begins at time $t = 0$, where a deterministic cost of cD_1 is incurred. Subsequently, the firm receives an instantaneous revenue of P_tD_0 from time $t = 0$ until T_1, at which point the capacity of the project is expanded to $D'_1 = D_0 + D_1$ and the firm earns a perpetual stream of stochastic revenues $P_tD'_1$.

We derive the NPV of the project by discounting the continuous cash flow over its lifetime. The discounted to time $t = 0$ expected NPV, $V(P, T_1)$, of this single-stage expansion conditional on the makespan of the project can be formulated as[4]

$$\begin{aligned} V(P, T_1) &= \mathbb{E}\left[\int_0^{T_1} e^{-rt} P_t D_0 dt + \int_{T_1}^{\infty} e^{-rt} P_t D'_1 dt - C_1 \middle| P, T_1 \right] \\ &= \frac{PD_0}{r-\alpha} + \frac{PD_1}{r-\alpha} e^{-(r-\alpha)T_1} - cD_1. \end{aligned} \tag{2}$$

The mean, variance, skewness and kurtosis of $V(P, T_1)$ are given, respectively, by

$$\mu = \mathbb{E}\left[V(P, T_1)\right] = \frac{PD_0}{r-\alpha} + \frac{PD_1}{r-\alpha} M_{T_1}(\alpha - r) - cD_1, \tag{3}$$

$$\sigma^2 = m_2, \quad \psi = \frac{m_3}{m_2^{3/2}}, \quad \epsilon = \frac{m_4}{m_2^2}, \tag{4}$$

where

$$m_k = \mathbb{E}\left[\left(V(P, T_1) - \mu\right)^k\right] = \left(\frac{PD_1}{r-\alpha}\right)^k \mathbb{E}\left[\left(e^{-(r-\alpha)T_1} - M_{T_1}(\alpha - r)\right)^k\right]$$

and $M_{T_1}(\delta) = \mathbb{E}\left[e^{\delta T_1}\right]$, $\delta \in \mathbb{R}$.

Proposition 1 *The cdf and pdf of the NPV of a single-stage project are given by*

$$G_V(v) = 1 - F_{T_1}\left(-\frac{1}{r-\alpha} \ln \frac{(r-\alpha)(v + cD_1) - PD_0}{PD_1}\right), \tag{5}$$

$$\begin{aligned} g_V(v) = &\frac{1}{(r-\alpha)(v + cD_1) - PD_0} f_{T_1} \\ &\left(-\frac{1}{r-\alpha} \ln \frac{(r-\alpha)(v + cD_1) - PD_0}{PD_1}\right), \end{aligned} \tag{6}$$

for $v \geq PD_0/(r-\alpha) - cD_1$.

[4] In what follows, we abbreviate the random variable "conditional expected NPV" in (2) to simply "NPV", whereas the "expected NPV" is constant and is given by (3).

Proof From (2), we have that

$$G_V(v) = \mathbb{P}\left(\frac{PD_0}{r-\alpha} + \frac{PD_1}{r-\alpha}e^{-(r-\alpha)T_1} - cD_1 \le v\right)$$
$$= \mathbb{P}\left(T_1 \ge -\frac{1}{r-\alpha}\ln\frac{(r-\alpha)(v+cD_1)-PD_0}{PD_1}\right),$$

from which (5) follows for $v \ge PD_0/(r-\alpha) - cD_1$. The density function (6) follows from differentiating (5) with respect to v. □

From Proposition 1, for a given probability distribution for $\tau_1 = T_1$, we derive the distribution of $V\big(P, T_1\big)$. In turn, this facilitates the evaluation of the risk associated with the project by looking at the left tail of its NPV distribution. For example, consider $\mathrm{VaR}_p(X) = -q_p^+(X)$, where $q_p^+(X) = \inf\{v \in \mathbb{R} : \mathbb{P}(X \le v) > p\}$ is the p-quantile of a random variable X, for $p \in (0,1)$, while $\mathrm{CVaR}_p(X)$ denotes the expectation of X given that it is larger than $\mathrm{VaR}_p(X)$. Given a closed-form expression for G_V, we can also obtain the VaR and CVaR of the project NPV.

Proposition 2 *For the NPV of the project at level p, we have that*

$$\mathrm{VaR}_p(V) = -\frac{PD_0}{r-\alpha} - \frac{PD_1}{r-\alpha}e^{-(r-\alpha)F_{T_1}^{-1}(1-p)} + cD_1$$
$$\text{and } \mathrm{CVaR}_p(V) = \frac{1}{p}\int_0^p \mathrm{VaR}_q(V)dq. \tag{7}$$

Proof By definition of VaR, we have that

$$\mathrm{VaR}_p(V) = -\inf\{v \in \mathbb{R} : \mathbb{P}(V \le v) > p\}$$
$$= -\inf\left\{v \in \mathbb{R} : 1 - F_{T_1}\left(-\frac{1}{r-\alpha}\ln\frac{(r-\alpha)(v+cD_1)-PD_0}{PD_1}\right) > p\right\}$$
$$= -\inf\left\{v \in \mathbb{R} : v > \frac{PD_0}{r-\alpha} + \frac{PD_1}{r-\alpha}e^{-(r-\alpha)F_{T_1}^{-1}(1-p)} - cD_1\right\},$$

from which the final result follows. The CVaR follows by definition. □

3.3 Multi-stage project

In this section, we generalise to a multi-stage project and formulate its NPV, before moving on to scheduling these stages in Sect. 4. The stochastic cash flow stream of the multi-stage project is shown in Fig. 2.

The NPV of a serial project with $n \ge 1$ stages is given by the sum of the NPVs of the various capacity expansions, i.e.

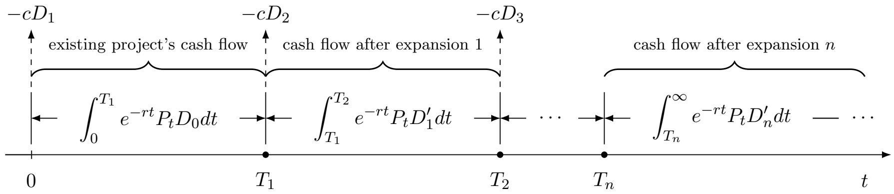

Fig. 2 Cash flow of multi-stage project

$$\begin{aligned} V(P, T_1, \dots, T_n) &= \sum_{j=1}^{n} \mathbb{E}\left[\int_{T_{j-1}}^{T_j} e^{-rt} P_t D'_{j-1} dt - C_j e^{-rT_{j-1}} \middle| P, T_{j-1}, T_j \right] \\ &\quad + \mathbb{E}\left[\int_{T_n}^{\infty} e^{-rt} P_t D'_n dt \middle| P, T_n \right] \\ &= \sum_{j=0}^{n} \mathbb{E}\left[\int_{T_j}^{\infty} e^{-rt} P_t D_j dt \middle| P, T_j \right] - \sum_{j=1}^{n} c D_j e^{-rT_{j-1}} \\ &= \frac{PD_0}{r-\alpha} + \sum_{j=1}^{n} V_j(P, T_{j-1}, T_j), \end{aligned} \tag{8}$$

where

$$V_j(P, T_{j-1}, T_j) \equiv V_j = \frac{PD_j}{r-\alpha} e^{-(r-\alpha)T_j} - cD_j e^{-rT_{j-1}} \tag{9}$$

corresponds to the increment of the NPV of the project's cash flows due to the jth capacity expansion, for $j \in \{1, 2, \dots, n\}$; also, $T_0 \equiv 0$.

