

ARTIFICIAL INTELLIGENCE-DRIVEN APPROACH TO IDENTIFY AND RECOMMEND THE WINNER IN A TIED EVENT IN SPORTS SURVEILLANCE

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Abstract

The proliferation of fractal artificial intelligence (AI)-based decision-making has propelled advances in intelligent computing techniques. Fractal AI-driven decision-making approaches are used to solve a variety of real-world complex problems, especially in uncertain sports surveillance situations. To this end, we present a framework for deciding the winner in a tied sporting event. As a case study, a tied cricket match was investigated, and the issue was addressed with a systematic state-of-the-art approach by considering the team strength in terms of the player score, team score at different intervals, and total team scores (TTSs). The TTSs of teams were compared to recommend the winner. We believe that the proposed idea will help to identify the winner in a tied match, supporting intelligent surveillance systems. In addition, this approach can potentially address many existing issues and future challenges regarding critical decisionmaking processes in sports. Furthermore, we posit that this work will open new avenues for researchers in fractal AI.

Keywords: Fractal AI; OWA; Multi-Criteria Decision-Making (MCDM); Data Analysis; Artificial Intelligence; Sports; Surveillance.

1. INTRODUCTION

The development of fractal artificial intelligence (fractal AI) has paved the way for accurate and efficient decision-making in several real-life domains by exploring data analytics and computational techniques.¹ This novel advancement has led to the development of intelligent machines that can extract latent information and self-similar patterns from complex systems.² These intelligent machines are used in various real-life applications, such as disease classification and diagnosis, image processing, video analytics,³ text mining and natural language processing, fraud and cybercrime detection,⁴ sentiment analysis,⁵ and recommender systems (RSs).^{6,7} Extending its applicability to other scenarios, the intelligent techniques are also used for decision-making in sports science, such as in the result prediction in a tennis match,⁸ ranking of players in football,⁹ and E-sports. Similar to other sports, these approaches have significantly contributed to cricket by identifying many decisionmaking aspects. To this end, researchers have suggested intelligent decision-making approaches for the classification of all-rounders,¹⁰ ranking and performance evaluation of players,¹¹ finding key performance indicators,¹² measuring batting parameters,¹³ selecting the best-playing XI for a particular match,¹⁴ and predicting the winner of a particular match.¹⁵

The application of these intelligent techniques in sports has improved the transparency and effectiveness of the critical decision-making process. The commercialization of sports has also led to the inception of various leagues of different sports. It has provided a lucrative opportunity for the younger generation to pursue their careers in sports, improved the skills of players, and increased the competition level. Because of the high level of competition, the winner in a game is decided by a narrow margin, and even many times the matches are tied, that is, the usual point or goal system fails to decide the winner. Some examples of tied matches are two competing teams scoring an equal number of goals in a football or hockey match, both teams scoring an equal number of runs in a cricket match, or both teams scoring equal points at the end of a basketball regulation. To cope with these situations and decide the winner in a tied match, different measures are adopted in different sports, such as free shoot-outs in football and hockey, multiple five-minute overtime until the winner is identified in basketball and a Super Over in cricket. However, these measures have some potential shortcomings; for example, in some specific scenarios, these approaches cannot provide satisfactory outcomes and fail to identify the winner, such as if both teams score equal goals in penalty shootouts in hockey/football or both teams score equal runs in a Super Over in cricket. Table 1 summarizes the condition for a match to qualify as a tie in different sporting scenarios, existing solutions for the tied match, and the limitations of the existing solutions.

We identified tied events in sports as a multicriteria decision-making (MCDM) problem that can be solved using intelligent decision-making techniques. As a case study, we considered a tie situation in a cricket match because of the popularity of cricket as a sport around the world.¹⁶ The increase in the number of nations playing cricket and becoming members of the International Cricket Council (ICC) indicates the recent inclination of many counties toward the sport. There were 30 members of the ICC in 2000, which has now climbed to 106, a rise of more than 350%. Surprisingly, the maximum number of members in the ICC is from Europe, and 33 countries from Europe play cricket. With the increasing number of participants, a new format in the game has also been introduced, known as T20 Internationals. This shorter format has attracted masses around the globe toward playing cricket. In particular, in India and its subcontinents (Pakistan, Sri Lanka, and Bangladesh), this sport has significantly contributed to their economy.¹⁷ According to a recent report, only T20 contributed a revenue of £5.983 billion.^a

To solve a tied cricket match, many approaches for deciding a winner have been adopted, such as dividing equal points between two teams, declaring a winner to a team having a better run rate in the series, and Super Over. All these solutions for a tied match have lacuna, such as deciding on a better run rate (RR) do not include the performance on a particular match for which the decision is to be made. Therefore, these rules have been replaced by new practices as they arrive. Moreover, the controversial decision of awarding the trophy at the end of the 2019 Cricket World Cup to England motivated us to find an appropriate solution for the tied match problem. To this end, we explored different aspects in which a cricket match is played, analyzed different factors that may impact the result of a cricket match, and proposed a solution for the identification of the winner in a tied cricket match. We followed a systematic approach that analyzed player performance in the form of the player score (PS) and team performance in different phases of the game to identify the winner. The PSs were calculated using a fuzzy-based MCDM operator. The technique for calculating the PS is efficient and easy

Sport	Condition for a Tied Match	Existing Solutions	Limitations
Football	Both teams scoring an equal number of goals	Free shoot-out	Takes extra time
Hockey	Both teams scoring an equal number of goals	Free shoot-out	Takes extra time
Basketball	Both teams scoring equal points at the end of the regulation time	Multiple five-minute overtime until the decision of the winner	Takes extra time
Cricket	Both teams scoring equal runs in their innings	Super Over	Takes extra time Failed in the 2019 Cricket World Cup Final

Table 1 Tied Matches in Different Sports.

