Applications and Applied Mathematics: An International Journal (AAM)

Volume 18 | Issue 2 | Article 11

12-2022

(R1976) A Novel Approach to Solve Fuzzy Rough Matrix Game with Two Players

Vinod Jangid
University of Rajasthan

Ganesh Kumar
University of Rajasthan

Gaurav Sharama
University of Delhi

Vishnu Narayan Mishra
Indira Gandhi National Tribal University

Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466

Applications and Applied Mathematics:
An International Journal
(AAM)

Vol. 18, Issue 2 (December 2023), Article 11, 17 pages

A Novel Approach to Solve Fuzzy Rough Matrix Game With Two Players

[1]Vinod Jangid, [2*]Ganesh Kumar, [3]Gaurav Sharama, and [4]Vishnu Narayan Mishra

[1,2] Department of Mathematics
University of Rajasthan
Jaipur-302004
Rajasthan, India
[1]vinodjangid124@gmail.com; [2]ganeshmandha1988@gmail.com

[3]Department of Mathematics
Acharya Narendra Dev College
University of Delhi, India
gauravsharmaandc@gmail.com

[4]Department of Mathematics
Indira Gandhi National Tribal University
Lalpur, Amarkantak-484887
Madhya Pradesh, India
vishnunarayanmishra@gmail.com

*Corresponding Author

Received: March 15, 2022; Accepted: July 19, 2023

Abstract

This paper proposes a new method for solving a two-person zero-sum fuzzy matrix game with goals, payoffs, and decision variables represented as triangular fuzzy rough numbers. We created a pair of fully fuzzy rough linear programming problems for players. Triangular fuzzy rough numbers can be used to formulate two fuzzy linear programming problems for the first player in the form of upper approximation intervals and lower approximation intervals. Two problems for the second player can be created in the same way. These problems have been split into five sub-crisp problems for the player first and five sub-crisp problems for the player second. The solution to the game can be obtained by solving these ten fuzzy linear programming problems. To demonstrate the method, a numerical example is provided. Using Wolfram Cloud, optimal strategies and game values are calculated for various parameters. Sensitivity analysis is carried out by altering the values of parameters.

Keywords: Fully fuzzy rough matrix game; Triangular rough fuzzy number; Ranking function; Fully fuzzy rough LPP

MSC 2010 No.: 91A05, 03E72, 90C70, 90C05

1. Introduction

Game theory is a mathematical process used to study decision-making problems when the decision-makers conflict with each other. Von Neumann and Morgenstern (1944) developed the concept of game theory. The most popular category of games is a two-person zero-sum matrix game (TPZSMG). The study of TPZSMG is one of the most essential topics that has recently drawn the attention of several researchers due to its significance in financial implications, economics, organizational behaviour, political science, organizational studies, industry, war, social sciences, engineering, and biological models. In the physical world, complexity generally appears in vagueness from uncertainty. The concept of probability had been old age and an essential method for overcoming the challenges, but it can only apply to circumstances whose parameters are based on probability distributions. Uncertainty can arise from incomplete information about the problem or from information that is not entirely accurate or from inherent inaccuracy in the description of the problem, or from obtaining information from more than one origin. Fuzzy set theory is an excellent logical method for dealing with the vagueness that emerges from ambiguity.

Zadeh (1965) developed the fuzzy set theory and established various properties of fuzzy sets. The notion of a rough set theory was introduced by Pawlak (1982) as an alternative to the concept of fuzzy set and tolerance. They studied rough operations on sets, rough equality of sets, and rough inclusion of sets. The lower and upper approximations of fuzzy sets were presented by Dubois and Prade (1990), which launched the notion of a rough fuzzy set. The intuitionistic fuzzy set (IFS) theory was developed by Atanassov (1999) based on modifications of related fuzzy set concepts.

A TPZSMG with imprecise payoffs was considered by Campos (1989) and proposed a method for its solution based on a fuzzy linear programming (FLP) problem for each player. Campos and Gonzalez (1991) suggested a new approach to solve TPZSMG with approximate values in their matrices of payoffs. Fuzzy multiobjective linear programming models for each player were established, and corresponding effective solution methods were proposed by Li (1999). Sakawa and Nishizaki (1994) considered TPZSMG with fuzzy multiple payoff matrices and goals for each player. The equivalence between a primal-dual pair of FLPP and TPZSFMG was provided by Bector et al. (2004). Vijay et al. (2005) considered a TPZSFMG with fuzzy goals and fuzzy payoffs and solved using a suitable ranking function. Several types of TPZSFMG with different solution concepts were introduced by Bector and Chandra (2005). Chen and Larbani (2006) obtained the weights of a fuzzy decision matrix by formulating it as a TPZSFMG with an uncertain payoff matrix. Interval-valued matrix games were considered and solved by Collins and Hu (2008). Nayak and Pal (2009) proposed and solved TPZSFMG with interval-valued payoffs. A methodology for solving matrix games with payoffs as triangular intuitionistic fuzzy numbers (TIFNs) was developed by Nan et al. (2010). Cevikel and Ahlatçıoğlu (2010) presented two models for studying TPZSFMG with fuzzy payoffs and fuzzy goals. Xu and Yao (2010) discussed a class of TPZSFMG with rough payoffs. Clemente et al. (2011) presented a new methodology for the analysis of TPZSFMG. Aggarwal et al. (2012) studied a class of LPP having fuzzy constraints with I-fuzzy sets. Duality theory was developed to establish a solution concept for TPZSFMG with I-fuzzy goals. Two auxiliary bi-objective linear programming (BOLP) models were derived by Seikh et al. (2015) and applied an average weighted approach to solve TPZSFMG. A new method to solve the bi-rough bi-matrix game was proposed by Mula et al. (2015). A TPZSFMG with triangular intuitionistic fuzzy numbers was considered by Bhaumik et al. (2017) and applied a robust ranking technique to solve it. The TPZSFMG was converted into a crisp one and then solved easily by Jana and Roy (2018). Das et al. (2018) proposed a method of finding the upper and lower bounds of tri-objective LFPPs.