Expressions for the true NPV density and distribution functions are not available in closed form in the multi-stage case, however we can obtain very accurate analytical approximations; these can then be used to compute the risk measures of the sequential capacity expansion. To this end, we fit a Pearson curve type based on the first four moments of the true, but otherwise unknown, distribution of $V(P, T_1, \dots, T_n)$. More specifically, the Pearson family of solutions $g_V(x)$ satisfies the differential equation

$$\frac{d \ln g_V(x)}{dx} = -\frac{a + x}{c_0 + c_1 x + c_2 x^2} \tag{10}$$

resulting in well-defined density functions. Solving Eq. (10) yields the general form of Pearson's density function

$$g_V(x) = \mathcal{C}\left(c_0 + c_1 x + c_2 x^2\right)^{-\frac{1}{2c_2}} \exp\left\{\frac{\left(c_1 - 2ac_2\right)\arctan\left(\frac{c_1+2c_2x}{\sqrt{4c_0c_2-c_1^2}}\right)}{c_2\sqrt{4c_0c_2 - c_1^2}}\right\}, \quad (11)$$

where $\mathcal{C}$ is the normalising constant and the parameters $\{a, c_0, c_1, c_2\}$ control the shape of the distribution. We estimate these based on the first four integer moments $\{\mu_1, \mu_2, \mu_3, \mu_4\}$ as

$$a = c_1 = \frac{\sqrt{\upsilon\gamma}(\theta + 3)}{10\theta - 12\gamma - 18}, \quad c_0 = \frac{(4\theta - 3\gamma)\upsilon}{10\theta - 12\gamma - 18}, \quad c_2 = \frac{2\theta - 3\gamma - 6}{10\theta - 12\gamma - 18}, \quad (12)$$

where

$$\upsilon = \mu_2 - \mu_1^2, \quad \gamma = \frac{\left(\mu_3 - 3\mu_1\mu_2 + 2\mu_1^3\right)^2}{\upsilon^3}, \quad \theta = \frac{\mu_4 - 4\mu_1\mu_3 + 6\mu_1^2\mu_2 - 3\mu_1^4}{\upsilon^2} \quad (13)$$

are the variance, squared skewness and kurtosis, respectively. We can classify the Pearson distribution family types as in Johnson et al. (1994) which is standard in the literature. First, we select a family according to the η-criterion proposed by Elderton and Johnson (1969): given $\sqrt{\gamma}$ and θ, we compute

$$\eta = \frac{\gamma(\theta + 3)^2}{4(4\theta - 3\gamma)(2\theta - 3\gamma - 6)}. \quad (14)$$

We distinguish between the *main* types corresponding to $\eta < 0$ (I), $0 < \eta < 1$ (IV) and $\eta > 1$ (VI); and the *transition* types $\eta = 0, \theta = 3$ (normal), $\eta = 0, \theta < 3$ (II), $\eta = \pm\infty$ (III), $\eta = 1$ (V) and $\eta = 0, \theta > 3$ (VII). Then, we can approximate the NPV distribution accordingly. Important advantages of the Pearson fitting approach, as we will demonstrate next, are its excellent results for different skewness-kurtosis $(\gamma^{1/2}, \theta)$ levels (see also Brignone et al. 2021), for varying number of stages and general assumptions for the distribution of τ_j. We feel that presenting more details about the proximity of distributions with shared moments is beyond the scope of this research; for more, see Akhiezer (1965, Corollary 2.5.4), Lindsay and Basak (2000, Theorems 1, 2) and Kyriakou et al. (2023). Given the Pearson fitted cdf of the project's NPV, $G_V(v)$, the VaR and CVaR follow:

$$\mathrm{VaR}_p(V) = -G_V^{-1}(p) \quad \text{and} \quad \mathrm{CVaR}_p(V) = \frac{1}{p}\int_0^p \mathrm{VaR}_q(V)dq. \quad (15)$$

To illustrate the Pearson curve fit, we set $P = 1$, $r = 0.1$, $\alpha = 0.08$, $\beta = 0.1$, $c = 30$, $D_0 \equiv 0$ and $D_j = 10$ for all $j \in \{1, 2, \dots, n\}$, and study the Pearson curve approximation for $\tau_j \sim \text{LogN}(m, s)$ and $\tau_j \sim \text{Weibull}(\lambda, \kappa)$.[5] For the sake of comparison, we assume that they share the same mean and variance, e.g. $e^{m+\frac{1}{2}s^2} = \lambda\Gamma(1 + 1/\kappa) = 10$ and $e^{2m+s^2}\left(e^{s^2} - 1\right) = \lambda^2\left[\Gamma(1 + 2/\kappa) - (\Gamma(1 + 1/\kappa))^2\right] = 28$, from which we obtain parameters $m = 2.18$, $s = 0.50$, $\lambda = 11.28$ and $\kappa = 1.96$. We compare the density approximations with the true simulation estimates in Fig. 3. The top and bottom panels of Table 1 also report the true mean, variance, skewness, kurtosis, $\text{VaR}_{0.05}$ and $\text{CVaR}_{0.05}$ of $V(P, T_1, \dots, T_n)$, along with the values corresponding to the Pearson curve fit and the associated absolute percentage errors.

Two comments are in order. It is obvious from Fig. 3 and Table 1 that the approximation of the NPV distribution by a Pearson curve fit is very accurate, regardless of the distribution of τ_j and the number of stages. While one can rely on Monte Carlo simulation estimates, using a Pearson approximation leads to an analytical expression that considerably reduces the computational effort and avoids unwanted simulation error. In particular, for the accuracies reported in Table 1 based on 10^7 simulation trials for the Monte Carlo estimates, we achieve a reduction in the computing time by a factor of 100 the least.

In addition, we recall that we have chosen the lognormal and Weibull parameter values for τ_j so that their mean and variance are matched. Nevertheless, the resulting variance, skewness and kurtosis of $V(P, T_1, \dots, T_n)$ vary significantly between the two distributions. This implies that the assumptions about the distribution and the higher moments of the duration variables can affect substantially the risk characteristics of the NPV and, therefore, the valuation and planning of a project.

4 Optimal scheduling of serial project

4.1 Impact of duration variability on expected NPV and VaR

In this section, we investigate the optimal order in which the stages of a serial project should be executed, as well as the factors affecting the NPV distribution and the downside risk of the project. Specifically, we study the impact of duration variability on the optimal sequence of a serial project and investigate the importance of risk considerations in stochastic project scheduling.

We start by showing how the expected NPV, μ, and VaR of a project with single capacity expansion depend on the variance of its makespan, τ_1, with the real-world example of the Hornsea offshore wind farm. More specifically, the construction (i.e. Stage 1) of the 1.2-gigawatt (GW) wind farm takes about three years with a cost around £4.2 billion (https://orsted.co.uk/energy-solutions/offshore-wind/our-wind-farms?gad=1&gclid=EAIaIQobChMIwrWkxLex_wIVDsXtCh1XsA_vEAAY

[5] Both the lognormal (Chen et al. 2018; Trietsch et al. 2012) and exponential distributions (Sobel et al. 2009) are commonly used to model activity duration in the project management literature. Here, we use a Weibull distribution which generalises the exponential distribution with an extra parameter that offers added flexibility. More results related to other distributions we have looked at can be made available upon request.