^ahttps://www.europeanbusinessreview.com/how-much-money-does-cricket-make-around-the-world/ (accessed on October 20, 2022)

to incorporate and may also be used to solve other decision-making problems in cricket, including analyzing the situation of an interrupted match, deciding the winner, and identifying the best player in the series. The main contributions of this study are summarized as follows:

- Formulation and generalization of tied events as decision-making problems. To the best of our knowledge, this study is the first of its kind to investigate winners in tied events.
- We propose a multi-tier framework to identify and recommend winners in tied events, considering cricket as a case study.
- The proposed approach selects important batting features and applies ordered weighted aggregation (OWA) for the PS calculation. The proposed approach reduces the subjectivity involved in the performance evaluation of players and is easy to implement.
- To reduce the complexity of feature extraction, an expert's opinion is considered to validate the feature selection procedure.

The remainder of this paper is organized as follows. Section 2 discusses related works. Section 3 describes the importance of MCDM and the proposed framework. Furthermore, it discusses the details of the data collection, pre-processing, feature selection, weight calculation, and PS calculation. Section 4 contains the experimental results, and Sec. 5 is devoted to the discussion and future directions. Finally, Sec. 6 concludes the paper.

2. PRELIMINARIES AND RELATED WORKS

In this section, we briefly discuss the techniques used in the proposed framework. A literature review related to the proposed approach is also presented.

2.1. OWA Operator

OWA is a weighted-sum MCDM operator proposed by Yagar.¹⁸ It reduces uncertainties in the decisionmaking process using the linguistic quantifier at least half, at most half, and as much as possible. Mathematically, OWA is defined as the *n*dimensional mapping over an *n*-dimensional vector of associated weights.

OWA: $\mathbb{R}^n \to \mathbb{R}$ score can be calculated as

OWA
$$(c_1, c_2, c_3, \dots, c_n) = \sum_{i=1}^n w_i d_i,$$
 (1)

where d_i is obtained by arranging the sequence of criteria c in descending order and $w_i \in W$.

OWA has been extensively used for various MCDM applications in sports, such as decisionmaking and human resource selection,¹⁹ ranking of players in football,⁹ talent identification, and enhancement in cricket.¹⁶ Khalid *et al.*¹¹ used OWA for the performance evaluation and ranking of players in cricket.

2.2. Related Works

The literature review suggests that researchers have used soft computing and machine learning approaches for performance analysis in sports, such as football, tennis, basketball, and cricket. Fractal AI has also been investigated for many decisionmaking problems in recent years. For instance, Li *et al.*²⁰ incorporated fractal AI in exploring the trajectory and tracking the ball for table tennis. Their work is instrumental in providing a potential solution for predicting complex structures. In cricket, these approaches are used for various purposes, such as performance evaluation and ranking, predicting the outcome of a match, finding all-time best cricketers, and classifying all-rounders.

Tavana et al.²¹ proposed a fuzzy inference system for player selection and team formation by evaluating a player's performance in multiplayer sports. They designed a two-step team formation model. In the first step, player performance is evaluated using a fuzzy-based MCDM approach, and the top performers are selected and included in the team. Alternative combinations are evaluated using a fuzzy inference system, and the best combination forms the team. In a case study, they applied their model to player selection and team formation in soccer. Similarly, Qader et al.²² designed an MCDM method for the selection of football players. They proved that the technique for order of preference by similarity to the ideal solution (TOPSIS) is a good and appropriate technique for the selection of players. The proposed approach was tested on sample data of 24 players collected from a school in Malaysia. Moreover, Salih $et\ al.^{23}$ developed an MCDM method called the fuzzy decision by opinion score method (FDOSM) to overcome challenges, such as inconsistency, vagueness, unnatural comparison, and normalization, and used this method to select the best players for different positions in a football team.

In cricket, Ahmed *et al.*²⁴ proposed an MCDM approach using a multi-objective genetic algorithm to evaluate the batting and bowling performance of players. The proposed approach was tested on a dataset of auctions of the Indian Premier League (IPL) 2011, and the results are highly promising. Amin¹³ proposed a two-stage method based on regression and OWA to measure and rank the batting parameters in the T20 cricket. They tested their approach on the data of 40 batsmen from the IPL 2011 and found that the batting strike rate $(B_t S_R)$ is the most important feature that suggests the rank of a batsman in the T-20 cricket. Winner identification and target prediction in matches interrupted by rain and bad light are always a topic of concern for limited cricket. In this direction, Ali²⁵ proposed a solution to combat this problem based on player ratings, rankings, and team scores (TSs) at the time of interruption. The proposed solution is less complex and more efficient than the well-known Duckworth/Lewis method.