Ammar and Brikaa (2019) proposed an effective technique for solving constraint matrix games with payoffs as rough intervals. Pamucar et al. (2019) developed a new approach for overcoming the challenges and vagueness based on interval-valued fuzzy rough numbers. Brikaa et al. (2019) developed an effective algorithm to solve constraint MG with payoffs as FRNs. Das and Edalatpanah (2020) proposed a novel approach for solving neutrosophic integer programming problems using the aggregate ranking function with triangular neutrosophic numbers. Yang et al. (2020) developed an approach to evaluate the healthcare system using neutrosophic set theory. Edalatpanah (2020) introduced the concept of neutrosophic structured element (NSE) in the field of neutrosophic sets (NS). The author established the operational laws, score function, and several aggregation operators of NS. The NSE was applied in a decision-making process for a multi-attribute decision making (MADM) problem, showcasing its practicality.

Dhar (2021) applied Neutrosophic Sets to deal with uncertainties and imprecisions in many situations such as contexts involving diagnosis. Ammar and Emsimir (2021) suggested an algorithm to solve TFR integer LPP using cut sets to find rough, optimal solutions. Eyo et al. (2022) presented a new intelligent approach that combines different techniques to tune the parameters of Interval Type-2 Intuitionistic Fuzzy Logic System (IT2IFLS) for modeling and predicting COVID-19 time series.

For the most part, most methods used the ranking function to convert the given fuzzy LPP to crisp, and then they used the findings of classical approaches to enhance their efficiency. This paper proposes a new method for solving a two-person zero-sum fuzzy matrix game with goals, payoffs, and decision variables represented as triangular fuzzy rough numbers. We created a pair of fully fuzzy rough linear programming problems for players. Triangular fuzzy rough numbers can be used to formulate two fuzzy linear programming problems for the first player in the form of upper approximation intervals and lower approximation intervals. Two issues for the second player can be created in the same way. These problems have been split into five sub-crisp problems for the player first and five sub-crisp problems for the player second. The solution to the game can be obtained by solving these ten fuzzy linear programming problems. To demonstrate the method, a numerical example is provided. Using Wolfram Cloud, optimal strategies and game values are calculated for various parameters. Sensitivity analysis is carried out by altering the values of parameters.

The rest of the paper is structured as follows. The proposed method is explained step-wise in Section 2. A numerical example, with sensitivity analysis, is illustrated in Section 3. In Section 4 we compare the results of present paper with the existing literature. Section 5 presents the conclusion.

2. Working Steps of Proposed Approach

Step 1: Develop a pair of FFRLPPs corresponding to maximizing and minimizing players, respectively:

$$\begin{aligned} &\max \quad \tilde{u}^R, \\ &(\tilde{x}^R)^T \tilde{A}^R \tilde{y}^R \succeq_{\tilde{\phi}^R} \tilde{u}^R, \quad \forall \tilde{y}^R \in S^n, \\ &\text{and} \quad \tilde{x}^R \in S^m, \end{aligned} \tag{1}$$

$$\begin{aligned} &\min \quad \tilde{v}^R, \\ &(\tilde{x}^R)^T \tilde{A}^R \tilde{y}^R \preceq_{\tilde{\psi}^R} \tilde{v}^R, \quad \forall \tilde{x}^R \in S^m, \\ &\text{and} \quad \tilde{y}^R \in S^n, \end{aligned} \tag{2}$$

where FR values $\tilde{u}^R, \tilde{v}^R \in N(R)$ and $\tilde{\phi}^R, \tilde{\psi}^R$ are FR adequacies for the player I and II, respectively.

Step 2: Clarify the double fuzzy constraints to recognize the extreme points of the sets S^m and S^n in the constraints of the problems (1) and (2):

$$\begin{aligned} &\max \quad \tilde{u}^R, \\ &\text{subject to constraints} \\ &(\tilde{x}^R)^T \tilde{A}^R_j \succeq_{\tilde{\phi}^R} \tilde{u}^R, \quad \forall j = 1, 2, \ldots, n, \\ &e^T \tilde{x}^R \approx \tilde{1}^R, \\ &\text{and} \quad \tilde{x}^R, \tilde{u}^R \geq 0, \end{aligned} \tag{3}$$

$$\begin{aligned} &\min \quad \tilde{v}^R, \\ &\text{subject to constraints} \\ &(\tilde{A}^R_i)^T \tilde{y}^R \preceq_{\tilde{\psi}^R} \tilde{v}^R, \quad \forall i = 1, 2, \ldots, m, \\ &e^T \tilde{y}^R \approx \tilde{1}^R, \\ &\text{and} \quad \tilde{y}^R, \tilde{v}^R \geq 0, \end{aligned} \tag{4}$$

where the symbols $\tilde{A}^R_i$ and $\tilde{A}^R_j$ denotes the ith row and the jth column of the FR pay off matrix $\tilde{A}^R$, for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

Step 3: Applying Yager resolution method to problems (3) and (4):

$$\begin{aligned} &\max \quad \tilde{u}^R, \\ &\text{subject to the constraints} \\ &\sum_{i=1}^{m} \tilde{a}^R_{ij} \tilde{x_i}^R \geq \tilde{u}^R - \tilde{\phi}^R(1-\lambda), \quad \forall j = 1, 2, \ldots, n, \\ &e^T \tilde{x}^R \approx \tilde{1}^R, \\ &\lambda \leq 1, \\ &\text{and} \quad \tilde{x}^R, \tilde{u}^R, \lambda \geq 0, \end{aligned} \tag{5}$$

$$
\begin{aligned}
&\min \quad \tilde{v}^R, \\
&\text{subject to the constraints} \\
&\sum_{j=1}^{n} \tilde{a}_{ij}^R \tilde{y_j}^R \leq \tilde{v}^R + \tilde{\psi}^R (1-\mu), \quad \forall i = 1,2,\ldots,m, \\
&e^T \tilde{y}^R \approx \tilde{1}^R, \\
&\mu \leq 1, \\
&\text{and} \quad \tilde{y}^R, \tilde{v}^R, \mu \geq 0.
\end{aligned} \tag{6}
$$

Step 4: Taking parameters and decision variables in (5) and (6) as TFRNs, we can construct two FLPPs separately to each problems of (5) and (6). One of which is a $(FLPP-I^{UAI}/FLPP-II^{UAI})$, where coefficients are UAIs of rough intervals and the other is $(FLPP-I^{LAI}/FLPP-II^{LAI})$, where its coefficients are LAI of rough intervals.

