Chapter

Creating a Culturally Responsive Mathematics Education: The Case of Gebeta Game in Ethiopia

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Abstract

This chapter presents an example of creating a culturally responsive mathematics education in Ethiopia using a locally available game called Gebeta. The study is framed using two theoretical frameworks: funds of knowledge and cultural commognition. To this end, an ethnographic study has been employed, which helps to describe, analyze, and investigate a particular group, culture, or community. The findings show that the game is well situated as a developed body of knowledge and skills in the culture. It has remarkable potential to foster mathematical thinking and communication at different levels of the school context. Specifically, in terms of mathematical thinking, the game has affordances to foster early numerical and algorithmic thinking. Educators and stakeholders involved in designing tasks and activities for the curriculum and syllabus should consider incorporating the Gebeta game and other culturally available activities to embed them as part of formal school mathematics in a meaningful way.

Keywords: culture, communication, cognition, funds of knowledge, Gebeta

1. Introduction

The content of the mathematics curriculum and classroom practices in schools used to be dominated by the view that mathematics is a value-free body of knowledge disassociated from its cultural and social context [1–4]. According to Stephen Lerman, the social turn in mathematics education has challenged such practices [5] since, for sociocultural theory learning theorists, learning is a social phenomenon, that is. knowledge is acquired in social settings [4–6]. While emphasizing the importance of a child as an independent learner, Vygotsky defined the "more knowledgeable other (MKO) as anyone who has a better understanding or a higher ability level than the learner, particularly regarding a specific task, concept, or process," [6]. Hence, learning happens with the presence of an MKO in a given social context, creating the opportunity for discourse.

Prahmana reflected that Freudenthal and Ubiratan D'Ambrosio had reacted to the movement in the USA in the teaching and learning of mathematics, called

"back to basics," which focused solely on mathematical procedures and rules [7]. As a result, D'Ambrosio initiated the idea of Ethnomathematics, while Freudenthal introduced realistic mathematics education (RME). The former focuses on "mathematics exploration from ideas, methods, and techniques a society uses to respond to its environment," and the latter "emphasizes the students' thinking levels and the mathematization process from surrounding phenomena" ([7], p. 257). RME uses real context from the students' surroundings. At the same time, ethnomathematics emphasizes values while reinventing the concept of mathematics, and both somehow consider the reality and the social and cultural context of the learners. If school mathematics should be rooted in these principles and values mentioned briefly, the curriculum (intended-implemented-attained) plays a vital role in helping teachers and learners realize it [4]. The challenge has been how to implement such ideas in practice. This chapter aims to contribute to such endeavors within Ethiopia and other countries where such emphasis is needed across the globe. Hence, we first define the Ethiopian context and the game under consideration in this study, which can be used in the teaching and learning of mathematics at different school levels in Ethiopia.

2. Setting and research question

In Ethiopia, one of the oldest countries in the world, there are 294 local games, according to the Cultural Sports Office of the country. Only 11 have been considered official cultural sports for competitions in the country. The Gebeta (ገበጣ) game is one of these 11 games that have obtained official status. It is played across towns and rural areas in the country. It has many different versions with various local rules. However, the nationally recognized game follows standard rules, similar to the internationally recognized version of Mancala. Studying such cultural activities for education, particularly mathematics education, is an ongoing concern for many educators in the country [4, 8, 9].

Such games and activities, which are locally available and accessible, provide out-of-school mathematical activities [10]. Kids engaged with such games at an early age are already exposed to different mathematical concepts that will be learned formally within the school curriculum. If school mathematics utilizes such opportunities to learn the subject, student learning could be optimized. The school mathematics curriculum and instruction must systematically embed such games [11]. The challenge is finding example activities and tasks rooted in the culture and situating those activities to fit within a given curriculum. Thorough investigation and rigorous work are needed if such locally available games and activities are to be integrated into the formal education system *via* the curriculum, syllabus, and textbook to inform the teaching and learning of mathematics. Partly, this need motivates us to fill the gap in Ethiopia's education system's curriculum and instruction. Therefore, we wanted to explore and investigate the potential of the Gebeta game to support mathematics teaching and learning in the country. So, we asked the following questions: a) To what extent is Gebeta an available cultural game in society? b) Which mathematical concepts, operations, and procedures are embedded in the Gebeta game? c) How can this be integrated into the education system? These questions shape the present research at hand, and this paper shows examples of the affordances found in the culture to attain a bigger picture.

3. Theoretical perspectives

The study about the availability and relevance of the Gebeta game in the culture and the opportunity embedded in the game for the teaching and learning of mathematical concepts and procedures should be grounded theoretically. For this reason, two different theoretical frameworks that complement each other for the chosen study are presented as follows.

3.1 Funds of knowledge (FoK)

Moll coined the term funds of knowledge (FoK) to refer to "the historically accumulated and culturally developed bodies of knowledge and skills" ([12], p. 133). They studied household knowledge to foster a participatory pedagogy that would enable educators to facilitate the transfer of knowledge between home and school contexts. As a result, they developed a framework with broader and more diverse knowledge, including agriculture, mining, medicine, economics, religion, and others. Given that FoK emanates from a given household or community, the list can also include different plays and games embedded in that household or community.