Moving forward, Jayanth *et al.*¹⁴ used a nonlinear kernel-based support vector machine (SVM) for winner and team predictions. The authors used k-means clustering and k-nearest neighbor (KNN) for ranking and grouping players, tested their model on the data of 2011 ICC Cricket World Cup matches, and evaluated their precision, accuracy, and recall. Similarly, Wickramasinghe¹⁵ proposed a machine learning model using naïve Bayes (NB), which predicts the winner of an ODI match after the completion of the first inning. They tested different feature selection techniques and found that univariate feature selection techniques yielded the highest accuracy of more than 85%. In another study, Premkumar *et al.*¹² used the factor analysis method to rank batsmen and bowlers in ODI cricket and successfully minimized the subjectivity through a proper feature selection and batsman score calculation. However, they did not consider the experience of a batsman in PS calculations and rankings. This limitation exposes a severe shortcoming of their approach as it weights experienced and inexperienced players equally.

Li *et al.*²⁶ proposed a two-step data-driven approach for the performance prediction of teams in basketball. The proposed approach uses multivariate logistic regression analysis to estimate the winning probability and match outcome. Then, it uses data envelopment analysis (DEA) to analyze player portfolios and schedule optimal playing positions for each player by analyzing historical data. Multiple experiments were performed to test the efficiency of the proposed approach, and the results are promising.

A summary of the related works is presented in Table 2. It shows the use of AI-based techniques in performance analysis and decision-making in various sports. However, studies addressing the issue of deciding a winner in a tied event in sports are lacking. Therefore, we propose a solution and the design

Reference	Prime Contribution	Technique/Tools	Data	Limitation
Tavana <i>et al.</i> ²¹	Player selection and team formation by evaluating players' performance in multi-player sports	Fuzzy inference system	Professional soccer team based in Tehran	The proposed approach of team formation and player selection lacks a good level of accuracy as it has been discussed in the paper
Ahmed <i>et al.</i> ²⁴	Performance evaluation of batsmen and bowlers; player ranking	Multi-objective genetic algorithm	Data of players who played in the IPL 2011	Considered very few features for player ranking
Amin et al. ¹³	Measure the most important ranking parameter in T20 and argued that the strike rate is the most important factor	Two-stage regression OWA	Data of players who played in the IPL 2011	Tested on a small dataset of 40 batsmen

Table 2Summary of the Related Works.

(Continued)

Reference	Prime Contribution	Technique/Tools	Data	Limitation
Ali ²⁵	Winner identification/target prediction in interrupted matches It is based on the player rating, ranking, and score of the teams at the time of interruption	Proposed different mathematical formulas for different kinds of interruption	Data of the match played between Sri Lanka and South Africa in the ICC Cricket World Cup	Did not validate the proposed approach by using real matches for all different situations
Qader $et \ al.^{22}$	Selection of football players	TOPSIS	A sample data of 24 players collected from a school in Malaysia	Tested on a very small dataset
Jayanth <i>et al.</i> ¹⁴	Team prediction, ranking, and grouping of players	Nonlinear kernel-based SVM, KNN, and <i>k</i> -means	Data of matches and players who played in the 2011 ICC Cricket World Cup	Ignored the experience
Wickramasinghe ¹⁵	Predicted the outcome of a match after the completion of the first inning Tested different feature selection approaches and argued that the univariate is the best with 85% accuracy	NB	Data of the ODI series played between September 30, 2018 and January 1, 2020	Player ranking is missing, which is one of the important indicators of team strength. Types of bowlers in the team were not considered for predicting the outcome of the match
Premkumar <i>et al.</i> ¹²	Rank batsmen and bowlers in cricket	Factor analysis method	The ICC rank list of December 31, 2015	Ignored the experience of the players
Salih <i>et al.</i> ²³	Selected best players for different positions in a football team	FDOSM	A sample data of 24 players collected from a school in Malaysia	They have chosen only one fuzzification one defuzzification technique without testing the results of other techniques Tested on a very small dataset
Y. Li <i>et al.</i> ²⁶	Performance prediction of teams in basketball	Multivariate logistic regression analysis, DEA	National Basketball Association and Golden State Warriors dataset	They have considered match-specific details, such as opposition strength and strategies, for predicting performance

Table 2(Continued)

of a framework to address this issue by considering cricket matching as a case study.

3. MCDM FRAMEWORK FOR DECIDING THE WINNER IN A TIED EVENT IN SPORTS

MCDM is primarily concerned with conceptualizing how to weigh different features that may have an impact on decision-making. However, when a deadlock-like situation in an event occurs, which we refer it to as a "tied event," happens, for example, an equal number of votes for a contestant in an election or a tied match in any sport, it becomes tough and needs an adequate approach to identify and recommend the winner. To address this issue, we consider cricket as a case study. Fortunately, the 2019 ICC Cricket World Cup is a real scenario for

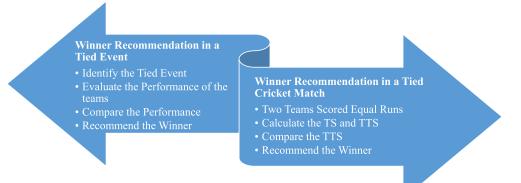


Fig. 1 Winner recommendation in a tied match.