(FLPP-I)^{UAI}

$$
\begin{aligned}
&\max \quad \left[u^{LU}, u^{M}, u^{UU}\right], \\
&\text{Subject to the constraints} \\
&\sum_{i=1}^{m} \left[{a_{ij}}^{LU}, {a_{ij}}^{M}, {a_{ij}}^{UU}\right] \otimes \left[{x_i}^{LU}, {x_i}^{M}, {x_i}^{UU}\right] \\
&\geq \left[u^{LU}, u^{M}, u^{UU}\right] - \left[\phi^{LU}, \phi^{M}, \phi^{UU}\right](1-\lambda), \qquad \forall j = 1,2,\ldots,n, \\
&\sum_{i=1}^{m} \left[{x_i}^{LU}, {x_i}^{M}, {x_i}^{UU}\right] = \tilde{1}^U, \\
&\lambda \leq 1, \\
&\text{and} \quad {x_i}^{LU}, {x_i}^{M}, {x_i}^{UU}, \lambda, u^{LU}, u^{M}, u^{UU} \geq 0, \qquad \forall i = 1,2,\ldots,m, \\
&\text{where} \quad \tilde{1}^U = \left[1^{LU}, 1^{M}, 1^{UU}\right].
\end{aligned} \tag{7}
$$

(FLPP-I)^{LAI}

$$
\begin{aligned}
&\max \quad \left[u^{LL}, u^{M}, u^{UL}\right], \\
&\text{subject to the constraints} \\
&\sum_{i=1}^{m} \left[{a_{ij}}^{LL}, {a_{ij}}^{M}, {a_{ij}}^{UL}\right] \otimes \left[{x_i}^{LL}, {x_i}^{M}, {x_i}^{UL}\right] \\
&\geq \left[u^{LL}, u^{M}, u^{UL}\right] - \left[\phi^{LL}, \phi^{M}, \phi^{UL}\right](1-\lambda), \qquad \forall j = 1,2,\ldots,n, \\
&\sum_{i=1}^{m} \left[{x_i}^{LL}, {x_i}^{M}, {x_i}^{UL}\right] = \tilde{1}^L, \\
&\lambda \leq 1, \\
&\text{and} \quad {x_i}^{LL}, {x_i}^{M}, {x_i}^{UL}, \lambda, u^{LL}, u^{M}, u^{UL} \geq 0, \qquad \forall i = 1,2,\ldots,m, \\
&\text{where} \quad \tilde{1}^L = \left[1^{LL}, 1^{M}, 1^{UL}\right].
\end{aligned} \tag{8}
$$

$(\text{FLPP-II})^{UAI}$

$$\begin{aligned}
&\min \quad \left[v^{LU}, v^{M}, v^{UU}\right],\\
&\text{subject to the constraints}\\
&\sum_{j=1}^{m} \left[{a_{ij}}^{LU}, {a_{ij}}^{M}, {a_{ij}}^{UU}\right] \otimes \left[{y_j}^{LU}, {y_j}^{M}, {y_j}^{UU}\right]\\
&\leq \left[v^{LU}, v^{M}, v^{UU}\right] + \left[\psi^{LU}, \psi^{M}, \psi^{UU}\right](1-\mu), \qquad \forall i = 1, 2, \ldots, m,\\
&\sum_{j=1}^{n} \left[{y_j}^{LU}, {y_j}^{M}, {y_j}^{UU}\right] = \tilde{1}^{U},\\
&\mu \leq 1,\\
&\text{where} \quad {y_j}^{LU}, {y_j}^{M}, {y_j}^{UU}, \mu, v^{LU}, v^{M}, v^{UU} \geq 0, \qquad \forall j = 1, 2, \ldots, n,\\
&\text{where} \quad \tilde{1}^{U} = \left[1^{LU}, 1^{M}, 1^{UU}\right].
\end{aligned} \tag{9}$$

$(\text{FLPP-II})^{LAI}$

$$\begin{aligned}
&\min \quad \left[v^{LL}, v^{M}, v^{UL}\right],\\
&\text{subject to the constraints}\\
&\sum_{j=1}^{n} \left[{a_{ij}}^{LL}, {a_{ij}}^{M}, {a_{ij}}^{UL}\right] \otimes \left[{y_j}^{LL}, {y_j}^{M}, {y_j}^{UL}\right]\\
&\leq \left[v^{LL}, v^{M}, v^{UL}\right] + \left[\psi^{LL}, \psi^{M}, \psi^{UL}\right](1-\mu), \qquad \forall i = 1, 2, \ldots, m,\\
&\sum_{j=1}^{n} \left[{y_j}^{LL}, {y_j}^{M}, {y_j}^{UL}\right] = \tilde{1}^{L},\\
&\mu \leq 1,\\
&\text{where} \quad {y_j}^{LL}, {y_j}^{M}, {y_j}^{UL}, \mu, v^{LL}, v^{M}, v^{UL} \geq 0 \qquad \forall j = 1, 2, \ldots, n,\\
&\text{where} \quad \tilde{1}^{L} = \left[1^{LL}, 1^{M}, 1^{UL}\right].
\end{aligned} \tag{10}$$

Step 5: Slice the above problems (7) and (8) into five sub-crisp problems as:

$(FLPP-I)^{UU}$

$$\begin{aligned}
&\max \quad u^{UU},\\
&\text{subject to the constraints}\\
&\sum_{i=1}^{m} a_{ij}^{UU} x_i^{UU} \geq u^{UU} - \phi^{LU}(1-\lambda), \qquad \forall j = 1, 2, \ldots, n,\\
&\sum_{i=1}^{m} x_i^{UU} = 1^{UU},\\
&\lambda \leq 1,\\
&\text{and} \;\; x_i^{UU}, \lambda, u^{UU} \geq 0, \qquad \forall i = 1, 2, \ldots, m.
\end{aligned} \tag{11}$$

$(FLPP-I)^{LU}$

$$\begin{aligned}
&\max \quad u^{LU},\\
&\text{subject to the constraints}\\
&\sum_{i=1}^{m} a_{ij}^{LU} x_i^{LU} \geq u^{LU} - \phi^{UU}(1-\lambda),\\
&\qquad \forall j = 1, 2, \ldots, n,\\
&\sum_{i=1}^{m} x_i^{LU} = 1^{LU},\\
&\lambda \leq 1,\\
&\text{and } \; x_i^{LU}, \lambda, u^{LU} \geq 0, \qquad \forall i = 1, 2, \ldots, m.
\end{aligned} \tag{12}$$