According to Bishop, playing is one of the six mathematical activities found in almost every culture [1]. It is possible to argue that the games available at home or in their community that interest children can be counted as their FoK [2]. Games provide opportunities for community participation, which is crucial to social learning and cognitive development [13]. Chesworth indicated a FoK approach to strengthening curriculum and pedagogical decisions informed by children's play choices and interests [14]. Of course, what interests children in play is a complex matter. Chesworth argues that "FoK offers an alternative mode for understanding children's interests within the context of their participation in the activities for their homes, classrooms, and communities" ([14], p. 296). Out-of-school mathematics is another terminology that emphasizes the nonformal mathematical-related activities learners engage in daily life [10].

3.2 Cultural commognition

Commognition is another interpretation of sociocultural learning theory focused on education. It comprises two words, communication and cognition, signifying the unity of interaction and thinking [15, 16]. This means that doing mathematics in this theoretical framework includes both thinking and communicating it. In this regard, culture can serve as a mediator for forming and developing competencies, such as problem-solving and critical thinking. Radford argued for the connection of mathematical thinking (cognition) to its historical and cultural situatedness and further stated: "I will suggest that thinking in general, and mathematical thinking (MT) in particular, are forms of reflective, mediated social praxis where the organization of individuals' sensuous cognitive processes is related to the meaning of things as they become objectified in practical and theoretical activity" ([17], p. 440).

Further, it was inspired by scholars, such as Brenner, who added: "Cognition to the formula for culturally relevant instruction in mathematics" ([18], p. 214). Turner synthesizes findings that integrate children's mathematical thinking with cultural FoK (CFoK) in mathematics instruction [19]. Therefore, cultural commognition refers to the mathematical thinking and communication situated in the given cultural

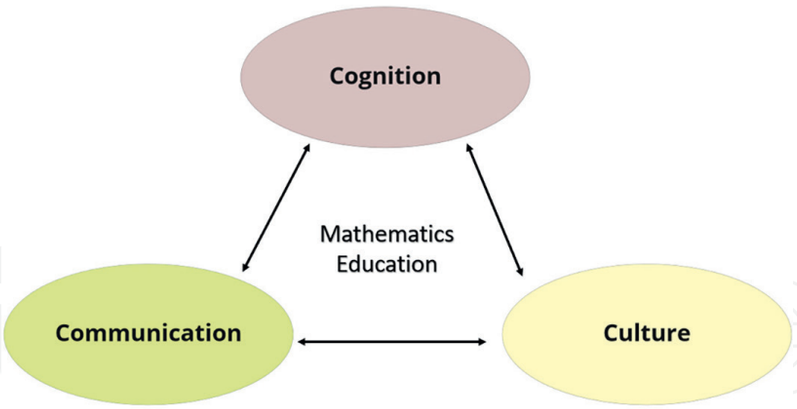

Figure 1.
Cultural commognition in mathematics education is inspired in Ref. [20] work of interconnection that involves society, cognition, and culture.

setting and context. Inspired by those who argued that mathematical production is subsidized by an interconnection involving society, cognition, and culture, this work presents the interconnection between communication, cognition, and culture [20], as shown in **Figure 1**.

3.3 Introducing Gebeta as an artifact

Culture-specific tools are integral in how learners organize and think about mathematical concepts and procedures [11]. "Gebeta" is one of the ancient African board games. It originated from ancient Ethiopians and was discovered by archeologists in Eretria, a place called Matara, which was then part of Ethiopia (see **Figure 1a** & **d**). Generally, it is also referred to as Mancala in other locations. However, the type of game described in this article is unique to Ethiopia. Tesfamicael argued that "it is difficult to find structured literature about the Gebeta game in the country." Nevertheless, some evidence was uncovered by archeologists who searched for relics in the northern part of the country.

They found a Gebeta-playing artifact and estimated that the game was practiced in Ethiopia from the sixth to seventh BC. The game can be played in both rural areas and towns. Players can dig 12 or 18 equal-sized small half-sphere-shaped holes in the ground and collect 48 small stones, beads, or seeds, placing four in each hole. Alternatively, the game can be made on a wooden material. **Figure 2** provides examples of 12-hole (homes) Gebeta artifacts, including both old and modern versions. Additionally, a digital version created using block programing, Scratch, is included.

3.4 Mathematical thinking (MT) *via* Gebeta game

MT is a complex matter to deal with. If one uses ChatGPT to get an answer to the question, "What is MT"? then ChatGPT provides a comprehensive definition as follows [21]:

> *Mathematical thinking is a cognitive process that involves analyzing, reasoning, problem-solving, and making connections using mathematical concepts and principles. It goes beyond simply performing calculations and involves deeper levels of understanding and manipulation of mathematical ideas.*

Figure 2.
a) Gebeta playground made on the ground b) virtual Gebeta made of scratch block programming. c) Commercial Gebeta (mancala) game boards available www.kodkod.com. d) Gebeta game board made in Zege monastery in Lake Tana.

To highlight the process aspect further, the ChatGPT definition provides ten aspects of MT: abstraction, logical reasoning, pattern recognition, problem-solving, creativity, symbolic manipulation, critical analysis, visualization, generalization, and communication. Stacey emphasized the importance of MT as it is a goal of schooling, a way of learning, and teaching mathematics [22]. Furthermore, Stacey explained MT using two pairs of processes: specializing vs. generalizing and conjecturing vs. convincing [22]. Additionally, aside from the process aspect, there is a content-specific aspect of MT, such as numerical, algebraic, geometrical, statistical, stochastic, and computational (algorithmic) thinking. The curriculum mainly encompasses these content aspects of MT. A robust curriculum should also consider the process aspect. In connection to the Gebeta game, early numerical thinking (ENT) and algorithmic thinking (AT) are evidenced and highlighted. Therefore, this study primarily focuses on these two aspects.