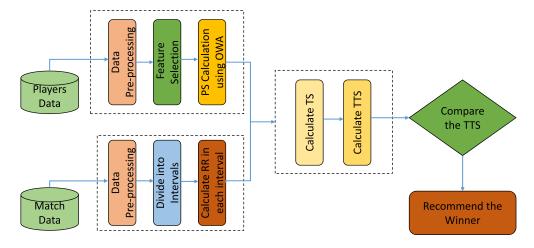


Fig. 2 Architecture of the proposed framework for the recommendation of a winner in a tied match.

the above problems, which have been extensively addressed in this study. Figure 1 presents the general framework for identifying and recommending the winner in a tied match. In the first step, we identified whether a match is tied. In the case of a tied match, team performance was evaluated by calculating the TS and total team score (TTS). Subsequently, the performances of both teams were compared to identify and recommend the winner. Moreover, the framework specifically for our case was elaborated with the help of Fig. 2 and is discussed in the following sections.

3.1. Cricket as a Case Study

ODI cricket is also known to be limited cricket. It is played between two teams. Each team has 11 players, receives an inning of 50 overs to bat and sets the target for another team, or chases the target set by the team that batted first. The team that scores at least one run more than its counterpart in the stipulated 50 overs before losing all 10 wickets is declared the winner of the match. Sometimes, runs scored by both teams are equal, which is called a tied match, and the decision of the winner becomes suspicious. Experts have suggested some methods to decide the winner in a tied cricket match, such as comparing the RR of both teams or comparing the number of wickets taken by both teams. However, such methods were rejected because of their various limitations.

Recently, the ICC introduced Super Over for deciding a winner in a tied match in limited cricket. In Super Over, every team is asked to bat for one extra over (an over consists of six balls), and the team scoring more runs is recommended as the winner. However, what happens next if there is a tie in the Super Over? No satisfactory solution for this has been proposed by the ICC.

A recent incident in the final match of the 2019 ODI World Cup played between England and New Zealand was the same. Both teams scored an equal number of runs in the Super Over. The winner in the match was decided on the basis of the number of boundaries scored, which was proven to be one of the most controversial decisions that the ICC had made to declare a winner in a mega event like the Cricket World Cup. The decision was criticized by many people around the globe, including cricketers, movie legends, and fans. This matter motivated us to explore a better way to identify winners in tiedlimited-over cricket matches. Therefore, we developed an efficient and promising approach that is easy to incorporate in solving the above issue.

In this direction, we propose a multi-tier framework for recommending the winner in a tied cricket match. The proposed framework considers player ranking in terms of the PS, evaluates the team performance in terms of the scoring rate and batsmen out in different intervals of a match, then calculates the TS in each interval, and aggregates the TS to find the TTS. Finally, the TTS of both teams was compared to identify and recommend the winner. The steps of the proposed framework are discussed as follows.

3.1.1. Data

The first step in the proposed framework is to collect the player and match data that are available on the official websites of the ICC^b and ESPN.^c These data were saved in the appropriate format. The features of the data representing the batting ability of players are presented in Table 3. The given features show variable correlations with batting performance. However, considering all these features to calculate the PS increases computational complexity. In addition, some features have an impact on others, so reusing them will be redundant. Therefore, we analyzed the impact of different batting features on the performance of players and selected important features for calculating the PSs. The details of the feature selection and performance evaluation are explained in the following sections.

3.1.2. Feature selection and extraction

We selected important batting features to calculate the PS with reduced subjectivity and transparency. The batting average (BA) is an important indicator of player performance, and many researchers have considered the BA as the sole feature for evaluating performance and ranking players. However, ODI cricket is played under certain constraints, such as a fixed number of overs in an inning and restrictions on wide balls, as compared to test cricket. Therefore, a batsman with a high BA and consuming many balls is unsuitable for this format. Hence, the $B_t S_R$ is another significant feature selected for the performance evaluation. Experience is an important aspect of all domains of life. A player who has played more matches and scored more runs in his career has the experience of different playing conditions and is more capable of overcoming a pressure

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Sl. No	Feature	Data Type	Description
1	Name	String	Name of player
2	Matches	Integer	Number of matches played in career
3	Innings	Integer	Number of innings in which the player batted
4	Not out	Integer	Number of not-out innings
5	Runs	Integer	Total runs scored in a career
6	$_{ m HS}$	Integer	Highest score
7	BA	Float	BA
8	$_{\mathrm{BF}}$	Integer	Balls faced in career
9	$B_t S_R$	Float	$\mathrm{B}_t\mathrm{S}_R$
10	Hundred	Integer	Number of hundreds scored
11	Fifty	Integer	Number of the fifties scored
12	Zeros	Integer	Number of times he got out on zeros
13	Fours	Integer	Fours hit in a career
14	Sixes	Integer	Sixes hit in career
		0	

Table 3 Data Description.

^bhttps://www.icc-cricket.com/about/members/ (accessed on October 20, 2021)

^chttps://www.espncricinfo.com/ (accessed on October 20, 2021)

Table 4FeaturesSelected and Extracted toCalculate the PS.