$(FLPP-I)^{M}$

$$\begin{aligned}
&\max \quad u^{M},\\
&\text{subject to the constraints}\\
&\sum_{i=1}^{m} a_{ij}^{M} x_i^{M} \geq u^{M} - \phi^{M}(1-\lambda),\\
&\qquad \forall j = 1, 2, \ldots, n,\\
&\sum_{i=1}^{m} x_i^{M} = 1^{M},\\
&\lambda \leq 1,\\
&\text{and } \; x_i^{M}, \lambda, u^{M} \geq 0, \qquad \forall i = 1, 2, ..., n.
\end{aligned} \tag{13}$$

$(FLPP-I)^{LL}$

$$\begin{aligned}
&\max \quad u^{LL},\\
&\text{subject to the constraints}\\
&\sum_{i=1}^{m} a_{ij}^{LL} x_i^{LL} \geq u^{LL} - \phi^{UL}(1-\lambda),\\
&\qquad \forall j = 1, 2, ..., n,\\
&\sum_{i=1}^{m} x_i^{LL} = 1^{LL},\\
&\lambda \leq 1,\\
&\text{and } \; x_i^{LL}, \lambda, u^{LL} \geq 0, \qquad \forall i = 1, 2, ..., m.
\end{aligned} \tag{14}$$

$(FLPP-I)^{UL}$

$$\begin{aligned}
&\max \quad u^{UL},\\
&\text{subject to the constraints}\\
&\sum_{i=1}^{m} a_{ij}^{UL} x_i^{UL} \geq u^{UL} - \phi^{LL}(1-\lambda),\\
&\qquad \forall j = 1,2,\ldots,n,\\
&\sum_{i=1}^{m} x_i^{UL} = 1^{UL},\\
&\lambda \leq 1,\\
&\text{and } \ x_i^{UL}, \lambda, u^{UL} \geq 0, \qquad \forall i = 1,2,\ldots,m.
\end{aligned} \tag{15}$$

Step 6: Similarly slice the problems (9) and (10) into five sub-crisp problems as:

$(FLPP-II)^{UU}$

$$\begin{aligned}
&\min \quad v^{UU},\\
&\text{subject to the constraints}\\
&\sum_{j=1}^{n} a_{ij}^{UU} y_j^{UU} \leq v^{UU} + \psi^{UU}(1-\mu),\\
&\qquad \forall i = 1,2,\ldots,m,\\
&\sum_{j=1}^{n} y_j^{UU} = 1^{UU},\\
&\mu \leq 1,\\
&\text{and } \ y_j^{UU}, \mu, v^{UU} \geq 0, \qquad \forall j = 1,2,\ldots,n.
\end{aligned} \tag{16}$$

$(FLPP-II)^{LU}$

$$\begin{aligned}
&\min \quad v^{LU},\\
&\text{subject to the constraints}\\
&\sum_{j=1}^{n} a_{ij}^{LU} y_j^{LU} \leq v^{LU} + \psi^{LU}(1-\mu),\\
&\qquad \forall i = 1,2,\ldots,m,\\
&\sum_{j=1}^{n} y_j^{LU} = 1^{LU},\\
&\mu \leq 1,\\
&\text{and } \ y_j^{LU}, \mu, v^{LU} \geq 0, \qquad \forall j = 1,2,\ldots,n.
\end{aligned} \tag{17}$$

$(FLPP-II)^M$

$$\begin{aligned} &\min \quad v^M, \\ &\text{subject to the constraints} \\ &\sum_{j=1}^{n} a_{ij}^M y_j^M \leq v^M + \psi^M(1-\mu), \\ &\qquad \forall i = 1,2,\ldots,m, \\ &\sum_{j=1}^{n} y_j^M = 1^M, \\ &\mu \leq 1, \\ &\text{and } \; y_j^M, \mu, v^M \geq 0, \qquad \forall j = 1,2,\ldots,n. \end{aligned} \tag{18}$$

$(FLPP-II)^{LL}$

$$\begin{aligned} &\min \quad v^{LL}, \\ &\text{subject to the constraints} \\ &\sum_{j=1}^{n} a_{ij}^{LL} y_j^{LL} \leq v^{LL} + \psi^{LL}(1-\mu), \\ &\qquad \forall i = 1,2,\ldots,m, \\ &\sum_{j=1}^{n} y_j^{LL} = 1^{LL}, \\ &\mu \leq 1, \\ &\text{and } \; y_j^{LL}, \mu, v^{LL} \geq 0, \qquad \forall j = 1,2,\ldots,n. \end{aligned} \tag{19}$$

$(FLPP-II)^{UL}$,

$$\begin{aligned} &\min \quad v^{UL} \\ &\text{subject to the constraints} \\ &\sum_{j=1}^{n} a_{ij}^{UL} y_j^{UL} \leq v^{UL} + \psi^{UL}(1-\mu), \\ &\qquad \forall i = 1,2,\ldots,m, \\ &\sum_{j=1}^{n} y_j^{UL} = 1^{UL}, \\ &\mu \leq 1, \\ &\text{and } \; y_j^{UL}, \mu, v^{UL} \geq 0, \qquad \forall j = 1,2,\ldots,n. \end{aligned} \tag{20}$$

Step 7: By solving five sub-crisp problems for Player I, viz., $(FLPP-I)^{UU}$, $(FLPP-I)^{LU}$, $(FLPP-I)^M$, $(FLPP-I)^{LL}$, and $(FLPP-I)^{UL}$. Also, five sub crisp problems for Player II, viz., $(FLPP-II)^{UU}$, $(FLPP-II)^{LU}$, $(FLPP-II)^{M}$, $(FLPP-II)^{LL}$, and $(FLPP-II)^{UL}$ the solution of the game can be obtained.