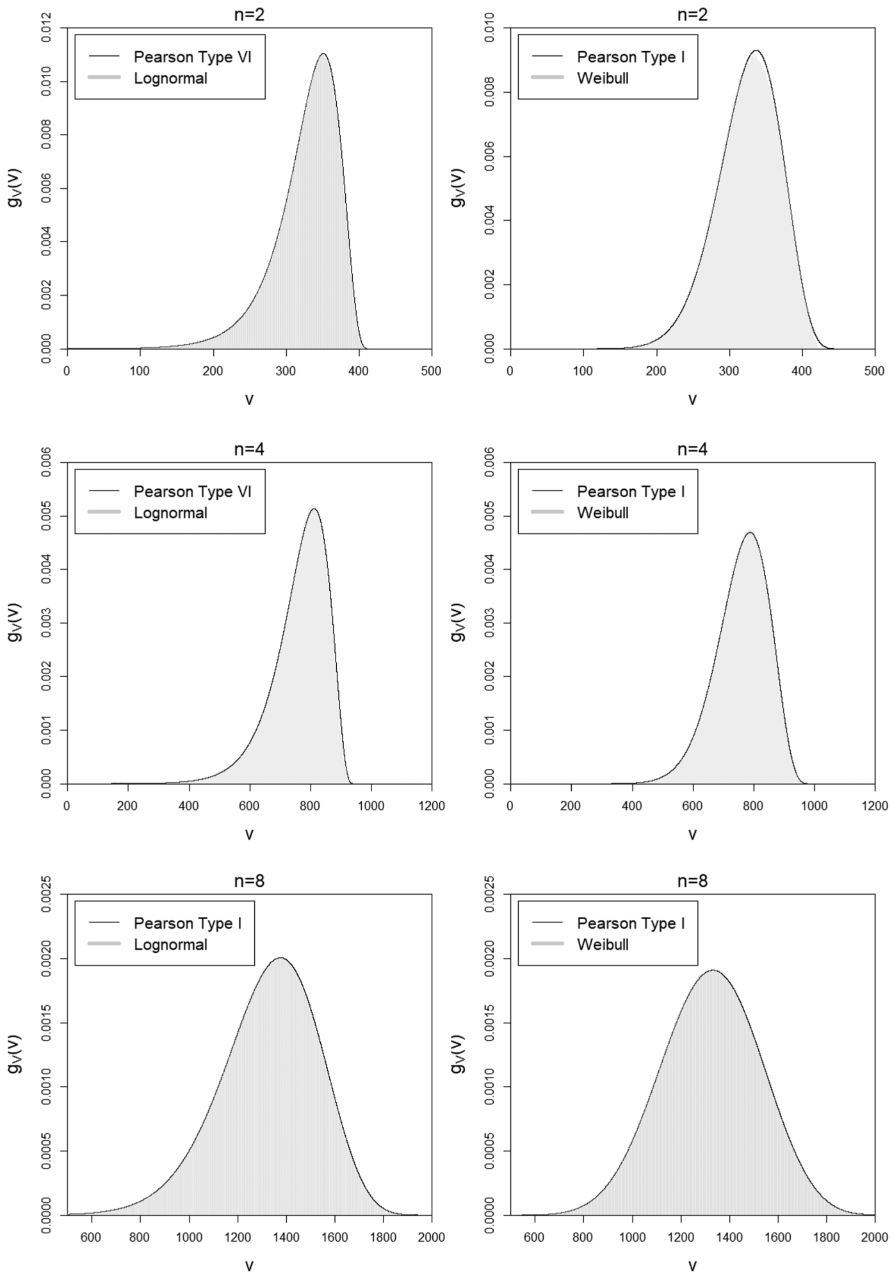

Fig. 3 Simulated and fitted Pearson pdf of NPV of n-stage capacity expansion for $\tau_j \sim \text{LogN}(m, s)$ (left panel) and $\tau_j \sim \text{Weibull}(\lambda, \kappa)$ (right panel) sharing same mean, 10, and variance, 28, with $m = 2.18$, $s = 0.50$, $\lambda = 11.28$ and $\kappa = 1.96$

Table 1 Simulation estimates and fitted-Pearson mean, variance, skewness, kurtosis, $VaR_{0.05}$ and $CVaR_{0.05}$ of NPV of n-stage capacity expansion for $\tau_j \sim \mathrm{LogN}(m, s)$ (upper panel) and $\tau_j \sim \mathrm{Weibull}(\lambda, \kappa)$ (lower panel) sharing same mean, 10, and variance, 28, with $m = 2.18$, $s = 0.50$, $\lambda = 11.28$ and $\kappa = 1.96$

Lognormal									
Stages	$n = 2$			$n = 4$			$n = 8$		
	Estimate	Pearson	Abs. error (%)	Estimate	Pearson	Abs. error (%)	Estimate	Pearson	Abs. error (%)
Mean	327.03	327.04	0.0	763.92	763.92	0.0	1327.57	1327.57	0.0
Var	1985.85	1981.25	0.2	8275.57	8274.28	0.0	41551.24	41551.02	0.0
Skew	− 1.3734	− 1.3720	0.1	− 1.1607	− 1.1588	0.1	− 0.4751	− 0.4750	0.0
Kurt	6.5072	6.5007	0.1	5.2344	5.2290	0.1	3.3347	3.3342	0.0
$VaR_{0.05}$	− 243.52	− 243.12	0.2	− 592.25	− 592.25	0.0	− 967.28	− 967.04	0.0
$CVaR_{0.05}$	− 205.33	− 205.32	0.0	− 521.53	− 522.27	0.1	− 854.34	− 854.16	0.0
Weibull									
Stages	$n = 2$			$n = 4$			$n = 8$		
	Estimate	Pearson	Abs. error (%)	Estimate	Pearson	Abs. error(%)	Estimate	Pearson	Abs. error (%)
Mean	325.31	325.31	0.0	759.93	759.93	0.0	1323.30	1323.30	0.0
Var	1846.15	1846.15	0.0	7444.58	7444.58	0.0	41237.06	41237.06	0.0
Skew	− 0.4563	− 0.4563	0.0	− 0.5591	− 0.5590	0.0	− 0.0694	− 0.0694	0.0
Kurt	3.1260	3.1260	0.0	3.2644	3.2642	0.0	2.7969	2.7969	0.0
$VaR_{0.05}$	− 249.98	− 249.87	0.0	− 603.79	− 604.31	0.1	− 984.84	− 984.21	0.1
$CVaR_{0.05}$	− 227.13	− 226.61	0.2	− 556.77	− 556.95	0.0	− 903.11	− 903.10	0.0

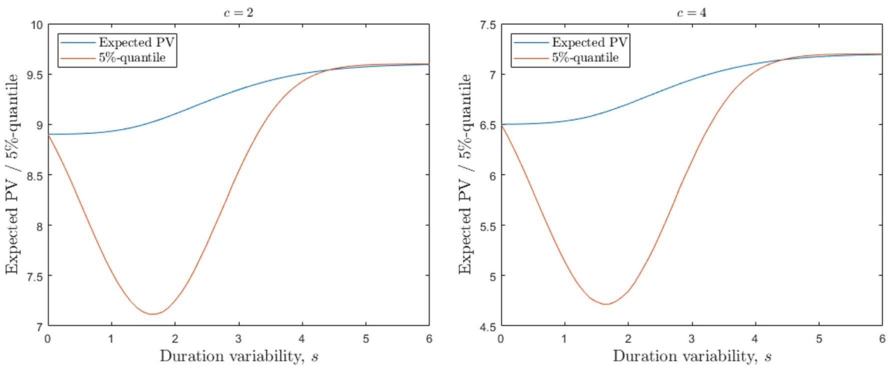

Fig. 4 Expected NPV and 5%-quantile of single capacity expansion as function of duration variability s when $\tau_1 \sim \mathrm{LogN}\left(\ln 3 - s^2/2, s\right)$, for $c = 2$ and 4

ASAAEgLAc_D_BwE). Hence, we set $D_0 \equiv 0$ GW, $D_1 = 1.2$ GW, $P = £0.2$ billion per GW, $r = 10\%$ per year, $\alpha = 8\%$ per year and $\beta = 10\%$ per year. Assuming that $\tau_1 \sim \mathrm{LogN}(m, s)$, then for $\mathbb{E}[\tau_1] = e^{m+\frac{1}{2}s^2} = \tau$ we get that $m = \ln \tau - s^2/2$. Thus, $\mathrm{Var}[\tau_1] = \left(e^{s^2} - 1\right)e^{2m+s^2} = \tau^2\left(e^{s^2} - 1\right)$, which is increasing with s for given τ. For $\tau = 3$ years, we present in Fig. 4 μ as an increasing function of s for $c = £2$ billion and £4 billion per GW capacity installed. This implies that, for a fixed makespan expectation, a capacity expansion with higher duration variability is expected to be more profitable.