According to van de Walle and colleagues, developing early number concepts and number sense involves dealing with several activities and concepts such as counting, one-to-one correspondence, cardinality, comparing and ordering quantities, subitizing, counting on and counting back, developing verbal counting skills, numeral writing, recognition, and thinking about zero [23]. Furthermore, algorithmic thinking (AT) is being embedded into curricula in STEM courses [24–26] as AT has become necessary for every child to experience twenty-first century skills [25]. Of course, the broader concept is computational thinking (CT), and in recent years, Bocconi argues that educational stakeholders have advocated CT and related concepts, such as programing and AT, as abilities for all that is as fundamental as numeracy and literacy [26]. They provided ways AT and programing can be integrated into the Nordic curriculum. Lockwood presented the construct of AT in mathematics education comprehensively and used NCTM's AT definition as follows: "AT is a method of thinking and guiding thought processes that use step-by-step procedures, require inputs and produce outputs, require decisions about the quality and appropriateness of information coming in and information going out, and monitor the thought

processes as a means of controlling and directing the thinking process. In essence, algorithmic thinking is simultaneously a method of thinking and a means for thinking about one's thinking" ([24], p. 7). In this sense, the Gebeta game can foster some aspects of AT.

4. Methodology

This is a qualitative study, and it follows an ethnographic research approach as a methodology. According to Creswell & Poth, ethnographic research is "a qualitative design in which the researcher describes and interprets the shared and learned patterns of values, behaviors, beliefs, and language of a culture-sharing group" ([27], p. 90). In this study, the researchers engaged in extracting the shared values, experiences, and engagement with the Gebeta game in the Ethiopian context. Using cultural commognition and the theory of funds of knowledge as a lens, the researchers studied the affordances of the game through fieldwork, primary interviews, observations, questionnaires, documents, and video recordings to find answers to the research questions posed.

4.1 Data collection

4.1.1 Entry and organization access

During our previous engagement in investigating the curriculum implications of ethnomathematics to promote student learning in Ethiopia, one of the researchers approached different ministries and offices to gather data on the subject [4]. Fortunately, we found relevant information about activities considered a cultural sport officially for computation with the Ethiopian Cultural Sports Federation. Capitalizing on this networking during the summer of 2021, one of the present researchers reached out to this office. Fortunately, the people were more open and collaborative, and they showed interest in our work. After a few telephone conversations, we held the first meeting with the head of the cultural sports division and his staff. They explained their work about cultural sports and provided us with a soft copy of the guiding documents about cultural sports in the country. This document is used as a resource for the data, partly.

4.1.2 Observation and video data

On the second day, two players aged 15 and 17 from the Gebeta game team in one of the subregions of Addis Ababa city were invited to demonstrate the game at the national level. An official referee was also present to officiate the game between the two young players. Initially, the official explained the rules and procedures with great enthusiasm. Subsequently, the two young players were allowed to compete. We were granted permission to video record the game, and after its completion, we conducted a follow-up questionnaire with the players and the referee. The researcher documented the session and made field notes.

Another video data is recorded in the northern-western city called Gondar to identify and locate a culture-sharing group for this ethnographic study [27]. The kids were asked to play the Gebeta game as they usually do. Otherwise, Gebeta is a game played spontaneously during the summer holidays. The video is recorded by a volunteer who

is originally from the area. The goal is to situate the study within the context of the culture and investigate if Gebeta is one aspect of CFoK for the learners in the country.

4.1.3 Small-scale survey on social media

To further strengthen the investigation of whether Gebeta is one of the accessible and meaningful cultural FoK for learners in Ethiopia, an additional small-scale survey was conducted on social media using Google Forms. Such an online survey will result in a sample of unknown representativeness, but it is more cost-effective and time-saving than in-person surveys and postal questionnaires [28]. The participants were not from a particular region or part of the country. They were recruited through personal networks, a mathematics education network in Ethiopia, *via* Telegram and Facebook Messenger. Most of those who participated and responded were highly educated (minimum master's degree in one field).

4.2 Data analysis

Ethnographic research follows three types of data analysis, according to Creswell & Poth: description, analysis, and interpretation of the culture-sharing group [27]. As a starting point, the researchers described the cultural context of the Gebeta game in Ethiopia. The story is told in a way that helps us understand the game within the given cultural settings in the community, society, and country at large. Furthermore, we interpret the data by examining different aspects of the game's influence on mathematics education. Finally, analysis, the quantitative side of qualitative study, is employed to highlight specific material introduced in the description of this cultural game [27]. In search of patterns of evidence for CFoK, the data will be triangulated by comparing and testing information from different sources. The conceptual frameworks presented above, CFoK and cultural commognition, generally guide the data analysis.

4.3 Ethical consideration

The study was conducted by obtaining informed consent and ensuring confidentiality [28]. The researchers obtained informed consent from the participants by providing information about the research process. Before collecting the data, permission to responsibly collect personal data was obtained from Norwegian Center for Research Data (NSD) in Norway. The participating authorities and persons in Ethiopia were also duly informed, and efforts were made to minimize or avoid using personal data. The images used were carefully selected to ensure they did not harm the participants despite their consent to participate in the research.