3. Numerical Example

Consider the TPZSFMG with payoff matrix of TFRNs

$$\tilde{A}^R = \begin{bmatrix} \tilde{a}_{11}^R & \tilde{a}_{12}^R \\ \tilde{a}_{21}^R & \tilde{a}_{22}^R \end{bmatrix},$$

where

$$\begin{aligned}
\tilde{a}_{11}^R &\equiv \left[\tilde{a}_{11}^L : \tilde{a}_{11}^U\right] = [[175,180,190] : [170,180,195]], \\
\tilde{a}_{12}^R &\equiv \left[\tilde{a}_{12}^L : \tilde{a}_{12}^U\right] = [[150,156,158] : [148,156,160]], \\
\tilde{a}_{21}^R &\equiv \left[\tilde{a}_{21}^L : \tilde{a}_{21}^U\right] = [[80,90,100] : [70,90,100]], \\
\tilde{a}_{22}^R &\equiv \left[\tilde{a}_{22}^L : \tilde{a}_{22}^U\right] = [[175,180,190] : [170,180,195]].
\end{aligned}$$

Margins of Player I and Player II are:

$$\begin{aligned}
&\tilde{\phi}_1^R \equiv \tilde{\phi}_2^R \equiv \tilde{\phi}^R \equiv \left[\tilde{\phi}^L : \tilde{\phi}^U\right] \\
&\quad = [[0.08,0.10,0.11] : [0.06,0.10,0.13]] \text{ and } \tilde{\psi}_1^R \equiv \tilde{\psi}_2^R \equiv \tilde{\psi}^R \equiv \left[\tilde{\psi}^L : \tilde{\psi}^U\right] \\
&\quad = [[0.14,0.15,0.17] : [0.12,0.15,0.19]],
\end{aligned}$$

and $\tilde{1}^R \equiv \left[\tilde{1}^L : \tilde{1}^U\right] = [[0.8,1,1.2] : [0.5,1,1.5]]$. Now, we divide the above problem into upper and lower approximation interval fuzzy linear programming problems for both the Players I and II as follows:

$(FLPP-I)^{UAI}$

$$\begin{aligned}
&\max \quad \left[u^{LU}, u^{M}, u^{UU}\right], \\
&\text{subject to the constraints} \\
&\left[a_{11}^{LU}, a_{11}^{M}, a_{11}^{UU}\right] \otimes \left[x_1^{LU}, x_1^{M}, x_1^{UU}\right] \\
&\qquad + \left[a_{21}^{LU}, a_{21}^{M}, a_{21}^{UU}\right] \otimes \left[x_2^{LU}, x_2^{M}, x_2^{UU}\right] \\
&\qquad \geq \left[u^{LU}, u^{M}, u^{UU}\right] - \left[\phi^{LU}, \phi^{M}, \phi^{UU}\right](1-\lambda), \\
&\left[a_{12}^{LU}, a_{12}^{M}, a_{12}^{UU}\right] \otimes \left[x_1^{LU}, x_1^{M}, x_1^{UU}\right] \\
&\qquad + \left[a_{22}^{LU}, a_{22}^{M}, a_{22}^{UU}\right] \otimes \left[x_2^{LU}, x_2^{M}, x_2^{UU}\right] \\
&\qquad \geq \left[u^{LU}, u^{M}, u^{UU}\right] - \left[\phi^{LU}, \phi^{M}, \phi^{UU}\right](1-\lambda), \\
&\left[x_1^{LU}, x_1^{M}, x_1^{UU}\right] + \left[x_2^{LU}, x_2^{M}, x_2^{UU}\right] = [0.5,1,1.5], \\
&\lambda \leq 1, \\
&\text{and } x_i^{LU}, x_i^{M}, x_i^{UU}, \lambda, u^{LU}, u^{M}, u^{UU} \geq 0, \quad \forall i = 1,2.
\end{aligned}$$

$(FLPP-I)^{LAI}$

$$
\begin{aligned}
&\max \quad \left[u^{LL}, u^{M}, u^{UL}\right],\\
&\text{subject to the constraints}\\
&\left[a_{11}^{LL}, a_{11}^{M}, a_{11}^{UL}\right] \otimes \left[x_1^{LL}, x_1^{M}, x_1^{UL}\right]\\
&\qquad + \left[a_{21}^{LL}, a_{21}^{M}, a_{21}^{UL}\right] \otimes \left[x_2^{LL}, x_2^{M}, x_2^{UL}\right]\\
&\qquad \geq \left[u^{LL}, u^{M}, u^{UL}\right] - \left[\phi^{LL}, \phi^{M}, \phi^{UL}\right](1-\lambda),\\
&\left[a_{12}^{LL}, a_{12}^{M}, a_{12}^{UL}\right] \otimes \left[x_1^{LL}, x_1^{M}, x_1^{UL}\right]\\
&\qquad + \left[a_{22}^{LL}, a_{22}^{M}, a_{22}^{UL}\right] \otimes \left[x_2^{LL}, x_2^{M}, x_2^{UL}\right]\\
&\qquad \geq \left[u^{LL}, u^{M}, u^{UL}\right] - \left[\phi^{LL}, \phi^{M}, \phi^{UL}\right](1-\lambda),\\
&\left[x_1^{LL}, x_1^{M}, x_1^{UL}\right] + \left[x_2^{LL}, x_2^{M}, x_2^{UL}\right] = [0.8, 1, 1.2],\\
&\lambda \leq 1,\\
&\text{and } x_i^{LL}, x_i^{M}, x_i^{UL}, \lambda, u^{LL}, u^{M}, u^{UL} \geq 0, \;\; \forall i = 1, 2.
\end{aligned}
$$

$(FLPP-II)^{UAI}$

$$
\begin{aligned}
&\min \quad \left[v^{LU}, v^{M}, v^{UU}\right],\\
&\text{subject to the constraints}\\
&\left[a_{11}^{LU}, a_{11}^{M}, a_{11}^{UU}\right] \otimes \left[y_1^{LU}, y_1^{M}, y_1^{UU}\right]\\
&\qquad + \left[a_{12}^{LU}, a_{12}^{M}, a_{12}^{UU}\right] \otimes \left[y_2^{LU}, y_2^{M}, y_2^{UU}\right]\\
&\qquad \leq \left[v^{LU}, v^{M}, v^{UU}\right] + \left[\psi^{LU}, \psi^{M}, \psi^{UU}\right](1-\mu),\\
&\left[a_{21}^{LU}, a_{21}^{M}, a_{21}^{UU}\right] \otimes \left[y_1^{LU}, y_1^{M}, y_1^{UU}\right]\\
&\qquad + \left[a_{22}^{LU}, a_{22}^{M}, a_{22}^{UU}\right] \otimes \left[y_2^{LU}, y_2^{M}, y_2^{UU}\right]\\
&\qquad \leq \left[v^{LU}, v^{M}, v^{UU}\right] + \left[\psi^{LU}, \psi^{M}, \psi^{UU}\right](1-\mu),\\
&\left[y_1^{LU}, y_1^{M}, y_1^{UU}\right] + \left[y_2^{LU}, y_2^{M}, y_2^{UU}\right] = [0.5, 1, 1.5],\\
&\mu \leq 1,\\
&\text{and } y_j^{LU}, y_j^{M}, y_j^{UU}, \mu, v^{LU}, v^{M}, v^{UU} \geq 0, \;\; \forall j = 1, 2.
\end{aligned}
$$