Of particular interest is the U-shaped 5%-quantile curve, from which we observe that the risk exposure of the expansion, $\mathrm{VaR}_{0.05}(V) = -q^+_{0.05}(V)$, is surprisingly low when the duration variance is large. This counter-intuitive result can be attributed to the VaR_p of the project's NPV, which is determined by the quantile of τ_1 (see Proposition 2); for lognormal τ_1, the quantile as a function of s increases initially and then starts to decrease. For $\mathbb{E}[\tau_1] = \tau$ held fixed, the skewness of τ_1 given by $\left(e^{s^2} + 2\right)\sqrt{e^{s^2} - 1}$ increases in s, so that a higher duration variability implies a shorter makespan with increasing concentration around small values. This implies that high duration variability can benefit both risk-neutral and risk-averse decision-makers; therefore, variance reduction is not always necessary, especially when the duration variability is moderate to high.

Next, we add one more stage and consider two capacity expansions of equal size and cost to demonstrate the implications of risk aversion for optimal scheduling. We assume that each stage of the project has the same expected duration τ but different duration variability s. Therefore, the question now arises: how should the firm determine the order of execution for each stage to achieve a higher (lower) project value (risk exposure)?

Assuming that $\tau_1 \sim \mathrm{LogN}(\ln \tau - 1/2, 1)$, $\tau_2 \sim \mathrm{LogN}(\ln \tau - s^2/2, s)$ and $D_1 = D_2 = 1.2$ GW, Fig. 5 illustrates how μ and the 5%-quantile of the project depend on s, for each of the two possible ways of scheduling. As we are moving beyond the single-stage expansion, for the necessary computations we resort to

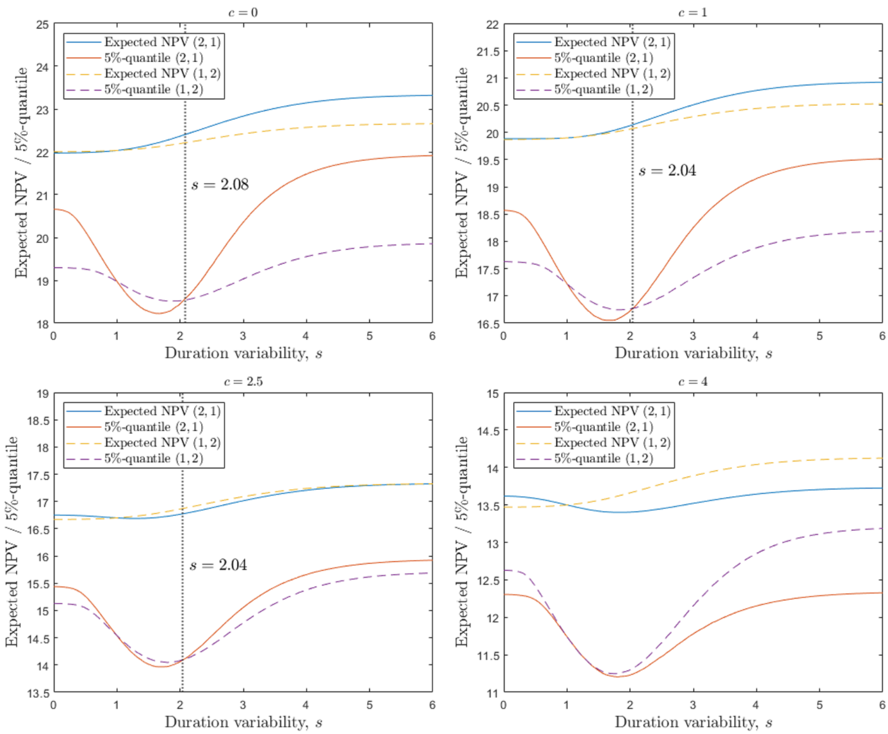

Fig. 5 Expected NPV and 5%-quantile of two-stage project as function of duration variability s when $\tau_1 \sim \text{LogN}(\ln 3 - 1/2, 1)$ and $\tau_2 \sim \text{LogN}\left(\ln 3 - s^2/2, s\right)$, and stage 2 is executed before stage 1 (solid lines) or vice versa (dashed lines)

the relevant expression in (15) based on the Pearson curve fit. Consistent with Fig. 4, we find that executing the stage with high duration variability first, that is, τ_2 for $s > 1$ (blue solid line) can result in larger expected NPV but only when c is small (e.g. $c \leq 2$). As c increases, this strategy is no longer appropriate. Indeed, μ eventually decreases with s as shown in the bottom right panel of Fig. 5. This happens because the discount factor of the project's cost, $\mathbb{E}[e^{-r\tau_2}]$, increases faster than that of the revenue, $\mathbb{E}[e^{-(r-\alpha)\tau_2}]$, as s rises. Consequently, duration variability is desirable for risk-neutral decision-makers when the cost is relatively low, as their preferences are independent of the risk associated with the schedule, but it can be harmful to the project's value if capacity expansions are costly.

On the other hand, executing first the stage with lower duration variability, that is, $\tau_1 \sim \text{LogN}(\ln 3 - 1/2, 1)$, when the other stage has duration $\tau_2 \sim \text{LogN}\left(\ln 3 - s^2/2, s\right)$ with $s > 1$ (purple dashed line), does not guarantee lower risk exposure, particularly when the cost of each expansion is low compared to its revenues. Whereas this scheduling strategy can be quite safer if s is low-to-moderate, it performs poorly in terms of both the project's value and risk exposure if s is large (magenta line appears above the purple line, despite the smaller variance of τ_1 and stage 1 being implemented first). Again, this follows from the positively skewed distribution of τ_2 with increasing concentration around smaller values as s increases. In particular, the 95%-quantile linked to τ_2 becomes smaller if $s > 1.67$, thus indicating that the firm should execute stage 2 first despite its increasing duration variability. However, if the capacity expansions are costly (e.g. $c \geq 4$), then high duration variability becomes undesirable as it always leads to lower expected NPV and higher risk. Therefore, the decision-maker should always execute stages with lower duration variability first (see the bottom right panel of Fig. 5).

Consequently, our results indicate that duration variability can affect significantly the optimal sequence of stages for both risk-neutral and risk-averse decision-makers; moreover, its impact depends on the level of expansion cost. Indeed, for all stages with same expected duration and capacity, risk-neutral decision-makers should always execute the stages with higher duration variability earlier if the cost is relatively low. However, under risk aversion, the optimal sequence of stages is less obvious as it depends on the trade-off between maximising the expected NPV and minimising the downside risk of a project. For example, a risk-averse decision-maker may be willing to bear a slightly higher risk in exchange for a larger expected NPV, and vice versa. Moreover, this trade-off can be even more complicated due to the fact that high duration variability is not always harmful. Due to the technological uncertainty reflected in the makespan of a project, it is implied that risk considerations have to be incorporated in stochastic project scheduling.