5. Findings

5.1 Gebeta is part of the cultural in the family, community, and society

During the visit to the Ethiopian Cultural Sports Federation office, the officials claimed that there are 293 registered cultural games in the country. This excerpt from the document shows the office's mandate to organize and structure scientifically. Furthermore, the document reveals that only 11 of these games have been adopted as

national cultural sports games. Among these 11, the Gebeta game is included. This information is depicted in **Figure 3**, which includes a text excerpt from the guiding document about cultural sports in the country.

During the videotaped Gebeta game at the head office, two young players from the sub-city of Arada in Addis Ababa played against each other, officiated by a referee who has been working on cultural sports for 30 years. They were all dressed according to the cultural context of the game. The head of cultural sports opened the demonstration. He said, "Today, as promised, we have invited players from one of the sub-cities in Addis Ababa called Arada. They are here with a referee, who is also a game instructor. He leads a cultural sports project in the Arada area. He will take responsibility for officiating and explaining the rules and cultural interpretations of the game today. The two young people are here because they are among the best, and they are the ones who know all the four types of Gebeta game (see **Figure 4**)."

After that, the referee took over and introduced himself first. Then, he continued with a deeper explanation of the country's history and development of the cultural sport. In the Room was also another officer who has facilitated this meeting.

Because Ethiopia's cultural sports games are growing over time, the need to organize and synchronize these games scientifically has increased. This is to ensure that they can be widely distributed and enjoyed by society. In 1967, the Sports Commission was established by a decree, and it was decided that the cultural sports games would fall under this commission. Mr. Tefera Mekonnen, a member of the commission's staff, was responsible for traveling across the provinces to gather approximately 293 cultural sports games. Among these, the commission selected games that were considered nationally acceptable and established rules and regulations for the competitions. Currently, these games are played throughout Ethiopia in competitive situations.

Figure 3.
Extract from the cultural sports federation guiding document (Amharic text translated to English by one of the researchers).

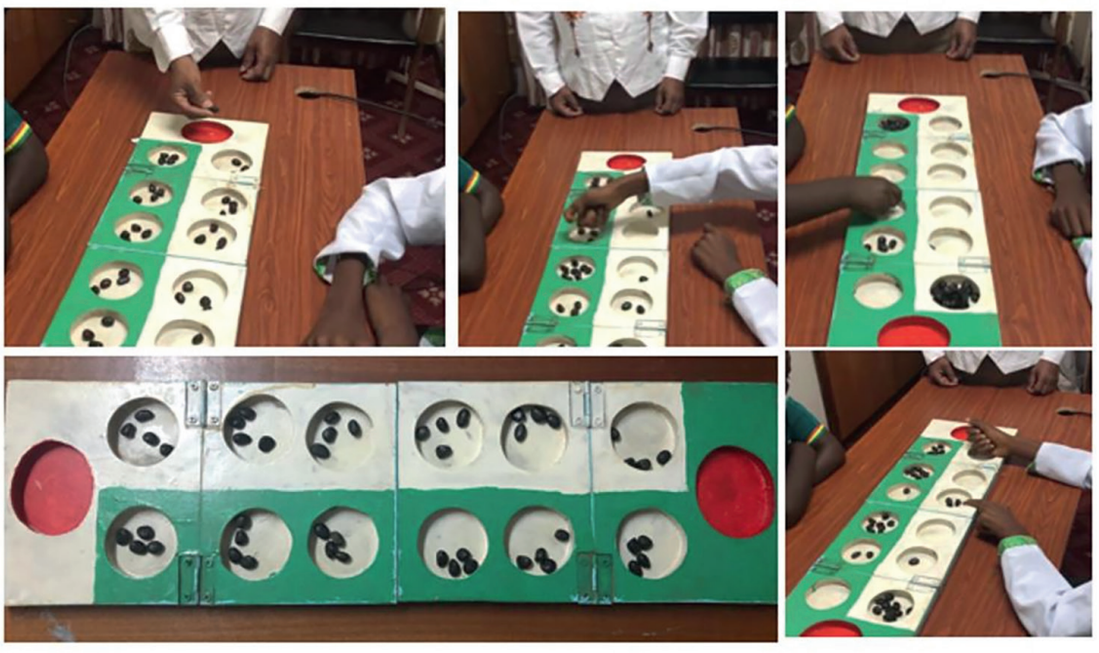

Figure 4.
The referee, the two teenage players from the Arada sub-city of Addis Ababa, and the official game board of 84 cm x 20 cm with 12 holes/homes and two storing holes (homes).

> *"I am …; I have contributed to a cultural sport for over 30 years. Whenever I mention cultural sports in Ethiopia, I always want to recognize the contribution of the late Mr. Tefera Mekonnen, who tremendously contributed to establishing this work in the country. He is the one who traveled throughout the country around 45 years ago, sitting on a horse and mule, and traveling on foot, collecting information about these games. I am always grateful that he showed me the way and recognized and entrusted me with this work to continue to the next generation; that is why I am working hard to realize that. I am training, teaching, as well as leading projects that involve young people. Today, two young teenagers from Arada are here to demonstrate the game."*

Furthermore, the referee stated, "the game usually starts with greetings and handshakes. However, due to coronavirus concerns, the players will now begin with an elbow bump greeting." Both the players and the referee have already donned cultural clothes. As the game commences, he also explains which versions of the game will be demonstrated, focusing on grammar, spelling, and punctuation errors.