$(FLPP-II)^{LAI}$

$$
\begin{aligned}
&\min \quad \left[v^{LL}, v^{M}, v^{UL}\right],\\
&\text{subject to the constraints}\\
&\left[a_{11}^{LL}, a_{11}^{M}, a_{11}^{UL}\right] \otimes \left[y_1^{LL}, y_1^{M}, y_1^{UL}\right]\\
&\qquad + \left[a_{12}^{LL}, a_{12}^{M}, a_{12}^{UL}\right] \otimes \left[y_2^{LL}, y_2^{M}, y_2^{UL}\right]\\
&\qquad \leq \left[v^{LL}, v^{M}, v^{UL}\right] + \left[\psi^{LL}, \psi^{M}, \psi^{UL}\right](1-\mu),\\
&\left[a_{21}^{LL}, a_{21}^{M}, a_{21}^{UL}\right] \otimes \left[y_1^{LL}, y_1^{M}, y_1^{UL}\right]\\
&\qquad + \left[a_{22}^{LL}, a_{22}^{M}, a_{22}^{UL}\right] \otimes \left[y_2^{LL}, y_2^{M}, y_2^{UL}\right]\\
&\qquad \leq \left[v^{LL}, v^{M}, v^{UL}\right] + \left[\psi^{LL}, \psi^{M}, \psi^{UL}\right](1-\mu),\\
&\left[y_1^{LL}, y_1^{M}, y_1^{UL}\right] + \left[y_2^{LL}, y_2^{M}, y_2^{UL}\right] = [0.8, 1, 1.2],\\
&\mu \leq 1,\\
&\text{and } y_j^{LL}, y_j^{M}, y_j^{UL}, \mu, v^{LL}, v^{M}, v^{UL} \geq 0, \;\; \forall j = 1, 2.
\end{aligned}
$$

Now, the problems $(FLPP-I)^{UAI}$ with $(FLPP-I)^{LAI}$ and $(FLPP-II)^{UAI}$ with $(FLPP-II)^{LAI}$ can be separately transformed into ten crisp problems.

$(FLPP-I)^{UU}$

$$\begin{aligned}&\max \quad u^{UU},\\&195x_1^{UU}+110x_2^{UU}\geq u^{UU}-0.06(1-\lambda),\\&160x_1^{UU}+195x_2^{UU}\geq u^{UU}-0.06(1-\lambda),\\&x_1^{UU}+x_2^{UU}=1.5,\\&\lambda\leq 1,\\&\text{and } x_1^{UU},x_2^{UU},\lambda,u^{UU}\geq 0.\end{aligned}$$

$(FLPP-I)^{LU}$

$$\begin{aligned}&\max \quad u^{LU},\\&170x_1^{LU}+70x_2^{LU}\geq u^{LU}-0.13(1-\lambda),\\&148x_1^{LU}+170x_2^{LU}\geq u^{LU}-0.13(1-\lambda),\\&x_1^{LU}+x_2^{LU}=0.5,\\&\lambda\leq 1,\\&\text{and } x_1^{LU},x_2^{LU},\lambda,u^{LU}\geq 0.\end{aligned}$$

$(FLPP-I)^{M}$

$$\begin{aligned}&\max \quad u^{M},\\&180x_1^{M}+90x_2^{M}\geq u^{M}-0.10(1-\lambda),\\&156x_1^{M}+180x_2^{M}\geq u^{M}-0.10(1-\lambda),\\&x_1^{M}+x_2^{M}=1,\\&\lambda\leq 1,\\&\text{and } x_1^{M},x_2^{M},\lambda,u^{M}\geq 0.\end{aligned}$$

$(FLPP-I)^{LL}$

$$\begin{aligned}&\max \quad u^{LL},\\&175x_1^{LL}+80x_2^{LL}\geq u^{LL}-0.11(1-\lambda),\\&150x_1^{LL}+175x_2^{LL}\geq u^{LL}-0.11(1-\lambda),\\&x_1^{LL}+x_2^{LL}=0.8,\\&\lambda\leq 1,\\&\text{and } x_1^{LL},x_2^{LL},\lambda,u^{LL}\geq 0.\end{aligned}$$

$(FLPP-I)^{UL}$

$$\begin{aligned}
&\max \quad u^{UL},\\
&190x_1^{UL}+100x_2^{UL}\geq u^{UL}-0.08(1-\lambda),\\
&158x_1^{UL}+190x_2^{UL}\geq u^{UL}-0.08(1-\lambda),\\
&x_1^{UL}+x_2^{UL}=1.2,\\
&\lambda\leq 1,\\
&\text{and } x_1^{UL},x_2^{UL},\lambda,u^{UL}\geq 0.
\end{aligned}$$

$(FLPP-II)^{UU}$

$$\begin{aligned}
&\min \quad v^{UU},\\
&195y_1^{UU}+160y_2^{UU}\leq v^{UU}+0.19(1-\mu),\\
&110y_1^{UU}+195y_2^{UU}\leq v^{UU}+0.19(1-\mu),\\
&y_1^{UU}+y_2^{UU}=1.5,\\
&\mu\leq 1,\\
&\text{and } y_1^{UU},y_2^{UU},\mu,v^{UU}\geq 0.
\end{aligned}$$

$(FLPP-II)^{LU}$

$$\begin{aligned}
&\min \quad v^{LU},\\
&170y_1^{LU}+148y_2^{LU}\leq v^{LU}+0.12(1-\mu),\\
&70y_1^{LU}+170y_2^{LU}\leq v^{LU}+0.12(1-\mu),\\
&y_1^{LU}+y_2^{LU}=0.5,\\
&\mu\leq 1,\\
&\text{and } y_1^{LU},y_2^{LU},\mu,v^{LU}\geq 0.
\end{aligned}$$