4.2 Risk management and optimal scheduling

To address the trade-off between the expected NPV and risk exposure of a serial project due to the different ways of scheduling, we incorporate risk measures, such as the VaR and CVaR, into the stochastic project scheduling problem. To this end, we define the symmetric group S_n on the set $N = \{1, 2, \dots, n\}$ and $\boldsymbol{\pi} \in S_n$ a permutation of N (i.e. a bijection from N to N itself). In our context, a permutation $\boldsymbol{\pi} = (\pi(1), \dots, \pi(n))$ encompasses the sequence of stages of a serial project; for any $j, k \in N$, $\pi(j) = k$ means that stage k is the jth term of the sequence. Also, we introduce $\omega \in [0, 1]$ which reflects the risk appetite of a decision-maker, with large (small) ω corresponding to high (low) risk aversion. Thus, the optimal sequence of stages $\boldsymbol{\pi}^*_\omega$, which maximises a combination of the expected NPV and the risk measure of a serial project, can be formulated as

$$\boldsymbol{\pi}^*_\omega = \underset{\boldsymbol{\pi} \in S_n}{\operatorname{argmax}}\, V(\boldsymbol{\pi}), \tag{16}$$

where

Table 2 Optimal schedule of two-stage project with duration variability s for decision-makers with risk appetite ω obtained from mean-risk model (16), for $\mathcal{R} \equiv$ VaR$_{0.05}$ and $c = 3$ (upper panel) or 2 (lower panel)

$c = 3$									
	Expected NPV μ		VaR$_{0.05}$		Optimal Sequence $\boldsymbol{\pi}^*_\omega$				
	$\pi = (1, 2)$	$\pi = (2, 1)$	$\pi = (1, 2)$	$\pi = (2, 1)$	$\omega = 0$	$\omega = 0.25$	$\omega = 0.5$	$\omega = 0.75$	$\omega = 1$
$s = 0.5$	15.6090	15.6858	−14.1864	−14.2613	(2, 1)	(2, 1)	(2, 1)	(2, 1)	(2, 1)
$s = 1.5$	15.6938	15.6031	−13.1931	−13.1262	(1, 2)	(1, 2)	(1, 2)	(1, 2)	(1, 2)
$s = 2.2$	15.8407	15.6771	−13.2537	−13.2625	(1, 2)	(1, 2)	(1, 2)	(1, 2)	(1, 2)
$s = 3.0$	16.0228	15.8501	−13.9066	−13.9731	(1, 2)	(1, 2)	(1, 2)	(1, 2)	(1, 2)

$c = 2$									
	Expected NPV μ		VaR$_{0.05}$		Optimal Sequence $\boldsymbol{\pi}^*_\omega$				
	$\pi = (1, 2)$	$\pi = (2, 1)$	$\pi = (1, 2)$	$\pi = (2, 1)$	$\omega = 0$	$\omega = 0.25$	$\omega = 0.5$	$\omega = 0.75$	$\omega = 1$
$s = 0.5$	17.7423	17.7852	−15.8959	−16.2493	(2, 1)	(2, 1)	(2, 1)	(2, 1)	(2, 1)
$s = 1.5$	17.8268	17.7876	−15.0290	−14.8855	(1, 2)	(1, 2)	(1, 2)	(1, 2)	(1, 2)
$s = 2.2$	17.9748	17.9397	−15.0560	−15.1548	(1, 2)	(1, 2)	(2, 1)	(2, 1)	(2, 1)
$s = 3.0$	18.1547	18.1763	−15.6240	−16.1191	(2, 1)	(2, 1)	(2, 1)	(2, 1)	(2, 1)

$\pi = (1, 2)$: execute stage 1 followed by stage 2; $\pi = (2, 1)$: execute stage 2 followed by stage 1

$$\begin{aligned} V(\boldsymbol{\pi}) = (1-\omega)\mathbb{E}\left[\sum_{j=1}^{n}\frac{PD_{\pi(j)}}{r-\alpha}e^{-(r-\alpha)\sum_{k=1}^{j}\tau_{\pi(k)}} - \sum_{j=1}^{n} cD_{\pi(j)}e^{-r\sum_{k=0}^{j-1}\tau_{\pi(k)}}\right] \\ - \omega\mathcal{R}\left(\sum_{j=1}^{n}\frac{PD_{\pi(j)}}{r-\alpha}e^{-(r-\alpha)\sum_{k=1}^{j}\tau_{\pi(k)}} - \sum_{j=1}^{n} cD_{\pi(j)}e^{-r\sum_{k=0}^{j-1}\tau_{\pi(k)}}\right), \end{aligned} \tag{17}$$

$\pi(0) = \tau_0 = 0$ and $\mathcal{R}(\cdot)$ is a risk measure such that a larger value of it implies higher risk. In particular, we consider $\mathcal{R} \in \{\text{VaR}_p, \text{CVaR}_p\}$ to account for the left tail of the NPV distribution of a project, which we evaluate based on (15). Given a p level of confidence, ω controls the weights of the expected NPV and the risk exposure in this mean-risk model. A decision-maker is assumed to be risk-neutral if $\omega = 0$, in which case the second part of (17) vanishes and (16) reduces to the expected NPV maximisation model. In the next section, we obtain the optimal schedule of a serial project under various combinations of duration variability and decision-maker's risk appetite, and show how the results based on either the mean-VaR or mean-CVaR model differ.

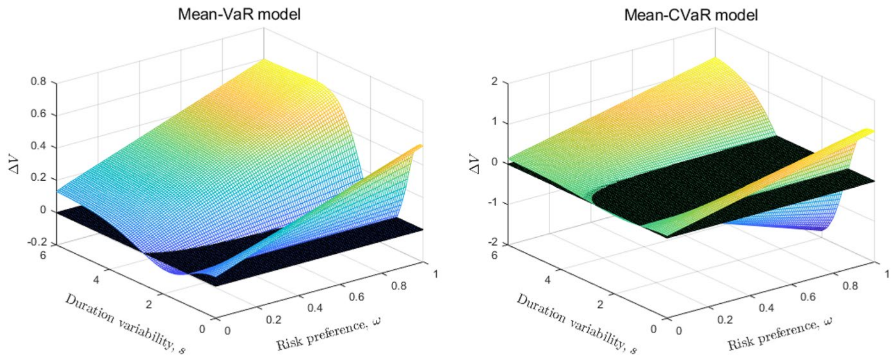

Fig. 6 Left panel: difference between mean-VaR model with schedule $\pi = (2, 1)$ and $\pi = (1, 2)$ for varying duration variability s and risk preference ω when $c = 2$. Right panel: same as left based, instead, on mean-CVaR model

5 Project scheduling: numerical illustration

We revisit, first, the example in Fig. 5, where a firm considers scheduling the two phases of an offshore wind farm, each with capacity of 1.2 GW. The upper panel of Table 2 reports the expected NPV and $\text{VaR}_{0.05}$ of the two-stage project for $c = \{2, 3\}$ and $s \in \{0.5, 1.5, 2.2, 3\}$. The optimal sequences of stages for decision-makers with different attitudes towards risk are also obtained based on the mean-risk model (16)–(17) with $\mathcal{R} \equiv \text{VaR}_{0.05}$. Consistent with the bottom right panel of Fig. 5, we have $\pi^*_\omega = (2, 1)$ if $s < 1$ and $\pi^*_\omega = (1, 2)$ if $s > 1$, for any $\omega \in [0, 1]$. Therefore, our results indicate that decision-makers should always execute the stage with lower duration variability first when the cost of each stage is high.

Next, we examine whether the aforementioned scheduling strategy is still optimal if the expansion cost is lower, e.g. $c = 2$. The lower panel of Table 2 confirms that both the duration variability of each capacity expansion and the decision-makers' risk preferences can affect significantly the optimal schedule of a project in this case. Taking $\omega = 0.5$ as an example, we obtain $\boldsymbol{\pi}^*_{0.5} = (2, 1)$ if $s < 1$ or $s > 2.14$, whereas $\boldsymbol{\pi}^*_{0.5} = (1, 2)$ if $1 < s < 2.14$. This implies that a risk-averse decision-maker with $\omega = 0.5$ may choose to reduce the risk exposure of the project by a significant amount without foregoing too much revenue when $s > 2.14$.