> *"The 12-homes Gebeta game has four different types: 1-kinchibosh (*ቅንጭቦሽ*), 2-kinchibosh, 3-kinchibosh, and 4-effisosh (*እፍሶሽ*). There are different versions Gebeta game but are only focused on those that are nationalized by establishing rules and regulations of the game scientifically. Of course, we could have worked harder to recognize and register these cultural sports at the Olympic level. Even we need people who work on it so that it is played using technology."*

Hence, first, 1-kinchibosh was demonstrated. Mr. Yaregal said, "First, we should decide who to start. We do it using a draw by flipping a coin. What do you prefer, Anbesa (አንበሳ meaning lion) or Sew (ሰው meaning man)?" Then, he flipped the coin, and the one dressed green got the chance to start the game. After he took his round, the second player chose a home (hole) to start and moved the stones further. We asked him why he chose to start there. He explained, and Mr. Belayneh interjected in between and provided the different strategies to win the game. After the first round, they continued demonstrating the other three game versions. The detail is not the focus of this article, and we return to it later on the mathematics, language, cultural representations, and interpretations involved in the game. In addition to this official data, other evidence collected to establish that Gebeta is among the CFoK was from the survey. The result of the small-scale survey is summarized in **Table 1**.

5.2 Scaffolding ´Gondar's Gebeta´ game

Gebeta game has many versions. Even the 12 homes Gebeta game has four different ones, as mentioned above. The kind of Gebeta game that is the focus of this work is different from the official cultural sports, but the one that one of the researchers used to play before he started elementary school. It is played around Gondar, the northwestern part of the country; we call it Gondar's Gebeta game. It might also be known in other parts of the country; further, investigation is needed. The rationale for presenting this version of the game is the following: i) its simplicity to be understood and played, ii) its contribution to mathematical cognition, iii) it is known to us at an early age before school, and iv) its availability within our proximity and how we enjoyed it.

Questions	Response
Do you know the game called Gebeta (ገበጣ)?	95% of the 22 who participated in the survey responded YES.
Have you played it before?	81.8% of the 22 who participated in the survey responded YES. 18.2% said NO.
Did you play it by making holes in the ground and collecting pebbles (small stones), or you played on a material made for the Gebeta game?	By making holes in the ground = 63.6%. On a material made for the Gebeta game, 34.4%
Why did you play the game?	Entertainment (most answers), for leisure, Competition, fun and interesting, to enjoy, interesting to play, it was a tradition in my place I was born, just to spend my time, it is one type of game that I used to play with my siblings because it a good game.
When did you start playing it? At which age?	12, 10, almost at 7 when I was a kid, when I was a kid, 5, 7, 12, 14, at 7, around 10, maybe in early 20, 15, before 35 years, I think as a kid, when I was 5, 14.
Where was the first time you played it?	At birthplace at rural area, in the rural area, in rural areas, northwest & central, Ethiopia (two times), in my birthplace, anywhere, home, in my home town Gondar (town), in my birth village, Lay Armachiho, Tikildingay, Dessie, Ethiopia, cannot remember, neighborhood, Debrezeit (town).

Table 1.
Summary of the small-scale survey questions and responses about the Gebeta game in Ethiopia.

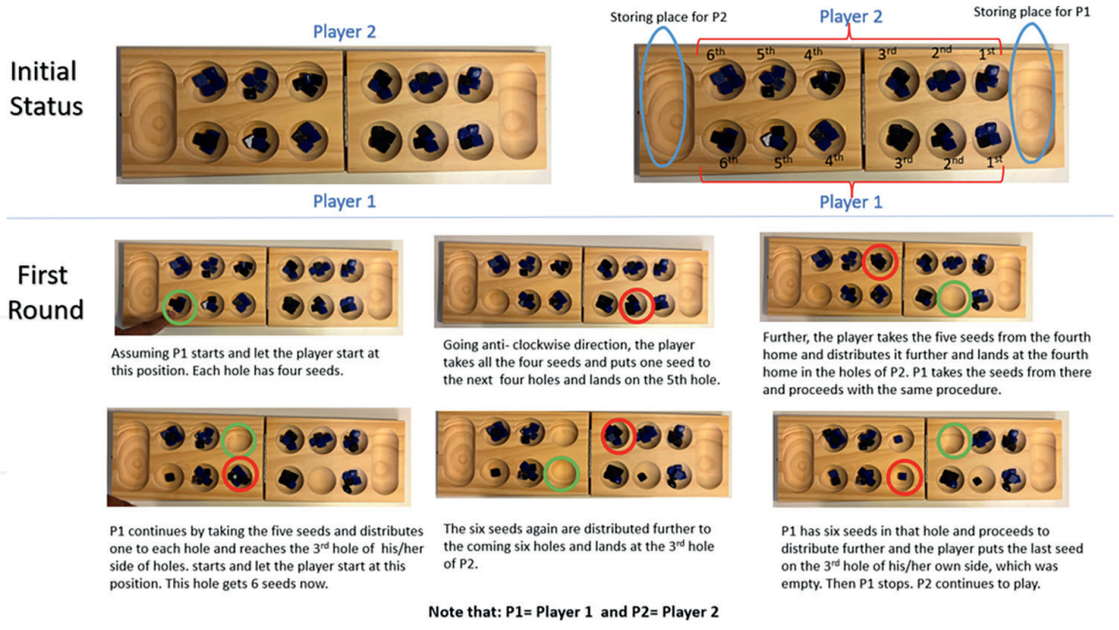

Figure 5.
The initial status of the two-player playing the Gebeta game and the first round play by player 1.