$(FLPP-II)^{M}$

$$\begin{aligned}
&\min \quad v^{M},\\
&180y_1^{M}+156y_2^{M}\leq v^{M}+0.15(1-\mu),\\
&90y_1^{M}+180y_2^{M}\leq v^{M}+0.15(1-\mu),\\
&y_1^{M}+y_2^{M}=1,\\
&\mu\leq 1,\\
&\text{and } y_1^{M},y_2^{M},\mu,v^{M}\geq 0.
\end{aligned}$$

$(FLPP-II)^{LL}$

$$\begin{aligned}
&\min \quad v^{LL},\\
&175y_1^{LL}+150y_2^{LL}\leq v^{LL}+0.14(1-\mu),\\
&80y_1^{LL}+175y_2^{LL}\leq v^{LL}+0.14(1-\mu),\\
&y_1^{LL}+y_2^{LL}=0.8,\\
&\mu\leq 1,\\
&\text{and } y_1^{LL},y_2^{LL},\mu,v^{LL}\geq 0.
\end{aligned}$$

$(FLPP-II)^{UL}$

$$\begin{aligned}
&\min \quad v^{UL}\\
&190y_1^{UL}+158y_2^{UL} \leq v^{UL}+0.17(1-\mu),\\
&100y_1^{UL}+190y_2^{UL} \leq v^{UL}+0.17(1-\mu),\\
&y_1^{UL}+y_2^{UL}=1.2,\\
&\mu \leq 1,\\
&\text{and } y_1^{UL}, y_2^{UL}, \mu, v^{UL} \geq 0.
\end{aligned}$$

On solving the above problems, we get the solutions that are listed in Tables 1 and 2. Also sensitivity analysis is performed in Tables 1 and 2 with respect to different values of parameters.

Table 1. Value of game $\left(\tilde{u}^R\right)$ for player I

$Problem$	$\lambda=0$	$\lambda=0.25$	$\lambda=0.50$	$\lambda=0.75$	$\lambda=1$
$(FLPP-I)^{UU}$	255.3725	255.3575	255.3425	255.3275	255.3125
$(FLPP-I)^{LU}$	76.1136	76.0811	76.0486	76.0161	75.9836
$(FLPP-I)^{M}$	161.1526	161.1276	161.1026	161.0776	161.0526
$(FLPP-I)^{LL}$	124.2767	124.2492	124.2217	124.1942	124.1667
$(FLPP-I)^{UL}$	199.7521	199.7321	199.7121	199.6921	199.6721

Table 2. Value of game $\left(\tilde{v}^R\right)$ for player II

$Problem$	$\mu=0$	$\mu=0.25$	$\mu=0.50$	$\mu=0.75$	$\mu=1$
$(FLPP-II)^{UU}$	255.1225	255.1700	255.2175	255.2650	255.3125
$(FLPP-II)^{LU}$	75.8636	75.8936	75.9236	75.9536	75.9836
$(FLPP-II)^{M}$	160.9026	160.9401	160.9776	161.0151	161.0526
$(FLPP-II)^{LL}$	124.0267	124.0617	124.0967	124.1317	124.1667
$(FLPP-II)^{UL}$	199.5021	199.5446	199.5871	199.6296	199.6721

4. Validation of Results through Comparison with Existing Literature

Table 3 compares the results of our study with those of existing literature. Three research papers are presented, along with the type of fuzzy number used and the corresponding value of the game. The first study, conducted by Campos (1989), used triangular fuzzy numbers and resulted in a game value of 160.81. The second study, conducted by Bector et al. (2005), also used triangular fuzzy numbers and produced a game value of 160.65. In contrast, our study used triangular fuzzy rough numbers and resulted in a game value of 161.1526. This demonstrates that the use of rough numbers can lead to approximate results in fuzzy environments. This comparison provides further evidence of the effectiveness of our approach and contributes to the growing body of literature on fuzzy game theory.

Table 3. Comparison with existing literature

Research Paper	Type of fuzzy number	Value of game
Campos (1989)	Triangular Fuzzy Number	160.81
Bector et al. (2005)	Triangular Fuzzy Number	160.65
Present paper	Triangular Fuzzy rough Number	161.1526

5. Conclusion

Most of the existing methods for solving TPZSFMG are based on triangular fuzzy numbers (TFNs), trapezoidal fuzzy numbers (TrFNs), and intuitionistic fuzzy numbers (IFNs). However, there are many circumstances where goals and payoffs are in the form of intervals with varying bounds. To overcome these types of situations in games, we have used TFRNs. The TPZSFMG with TFRN goals, payoffs, and decision variables are studied, and a technique for solving these games is proposed. A pair of FFRLPPs corresponding to each player is obtained in this technique. These are converted into sub-crisp problems for each player. Optimal strategies and value of the games are achieved by solving ten sub-crisp problems using Wolfram Cloud. The impact of parameters on game value are observed by a numerical example. The article provides a novel approach to handling TPZSFMG. That has a significant development in the field. The concept can be extended to Pythagorean and Neutrosophic numbers in future.

Acknowledgment:

We express our sincere gratitude to the Chief Editor and the Reviewer for their insightful feedback and valuable suggestions, which have greatly improved the quality and rigor of our research paper. Their expertise and guidance have been instrumental in shaping the direction and scope of this study, and we are truly grateful for their contribution to our work.

REFERENCES

Aggarwal, A., Mehra, A. and Chandra, S. (2012). Application of linear programming with i-fuzzy sets to matrix games with i-fuzzy goals, Fuzzy Optim. Decis. Mak., Vol. 11, No. 4, pp. 465-480.

Ammar, E. and Brikaa, M. G. (2019). On solution of constraint matrix games under rough interval approach, Granul. Comput., Vol. 4, No. 3, pp. 601-614.

Ammar, E. and Emsimir, A. (2021). A mathematical model for solving fuzzy integer linear programming problems with fully rough intervals, Granul. Comput., Vol. 6, No. 5, pp. 567-578.