Sl. No.	Feature	Description/Formula
1	Matches	Number of matches played
2	Runs	Runs scored in career
3	BA	$\frac{\text{Runs}}{\text{Innings}-\text{Not Out}}$
4	$\mathbf{B}_t \mathbf{S}_R$	$\frac{\text{Runs}}{\text{BF}} \times 100$
5	$M_s R_t$	$\frac{\text{Fifties} + \text{Hundreds}}{\text{innings}}$

situation and winning a match. Therefore, the number of matches played and runs scored in a career were also selected as two features to calculate the PS. The consistency of players playing big innings and a score of 50-plus runs frequently is an indicator of player greatness and capability. A feature named milestone ratio $(M_s R_t)$ was extracted using the number of fifties and hundreds scored and the number of innings played. The formulas for $M_s R_t$ are presented in Table 4.

To validate the extracted features, we adopted the approach discussed by Shahab *et al.*²⁷ and contacted many individuals to obtain suggestions from experts. We received 50 expert suggestions regarding the feature selection. The familiarity of these experts, who are academicians, computer science graduates, research scholars and cricket-familiar individuals, is shown in Fig. 3. Familiarity is indicated on a scale of 1 to 5, where 1 is for a cricket naïve and 5 is for a cricket lover with great familiarity of the sport. These experts were requested to provide features for deciding the performance of a team in a cricket match. All the features for which at least three experts agree were retained. Coincidently, we received a precision of 0.84 for the above five features, which are considered to calculate the PS.

Other features, such as the number of not-out, number of fours and sixes, and highest score, were ignored as most of the experts have their suggestions against these features, and no more than two experts have favorable opinions for these features. In addition, precise feature selection would help reduce the curse of dimensionality and improve the performance of the aggregation function. Table 4 provides a description and formula of the features considered for calculating the PS.

3.1.3. PS calculation for the performance evaluation

Player performance was evaluated in terms of the PS calculated using OWA. A very important aspect of OWA is that criterion c is not associated with any weight, but weights are associated with a specific order of criteria. This was achieved by sorting and rearranging the criteria in descending order. The OWA weights were calculated using three different quantifiers, i.e. at least half, at most half, and as many as possible. For details on calculating weights and understanding how different linguistic quantifiers behave, please see Refs. 28 and 29. The calculation process for OWA is illustrated using an example. For example, if the number of criteria n = 5, then for as many quantifiers as possible, the parameters are a = 0.3, b = 0.8, and the corresponding weight vector W obtained using the fuzzy linguistic quantifier is given by the following equation:

$$W = \begin{pmatrix} 0.0\\ 0.2\\ 0.4\\ 0.4\\ 0.0 \end{pmatrix}.$$
 (2)

Then, the OWA score can be calculated as follows:

 $\begin{aligned} & \text{OWA}(0.97 \quad 0.89 \quad 0.76 \quad 0.83 \quad 0.71) \\ &= 0.0 * 0.97 + 0.2 * 0.89 + 0.4 * 0.83 \\ &\quad + 0.4 * 0.76 + 0.0 * 0.71 = 0.746. \end{aligned}$

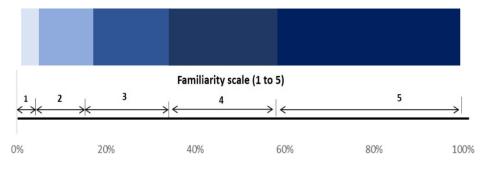


Fig. 3 Distribution of experts over a 1–5 scale for their familiarity with cricket.

Similarly, for at-most-half quantifiers, the parameters are a = 0 and b = 0.5, and the corresponding weight vector W and OWA score are given as follows:

$$W = \begin{pmatrix} 0.2 \\ 0.4 \\ 0.4 \\ 0.0 \\ 0.0 \end{pmatrix},$$

$$OWA(0.97 \quad 0.89 \quad 0.76 \quad 0.83 \quad 0.71)$$

$$= 0.2 * 0.97 + 0.4 * 0.89 + 0.4 * 0.83$$

$$+ 0.0 * 0.76 + 0.0 * 0.71 = 0.81. \quad (3)$$

For the at-least-half quantifier, the parameters are a = 0.5 and b = 1, and the weight vector W and OWA score are as follows:

$$W = \begin{pmatrix} 0.0\\ 0.0\\ 0.2\\ 0.4\\ 0.4 \end{pmatrix}$$

$$OWA(0.97 \quad 0.89 \quad 0.76 \quad 0.83 \quad 0.71)$$

= 0.0 * 0.97 + 0.0 * 0.89 + 0.2 * 0.83
+ 0.4 * 0.76 + 0.4 * 0.71 = 0.91. (4)

3.1.4. Winner recommendation algorithms

The primary objective of this study is to identify which team has played better cricket in a tied cricket match and accordingly recommend the winner. For this purpose, we propose two algorithms. Algorithm 1 is a multi-tier algorithm that considers the team strength by calculating the PS of all players who played the match, and each inning is divided into five intervals with a maximum of 10 overs. The algorithm analyzed the performance of the team in each interval by calculating the TS. The TS was calculated by considering the RR and players in each interval. The TS of all intervals was aggregated to compute the TTS, which is used for decision-making in Algorithm 2. If the match is tied, that is, at the end of the match, if runs scored by both teams are equal, then Algorithm 2 compares the TTS of both teams and a team with a higher TTS is recommended as the winner.