Assume there are only two players (see **Figures 5**–7). At the start, each player has six equal holes called homes (in the local language, ቤት bet). Each hole contains four seeds, which count as one completed home. The players aim to own as many holes/homes as possible. To achieve this, players must empty their holes and then rebuild them to contain four seeds again, thus completing a home. The game is played in multiple rounds until one player loses some or all of their initial homes. The first player

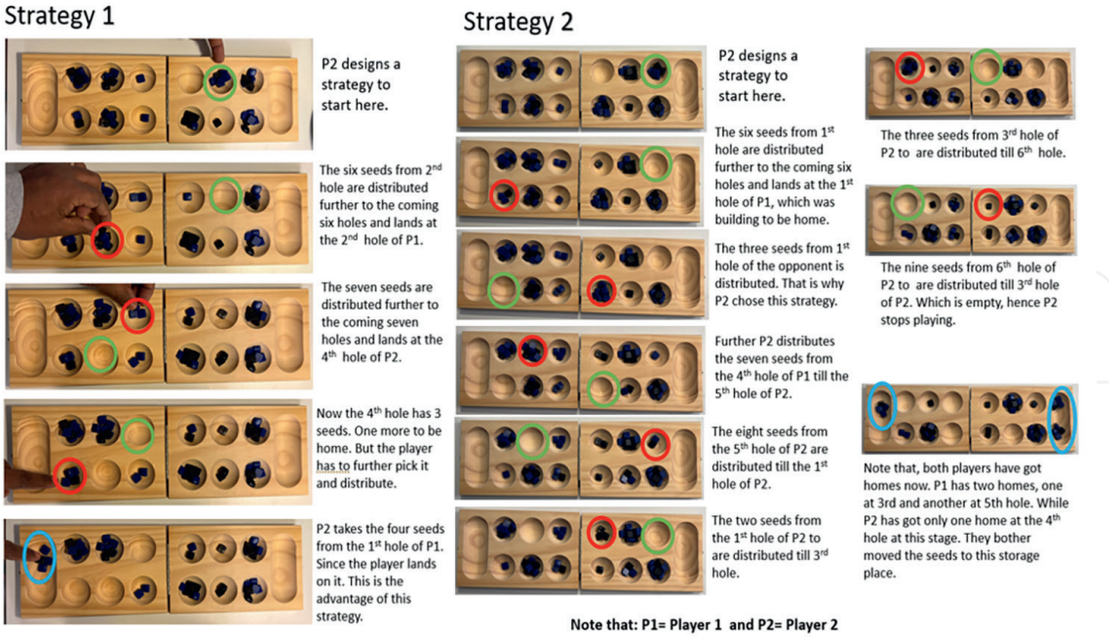

Figure 6.
Two possible strategies by player 2.

Figure 7.
Kids playing the Gebeta game on the ground in Gondar.

begins at one of their homes, picks up all four seeds, moves around in an anticlockwise direction, and distributes one seed to each subsequent hole until all seeds have been distributed. The player then collects all the seeds from the last hole they placed seeds in. This process continues until the player reaches an empty hole, at which point they stop playing and pass the turn to the other player.

The second player follows the same procedure starting with a hole of their choice and picking up all the seeds in that hole. As the players continue to go around and around, if a hole becomes filled with four seeds again, the player who owns that hole immediately collects the four seeds and claims their first home. However, if a player accidentally adds another seed to a hole that already contains four seeds, they are unable to pick up the four seeds. On the other hand, if a player can put his last pebble

on the hole/homes of the other side, which has three, then that player collects that home for him/herself. Actually, that is one of the ways a player can win more homes. Another strategy is to be able to attack the holes, which are on the way to four seeds (complete homes). The game can be completed if one of the players has more or even all homes. Hence, the game involves strategic thinking to gain more homes. The first round for each player is simulated as follows. Let us assume two players, players 1 and 2, are playing. Assume that player 1 starts and **Figure 5** provides the simulation of the player's steps starting from the initial step.

Player 2 starts his/her turn. The game demands the player to design strategies to optimize the win, that is. to acquire more homes. **Figure 6** provides two different strategies that Player 2 can follow, resulting in differing outcomes. In strategy 1, at the end of the first round, the player obtains one home, while the other player gets nothing. On the other hand, in strategy 2, Player 1 gains two homes, and Player 2 only gets one home. Which strategy is better in the long run? Was there another strategy that surpasses the aforementioned two? The answer to these questions depends on various variables. However, the Gebeta game undeniably requires critical and strategic thinking.

Figure 7 showcases versions of the Gebeta game played by kids in Gondar. In the video, the kids surrounding the two players can be heard suggesting different strategies. Sometimes, competitive players may not allow external suggestions, while in other cases, they may tolerate them.

6. Discussion

6.1 Gebeta game as a cultural funds of knowledge

The Gebeta game is considered one of the cultural games, as evidenced in both the national document of the cultural sports federation in Ethiopia and the response from participants in a small-scale survey conducted *via* social media. Additionally, children playing the Gebeta game in Gondar, as described above, further supports its cultural significance. The survey results showed that 95% of respondents were aware of the game, and 81.2% had played it between the ages of 5 and 15. The game is predominantly played in rural areas and towns, usually on the ground, for entertainment, fun, and competition. This indicates that the Gebeta game is a prevalent form of cultural knowledge in most communities in Ethiopia. Children engage in this game at an early age, regardless of whether they live in cities or rural areas. This aligns with Moll and colleagues' definition of FoK (Funds of Knowledge) as a historically accumulated and culturally developed body of knowledge and skills within a household or community [12]. Clark demonstrated how nonschool activities that draw on children's FoK can enhance problem-solving abilities [29]. In this context, we argue that Gebeta is one of those cultural activities that can be considered as part of learners' FoK. The game incorporates various concepts and procedures of school mathematics. If adequately studied and integrated into the school curriculum and instruction in Ethiopia, it has the potential to enhance mathematical learning.