Aiming to shed more light on the results, we present in Fig. 6 the scenarios in which a schedule $\boldsymbol{\pi}$ optimises a mean-risk model. The left plot illustrates the difference between the mean-VaR model (17) with $\boldsymbol{\pi} = (2, 1)$ and $\boldsymbol{\pi} = (1, 2)$, i.e. $\Delta V = V((2, 1)) - V((1, 2))$, for different combinations of duration variability s and risk preference ω: positive differences mean $\boldsymbol{\pi}^*_\omega = (2, 1)$; negative differences correspond to $\boldsymbol{\pi}^*_\omega = (1, 2)$; intersections between the surface and the xy-plane refer to transitioning optimal results. Indeed, for $c = 2$, it can be observed that decision-makers prefer to execute stage 2 first if $s > 2.76$.

By analogy, the right plot shows the results corresponding to $\mathcal{R} \equiv \text{CVaR}_{0.05}$ in (17). Here, we observe that a risk-averse decision-maker is more likely to execute stage 1 first, when the duration variability of stage 2 is of a moderate level, i.e.

Table 3 Optimal schedule of three-stage project (upper panel) and four-stage project (lower panel) for decision-makers with risk appetite ω, unit cost c, and $D_j = 1.2$

$n = 3$										
	$c = 6$					$c = 2$				
	$\omega = 0$	$\omega = 0.25$	$\omega = 0.5$	$\omega = 0.75$	$\omega = 1$	$\omega = 0$	$\omega = 0.25$	$\omega = 0.5$	$\omega = 0.75$	$\omega = 1$
$s = 1.5$	(2, 1, 3)	(1, 2, 3)	(1, 2, 3)	(1, 2, 3)	(1, 2, 3)	(3, 1, 2)	(1, 2, 3)	(1, 2, 3)	(1, 2, 3)	(1, 2, 3)
$s = 2.5$	(2, 1, 3)	(1, 2, 3)	(1, 2, 3)	(1, 2, 3)	(1, 2, 3)	(3, 1, 2)	(3, 1, 2)	(1, 3, 2)	(1, 3, 2)	(1, 3, 2)
$s = 3.0$	(2, 1, 3)	(1, 2, 3)	(1, 2, 3)	(1, 2, 3)	(1, 2, 3)	(3, 1, 2)	(3, 1, 2)	(3, 1, 2)	(3, 1, 2)	(3, 1, 2)
$s = 4.0$	(2, 1, 3)	(1, 2, 3)	(1, 2, 3)	(1, 2, 3)	(1, 2, 3)	(3, 1, 2)	(3, 1, 2)	(3, 1, 2)	(3, 1, 2)	(3, 1, 2)
$n = 4$										
	$c = 8$					$c = 2$				
	$\omega = 0$	$\omega = 0.25$	$\omega = 0.5$	$\omega = 0.75$	$\omega = 1$	$\omega = 0$	$\omega = 0.25$	$\omega = 0.5$	$\omega = 0.75$	$\omega = 1$
$s = 1.5$	(3, 2, 1, 4)	(1, 3, 2, 4)	(1, 2, 3, 4)	(1, 2, 3, 4)	(1, 2, 3, 4)	(4, 2, 1, 3)	(1, 2, 3, 4)	(1, 2, 3, 4)	(1, 2, 3, 4)	(1, 2, 3, 4)
$s = 2.5$	(3, 2, 1, 4)	(1, 2, 3, 4)	(1, 2, 3, 4)	(1, 2, 3, 4)	(1, 2, 3, 4)	(4, 2, 1, 3)	(4, 1, 2, 3)	(1, 4, 2, 3)	(1, 4, 2, 3)	(1, 4, 2, 3)
$s = 3.0$	(3, 2, 1, 4)	(1, 2, 3, 4)	(1, 2, 3, 4)	(1, 2, 3, 4)	(1, 2, 3, 4)	(4, 2, 1, 3)	(4, 1, 2, 3)	(4, 1, 2, 3)	(4, 1, 2, 3)	(4, 1, 2, 3)
$s = 4.0$	(3, 2, 1, 4)	(1, 3, 2, 4)	(1, 2, 3, 4)	(1, 2, 3, 4)	(1, 2, 3, 4)	(4, 2, 1, 3)	(4, 1, 2, 3)	(4, 1, 2, 3)	(4, 1, 2, 3)	(4, 1, 2, 3)

For $n = 3$, $\tau_1 \sim \mathrm{LogN}(\ln 3 - 0.09/2, 0.3)$, $\tau_2 \sim \mathrm{LogN}(\ln 3 - 1/2, 1)$ and $\tau_3 \sim \mathrm{LogN}\left(\ln 3 - s^2/2, s\right)$; for $n = 4$, $\tau_1 \sim \mathrm{LogN}(\ln 3 - 0.09/2, 0.3)$, $\tau_2 \sim \mathrm{LogN}(\ln 3 - 0.64/2, 0.8)$, $\tau_3 \sim \mathrm{LogN}(\ln 3 - 1/2, 1)$ and $\tau_4 \sim \mathrm{LogN}\left(\ln 3 - s^2/2, s\right)$

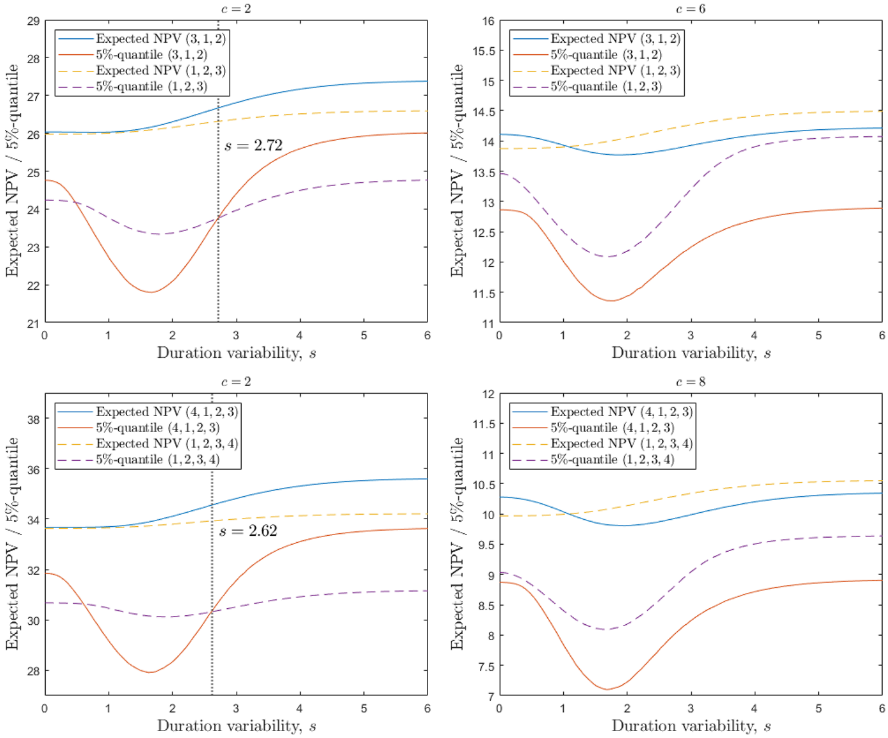

Fig. 7 Expected NPV and 5%-quantile of three-stage project (upper panel) as function of duration variability s for schedule $\pi = (3, 1, 2)$ (solid lines) and $\pi = (1, 2, 3)$ (dashed lines); and of four-stage project (lower panel) as function of duration variability s for schedule $\pi = (4, 1, 2, 3)$ (solid lines) and $\pi = (1, 2, 3, 4)$ (dashed lines)

$1 < s < 3.72$., still larger than that of stage 1. This can be attributed to CVaR being a more conservative risk measure than VaR, rendering a same expected NPV less attractive to decision-makers. Similar (unreported) results are obtained for $p < 0.05$.