Algorithm 1. Calculating the TTS.

 $TSi = Team \ score \ in \ Interval \ I$ $RRi = Run \ rate \ in \ Interval \ I$ $RSi = Runs \ scored \ in \ Interval \ I$ $Oi = Overs \ bowled \ in \ Interval \ I$

TTS = 0;Wickets = 0; For intervals 1 to 5; While (wickets < 10) {

 $RRi = \frac{RSi}{Oi};$

If the number of wickets falls in Interval I is k;

$$Wickets = wickets + k;$$

$$TS_i = RR_i - \sum_{j=0}^{k} (PSj);$$

$$TTS = \sum_{i=1}^{5} (TSi);$$

}

Algorithm 2. Recommending the Winner.
TTS(A) = TTS of Team A;
TTS(B) = TTS of Team B;
Compare the TTSs of Team A and Team B
If $(TTS(A) > TTS(B))$
Winner = Team A;
Else
Winner = Team B;

4. EXPERIMENTAL RESULTS

To test and validate our framework, we extracted the data from a tied match of the 2019 World Cup Final played between England and New Zealand. The experiments were performed on an Intel(R)Core(TM) i5-9300H CPU @ 2.40 GHz machine with 8 GB RAM, 500 hard disks, and Windows 10, 64-bit operating system. We used the Jupyter Notebook³⁰ based on Python 3.8.4. Several Python libraries, such as Numpy, Pandas, and Matplotlib, were used to perform the experiments. The Jupyter Notebook is a highly efficient tool used in data science for feature engineering, exploratory data analysis, prototyping, and model comparison and deployment.³¹ We extracted the player data from both teams and saved the data in csv format, preprocessed the data to remove noise, and then selected and extracted

Sl. No.	Player Name	At Most Half	As Many as Possible	At Least Half
1	Martin Guptill	0.799	0.757	0.561
2	Henry Nicholls	0.586	0.302	0.177
3	Kane Williamson	0.802	0.675	0.539
4	Ross Taylor	0.978	0.846	0.596
5	Tom Latham	0.608	0.411	0.290
6	James Neesham	0.624	0.282	0.158
7	Colin de Grandhomme	0.605	0.214	0.104
8	Mitchell Santner	0.565	0.267	0.117
9	Matt Henry	0.496	0.158	0.055
10	Trent Boult	0.4362	0.159	0.045
11	Lockie Ferguson	0.258	0.094	0.033

Table 5 PSs of New Zealand Calculated using Different Quantifiers.

Table 6 PSs of England Calculated using Different Quantifiers.

Sl. No.	Player Name	At Most Half	As Many as Possible	At Least Half
1	Jason Roy	0.772	0.472	0.358
2	Jonny Bairstow	0.787	0.437	0.313
3	Joe Root	0.831	0.671	0.531
4	Eoin Morgan	0.906	0.789	0.568
5	Ben Stokes	0.712	0.448	0.323
6	Jos Buttler	0.840	0.586	0.405
7	Chris Woakes	0.578	0.323	0.165
8	Liam Plunkett	0.581	0.265	0.115
9	Jofra Archer	0.235	0.037	0.013
10	Adil Rashid	0.586	0.261	0.111
11	Mark Wood	0.415	0.109	0.034

important features. We then applied OWA to calculate the PSs using the three different quantifiers discussed above. Tables 5 and 6 present the calculated PSs of the New Zealand and England teams, respectively, using different quantifiers. Subsequently, we applied our proposed algorithms to the observed match data, and the TTS was calculated and compared to recommend the winner. The statistics on performance, such as the RR, players out, and the sum of the players' scores using different quantifiers, namely, at most half, as many as possible, and at least half in each interval, are presented in Table 7 for New Zealand and Table 8 for England. The performance statistics show that the RR of both teams varied in each interval, and it was the highest for both teams in the last interval, which is a normal trend in limited-over cricket.

The match statistics of England inning show that after a promising start in the first interval, England lost momentum in the second interval as their RR slowed down, and they also lost two of their important players in the second interval. This was a

Interval	Over Range	$\mathbf{R}\mathbf{R}$	Players Out	Sum of Players' Scores Out in an Inter		
				At Least Half	As Many as Possible	At Most Half
I-1	1 to 10	3.3	M Guptil	0.561	0.757	0.799
I-2	11 to 20	5.8		0	0	0
I-3	21 to 30	3.5	Williamson, Nicholls	0.716	0.977	1.388
I-4	31 to 40	5.3	Tailor, Neesham,	0.754	1.128	1.602
I-5	41 to 50	6.2	Grandhomme, Latham, Henry	0.449	0.783	1.709

Table 7 Match Statistics of New Zealand.