The Vygotskian perspective of learning advocates learning as participation in cultural activities [6, 13]. The Gebeta game is inherently suited for such learning since it is accessible to kids at an early age. It can be played by more than two players (if three players, they can share four homes at the start). One knowledgeable individual can train the other(s). That is to say, the more knowledgeable other (MKO), according

to Vygotsky, helps the novice to learn the game [6, 13]. The MKO can be a father, mother, sibling, or friend with whom the village kids frequently interact. The MKO and those learning the game communicate different strategies, which occur in the most natural context of the culture. In the video, the kids from Gondar demonstrate that others suggest the best strategies for the players. This demonstrates participation and communication of winning strategies at every level as the game can progress in various ways at any given level. Finding the best ways to optimize the chances of winning requires abstraction, pattern recognition, critical and strategic thinking, conjecturing, and convincing, which are the process aspects of MT [21, 22].

6.2 Gebeta game as a cultural commognition artifact

Commognition signifies the unity of interaction and thinking [15, 16], and a cultural artifact, such as the Gebeta game, could stimulate it. In particular, the Gondar Gebeta game presented in this work can potentially mathematically engage kids at age 5 and 6 [4, 11]. We are doing more profound research with kids in Ethiopia and Norway to see which aspects of mathematical thinking, language, and communication are visible in both contexts. In this regard, as depicted in **Figure 8**, the potential of the Gebeta game to facilitate both the mathematical thinking and the communication aspect could be investigated very well. The question could be which mathematical thinking is most pronounced. Mathematical thinking (cognition) is a cognitive process by which students come to understand mathematical concepts and procedures. The process includes problem-solving, reasoning and proof, communications, connections, and representation, according to [30]. If mathematics should be thought of as a human activity, games, such as Gebeta, can facilitate such a learning process. In the game, many mathematical activities are embedded. These are presented as follows:

6.2.1 Gebeta and early numerical thinking (ENT)

Counting: For starters, it is basically a counting activity. One has to count how many seeds a home has to play the game till the end of the game. A home can have 1, 2, 3, 4, ..., 11, or more seeds. It can be empty also, representing the concept of zero. As counting is the basis of numerical thinking [11, 23], playing this game at an early age repeatedly could boost skills in early numeracy.

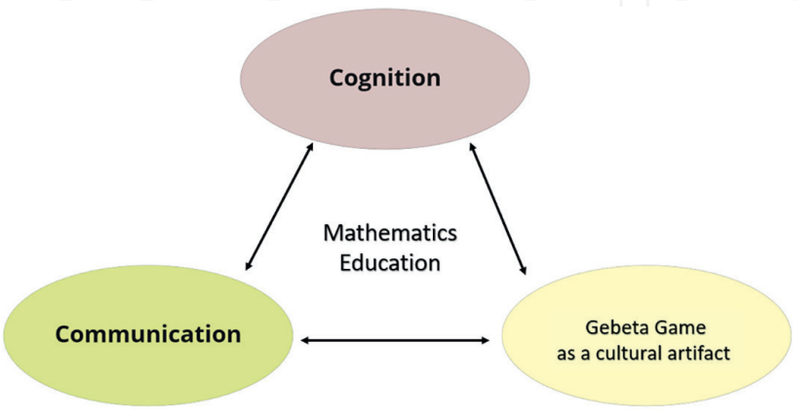

Figure 8.
Gebeta game as an artifact for commognition in mathematics.

One-to-one corresponding: During playing, one first counts how many seeds each home holds and decides to start at one of the homes. Then the players grab the seeds in one home and distribute the seeds to consecutive homes (holes). This is done using the one-to-one corresponding principle: For each home, you put one seed. Of course, this is a subtraction activity too. From the number of seeds in the hand, you reduce one each step forward. At the same time, it is an additional activity to the number of seeds in each hole/home.

Concept of zero: While at the start, one grabs all four seeds (see **Figure 1**). That home now has 4–4 = 0 seeds, and the following four homes gain one more seed, 4 + 1 = 5. Then, one empties the fifth hole/home and adds one seed for the next five holes. Making homes empty is an integral part of the game. In doing that, one communicates the concept of zero several times in a single game round. Hence, the concept of zero is visibly demonstrated *via* an empty hole/home so that preschoolers understand its meaning.

Subitizing, cardinal, and ordinal numbers: These skills are highly involved if one intends to win more homes in the game. Each player continuously engages in subitizing (perceptual or conceptual) activity from the start. For example, a home can have 5, 6, or 7 seeds. Then one has to update as the game proceeds instantly. For instance, in strategy 2 (see **Figure 2**), Player 2 begins at that house with six seeds, intending to annul the potential of being a home for the opponent player. It means that by counting the number of seeds to reach that home, Player 2 can instantly recognize the six seeds and use the one-to-one correspondence principle with the order of the homes to reach that particular home of Player 1. In addition, the concept of the ordinal number, the sixth home, is embedded in the thinking process.