Next, the upper and lower panels of Table 3 show examples of the optimal schedule of three-stage and four-stage capacity expansions, respectively. As the number of stages increases, the optimal sequence of stages becomes more ambiguous. However, we can still observe that risk-averse decision-makers prefer to execute stages with lower duration variability first if the capacity expansions are very costly (e.g. $c = 6$ or 8). On the other hand, if the cost of each stage is relatively low (e.g. $c = 2$), it can be optimal for decision-makers to execute the stage with the highest duration variability first (shaded grid), which is also consistent with our previous results. Furthermore, we take a closer look at the two schedules that appear most frequently in Table 3 for each panel: $\pi = (3, 1, 2)$ and $\pi = (1, 2, 3)$ for $n = 3$; $\pi = (4, 1, 2, 3)$ and $\pi = (1, 2, 3, 4)$ for $n = 4$. We present in the upper and lower panels of Fig. 7 the expected NPV and 5%-quantile of the three-stage and four-stage project, respectively. Results suggest that, for projects with more stages, duration variability is still undesirable when the cost is high, while it can be beneficial if the cost is low due to higher expected NPV and lower risk exposure.

In summary, the managerial insights of our results are threefold. First, for stages with equal size and expected duration, duration variability is unwanted when capacity expansions are costly, as this leads to lower expected NPV and higher risk of the project. Therefore, in this case, it is optimal for decision-makers (with any risk preference) to execute the stages with lower duration variability first. Second, we find that a project with higher duration variability can have larger expected NPV and is not always associated with worse downside risk if the expansion cost is relatively low. This suggests that, although variance reduction may be beneficial for scheduling a (resource-constrained) project with the aim of minimising its makespan, it can be harmful to the project financially. Finally, we demonstrate that the optimal schedule under risk aversion depends not only on decision-makers' risk preferences, but also on the level of duration variability and the cost of each stage.

6 Concluding discussion

In this paper, we develop a risk assessment and optimal scheduling framework for sequential capacity expansion under output price and technological uncertainty. We derive analytically the distribution, VaR and CVaR of the project's NPV. Our work showcases the potentially positive impact of duration variability on scheduling stochastic projects, emphasising the importance of risk considerations. Through a mean-risk model, we explore the trade-off between maximising expected NPV and minimising downside risk in serial projects, providing optimal investment strategies for risk-averse decision-makers.

We show that both the duration variability and the decision-makers' risk preferences can significantly affect the optimal sequence of stages of a serial project and that this also depends on the capacity expansion cost. More specifically, if the expansion cost of each stage is high, the duration variability is detrimental to a project's NPV and risk exposure. Hence, decision-makers should prioritise the execution of stages with lower duration variability. However, if the cost is relatively low, it can be optimal for risk-neutral decision-makers to execute the stages with higher duration variability first due to larger expected NPV. Taking also into account the decision-makers' attitudes towards risk, we find that executing stages with lower duration variability earlier does not guarantee lower risk exposure. In contrast to the intuition that increasing uncertainty entails greater risk exposure, our results suggest that higher duration variability may not lead to higher downside risk. Instead, it can be beneficial not only for risk-neutral but also for risk-averse decision-makers.

This counter-intuitive result of non-monotonic relationship between the VaR of a project's NPV and its duration variability arises when the skewness of the duration increases with respect to its variance. Indeed, in such a case, high duration variability implies shorter makespan with increasing concentration around small values and that the project is expected to have larger profit and lower risk exposure (if expansion cost is low). Consequently, investing in variance reduction is only recommended when the duration variability of each stage is low. However, if the skewness of the duration distribution decreases with growing variance or the project is particularly costly, high duration variability can cause an opposite effect, which is unfavourable for both risk-neutral and risk-averse decision-makers.

Hence, our study holds significant implications for investment under technological uncertainty when the true distribution of a project's makespan is unknown. Neglecting or underestimating this uncertainty can lead to inappropriate project scheduling and, therefore, lower NPV or greater downside risk. Directions for future research may include studying potential effects of the volatility of the price dynamics on the risk measures of the project, or the development of a real options framework to allow for discretion over investment timing (Heydari and Siddiqui 2010; Jeon 2021).[6] The objective would be to investigate how the managerial flexibility influences the distribution of the NPV and the risk measures of a serial project. Also, a computational comparison of different approximation methods and an algorithmic study on more elaborated project scheduling models taking risk aversion into account can also be meaningful extensions of this work.

Appendix A. The case of managerial investment flexibility

In what follows, we develop a real options framework which incorporates the firm's discretion over investment timing. In this case, the firm is not obligated to invest immediately in the next capacity expansion after each stage is completed. Instead, it has the option to delay the next investment while waiting for more favourable price conditions (refer also to Heydari and Siddiqui (2010) for a study of the optimal interruption policy of multiple-exercise interruptible load contracts).

We denote by $T_i = \sum_{j=1}^{i}(w_j + \tau_j)$ the completion time of stage i, which now includes both waiting times $\{w_j\}_{j=1}^{i}$ and construction times $\{\tau_j\}_{j=1}^{i}$ of all stages up to i. Let $P^{(i)}$ be the investment threshold of stage i and $P^{(i)^*}$ the optimal investment threshold. Hence, w_i is the random first-passage time of the price process through the investment threshold from below, i.e. $w_i = \inf\left\{t \geq 0 : P_{T_{i-1}+t} \geq P^{(i)}\right\}$.

Figure 8 illustrates the cash flows of a single-stage capacity expansion when the firm has discretion over investment timing. We assume that the initial output price of the project is too low to justify immediate investment; therefore, the firm must defer it.

The firm's expected option value is determined via backward induction. Therefore, we first assume that the project is already active and accrues stochastic revenues. The conditional expected NPV of the project is given by

$$V\left(P, \tau_1\right) = \frac{PD_0}{r-\alpha} + \frac{PD_1}{r-\alpha}e^{-(r-\alpha)\tau_1} - cD_1. \tag{18}$$

[6] We do provide a real options framework in A for risk assessment and optimal scheduling of a two-stage project, granting the firm the flexibility to postpone the investment before each stage.

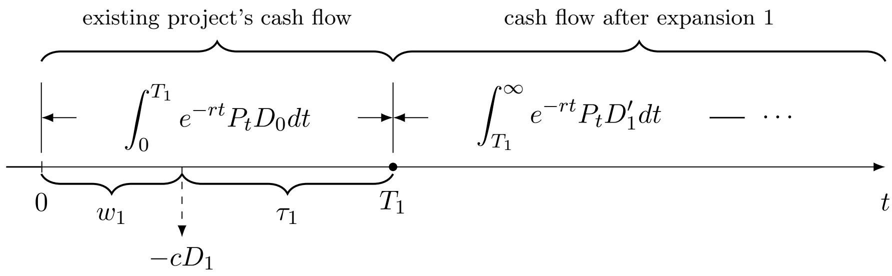

Fig. 8 Single-stage capacity expansion

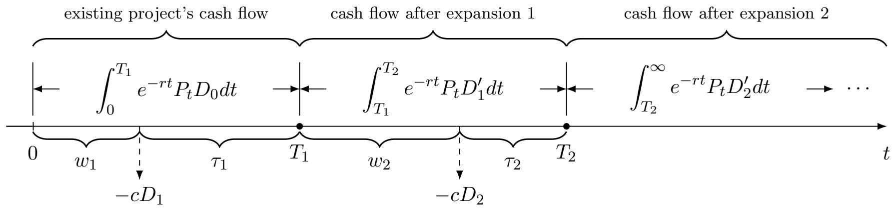

Fig. 9 Two-stage capacity expansion

Moving backwards, we assume that the initial output price is too low to justify immediate investment, so the firm must wait for a period of time, w_1. The firm's optimisation objective is