Interval	Over Range	$\mathbf{R}\mathbf{R}$	Players Out	Sum of Player	m of Players' Scores Out in a	
				At Least Half	As Many as Possible	At Most Half
I-1	1 to 10	3.9	Roy,	0.358	0.472	0.772
I-2	11 to 20	3.4	Root, Bairstow	0.844	1.108	1.618
I-3	21 to 30	4.2	Morgan,	0.568	0.789	0.906
I-4	31 to 40	5.5		0	0	0
I-5	1 to 10	7.1	Butler, Woakes, Plunkett, Archer, Adil, Mark wood	0.843	1.581	3.236

 Table 8
 Match Statistics of England.

huge setback that they had never overcome. Table 8 shows that England's performance improved in the third and fourth intervals, in which they not only improved their RR but lost only one wicket in the third interval, and they did not lose any wicket in the fourth interval. The TTS difference of the second interval was so high that after a relatively improved performance in the third and fourth intervals, England's TTS was below New Zealand's TTS after the fourth interval. To overcome the target, England scored with a high RR in the last interval, but in the process, they lost six wickets because their TTS was lower than that of New Zealand at the end of the match. The same trend is observed in Figs. 4, 5 and 6. In Fig. 5, the PS using the as-many-as-possible quantifier is considered to calculate the TS and TTS and in Fig. 6; the PS calculated using the at-most-half quantifier for obtaining the TS and TTS is shown. We discuss the TTS trend, which considers the at-least-half quantifier for the PS calculation. It is the most highly rated and favored PS calculation and ranking technique by cricket experts.¹¹

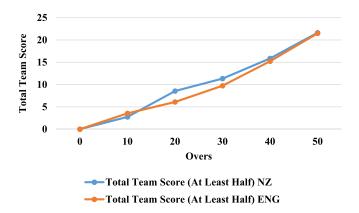


Fig. 4 Performance comparison when the PS is calculated using the at-least-half quantifier.

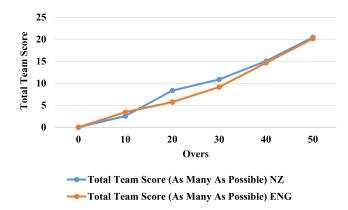


Fig. 5 Performance comparison when the PS is calculated using the as-many-as-possible quantifier.

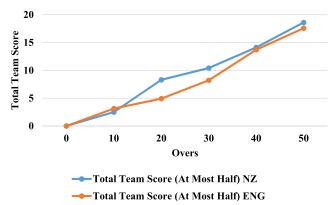
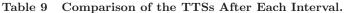


Fig. 6 Performance comparison when the PS is calculated using the at-most-half quantifier.

Table 9 presents the TTS of both teams after each interval, which is calculated using Algorithm 1, and after incorporating the PS with three different quantifiers, namely, at most half, as many as possible, and at least half, which has been discussed in the previous section. Figure 7 is a graphical representation of Table 9, in which the PS calculated using the at-least-half quantifier is considered for the calculation of the TS and TTS. The blue

Overs		ГS st Half)		ΓTS γ as Possible)	_	ГS st Half)
	NZ	ENG	NZ	ENG	NZ	ENG
10	2.739	3.542	2.543	3.428	2.501	3.128
20	8.539	6.098	8.343	5.72	8.301	4.91
30	11.323	9.73	10.866	9.131	10.413	8.204
40	15.869	15.23	15.038	14.631	14.111	13.704
50	21.62	21.487	20.455	20.15	18.602	17.568



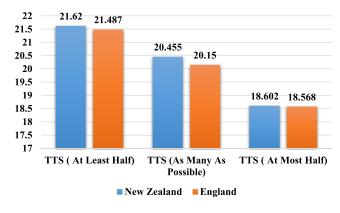


Fig. 7 Comparison of the TTS using different quantifiers to calculate the PS.

line represents the performance of the New Zealand team, and the red line represents the performance of the England team in terms of the TTS. The graph shows that the TTS of New Zealand was lower than that of England in the first interval. After the first interval, the TTS of New Zealand grew very fast and was always better than the TTS of England until the end of the match. The reason can be seen in Tables 7 and 8, which show that New Zealand followed a cautious approach to scoring runs and lost one of their players with a high PS in the first interval, whereas England scored briskly in the first interval and lost a player with a lower PS than New Zealand. After a slow and cautious start in the first interval, New Zealand scored with a high RR and without losing any wicket in the second interval. This situation boosted their TTS to a greater height, after which their TTS grew at a steady rate in the third and fourth intervals and again grew rapidly in the last interval. This is the ideal approach for batting an ODI match.

The trend is almost similar when the PS calculated using different quantifiers is considered for the TS and TTS calculations. Figure 8 shows that at the end of the match, the TTS of New Zealand is better than that of England when the PS calculated using

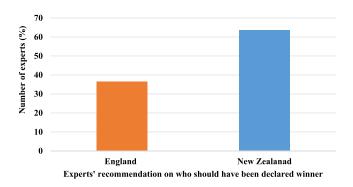


Fig. 8 Graphical representation of the percentage of experts — "for and against" the decision of ICC for winning the 2019 World Cup final match.

three different quantifiers is considered. Hence, the TTS calculated and compared with the proposed algorithms showed that New Zealand played better cricket than England and should be recommended as the winner of this particular match.

5. DISCUSSION AND FUTURE DIRECTION

5.1. Performance Evaluation

The decision-making process in every domain of life is tricky but plays a vital role in many situations.