The 12-home Gondar's Gebeta game involves a maximum of 48 seeds, which is 12 times 4. Repeated addition of four represents multiplication. Each player has six homes in this game, so 6 x 4 = 24 seeds. When you win a home, you have 1 x 4 = 4 seeds. If you have two homes, you have 2 x 4 = 8 seeds, and so on. One can also think that if there are two players, they divide the 12 homes by two, so 12 ÷ 2 = 6 homes each. The 48 seeds are divided into 12 homes, so 48 ÷ 12 = 4. To win a home, the players must first empty the hole (zero seed) and then put in four seeds to build it. This game uses a base-five counting system in addition to the base-ten counting system. Different arithmetic operations are embedded from the outset. If played many times with a grown-up person, kids can develop advanced numerical thinking even before they start formal school.

6.2.2 Gebeta and algorithmic thinking

Adopting NCTM's definition of AT [24], we can say that the Gebeta game is inherently algorithmic. It is a game in which one has to decide where to start and where to land for many rounds, following step-by-step procedures, and requires a strategy to win using the game's rules. According to Bishop, playing is considered one of the mathematical activities [2]. This game challenges players to be creative to win homes. One should have good counting strategies for repeatedly determining where to start and finish. The game can even be programmed using block programming tools, such as Scratch (See **Figure 9**) or text programing, such as Python. **Figure 8** shows the adoption of the Mancala game, which one can find through a Google search, to a Gebeta game used in this study.

There are two different activities here. First, it is possible to create a programing activity that simulates the game. Another, it is possible to make applications that can

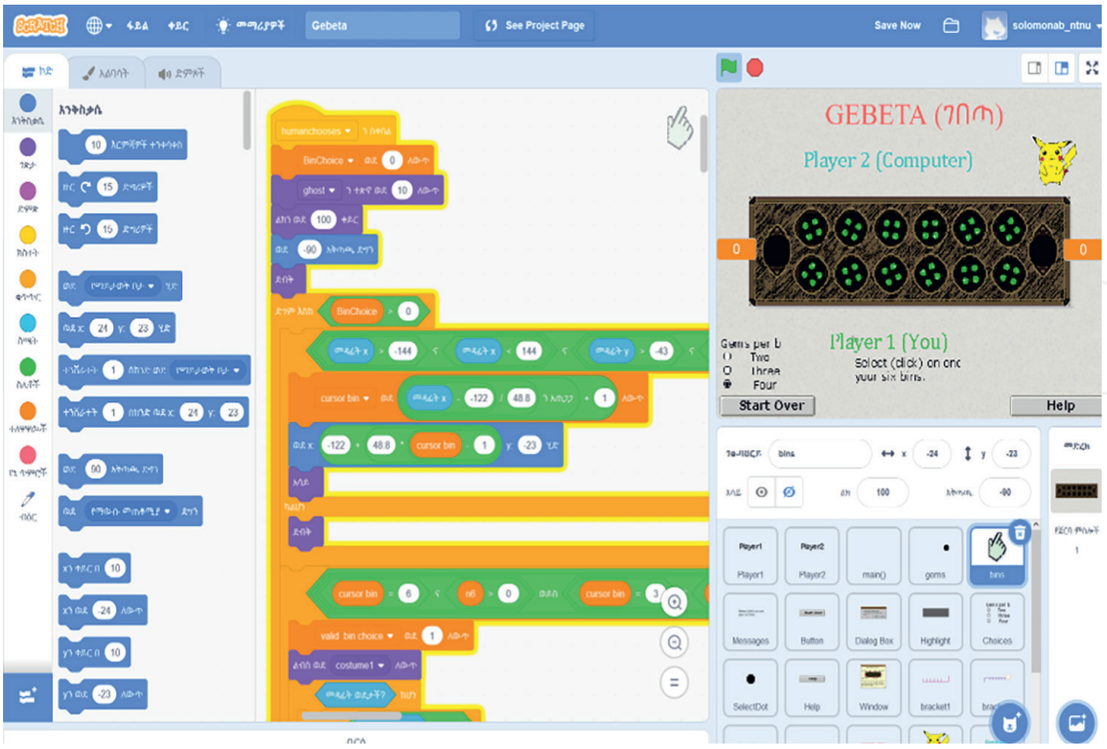

Figure 9.
Example of block programming to Gebeta game to foster algorithmic thinking.

help play the game on computers and mobile applications. That makes the game to be available virtually, and kids can access it on their mobile or PC.

6.3 Conclusion and implications

The Gebeta game is played in towns and rural areas and is integrated nationally as a cultural sports competition, as evidenced above. However, there needs to be clear evidence that it is used as part of the mathematics curriculum in the country. Gebeta games can be played without a proper game board, as shown in **Figure 6**. The availability and accessibility of the Gebeta game for everyone could be considered an advantage in considering it as part of the resources for mathematics education. The variations of the game, from simple to complex (such as the Gondar Gebeta game, 1-kinchibosh, 2-kinchibosh, 3-kinchibosh, and 4-kinchibosh), allow for learning the different mathematical concepts embedded in the game, as discussed above. Hence, the Gebeta game can be used as a CFoK (Cultural Form of Knowledge) and for cultural cognition. What is left is to design activities and tasks that can be integrated into the mathematics curriculum and syllabus in the country. This demands those who are engaged in designing tasks, instructional materials, and textbooks to search, for example, the Gebeta game and others, such as the 293 cultural games mentioned in the National Cultural Sports Federation document and embed the activities and tasks that can foster mathematical cognition and communication across the school mathematics curriculum. There has been a movement among education stakeholders that indigenous knowledge and practices have to be part of the formal school system, as evidenced in the new curriculum of the country [31]. However, implementing it is challenging, and this work provides one particular example within mathematics education.

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