$$\begin{aligned} F(P,\tau_1) &= \sup_{w_1\in\mathcal{S}} \mathbb{E}\left[\int_0^{w_1+\tau_1} e^{-rt}P_tD_0dt + \int_{w_1+\tau_1}^{\infty} e^{-rt}P_tD_1'dt - C_1e^{-rw_1}\middle|P,\tau_1\right] \\ &= \sup_{w_1\in\mathcal{S}} \mathbb{E}\left[\int_0^{\infty} e^{-rt}P_tD_0dt\middle|P\right] \\ &\quad + \mathbb{E}\left[\left(\int_{\tau_1}^{\infty} e^{-rt}P_{w_1+t}D_1dt - cD_1\right)e^{-rw_1}\middle|P,\tau_1\right] \\ &= \max_{P^{(1)}>P} \frac{PD_0}{r-\alpha} + \left(\frac{P}{P^{(1)}}\right)^{\rho}\left(\frac{P^{(1)}D_1}{r-\alpha}e^{-(r-\alpha)\tau_1} - cD_1\right), \end{aligned} \tag{19}$$

where $\mathcal{S}$ is the set of stopping times of the filtration generated by the price process. Note that the last equality follows from the stochastic discount factor $\mathbb{E}[e^{-rw_1}|P] = \left(P/P^{(1)}\right)^{\rho}$ ((Dixit and Pindyck 1994, p. 315)), with $\rho > 1$ the positive root of $\beta^2x(x-1)/2 + \alpha x - r = 0$. By applying the first-order necessary condition (FONC) to the unconstrained optimisation problem (19), we obtain the optimal investment threshold of the first capacity expansion:

$$P^{(1)^*} = \frac{\rho}{\rho - 1} c(r - \alpha) e^{(r-\alpha)\tau_1}. \tag{20}$$

We now consider the two-stage capacity expansion as illustrated in Fig. 9.

Starting with the second capacity expansion, we first consider the case that once the construction of the first stage is completed at T_1, the price process is high enough for immediate investment, i.e. $P^{(2)} = P_{T_1+w_2} \leq P_{T_1}$. The conditional expected NPV of the second expansion (discounted to time $T_1 = w_1 + \tau_1$) is given by

$$V\left(P_{T_1}, \tau_2\right) = \frac{P_{T_1} D_2}{r - \alpha} e^{-(r-\alpha)\tau_2} - cD_2. \tag{21}$$

Next, if $P^{(2)} > P_{T_1}$, that is, the firm cannot invest directly in the second stage and must wait, the maximised value of the option to invest in the second stage is given by

$$\begin{aligned} F^{(2)}\left(P_{T_1}, \tau_2\right) &= \sup_{w_2 \in \mathcal{S}} \mathbb{E}\left[\int_{w_2+\tau_2}^{\infty} e^{-rt} P_{T_1+t} D_2 dt - C_2 e^{-rw_2} \middle| P_{T_1}, \tau_2 \right] \\ &= \max_{P^{(2)} > P_{T_1}} \left(\frac{P_{T_1}}{P^{(2)}} \right)^{\rho} \left(\frac{P^{(2)} D_2}{r - \alpha} e^{-(r-\alpha)\tau_2} - cD_2 \right). \end{aligned} \tag{22}$$

The optimal investment threshold of the second expansion is then

$$P^{(2)^*} = \frac{\rho}{\rho - 1} c(r - \alpha) e^{(r-\alpha)\tau_2}. \tag{23}$$

Working backwards to the first stage, if it is still optimal to wait, i.e. $P^{(1)^*} \geq P$, the maximised option value of the first capacity expansion is

$$\begin{aligned} &F^{(1)}\left(P, \tau_1, \tau_2\right) \\ &\quad = \sup_{w_1 \in \mathcal{S}} \mathbb{E}\left[\int_{w_1+\tau_1}^{\infty} e^{-rt} P_t D_1 dt - C_1 e^{-rw_1} + e^{-rT_1} \mathbb{E}\left[F^{(2)}\left(P_{T_1}, \tau_2\right) \middle| P_{T_1}, \tau_2 \right] \middle| P, \tau_1, \tau_2 \right] \\ &\quad = \max_{P^{(1)} > P} \left(\frac{P^{(1)} D_1}{r - \alpha} e^{-(r-\alpha)\tau_1} - cD_1 + e^{-r\tau_1} \mathbb{E}\left[F^{(2)}\left(P_{T_1}, \tau_2\right) \middle| P_{T_1}, \tau_2 \right] \right) \left(\frac{P}{P^{(1)}} \right)^{\rho}, \end{aligned} \tag{24}$$

where the conditional expectation of the option value of the second expansion, given the information at time T_1, depends on whether or not the second stage is executed immediately, i.e. whether or not $P^{(2)^*} \leq P_{T_1}$:

Table 4 Optimal scheduling of a two-stage project, when the firm has the option to delay the next investment, for decision-makers with risk appetite ω and duration variability s, where $\tau_1 \sim \mathrm{LogN}(\ln 3 - 1/2, 1)$ and $\tau_2 \sim \mathrm{LogN}\left(\ln 3 - s^2/2, s\right)$. $\pi = (1,2)$: execute stage 1 followed by stage 2; $\pi = (2,1)$: execute stage 2 followed by stage 1

$c = 2$	Option Value		$\mathrm{VaR}_{0.05}$		Optimal Sequence $\boldsymbol{\pi}^*_\omega$				
	$\pi = (1,2)$	$\pi = (2,1)$	$\pi = (1,2)$	$\pi = (2,1)$	$\omega = 0$	$\omega = 0.25$	$\omega = 0.5$	$\omega = 0.75$	$\omega = 1$
$s = 0.5$	17.7865	17.8093	−14.8091	−14.6098	(2, 1)	(1, 2)	(1, 2)	(1, 2)	(1, 2)
$s = 1.5$	17.8716	17.8171	−14.4123	−13.7516	(1, 2)	(1, 2)	(1, 2)	(1, 2)	(1, 2)
$s = 2.0$	17.9705	17.9143	−14.4424	−14.0803	(1, 2)	(1, 2)	(1, 2)	(1, 2)	(1, 2)
$s = 2.5$	18.0860	18.0576	−14.5931	−14.8854	(1, 2)	(2, 1)	(2, 1)	(2, 1)	(2, 1)
$s = 3.0$	18.1942	18.2050	−14.8093	−15.7409	(2, 1)	(2, 1)	(2, 1)	(2, 1)	(2, 1)

$$\begin{aligned} &\mathbb{E}\left[F^{(2)}\left(P_{T_1}, \tau_2\right) \middle| P_{T_1}, \tau_2\right] \\ &= \left(\frac{P^{(2)^*} D_2}{r-\alpha} e^{-(r-\alpha)\tau_2} - cD_2\right)\left(\frac{P_{T_1}}{P^{(2)^*}}\right)^{\rho} \times \mathbb{P}\left(P^{(2)^*} > P_{T_1}\right) \\ &\quad + \left(\frac{P_{T_1} D_2}{r-\alpha} e^{-(r-\alpha)\tau_2} - cD_2\right) \times \mathbb{P}\left(P^{(2)^*} \leq P_{T_1}\right). \end{aligned} \tag{25}$$

In Table 4, we present the optimal scheduling of a two-stage project when the firm has the option to delay the investment for each stage. Our results confirm that executing the stage with higher duration variability is not always harmful.

We note that, while the generalised multi-stage problem does not admit an analytical solution, it is, nevertheless, possible to solve it numerically to obtain the optimal scheduling of the serial project.

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