Ref.	Matches	Runs	BA	$\mathbf{B}_t \mathbf{S}_R$	$(\mathbf{M}_{s}\mathbf{R}_{t})$
Ahmed <i>et al.</i> ²⁴	×	\checkmark	\checkmark	×	×
Amin et al. ¹³	×	×	×	\checkmark	×
Jayanth <i>et al.</i> ¹⁴	×	×	\checkmark	\checkmark	\checkmark
Premkumar <i>et al.</i> ¹²	\checkmark	\checkmark	×	\checkmark	×
Proposed Approach	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark

Table 10Features Included in Different Papers forEvaluating the Performance of Players.

However, to reduce subjectivity and bias in it, the following steps can be helpful³²:

- (1) Taking the consensus of a large group of people through voting.
- (2) Clarity in the selection of decision variables.
- (3) Clarity in mapping decision variables.
- (4) Assigning weights to the decision variables.

The PS calculation and winner decisions in a tied match are MCDM problems. We followed a multicriteria approach for calculating the PS and quantifying their attributes by aggregating several features. The different features that are considered in this study have an edge over other related works for two primary reasons: (i) No related work has experts' consensus for the features to be considered. (ii) Not all five features have been used by any existing work. Table 10 compares the number of features selected in our study with the total features considered in the literature. This shows that we have incorporated most features and that all these features are endorsed by 84% of the experts who participated in the survey. On the one hand, it proves that the features selected in the proposed approach are valid; on the other hand, it also provides a novel way of exploring players² performance and, hence, a mechanism for overall team evaluation. In addition, we normalized the features and assigned weights using a fuzzy linguistic quantifier. Subsequently, we used OWA to calculate the PS. All of these steps help reduce subjectivity.

Furthermore, the decision to declare England as a winner over New Zealand in the final match of the 2019 Cricket World Cup was something that many scholars and cricket legends would disagree with. In addition, we surveyed the opinions of many individuals regarding the ICC method for deciding the winner.

Approximately 50 experts are considered for this, and the percentage of people who go for and against the decision of the ICC is shown in Fig. 8. This indicated that the experts suggested New Zealand as a winner. With the proposed multitier aggregation approach based on the OWA operator, we argue that our approach coincides with the experts' suggestions. This supports our claim that the proposed approach can be considered a benchmark for decision-making problems in similar situations.

5.2. Future Challenges and Opportunities

In this section, we identify and discuss some open issues and possible areas in which the suggested approach can be applied. The proposed method is meant for deciding a winner in a cricket match when the game is tied, and a Super Over cannot be enough to decide the winner. There are several situations in which the approach can be applied in the same manner or with some modifications according to the requirement. In addition, it can be applied to other sports.

5.2.1. Deciding the best player of a match/series

In cricket, the decisions of the players of the match and series are based on the opinions of a few people. There is no formal method for deciding who will be awarded the best player award. On many occasions, experts have raised concerns about their choices. The proposed PS calculation approach can be used judiciously to determine the best player of a match/series.

5.2.2. Decision-making in interrupted matches

Matches interrupted by rain or bad light always create problems. Many methods have been suggested by different scientists to handle this problem, but owing to various limitations, they are rejected only after a few trials. The method suggested by Duckworth and Lewis, also known as the D/L method, is now being used for decision-making in interrupted matches. However, this method is complex. In the future, a robust algorithm using PSs for decision-making in interrupted matches can be designed to solve the problem of interrupted matches.

5.2.3. Application to other sports

For matches played in different sports, such as football and hockey, the suggested approach can be applied by selecting important features, and we may calculate the PS that can be used for winner decision-making, performance evaluation, and ranking of players. With the advent of advancements in AI, many organizing bodies, such as FIFA, are looking to bring in changes and apply computational intelligence for this purpose.

5.3. Limitations

Evidently, the ground truth is that nothing is perfect in this world. Despite its advantages and usefulness, this study also has some limitations. One is that the proposed approach was tested on a limited dataset of cricket only. Another limitation is related to the feature selection from the data. This study follows human intelligence and prior knowledge of feature selection rather than an automated algorithm.

6. CONCLUSION

The application of fractal AI has rapidly emerged in recent years for decision-making in complex situations. In this work, we suggest a method to decide a winner in a scenario where the performance of the competitors results in a tie. The primary intention is to identify and extract the factors influencing the performance of the subject and accordingly aggregate these features to determine the cumulative score that enables us to identify the winner on the fly during sports surveillance. We considered cricket as a case study because of its wide acceptance as a global event and real-life scenario to test the ordered weighted aggregation operator performance. Moreover, to the best of our knowledge, search and efforts, it is the first implementation of OWA for deciding the winner in a tied event, supporting intelligent surveillance for the sports domain.

In addition, the present protocol practiced by the ICC has recently received great criticism. By contrast, our approach comprehensively considers the aspects of playing a cricket match, players' performance, and their scores, which were calculated using fuzzy-based aggregation and overall team performance, aggregating the performance of all active players. The winner is recommended by following a multi-tier approach for data collection, feature selection and extraction, PS calculation, TS, and TTS calculation.

Furthermore, the proposed approach can help the decision-makers and stakeholders of the concerned issue to prevent dissatisfaction among viewers, supporters, and players. In addition, it can be used as a benchmark that would allow a fair process for identifying a winner in a tied event in general and in a cricket match in particular. It is envisaged that the proposed approach can serve as a benchmark for researchers actively searching for state-of-the-art AI-driven methods to find a decision-making solution for similar scenarios.

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