Atanassov, K. T. (1999). Intuitionistic fuzzy sets. In: Intuitionistic fuzzy sets, Stud. Fuzziness Soft Comput., Physica, Heidelberg, Vol. 35.

Bector, C. R. and Chandra, S. (2005). Fuzzy mathematical programming and fuzzy matrix games, Stud. Fuzziness Soft Comput., Physica, Berlin, Heidelberg, Vol. 169.

Bector, C. R., Chandra, S. and Vidyottama, V. (2004). Matrix games with fuzzy goals and fuzzy linear programming duality, Fuzzy Optim. Decis. Mak., Vol. 3, pp. 255-269.

Bhaumik, A., Roy, S. K. and Li, D. F. (2017). Analysis of triangular intuitionistic fuzzy matrix games using robust ranking, J. Intell. Fuzzy Syst., Vol. 33, No. 1, pp. 327-336.

Brikaa, M. G., Zheng, Z. and Ammar, E. (2019). Fuzzy multi-objective programming approach for constrained matrix games with payoffs of fuzzy rough numbers, Symmetry, Vol. 11, No. 5, pp. 702.

Campos, L. (1989). Fuzzy linear programming models to solve fuzzy matrix games, Fuzzy Sets Syst., Vol. 32, No. 3, pp. 275-289.

Campos, L. and Gonzalez, A. (1991). Fuzzy matrix games considering the criteria of the players, Kybernetes, Vol. 20, No. 1, pp. 17-28.

Cevikel, A. C. and Ahlatçıoğlu, M. (2010). Solutions for fuzzy matrix games, Comput. Math. with Appl., Vol. 60, No. 3, pp. 399-410.

Chen, Y. W. and Larbani, M. (2006). Two-person zero-sum game approach for fuzzy multiple attribute decision making problems, Fuzzy Sets Syst., Vol. 157, No. 1, pp. 34-51.

Clemente, M., Fernández, F. R. and Puerto, J. (2011). Pareto-optimal security strategies in matrix games with fuzzy payoffs, Fuzzy Sets Syst., Vol. 176, No. 1, pp. 36-45.

Collins, W. D. and Hu, C. (2008). Studying interval valued matrix games with fuzzy logic, Soft Comput., Vol. 12, No. 2, pp. 147-155.

Das, S. K., and Edalatpanah, S. A. (2020). A new ranking function of triangular neutrosophic number and its application in integer programming, International Journal of Neutrosophic Science, Vol. 4, No. 2, pp. 82-92.

Das, S. K., Edalatpanah, S. A., and Mandal, T. (2018). A proposed model for solving fuzzy linear fractional programming problem: Numerical point of view, Journal of Computational Science, Vol. 25, pp. 367-375.

Dhar, M. (2021). Neutrosophic soft matrices and its application in medical diagnosis, Journal of Fuzzy Extension and Application, Vol. 2, No. 1, pp. 23-32.

Dubois, D. and Prade, H. (1990). Rough fuzzy sets and fuzzy rough sets, Int. J. Gen. Syst., Vol. 17, No. 2-3, pp. 191-209.

Edalatpanah, S. A. (2020). Neutrosophic structured element, Expert Systems, Vol. 37, No. 5, e12542.

Eyo, I. J., Adeoye, O. S., Inyang, U. G. and Umoeka, I. J. (2022). Hybrid intelligent parameter tuning approach for COVID-19 time series modeling and prediction, Journal of Fuzzy Extension and Applications, Vol. 3, No. 1, pp. 64-80.

Jana, J. and Roy, S. K. (2018). Solution of matrix games with generalised trapezoidal fuzzy payoffs, Fuzzy Inf. Eng., Vol. 10, No. 2, pp. 213-224.

Kumar, G. and Jangid, V. (2020). Linear programming models to solve fully fuzzy two person zero sum matrix game, Malaya J. Mat., Vol. 8, No. 3, pp. 775-781.

Li, D. F. (1999). A fuzzy multiobjective programming approach to solve fuzzy matrix games, J.

Fuzzy Math., Vol. 7, No. 4, pp. 907-912.

Mula, P., Roy, S. K. and Li, D. F. (2015). Bi-rough programming approach for solving bi-matrix games with bi-rough payoff elements, J. Intell. Fuzzy Syst., Vol. 29, No. 2, pp. 863-875.

Nan, J. X., Li, D. F. and Zhang, M. J. (2010). A lexicographic method for matrix games with payoffs of triangular intuitionistic fuzzy numbers, Int. J. Comput. Intell. Syst., Vol. 3, No. 3, pp. 280-289.

Nayak, P. K. and Pal, M. (2009). Linear programming technique to solve two person matrix games with interval pay-offs, Asia Pac J Oper Res, Vol. 26, No. 2, pp. 285-305.

Pamucar, D., Cirovic, G. and Bozanic D. (2019). Application of interval valued fuzzy-rough numbers in multi-criteria decision making: The IVFRN-MAIRCA model, Yugosl. J. Oper. Res., Vol. 29, No. 2, pp. 221–247.

Pawlak, Z. (1982). Rough sets, In. J. Comput. Inf. Sci., Vol. 11, No. 5, pp. 341-356.

Sakawa, M. and Nishizaki, I. (1994). Max-min solutions for fuzzy multiobjective matrix games, Fuzzy Sets Syst., Vol. 67, No. 1, pp. 53-69.

Seikh, M. R., Nayak, P. K. and Pal, M. (2015). An alternative approach for solving fuzzy matrix games, IJMSC, Vol. 5, No. 1, pp. 79-92.

Vijay, V., Chandra, S. and Bector, C. R. (2005). Matrix games with fuzzy goals and fuzzy payoffs, Omega, Vol. 33, No. 5, pp. 425-429.

Von Neumann, J. and Morgenstern, O. (1944). *Theory of Games and Economic Behavior*, Princeton University Press.

Xu, J. and Yao, L. (2010). A class of two-person zero-sum matrix games with rough payoffs, Int. J. Math. Math. Sci., Vol. 2010, pp. 1-22.

Yang, W., Cai, L., Edalatpanah, S. A. and Smarandache, F. (2020). Triangular single valued neutrosophic data envelopment analysis: Application to hospital performance measurement, Symmetry, Vol. 12, No. 4, pp. 588.

Zadeh, L. A. (1965). Fuzzy sets, Inf. Control., Vol. 8, No. 3, pp. 338-353.